Homework 1 for Apma 507

Due Thursday - January 27, 2007

Problem 1. Using the definition of the limit of a sequence show that

$$\lim_{n\to\infty} \frac{4n+1}{5n-1} = \frac{4}{5}.$$

Problem 2. Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) := 2x + 3$. Use the ε and δ definition of continuity at a point to show that

$$\lim_{x\to 4} f(x) = 11.$$

Problem 3. Show that the equation $x^5 - x^4 + 3x^3 - 2x^2 + x - 1 = 0$ has at least a solution in the interval $[0, 1]$.

Problem 4. Let $f \in \mathcal{C}([a, b])$ be two times differentiable on (a, b). Also assume that there exist $x_0, x_1, x_2 \in [a, b]$ such that

$$a \le x_0 < x_1 < x_2 \le b \quad \text{and} \quad f(x_0) = f(x_1) = f(x_2) = 0.$$

Show that there exists $c \in (a, b)$ such that $f''(c) = 0$.

Problem 5. Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = 2x^3 + 5$. Show, using the definition of the derivative (that is the difference quotient), that $f'(1) = 6$.