

Corrections, Remarks and References

- On page 22, (2.31) defines a set of $m_0 + 1$ functions V_p for all $m_0 + 1$ failure patterns σ (including $\sigma = 0$ for the no-failure pattern), and their matching parameters k_{1j}^* , k_{2j}^* and k_{3j}^* are fixed at the values $k_{1j(i)}^*$, $k_{2j(i)}^*$ and $k_{3j(i)}^*$, for all $t \geq 0$, e.g., for the first function V_0 (the no failure case), k_{1j}^* , k_{2j}^* and k_{3j}^* are $k_{1j(0)}^*$, $k_{2j(0)}^*$ and $k_{3j(0)}^*$, for all $t \geq 0$; for the next function V_1 (if one failure occurred) or V_2 (if two failures occurred) or V_3 (if three failures occurred), k_{1j}^* , k_{2j}^* and k_{3j}^* are $k_{1j(1)}^*$, $k_{2j(1)}^*$ and $k_{3j(1)}^*$, for all $t \geq 0$; and so on.

With this definition, the functions V_p are continuous functions for all $t \geq 0$.

- On page 25, in (2.46), “ $(k_{3i}(t) - k_{3i(0)}^*)$ ” should be “ $(k_{3i}(t) - k_{3i(0)}^*)^2$ ”, and in (2.47), “ $(k_{3i}(t) - k_{3i(1)}^*)$ ” should be “ $(k_{3i}(t) - k_{3i(1)}^*)^2$ ”.
- On page 72, (3.73) should be

$$\alpha_{ij} = c\bar{A}^{n^*-1}b_j.$$

Remark: On pages 71 and 72, the proof of Lemma 3.2.1, for the case when $c\bar{A}^{n^*-1}b_i = \alpha_{ii} \neq 1$, can also be obtained with the following minor changes:

$$c(sI - \bar{A})^{-1}b_i P_m(s) = \alpha_{ii} \quad (64)$$

$$c(sI - \bar{A})^{-1}b_i = \frac{s^{n-n^*} + c_{n-n^*}s^{n-n^*-1} + \dots + c_2s + c_1}{s^n + a_n s^{n-1} + \dots + a_2s + a_1} \alpha_{ii} \quad (67)$$

$$c\bar{A}^k b_i = \frac{\alpha_{ii}}{\alpha_{ij}} c\bar{A}^k b_j = 0, \quad k = 0, 1, \dots, n^* - 2 \quad (72)$$

$$\alpha_{ij} = c\bar{A}^{n^*-1}b_j \quad (73)$$

$$c\bar{A}^{n^*-1}b_i = \frac{\alpha_{ii}}{\alpha_{ij}} c\bar{A}^{n^*-1}b_j \quad (74)$$

$$c\bar{A}^k b_i = \frac{\alpha_{ii}}{\alpha_{ij}} c\bar{A}^k b_j \quad (75)$$

$$c\bar{A}^k (b_i - \frac{\alpha_{ii}}{\alpha_{ij}} b_j) = 0, \quad k = 0, 1, \dots, n-1 \quad (76)$$

Using this result and applying (3.60) to $c(sI - \bar{A})^{-1}(b_i - \frac{\alpha_{ii}}{\alpha_{ij}} b_j)$, we obtain

$$c(sI - \bar{A})^{-1} \left(b_i - \frac{\alpha_{ii}}{\alpha_{ij}} b_j \right) = 0. \quad (77)$$

In view of (3.58), we see that (3.77) is equivalent to (3.59): $c(sI - A - b_i k_{1i}^{*T})^{-1} b_j = \alpha_{ij} W_m(s)$. This result is true for all $j = 1, 2, \dots, m$, $j \neq i$.

Remark: On pages 71 and 72, the proof of Lemma 3.2.1 is given based on a controllable canonical form realization (A, b_i, c) and the fact that (A, b_i, c) and (A, b_j, c) have the same relative degree n^* . The result of the lemma leads to

$$c(sI - A - b_i k_{1i}^{*T})^{-1} [b_i f_i + b_j u_j](t) = W_m(s) \alpha_{ii} [f_i](t) + W_m(s) \alpha_{ij} [u_j](t)$$

for two signals $f_i(t)$ and $u_j(t)$, as $c(sI - A - b_i k_{1i}^{*T})^{-1} b_i = W_m(s) \alpha_{ii}$ and $c(sI - A - b_i k_{1i}^{*T})^{-1} b_j = W_m(s) \alpha_{ij}$, for $\alpha_{ii} = c(A + b_i k_{1i}^{*T})^{n^* - 1} b_i$ and $\alpha_{ij} = c(A + b_i k_{1i}^{*T})^{n^* - 1} b_j$ as verified in the proof.

When (A, b_i, c) is not in the controllable canonical form realization but (A, b_i, c) and (A, b_j, c) have the same relative degree n^* , there is a transformation matrix T such that $A_c = T^{-1} A T$, $b_{ic} = T^{-1} b_i$ and $c_c = c T$ are in in the controllable canonical form realization, so that

$$\begin{aligned} & c(sI - A - b_i k_{1i}^{*T})^{-1} [b_i f_i + b_j u_j](t) \\ &= c_c (sI - A_c - b_{ic} k_{1ic}^{*T})^{-1} [b_{ic} f_i + T^{-1} b_j u_j](t), \end{aligned}$$

where $k_{1ic}^{*T} = k_{1i}^{*T} T$. Since (A_c, b_{ic}, c_c) and $(A_c, T^{-1} b_j, c_c)$ have the same relative degree and (A_c, b_{ic}, c_c) is the controllable canonical form realization, we also have

$$c(sI - A - b_i k_{1i}^{*T})^{-1} [b_i f_i + b_j u_j](t) = W_m(s) \alpha_{ii} [f_i](t) + W_m(s) \alpha_{ij} [u_j](t).$$

Remark: From Lemma 3.2.1, we can see that

$$c(sI - A - b_i k_{1i}^{*T})^{-1} b_i = \frac{1}{k_{2i}^* P_m(s)} = -c(sI - A - b_i k_{1i}^{*T})^{-1} b_j \frac{1}{k_{3i}^*}$$

have the same zeros. This can be verified as follows. From $c(sI - A - b_i k_{1i}^{*T})^{-1} b_i = \frac{1}{k_{2i}^* P_m(s)}$, it follows that there are $n - n^*$ poles of $\bar{A} = A + b_i k_{1i}^{*T}$ (those different from the zeros of $P_m(s)$) are made unobservable by k_{1i}^{*T} at $y(t) = cx(t)$ (that is, $(c, A + b_i k_{1i}^{*T})$ has $n - n^*$ unobservable poles), given that $(A + b_i k_{1i}^{*T}, b_i)$ is controllable as (A, b_i) is controllable and the state feedback gain k_{1i}^{*T} does change the controllability of $(A + b_i k_{1i}^{*T}, b_i)$ and (A, b_i) . This implies that

$$c(sI - A - b_i k_{1i}^{*T})^{-1} b_j = \frac{P_j(s)}{P_m(s)}$$

for some polynomial $P_j(s)$ of degree $n^* - 1$ or less.

From the condition that $cA^k b_i = 0$, $cA^k b_j = 0$, $k = 0, 1, \dots, n^* - 2$, we have

$$c\bar{A}^k b_j = c(A + b_i k_{1i}^{*T})(A + b_i k_{1i}^{*T})^{k-1} b_j = cA(A + b_i k_{1i}^{*T})^{k-1} b_j = \dots = cA^k b_j = 0$$

for $k = 0, 1, \dots, n^* - 2$, and from the additional condition that $cA^{n^* - 1} b_j \neq 0$, that

$$c\bar{A}^{n^* - 1} b_j = cA(A + b_i k_{1i}^{*T})^{n^* - 2} b_j = \dots = c(A^{n^* - 1} + A^{n^* - 2} b_i k_{1i}^{*T}) b_j = cA^{n^* - 1} b_j \neq 0.$$

From (3.60)–(3.62), the degree of $N(s)$ for $b = b_j$ is $n - n^*$, that is, the degree of $P_j(s)$ is 0.

This remark actually gives a simpler proof of Lemma 3.2.1.

The case when the degree of $P_j(s)$ is larger than 0 corresponds to the case when the relative degree of (A, b_j, c) is less than n^* , that is, when $c\bar{A}^k b_j \neq 0$ for some $k < n^* - 1$.

What if the relative degree of (A, b_j, c) is larger than n^* , e.g., $cA^{n^* - 1} b_j = 0$ in addition, so that

$$c\bar{A}^{n^* - 1} b_j = cA(A + b_i k_{1i}^{*T})^{n^* - 2} b_j = \dots = c(A^{n^* - 1} + A^{n^* - 2} b_i k_{1i}^{*T}) b_j = cA^{n^* - 1} b_j = 0,$$

in addition to $c\bar{A}^k b_j = c(A + b_i k_{1i}^{*T})^k b_j = 0$ for $k = 0, 1, \dots, n^* - 2$. In this case, it follows from (3.69): $c(\bar{A}^{n^*} + a_{n^*}^* \bar{A}^{n^* - 1} + a_{n^* - 1}^* \bar{A}^{n^* - 2} + \dots + a_2^* \bar{A} + a_1^* I) = 0$, that $c\bar{A}^{n^*} b_j = 0$. Similarly, $c\bar{A}^k b_j = 0$ for $k = n^* + 1, \dots, n - 1$, so that $N(s) = 0$ in (3.62) with $b = b_j$, that is, $P_j(s) = 0$ or $c(sI - A - b_i k_{1i}^{*T})^{-1} b_j = 0$. Thus, we have the result:

Proposition 1 Assume that (A, b_i) is controllable and there are $k_{1i}^* \in R^n$ and $k_{2i}^* \in R$ such that

$$c(sI - A - b_i k_{1i}^{*T})^{-1} b_i k_{2i}^* = W_m(s) = \frac{1}{P_m(s)}$$

where $P_m(s)$ is a monic polynomial of degree n^* . Then, (i) for any $b_j \in R^n$ such that the relative degree of (c, A, b_j) is $n_1^* < n^*$, there is a polynomial $P_j(s)$ of degree $n^* - n_1^*$ such that

$$c(sI - A - b_i k_{1i}^{*T})^{-1} b_j = \frac{P_j(s)}{P_m(s)};$$

and (ii) for any $b_j \in R^n$ such the relative degree of (c, A, b_j) is $n_2^* > n^*$, it follows that

$$c(sI - A - b_i k_{1i}^{*T})^{-1} b_j = \frac{0}{P_m(s)} = 0.$$

Note that $c\bar{A}^{n^*} b_j = 0$ means that

$$c\bar{A}^{n^*} b_j = cA(A + b_i k_{1i}^{*T})^{n^*-1} b_j = \dots = c(A^{n^*} + A^{n^*-1} b_i k_{1i}^{*T}) b_j = cA^{n^*} b_j + cA^{n^*-1} b_i k_{1i}^{*T} b_j = 0.$$

A direct proof of $cA^{n^*} b_j + cA^{n^*-1} b_i k_{1i}^{*T} b_j = 0$ may not be simple.

- On page 72, after (3.75), “ $k = n^*$ to $k = n - 1$ ” should be “ $k = n^*$ to $k = n - 1$ ”.
- On page 78, (3.112) should be

$$f_i^*(t) = \sum_{j \neq i} -\frac{\alpha_{ij}}{\alpha_{ii}} u_j(t).$$

- On page 79, the last sentence of the paragraph above (3.113) is so stated for the problem of “up to $m - 1$ ” actuator failures, which implies that (c, A, b_j) , $j = 1, 2, \dots, m$, all have relative degree n^* , a condition needed for the matching equation (3.116) (the corrected version; see below).

For the problem of totally less than $m - 1$ actuator failures, the condition that (c, A, b_j) , $j = 1, 2, \dots, m$, all have relative degree n^* needs to be explicitly stated.

- On page 80, (3.116) should be

$$c \left(sI - A - \sum_{j \neq j_1, \dots, j_p} b_j k_{11}^{*T} \right)^{-1} \left[\sum_{j \neq j_1, \dots, j_p} b_j f_1^* + \sum_{j=j_1, \dots, j_p} b_j u_j \right] (t) = 0.$$

Remark: In this equation, $\bar{A} \triangleq A + \sum_{j \neq j_1, \dots, j_p} b_j k_{11}^{*T}$ is the nominal closed-loop system characteristic matrix for a desired nominal value k_{11}^* of the gain vector k_{11} , such that

$$c \left(sI - A - \sum_{j \neq j_1, \dots, j_p} b_j k_{11}^{*T} \right)^{-1} \sum_{j \neq j_1, \dots, j_p} b_j = W_m(s) k_{21}^{*-1}$$

(by the closed-loop plant-model matching condition) and

$$c \left(sI - A - \sum_{j \neq j_1, \dots, j_p} b_j k_{11}^{*T} \right)^{-1} b_j = -W_m(s) k_{3j}^* k_{21}^{*-1}$$

(by Lemma 3.2.1), where

$$k_{3j}^* = -\frac{\alpha_{ij}}{\alpha_{ii}}, k_{21}^* = \frac{1}{\alpha_{ii}}$$

$\alpha_{ij} = c\bar{A}^{n^*-1}b_j$ and $\alpha_{ii} = c\bar{A}^{n^*-1}\sum_{j \neq j_1, \dots, j_p} b_j$ (as from the proof of Lemma 3.2.1).

The controller (3.113) can then be expressed as

$$v_1(t) = v_2(t) = \dots = v_m(t) = \theta^T \omega$$

for some parameter vector θ which contains all parameters of k_{11} , k_{21} and those in $f_1(t)$, and a corresponding vector signal $\omega(t)$. The same form of the error equation (3.101) can be obtained with the new notation $\bar{A} = A + \sum_{j \neq j_1, \dots, j_p} b_i k_{11}^* T$ and $\rho^* = 1/k_{21}^*$.

- On page 98, line 7, “ $\theta_1^{*T} \omega_1(t)$ ” should be “ $\theta_1^{*T} \frac{a(s)}{\Lambda(s)}$ ”.
- On page 98, line 10, “(3.7)” should be “(3.47)”.
- On page 98, line 14, “tracking” should be “tracks”.
- **Remark:** On page 46, for the conditions (2.166)–(2.168), if they hold for $q = 1$, then they also hold for $m > q \geq 1$. In other words, if any $m - q$ failures can be compensated, then any $m - i$ failures can be compensated for every $i = q + 1, q + 2, \dots, m - 1$ (and $i = m$, the no failure case).
- **Remark:** On page 99, line 5, $P_i(s)$ is a monic polynomial of degree $n^* - 1$, satisfies $\frac{\Lambda(s)P_m(s)}{P(s)} = P_i(s) + \frac{R(s)}{P(s)}$ for some polynomial $R(s)$ of degree $n - 1$ (such that $\theta_1^{*T} a(s) + \theta_{20}^* \Lambda(s) = -\theta_3^* R(s)$).
- On page 99 and page 100, use the notation “ ϕ_j^* ” to replace “ θ_j^* ” in (4.65), (4.66) and (4.70), and “ $\psi_j(t)$ ” to replace “ $\omega_j(t)$ ” in (4.65), (4.66), (4.69), (4.70), (4.71) and (4.73).
- On page 100, in (4.70), “ $A(s)$ ” is revised as

$$A(s) = [I_{q+1}, sI_{q+1}, \dots, s^{n-2}I_{q+1}]^T.$$

- On page 101, use the notation “ ϕ_j^* ” to replace “ θ_j^* ” in line 8, (4.78) and $\theta_{60}^* (\phi_1^*, \dots, \phi_m^*)$, “ $\psi_j(t)$ ” to replace “ $\omega_j(t)$ ” in line 8, (4.80) ($\psi_1(t), \dots, \psi_m(t)$), and “ $\omega_{60}(t)$ ” to replace “ $\bar{\omega}(t)$ ”.
- On page 101, “ $\omega_6(t)$ ” in (4.80) is revised as

$$\omega_6(t) = \left[\left(\frac{A(s)}{\Lambda(s)} [\psi_1](t) \right)^T, \dots, \left(\frac{A(s)}{\Lambda(s)} [\psi_m](t) \right)^T \right]^T.$$

- On page 101, “ $A(s)$ ” in (4.80) is revised as

$$A(s) = [I_{q+1}, sI_{q+1}, \dots, s^{n-2}I_{q+1}]^T.$$

- **Remark:** On page 102, after “... $\lim_{t \rightarrow \infty} (y(t) - y_m(t)) = 0$ ”, we note the following:

In the parametrization (4.79), the parameter vector dimensions can be reduced. For θ_{60}^* , since the first components of $\omega_i(t)$ (which is $\psi_i(t)$, under the new notation), $i = 1, 2, \dots, m$, are all equal to 1, the corresponding components in θ_{60}^* can be combined as one single parameter, with

the first components of $\omega_i(t)$ (or $\psi_i(t)$), $i = 2, \dots, m$, being deleted from $\bar{\omega}(t)$ (which is $\omega_{60}(t)$, under the new notation).

Similarly, for θ_6^* and $\omega_6(t)$, the first components of $\frac{A(s)}{\Lambda(s)}[\Psi_i](t)$, $i = 1, 2, \dots, m$, are all the same (which converges to the constant $\frac{1}{\Lambda(0)}$ and can be replaced by $\frac{1}{\Lambda(0)}$ in the design), and the corresponding components in θ_6^* can be combined as one single parameter, with the first components of $\frac{A(s)}{\Lambda(s)}[\Psi_i](t)$, $i = 2, \dots, m$, being deleted from $\omega_6(t)$. The $(k(q+1)+1)$ th components of $\frac{A(s)}{\Lambda(s)}[\Psi_i](t)$, $i = 1, 2, \dots, m$, are all the same (equal to 0), for $k = 1, \dots, n-2$, which can be deleted from $\omega_6(t)$, with the corresponding components in θ_6^* being also deleted from θ_6^* .

In the expression (4.53), a more compact form is with $q = q_j$ for each j , which can be similarly handled with a modification to the parametrization (4.79).

- On page 105, line 3, “ $G_{ij}(s)$, $i = 1, \dots, M$, $j = 1, \dots, n_i$, are $1 \times M$ vectors” should be “ $G_{ij}(s)$, $i = 1, \dots, M$, $j = 1, \dots, n_i$, are $M \times 1$ vectors”.
- On page 119, in (5.81) and (5.82), “-0.12879” should be “-0.012879” and “0.21326” should be “0.021326”.
- On page 120, in (5.83), “-0.12879” should be “-0.012879” and “-0.13” should be “-0.013”.
- On page 155, in line 5, change “We segment the elevator into” to “For (7.40), we segment the elevator into”, and in (7.61), change “ $B = [b_2, b_3]$ ” to “ $B = [b_1, b_2]$ ”.
- On page 156, in line 14, change “ $v(t)$ ” to “ $v(t) = v_0(t)$ ”.
- On page 120, in the line before (5.85), change “no-failure case” to “no-failure case with (5.82)”.
- On page 173, in equation (8.68), “ $0 < \gamma_\theta < \frac{1}{k_p^0}$ ” should be “ $0 < \gamma_\theta < 1$ ”.
- On page 174, equation (8.68) should be

$$\sum_{t=t_1}^{t_2} (y(t) - y_m(t))^2 \leq c_1 + c_2 \sum_{t=t_1}^{t_2} d^2(t) + c_3 \mu^2 (t_2 - t_1)$$

- **Remark:** On page 226, in (10.141), and after, the same notation “ θ ” is used to denote an unknown parameter vector, instead of the pitch angle in (10.138) until before (10.141).
- On page 258, line 16, and, on page 261, the last line, “Remark 11.5.1” should be “Remark 11.4.3”
- The example in Chapter 11, Section 11.4.4, is from
H. Xu and M. Mirmirani, “Robust adaptive sliding control for a class of MIMO nonlinear systems,” *Proceedings of the 2001 AIAA Guidance, Navigation and Control Conference*, Montreal, Canada, August 2001.

with the altitude fixed as a constant so that the system become a SISO one.

- On page 265, last line, “7–11” should be “7–8”.
- **Remark:** On page 266, the aircraft models are studied in different chapters:

- Boeing 737 longitudinal dynamics model (elevator/stabilizer failure)
Chapter 7
- Boeing 737 lateral-directional dynamics model (rudder/aileron failure)
Chapter 5
- Boeing 747 lateral-directional dynamics model (rudder failure)
Chapter 2, Chapter 3, Chapter 4, Chapter 8
- DC-8 lateral-directional dynamics model (aileron failure)
Chapter 6
- F-18 wing dynamics model (aileron failure)
Chapter 10
- Twin Otter longitudinal nonlinear dynamics model (elevator failure)
Chapter 10
- a hypersonic aircraft longitudinal nonlinear model (elevator failure).
Chapter 11

References of Our Recent Related Work

Journal Papers

- [1] X. D. Tang, G. Tao, L. F. Wang and J. A. Stankovic, “Robust and adaptive actuator failure compensation designs for a rocket fairing structural-acoustic model,” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 40, no. 4, pp. 1359–1366, October 2004.
- [2] X. D. Tang, G. Tao and S. M. Joshi, “Adaptive output feedback actuator failure compensation for a class of nonlinear systems,” *International Journal of Adaptive Control and Signal Processing*, vol. 19, pp. 419–444, 2005.
- [3] X. D. Tang, G. Tao and S. M. Joshi, “Virtual grouping based adaptive actuator failure compensation for MIMO nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1775–1780, 2005.
- [4] X. D. Tang, G. Tao and S. M. Joshi, “Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application,” acceptable for *Automatica*.

Conference Papers

- [1] X. D. Tang and G. Tao, “Adaptive compensation of actuator failures for nonlinear MIMO systems under relaxed design conditions,” *Proceedings of the 2003 American Control Conference*, pp. 5123–5128, Denver, CO, June 2003.
- [2] L. F. Wang, X. T. Tang, G. Tao and J. A. Stankovic, “Actuator failure compensation schemes for vibration control of a rocket fairing model,” *Proceedings of the 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, pp. 175–180, Washington, D.C., June, 2003.
- [3] E. F. Kececi, X. D. Tang and G. Tao, “Adaptive actuator failure compensation for concurrently actuated manipulators,” *Proceedings of the 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, pp. 411–416, Washington, DC, June 2003.
- [4] E. F. Kececi, X. D. Tang and G. Tao, “Adaptive actuator failure compensation for cooperating multiple manipulator systems,” *Proceedings of the 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, pp. 417–422, Washington, DC, June 2003.

- [5] J. T. Fei, S. H. Chen, G. Tao and S. M. Joshi, "A discrete-time adaptive actuator failure compensation controller," *Proceedings of the 5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes*, pp. 423–428, Washington, DC, June 2003.
- [6] X. D. Tang, G. Tao and S. M. Joshi, "Adaptive output feedback actuator failure compensation for a class of state-dependent nonlinear systems," *Proceedings of the 42nd IEEE Conference on Decision and Control*, pp. 1681–1686, Maui, Hawaii, 2003.
- [7] X. D. Tang, G. Tao and S. M. Joshi, "Adaptive output feedback design for actuator failure compensation using dynamic bounding: output tracking and an application," *Proceedings of the 2005 American Control Conference*, pp. 3186–3191, Portland, Oregon, 2005.
- [8] X. D. Tang, G. Tao and S. M. Joshi, "Adaptive output feedback design for actuator failure compensation using dynamic bounding: output regulation," *Proceedings of the 2005 American Control Conference*, pp. 3315–3320, Portland, Oregon, 2005.
- [9] G. Tao and S. M. Joshi, "Adaptive output feedback compensation of variant actuator failures," *Proceedings of the 2005 American Control Conference*, pp. 4862–4867, Portland, Oregon, 2005.
- [10] X. D. Tang, Y. Liu, G. Tao, and S. M. Joshi, "Adaptive failure compensation control of autonomous robotic systems for space exploration applications," the *2005 AIAA Infotech@Aerospace*, Washington, DC, September 2005.
- [11] Y. Liu, X. D. Tang, G. Tao, and S. M. Joshi, "Adaptive rudder failure compensation for aircraft flight control using engine differentials," the *2005 AIAA Infotech@Aerospace*, Washington, DC, September 2005.