Locally Unstable Neoclassical Dynamics*  

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Abstract  

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**JEL Codes:**
1 Introduction

Recent work by Beaudry, Galizia, and Portier (2017) argues that US real GDP can be accurately described as a locally unstable process, possibly surrounded by stable limit cycles. That paper develops a complicated model of unemployment that can produce such a situation. Here I present a much simpler model, based on the stochastic growth model with capacity utilization, that features multiple steady states with the calibrated steady state being locally unstable.

2 Model

The model is a simple stochastic growth model with elastic divisible labor. The representative household solves the dynamic program

$$v(k, z) = \max_{k', u, c, h, i} \left\{ \frac{\left(c (1 - h)^{\theta}\right)^{1 - \sigma}}{1 - \sigma} + \beta E \left[v(k', z')\right] \right\}$$

subject to the resource constraints

$$c + i \leq \exp(z) (uk)^{\alpha} h^{1 - \alpha}$$
$$k' \leq \left(1 - \frac{\delta}{\omega} u^\omega + a_2\right) k + \frac{a_1}{1 - \xi} i^{1 - \frac{\xi}{\xi}} k^{\frac{\xi}{\xi}}$$
$$u \leq 1.$$

Utilization of capital acts as in Greenwood, Hercowitz, and Huffman (1988), but with an upper bound that occasionally binds. TFP follows a stationary AR(1):

$$z' = \rho z + \sigma z \epsilon'$$
$$\epsilon' \sim N(0, 1).$$

3 Solution and Calibration

I solve the model globally, using PCHIP ($k$) and linear ($z$) splines to represent the functions, Gauss-Hermite quadrature to compute the expectations, and iterate on the equations that describe
the equilibrium; I use 1001 nodes for $k$, 51 for $z$, and 15 for the quadrature calculation, so the results are immune from numerical error. The parameters are calibrated to match capital’s share of income ($\alpha = 0.36$), a capital/output ratio of 12 ($\beta = 0.9909$), a consumption/output ratio of 0.75 ($\delta = 0.04$), steady hours of 0.33 ($\theta = 1.733$), and a steady state utilization rate of 0.82 ($\omega = 1.44$). The value of $\sigma$ is 2, a common value in the business cycle literature, $\xi = 0.23$ matches the elasticity of Tobin’s $q$ with respect to the investment/capital ratio, and $(a_1, a_2)$ are set to imply zero adjustment costs and zero marginal adjustment costs in the steady state. The TFP process has $\rho = 0.95$ and $\sigma_z = 0.0076$. Thus, none of the parameter choices are controversial, so the results are also not dependent on “weird parameters”.

Figure 1 shows the policy function $k' = g(k, 0) - k$, the change in the capital stock at the mean value of $z$. The function crosses 0 at two positive values of $k$; the calibrated steady state is the lower root, and the slope at that point is 1.01 > 1. There is also the standard unstable zero steady state. Although not shown for brevity, the functions for different values of $z$ also have the same basic shape and always have the “calibrated” steady state locally unstable.

Why does the model produce such a strange-looking policy function? For comparison, Figure 1 also shows the same functions but with (i) $u = \bar{u}$ (constant utilization at the steady state level) (ii) $\xi = \infty$ (no adjustment costs), this function displays only one interior steady state and the slope of the policy function is 0.979 (locally stable). Allowing for elastic utilization but no upper bound, I find that the slope of the policy function at the steady state gets closer to 1 (0.996), but stays locally stable. Adding the upper bound $u \leq 1$ does not change the slope. Thus, the key is to have elastic utilization and capital adjustment costs.

4 Conclusion

The economy studied here is Pareto efficient but still displays multiple steady states; the steady state that matches US data is locally unstable. However, the model does not display stable limit cycles, because it remains Pareto efficient. Adding distortions that are known to create limit cycles, such as external increasing returns to scale, may therefore represent a simpler structure to

\[1\] The upper bound on $u$ is crucial for solving the model near zero capital, where the slope of the policy function becomes infinite. Since I am interested in finding all the steady states, solving the model near zero is critical.
study the locally-unstable dynamics that describe US GDP.

References


Figures

Figure 1: Capital Law of Motion