The Wall Street Response to a Regulatory Sine Curve*

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*We thank seminar participants at the Federal Reserve Board, Carnegie Mellon University, University of Virginia, Peking University, Tsinghua PBOC School of Finance, the 2013 LAEF Conference "Accounting for Accounting in Economics", the 2014 Midwest Macroeconomics Meetings, and the 2014 SED Meetings for helpful comments. Young thanks the Bankard Fund for Political Economy at Virginia, the Batten Institute at the Darden School of Business, and LAEF at Santa Barbara for financial support. The views expressed herein are the authors' and do not necessarily reflect the opinions of the Board of Governors of the Federal Reserve System.

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Abstract

It is no coincidence that in the midst of a rally stock market and a strong economy, Congress approved in May 2018 to dismantle a significant part of Dodd Frank, the banking bill introduced in the wake of the Great Recession. The intensity of financial regulatory oversight in the U.S. has historically been characterized by a "regulatory sine curve": regulatory intensity increases after a recession and wanes as the economy returns to normalcy. To investigate the regulatory response to business cycles, we construct new indexes of financial regulatory intensity (FRI) using semantic search methodology, and find a strong, negative relationship between FRI and the aggregate state of the economy. We then embed a regulator-manager interaction with cyclical regulatory intensity in a Lucas asset pricing model to study the implications for stock market dynamics. Regulatory investigations play an important role in shaping information structure in the financial markets through two channels: first, investigations detect financial manipulation *ex post* and reveal hidden negative information; second, regulatory investigations impose adverse consequences for executives involved in manipulation and therefore deter managerial incentives to manipulate *ex ante*. Our calibration results suggest that these mechanisms can be quantitatively important in accounting for general asymmetries in stock returns.

Keywords: Cyclical financial regulation, Stock crash, Gradual boom and sudden crash.

JEL Classifications: G12, G30, K20.

"These banks are back to making record profits, but Washington insists on doing them more favors, even if it means raising the risk of another bailout."

— Sen. Elizabeth Warren, May 23, 2018.

1 Introduction

A decade after the global financial crisis tipped the United States into a recession, Congress agreed in May 2018 to free thousands of banks from the Dodd Frank rules, the regulatory regime put in place in the wake of the crisis to prevent another meltdown. The recent regulatory rollback is part of a general pattern: financial regulation and revelations of frauds appear cyclical.[1] Coffee (2013) offers a detailed recount of the securities laws initiated during or immediately following recessions, and argues that there is a "regulatory sine curve": regulatory intensity increases after a recession and wanes as the economy returns to normalcy.[2] Relatedly, the long boom of the 1990s was followed, first by recession, then by revelations of financial chicanery at many of America's largest companies. A wave of fraud revelations, such as Freddie Mac, Fannie Mae, Lehman, and AIG, again clustered at the beginning of the recent economic turmoil in 2008.

Regulatory actions play an important role in determining the information present in financial markets for two reasons. First, investigations detect financial misreporting *ex post* and reveal hidden negative information; thus, high regulatory activity in the present will act to improve information about the past, the present, and the future. Second, anticipated regulatory investigations will impose adverse consequences for executives involved in manipulation, and thus help limit managerial incentives to manipulate performance *ex ante*. Despite growing interest in the effects of financial regulation, the academic literature does not contain a sufficiently long time series characterization of financial regulatory behavior. In this paper, we aim to investigate the regulatory response to business cycles as well as its implications for stock market dynamics.

[1] Timothy Geithner, "Stress Test," 2015.

[2] "The Political Economy of Dodd-Frank" by John Coffee, *Cornell Law Review*.

We begin by constructing a new indexes of financial regulatory intensity (FRI) using a semantic search approach to measure the frequency of newspaper coverage on financial regulatory actions. We also construct a subindex of FRI that specifically captures regulatory actions targeted at financial fraud detection. Both FRI and the subindex have a strong time trend, suggesting that over the last century, an increasingly larger fraction of newspaper articles have references to financial regulatory actions, possibly because the financial industry has been growing bigger as a fraction of the overall economy. The indexes exhibit a strong negative relationship with the aggregate state of the economy: they spike during the Great Depression, the post dot-com bubble recession in 2001, and the recent Great Recession; more generally, the indexes are typically higher during NBER recession than in normal times and are negatively correlated with the GDP growth rate.

We then propose a model to study the implications of cyclical regulatory intensity for stock market dynamics, based on the idea that regulatory investigations both deter managerial incentives to manipulate information *ex ante* and reveal fraud when manipulation occurs *ex post*. In our model, managers have an incentive to portray an unjustified future success and distort the information content of financial statements, and regulators conduct investigations to detect such corporate frauds with a tendency to leave more discretion to managers in booms and investigate more intensively during downturns.

We conduct this exercise within a Lucas asset-pricing model that is standard in all aspects, except that a manager reports the firm's earnings, which may be subsequently audited by regulators. In particular, a regulator-manager model is embedded in a simple variant of the Lucas asset-pricing model. There is a continuum of newly hired managers in each period, each operating a firm, and they are paid with part of the reported earnings as compensation. Managers may possibly have discretion over the earnings reported to the investors, which are paid to investors as dividends. Manipulation occurs when reported earnings differ from the true value. Regulators investigate a fraction of the firms each period, and if manipulation is detected, investors bear monetary losses depending on the scale of accumulated manipulation. The key feature we focus on here is that the fraction of firms being investigated depends on the aggregate state of the economy. The investors are assumed to be risk-neutral; thus the price of a firm in each period is given by discounted expected future dividends net of executive compensation and financial losses associated with manipulation.

In our model of asset pricing, cyclical regulatory actions deliver countercyclical crash risk through two reinforcing mechanisms. A direct impact of cyclical regulation on stock dynamics works through information revelations: the lack of investigations in good times leave financial information manipulation undetected and renders stock fluctuations fairly mild; while strengthened regulation during economic downturns reveals accumulated hidden negative information and penalizes waves of frauds, causing stocks to plummet. Equally importantly, there is also an indirect impact of financial regulation by affecting managerial incentives. Loosened regulation during booms helps fuel managerial incentives to paint a successful yet false picture of firm performance. The increased noises in reports further mute stock movements in good times, and cause the downturns even sharper when the accumulated losses all come out at once in bad times. Although most stock crashes occurred in recessions, we argue that the seeds were actually sown during booms.

Cyclical regulatory activities contribute to gradual booms and sudden crashes in the financial markets also through both information revelation and incentive distortions. The asymmetric regulatory responses to business cycles cause more negative information to be revealed in large lump during bad times, resulting in large, negative returns during episodes of weak economic conditions and worsening stock market performances. As limited investigations and subsequent revelations during upturns leave uncertainties in financial information unresolved, investors discount the seemingly excellent and yet potentially fraudulent performance, slowly updating their beliefs about corporate outlooks in booms. In addition, loosened regulatory actions fuel managerial incentives to manipulate and corrode reporting quality in booms, further mitigating investors' response to positive corporate news in good times and causing more aggressive manipulation only to be revealed and reversed during periods of stress when investigations become intense.

Finally, we study the pattern of asymmetry in stock prices more formally, using tests of "time irreversibility" developed by Ramsey and Rothman (1996). Time irreversibility is generated either by (i) nonlinear data-generating processes or (ii) non-Gaussian innovations; the tests examine whether the bicovariances between current and past realizations are symmetric (a bicovariance is the covariance between a random variable and the square of another). We find evidence of irreversibility in stock prices both in the data and in the model, and the pattern of bicovariance signs are roughly consistent as well. In particular, we find evidence of the "slow-boom sudden-crash" dynamics that are the focus of Veldkamp (2005) and van Nieuwerburgh and Veldkamp (2006). In those papers, slow booms and sudden crashes are the result of procyclical information – with lots of firms producing during expansions, agents can obtain a better estimate of average TFP. In contrast, in our model information about firms is of lower quality during expansions, because managers are more willing/able to actively hide idiosyncratic failures.

Although ours is the first article that we are aware of that ties changing regulatory actions over the business cycle to asset price movements, there are a number of articles that are related to the spirit of our analysis, which is the role of institutions in monitoring and mitigating information asymmetries. A recent paper by Chen and Zha (2015) estimates the time-varying monitoring efficiency modeled in Greenwood, Sanchez, and Wang (2010), that is, banks' ability to detect if borrowing firms misreport. A growing body of work examines "credit cycles"— the idea that banks and other credit suppliers engage in behavior that exacerbates business cycle effects, making credit even tighter in recessions and looser in expansions (Bernanke and Gertler 1989, Kiyotaki and Moore 1997, and Ruckes 2004). Albuquerque and Wang (2008) studies the asset pricing and welfare implications of imperfect investor protection and assesses the magnitude of both the loss of investor welfare and the reduction in market value due to managerial over-investment. None of these articles, however, address cyclical patterns of policy responses, which is our key focus.

The rest of the paper proceeds as follows. Section 2 provides new evidence on the cyclical properties of regulatory investigations and stock return crashes. Section 3 discusses the problem of a regulator determining investigation intensities, having in mind how their policies influence the behavior of managerial manipulation. The one-period model highlights the link between information manipulation, regulatory intensities, and business cycles. Section 4 embeds this regulator-manager interaction into a dynamic infinite horizon economy to examine the implied properties for asset pricing. Section 5 presents the results and mechanisms of a calibrated version of the model. Section 6 concludes. We provide additional evidence on cyclical regulatory behavior and all technical details in the Appendix.

2 Cyclical Patterns of Financial Regulation and Stock Crashes

In this section we will discuss the cyclical patterns of financial regulation and provide new evidence on the cyclical properties of stock returns.

2.1 Financial Regulation and the Business Cycle

Casual observations suggest that financial regulation tends to become tightened during economic downturns.[3] Coffee (2013) offers a detailed recount of the securities laws initiated during or immediately following recessions, and argues that there is a "regulatory sine curve": regulatory intensity increases after a recession and wanes as the economy returns to normalcy.[4] For example, the Glass-Steagall Act was passed in 1932, during the Great Depression, but was gradually repealed over several decades, culminating with the Graham-Leach-Bliley Act of 1999 while the U.S. economy was experiencing the Great Moderation.

In addition to the rule-making of financial regulation, the implementation and enforcement of existing rules are at the discretion of federal regulators and may vary systematically over time, which may arguably have more direct, real-time information implications. It has

[3] Timothy Geithner, "Stress Test," 2015.

[4] "The Political Economy of Dodd-Frank" by John Coffee, *Cornell Law Review*.

been argued that a boom encourages and conceals financial fraud and misrepresentation by firms, which are then revealed by the ensuing recession.[5] The long boom of the 1990s was followed, first by recession, then by revelations of financial chicanery at many of America's largest companies. A wave of fraud revelations, such as Freddie Mac, Fannie Mae, Lehman, and AIG, again clustered at the beginning of the recent economic turmoil in 2008. Bertomeu and Magee (2011) show that quality of financial reporting and probability of revelations are minimal prior to a recession and increases during and after a recession. Empirical evidence for cyclical bias in bank examination, both in terms of frequency and rating stringency, is also mounting (see Syron 1991, Berger, Kyle, and Scalise 2001, and Laeven and Majnoni 2003). These observations motivate us to construct new indexes that separate out the implementation aspect of regulatory intensity from the overall references of financial regulation.

2.1.1 An Indicator of Financial Regulatory Intensity

The main objective of this section is to construct an indicator of financial regulatory intensity to analyze how it evolves over time. Existing datasets on financial regulatory actions only cover a relatively short period (typically starting from the late 1990s) and are piecemeal in nature. To investigate whether the intensity of financial regulation is systematically related to the underlying economic conditions, we construct a new index of the intensity of financial regulation using a text-analysis approach that measures the frequency of newspaper coverage on financial regulatory actions, and examine its evolution since 1900.

Using the ProQuest Newsstand and historical archives, we construct an index by searching from January 1900 onward for articles containing 'regulation' or 'Regulatory', 'financial' or 'finance', and one or more of the following terms: 'congress'; 'federal reserve/fed'; 'SEC/Security and Exchange Commission'; 'OCC/Cffice of the Controller of the Currency'; 'FDIC/Federal deposit insurance corporation" 'Stock exchange'. To meet our criteria, articles must include all three categories pertaining to financial regulation. To gauge how

[5] See Povel, Singh, and Winton (2007), Brunnermeier et al. (2009), and McDonnell (2013).

stringent existing regulatory rules are enforced, we also construct a sub-index specifically for implementation of financial regulation that search for articles containing, in addition to the above seach terms, one or more of the following terms: ‘enforcement’; ‘lawsuit’; ‘penalties’; ‘fines’; ‘investigation’; ‘examination’; ‘civil charges’; ‘criminal charges’; ’manipulation’; ’fraud’; and ‘supervision’. We do this for every day’s issue of the Washington Post, Wall Street Journal, and New York Times. The indexes are normalized by the total number of news articles in each newspaper, and are aggregated by summing the resulting series and scaling them to have a mean of 100 over the sample. Appendix A provides a detailed description of the index construction.

Figure 1 displays our Financial Regulation Indexes. The first striking feature is the strong time trend. It suggests that over the last century, an increasingly larger fraction of newspaper articles have references to financial regulation, possibly because the financial industry has been growing bigger as a fraction of the entire economy during the last century. The second feature is that the indexes co-move with the business cycle. For example, they spike during the Great Depression, the post dot-com bubble recession in 2001, and the recent Great Recession. More generally, the indexes are typically higher during NBER recessions than in normal times. Figure 2 and Figure 3 plot the Financial Regulation Index (FRI) and the FRI subindex against the GDP growth rate. They demonstrate a strong negative correlation, with a correlation coefficient of -0.566 after 1985 for the overall index and -0.603 for the subindex. We also report in Table 1 the results from binary regressions of FRI and FRI subindex on several measures of the aggregate state of the economy at the annual frequency from 1960 through 2016.

The methodology used to compute the FRI is similar to the one Baker, Bloom, and Davis (2017) followed to construct EPU. While we both use a semantic search approach to identify relevant newspaper articles, the set of words used in the searches is dramatically different, and the two indexes measure different economic variables and are characterized by distinct features. Our Financial Regulation Index capture a first-moment concept: the level of intensity in financial regulatory actions, and the second-moment movement, i.e.,

Table 1: The relationship between FRI/FRI subindex and the aggregate economy

	GDP growth	Aggregate earnings	Weighted average earnings per share
FRI	-9.098^{***}	-8.15^{***}	-264.099^{***}
FRI subindex	-7.284^{***}	-10.29^{***}	-366.384^{***}

the "uncertainty" aspect is at the very heart of Baker, Bloom, and Davis' EPU index. For example, EPU spiked in response to 9/11 event, Gulf War II, debt ceiling dispute of 2011, and the 2012 Fiscal Cliff Dispute, all of which FRI's are not response to. While national security threats or monetary policy shocks generate uncertainty about government policy overall, they do not have direct implications for the level of financial regulation.

2.2 Stock Returns and the Business Cycle

In this part we document a systematic relationship between recessions and the probability of stock crashes. Here, we present two measures of a stock crash. The first is taken from Barro and Ursua (2009) — a crash is a realized real return of less than −25 percent. In Figure (4), we see that, while the correlation of crash risk (frequency of crashes) with NBER recessions is low (0.07), there are pronounced spikes during each recession; the low correlation is then attributable to the smaller variation during expansions.

Figure (5) uses a measure of crash risk from Jin and Myers (2006), denoted COUNT, that captures the measured number of returns that exceed k standard deviations below the mean, with k chosen to generate frequencies of 0.1 percent in a log-normal distribution.[6] A high value of COUNT indicates high crash risk. We can see that not only is crash risk positively correlated with NBER recessions (0.39), it also tends to spike at the onset of recessions. If we look at GDP growth, we find a similar story — the correlation between crash risk and output is −0.42.

[6] Using a threshold of 0.01 percent does not change the correlation.

In discussion of the recent financial crisis of 2008-2009, much is made of the apparent coexistence of the economic downturn and increased stock crash risk. We show that such coexistence is not new — changes in the crash risk in the U.S. stock market have been coinciding with changes in the real economy.

We also want to point here to two other features of stock returns. First, the data display negative skewness (-0.139), showing that big downward movements (crashes) are more likely than big upward movements. Second, volatility is countercyclical – volatility in recessions is on average 1.39 times as large as in expansions.

3 Regulator-manager interaction

Here we present a simple model that highlights mechanisms that could give rise to countercyclical regulatory action, which in turn affects the percentage of firms committing frauds. The key parameter will govern the choice of regulatory intensity – how often regulators detect fraudulent earnings reports. This model will micro-found the cyclical bias in both regulatory detections of frauds and the prevalence of frauds that will be embedded in an asset pricing framework.

We assume that there is a continuum of firms, each being run by a manager. Firms' true earnings are jointly determined by an aggregate and an idiosyncratic state in each period. There are two values for the aggregate state $a \in \{g, b\}$ with $g > b$ ("good" and "bad"), and two values for the idiosyncratic state $y \in \{h, l\}$ with $h > l$ ("high" and "low"). Each firm's earnings are then ay; we assume a law of large numbers holds for the idiosyncratic state conditional on the aggregate state. The aggregate state is perfectly and costlessly observed by both the regulator and the firm manager, whereas the idiosyncratic state is the private information of the manager; manipulation occurs when the manager reports a high value of y when the true value is low.[7]

[7] Feroz, Park, and Pastena (1991) document that most SEC enforcement actions are aimed at overstatements, the average amount of restatements is negative, and over 75 percent of restatements are negative. Our model thus focuses on upward manipulation.

States realize: a (public info) y (private info)	Regulator commits to investigation frequency: τ	Manager reports: r	Possible investigations take place	Actual cash flows

The timeline in the above figure chronicles the sequence of events in the model. At the beginning of each period, an aggregate state (a) is perfectly revealed to all agents, and each firm's idiosyncratic state (y) is privately observed by the manager. The regulator commits to a regulatory policy in the current period, which boils down to a frequency of investigations (τ). Each manager subsequently makes a report of the idiosyncratic state (r), and investigation follows if there is any. We assume (without modeling the details such as credit conditions and collateral constraints) that the manager is unable to hide manipulation and will get caught if investigated in bad aggregate state (i.e., $a = b$). When the aggregate state is good ($a = g$), with probability $1 - \varepsilon$, the manager is able to finance the discrepancy in the report and conceal manipulation when investigated (and therefore escape punishment); while with probability ε this financing will not be possible and will be charged with fines $F(m)$. We denote the report of y by $R(y) = r \in \{h, l\}$.

We build in explicitly, therefore, a state-dependent probability of detection given investigation, by directly assuming it to be higher during recessions. Managers have incentives to manipulate performance by hiding temporary losses to avoid disclosing negative information about the firm. During booms, cash flows from corporate operations and external credit are readily available to absorb losses and thus prevent fraud from being revealed. In downturns, however, liquidity cushion tends to evaporate and managers may lose access to these funds, finding it more difficult to obscure reporting discrepancies.

The regulator chooses the frequency of investigations τ to minimize the prevalence of frauds subject to the cost of investigation, given by $C(\tau) = C\frac{\tau^2}{2}$, where $C \geq 0$. Managers differ according to the private utility they receive from reported earnings — a report of r delivers θar utils to the mangers, where $\theta \sim UNI\,[0,1]$. We view this assumption as a simple shorthand for a wide variety of reasons that managers may have different valuations of manipulating earnings, including variation in pay-performance sensitivity, preferences, and the complexity of business operations. The value of θ is the private information of the manager.

The Bayesian Nash equilibrium is characterized by $\tau(g) < \tau(b)$ and $x(g) > x(b)$ (solved in Appendix A). Managerial incentives to misreport vary with the aggregate state (a) for several reasons. First, the variation in τ and $p(a)$ implies that detection is lower in good times; then there will be a strong incentive to inflate earnings when a is high. Second, the private benefit from manipulation is increasing in a, which is an abstraction of factors such as high stock valuation associated with their option compensation and strategic motives when competitors are all reporting high earnings. These implications are consistent with the empirical literature. Wang, Winton, and Yu (2007) find that the effect of industry investment on fraud propensity is strongly positive. Cohen and Zarowin (2012) find that the tendency of firms to manipulate earnings upward to beat benchmarks is positively correlated with market-wide conditions. Both facts imply that, when the economy is booming, it is likely that more firms are misstating their earnings upward.

Our model generates the following features relevant for asset pricing. First, cyclical patterns in managerial manipulation emerge in response to a cyclical bias in regulatory actions: a boom encourages and conceals frauds. Second, rational investors who are informed about the regulator-managers interaction are uncertain about whether a particular report has been inflated. That is, investors can perfectly infer x given the equilibrium regulatory policy, but they cannot correctly gauge firms' idiosyncratic state. We show in the next section that the relationship between investigation intensity and manipulation frequency — together with the unrevealing financial reporting caused by manipulation — has considerable implications for the dynamics of financial markets.

4 Cyclical Regulation and Asset Pricing

We now embed the mechanisms identified in Section 3 into a dynamic model of asset pricing. We introduce risk-neutral investors who price stocks based on earnings reports, understanding that these reports involve incentives to misreport and investigation probabilities that move cyclically. Specifically, three central features generated in our model of regulation will be embedded in an infinite horizon economy to examine the implied business cycle properties of stock returns: (i) regulatory investigations that reveal hidden negative information are cyclical; (ii) manipulation tendencies are influenced by cyclical regulatory behavior and exhibit cyclicality; (iii) investors can infer the likelihood of manipulation (x) given the equilibrium regulatory policy, but cannot unambiguously gauge the true state of each firm. Note that x_a and λ_a were endogenous in the previous section, here we adopt a reduced-form approach in the pricing analysis using calibrated values of x_a and λ_a.

4.1 Setup

The economy is populated by a continuum of managers who are hired by investors to operate firms; these managers make reports regarding earnings just as in Section 3. The aggregate state a again takes on two values, g and b, with a transition matrix

$$\Pr\left(a_{t+1} = j | a_t = i\right) = \pi_{ij}.$$

Individual firm productivity takes on two values, h and l, with a transition matrix

$$\Pr\left(y_{t+1} = j | y_t = i\right) = \varpi_{ij}.$$

We will assume both processes are positively autocorrelated ($\pi_{hh} > \pi_{hl}$ and $\pi_{ll} > \pi_{lh}$) and symmetric ($\pi_{hh} = \pi_{ll}$). Total earnings are given by $a_t y_t$.

The timeline of the model events in each period is described in the figure below. Each period begins with the realization of a_t, which again is public information (observable to managers, regulators, and investors). The idiosyncratic value y_t is privately observed by the managers, who then make their report r_t. We simplify the regulator's action by assuming directly the probability of detection (given manipulation) $\lambda(a_t)$, where $\lambda(g) < \lambda(b)$; note that this probability includes both the intensity of regulation $\tau(a_t)$ and the success probability $p(a_t)$.

States realize: a_t (public info) y_t (private info)	Regulator commits to investigation frequency: τ_t	Manager reports r_t	Asset price based on a_t, r_t and others	Possible investigations take place; investors bear losses	Dividends are paid

4.2 Investor Learning and Price Formation

Investors are risk neutral and own the firms; the price of firm equity is then given by the present value of expected future dividends net of expected financial loss associated with detected manipulation. For notational convenience, we denote reports by $\widetilde{h}$ and $\widetilde{l}$ to distinguish them clearly from realized values h and l. Recall that $R(h) = \widetilde{h}$ (high productivity always generates a high report), $R(l) = \widetilde{h}$ with probability $x(a)$, and $R(l) = \widetilde{l}$ with probability $1 - x(a)$ (low productivity generates manipulation if $\theta \geq \widetilde{\theta}$). Calculating the posterior probability that actual earnings were high involves assessing the entire history of reports, back to the most recent detection, because the financial losses that investors bear are proportional to the number of misreported earnings; thus, when the manager makes a report, investors reassess all past reports as well as the current one. The financial loss borne by investors captures the observation that SEC enforcement actions and restatement announcements are associated with drastic market reactions and have negative impact on firms' future prospects (e.g., stock returns on average fall by about 10 percent around earnings restatements in the data). The SEC has collected over $10 billion penalties in fraud cases since 2002, and the amount of settlement fines has been growing over time. The loss of confidence in corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may force firms to forgo investment and accelerate layoffs.

We denote the current stock price by q_t. The relevant information in the report history can be summarized by a small set of state variables. In particular, we can price assets using the following set of state variables (for detailed examples of what each state variable represents, see Appendix B):

- a: the current aggregate productivity;
- γ: the conditional probability (based on information in the current report) that $y_t = h$;
- Z: the expected number of periods involving false reports between the last detection and the most recent low report (note that $Z = 0$ if there is no low report between the last detection and the current period);
- N: the number of consecutive high reports since the more recent of the last low report or the last detection;
- r: the current report;
- $\overline{y}$: the actual value of y realized before the series of N consecutive high reports.

Thus, $q_t = Q(a_t, \gamma_t, Z_t, N_t, r_t, \overline{y}_t)$.[8] Current idiosyncratic productivity is revealed in two circumstances. First, low reports are known to be truthful, meaning investors know the current value of y_t. Second, fraud detection occurs, revealing the entire history of y_t. The distinction will be important for pricing assets — a low report implies low productivity (albeit truthful) today, which will lead to downward revisions of past reports, because y_t is positively autocorrelated; while detection reveals *all* past reports. The law of motion for γ follows Bayes' rule:

$$\gamma_{t+1}^{a} = \begin{cases} \frac{\gamma\varpi_{hh}+(1-\gamma)\varpi_{lh}}{\mu_a} & r_{t+1} = \widetilde{h} \\ 0 & r_{t+1} = \widetilde{l}, \end{cases}$$

[8]In Sun (2014), the information in a history of possibly manipulated reports is characterized by a similar set of state variables. We extend Sun (2014) to incorporate state-dependent manipulation and detection, that is, both the likelihood of manipulation and that of detection vary systematically with the aggregate state.

where

$$\mu_a = \gamma\varpi_{hh} + (1-\gamma)\,\varpi_{lh} + \gamma\,(1-\varpi_{hh})\,x\,(a) + (1-\gamma)\,(1-\varpi_{lh})\,x\,(a)$$

is the conditional probability that a manager makes a high report next period in aggregate state a.

A high report in aggregate state a_t implies

$$Q\left(a_t, \gamma_t, Z_t, N_t, \widetilde{h}, \overline{y}_t\right) = a_t\widetilde{h} + (1-\lambda\,(a_t))\,W_{na}^{\widetilde{h}} + \lambda\,(a_t)\,W_{da}^{\widetilde{h}},$$

where W_{na}^{r} is the expected future value of the stock if detection does not occur in the current period and W_{da}^{r} is the expected value if detection takes place. Both future values are conditioned on the report and the current aggregate state. If no detection takes place, then

$$\begin{aligned} W_{na}^{\widetilde{h}} = {} & \beta\pi_{ag}\left[\mu_g Q\left(g, \gamma_{t+1}^{g}, Z_t, N_t+1, \widetilde{h}, \overline{y}_t\right) + (1-\mu_g)\,Q\left(g, 0, Z_t, N_t+1, \widetilde{l}, \overline{y}_t\right)\right] + \\ & \beta\pi_{ab}\left[\mu_b Q\left(b, \gamma_{t+1}^{b}, Z_t, N_t+1, \widetilde{l}, \overline{y}_t\right) + (1-\mu_b)\,Q\left(b, 0, Z_t, N_t+1, \widetilde{l}, \overline{y}_t\right)\right]. \end{aligned}$$

In contrast, if the firm is being investigated, then

$$\begin{aligned} W_{da}^{\widetilde{h}} = {} & -\kappa\left[Z_t + f\,(N_t+1, \overline{y}_t)\right] + \\ & \beta\pi_{ag}\left[\begin{array}{c} \gamma\left[\xi_{1g} Q\left(g, \frac{\varpi_{hh}}{\xi_{1g}}, 0, 0, \widetilde{h}, h\right) + (1-\xi_{1g})\,Q\left(g, 0, 0, 0, \widetilde{l}, h\right)\right] + \\ (1-\gamma)\left[\xi_{2g} Q\left(g, \frac{\varpi_{lh}}{\xi_{2g}}, 0, 0, \widetilde{h}, l\right) + (1-\xi_{2g})\,Q\left(g, 0, 0, 0, \widetilde{l}, l\right)\right] \end{array}\right] + \\ & \beta\pi_{ab}\left[\begin{array}{c} \gamma\left[\xi_{1b} Q\left(b, \frac{\varpi_{hh}}{\xi_{1b}}, 0, 0, \widetilde{h}, h\right) + (1-\xi_{1b})\,Q\left(b, 0, 0, 0, \widetilde{l}, h\right)\right] + \\ (1-\gamma)\left[\xi_{2b} Q\left(b, \frac{\varpi_{lh}}{\xi_{2b}}, 0, 0, \widetilde{h}, l\right) + (1-\xi_{2b})\,Q\left(b.0, 0, 0, \widetilde{l}, l\right)\right] \end{array}\right]. \end{aligned} \tag{1}$$

Here, ξ_{1a} is the conditional probability of getting a high report tomorrow when the aggregate state tomorrow is a, given the current idiosyncratic state is high, and ξ_{2a} is the probability of the high report tomorrow given the current idiosyncratic state is low:

$$\begin{aligned} \xi_{1a} &= \varpi_{hh} + (1-\varpi_{hh})\,x\,(a)\,, \\ \xi_{2a} &= \varpi_{lh} + (1-\varpi_{lh})\,x\,(a)\,. \end{aligned}$$

The first term in Equation (1) is the expected monetary loss for detected manipulation, composed of three parts. κ is the financial loss per unit of misstated earnings. As noted above, Z_t is the expected number of periods involving manipulation between the last detection and the most recent low report. $f(N_t + 1, \overline{y}_t)$ is the expected number of falsified reports amoung the $N_t + 1$ consecutive high reports that have occurred since the last low report or the last detection, whichever is more recent.[9]

If the current report is low the price is simpler, since it does not involve the possibility of current manipulation and therefore does not depend on γ:

$$Q(a_t, 0, Z_t, N_t, \overline{y}_t) = a_t\widetilde{l} + (1 - \lambda(a_t)) W_{na}^{\widetilde{l}} + \lambda(a_t) W_{da}^{\widetilde{l}};$$

note that even though the current report is low, the firm may well have misrepresented past earnings and therefore may still be investigated. The two expected value terms are

$$W_{na}^{\widetilde{l}} = \beta\pi_{ag}\left[\xi_g Q\left(g, \frac{\varpi_{lh}}{\xi_g}, Z_t, 0, \widetilde{h}, l\right) + (1 - \xi_g) Q\left(g, 0, Z_t, 0, \widetilde{l}, l\right)\right] +$$
$$\beta\pi_{ab}\left[\xi_b Q\left(b, \frac{\varpi_{lh}}{\xi_b}, Z_t, 0, \widetilde{h}, l\right) + (1 - \xi_b) Q\left(b, 0, Z_t, 0, \widetilde{l}, l\right)\right]$$

and

$$\begin{aligned} W_{da}^{\widetilde{l}} = {} & -\kappa\left[Z_t + f(N_t, \overline{y}_t)\right] + \\ & \beta\pi_{ag}\left[\xi_g Q\left(g, \frac{\varpi_{lh}}{\xi_g}, 0, 0, \widetilde{h}, l\right) + (1 - \xi_g) Q\left(g, 0, 0, 0, \widetilde{l}, l\right)\right] + \\ & \beta\pi_{ab}\left[\xi_b Q\left(b, \frac{\varpi_{lh}}{\xi_b}, 0, 0, \widetilde{h}, l\right) + (1 - \xi_b) Q\left(b, 0, 0, 0, \widetilde{l}, l\right)\right], \end{aligned}$$

where

$$\xi_a = \varpi_{lh} + (1 - \varpi_{lh}) x(a)$$

is the conditional probability of a high report tomorrow (note that since current productivity is known to be low, we do not need two different probabilities here). It is a straightforward application of the Banach fixed point theorem to show that Q exists and

[9] The derivation of f immediately follows that in Sun (2014), and is derived in Appendix B for clarity.

is unique. We define the gross return on a stock as reported earnings minus penalties (interpreted as the dividend) plus the current stock price divided by the lagged stock price.

These expressions permit us to compute the price recursively. We now proceed to show the mechanisms and results of the model in a calibrated exercise.

5 Quantitative Exercise

In this section we use numerical methods to study the implications of our model for stock prices. We first present our calibration and display some of the equilibrium functions. Then we discuss the model's implications for stock crashes, countercyclical volatility, and return skewness. Finally we turn to a study of time irreversibility.

5.1 Calibration

To calibrate our model we simulate a panel of 500 firms for 2000 periods; we choose the model parameters to match a specified set of facts using this simulated panel. We calibrate the Markov process of the aggregate state following Krusell and Smith (1998). Given the aggregate state process, we use Tauchen method to calibrate the transition probabilities and binary levels of the idiosyncratic productivity to match the mean and standard deviation of the S&P Composite deflated scaled earnings. The discount factor β plays no important role, so we pick $\beta = 0.98$ (a quarterly interest rate of 2 percent). For κ, the cost of being detected manipulating earnings, Karpoff, Lee, and Martin (2008) estimate that for each dollar of misreported earnings, a firm loses \$4.08 dollars when manipulation is revealed, so we set $\kappa = 4.08\,(h - l)$. We use the average value of firms subject to legal action during NBER expansions and recessions to set $\lambda(g)$ and $\lambda(b)$. To set $x(a)$ and $x(b)$, we follow the results in Cohen and Zarowin (2012). Table 1 summarizes the parameter choices.

Table 1

Calibrated Parameter Values

Parameter	Description	Value
g	Level of good aggregate state	1.01
b	Level of bad aggregate state	0.99
h	Level of high idiosyncratic state	1.064
l	Level of low idiosyncratic state	0.936
π_{gg}	Persistence of good aggregate state	0.975
ϖ_{hh}	Persistence of good idiosyncratic state	0.748
β	Discount Factor	0.980
κ	Monetary Loss	3.146
$\lambda(g)$	Detection probability in good state	0.024
$\lambda(b)$	Detection probability in bad state	0.037
$x(g)$	Manipulation likelihood in good state	0.060
$x(b)$	Manipulation likelihood in bad state	0.040

Figure (6) shows the function $f(N, \overline{y})$; the expected number of inflated reports rises monotonically with N for an initial low report, and displays a small downward segment between $N = 1$ and $N = 2$ for an initial high report. Figures (7)-(9) show how the pricing function Q varies with (γ, Z, N) across different reports. As the monetary loss associated with detected manipulation is a linear function of the number of restated financial statements, the price in response to a high report is linearly increasing in γ and linearly decreasing in Z. The price in response to a low report is also linearly decreasing in Z, with γ updated to 0.

5.2 Countercyclical Information

We discuss briefly here the central mechanism in the model, since it will underlie all of the facts we discuss below. During good times, enforcement is low $(\lambda(g) < \lambda(b))$ and therefore the incentive to misreport is high ($x(g) > x(b)$); furthermore, booms are persistent ($\pi_{gg} > \pi_{gb}$). Thus, during a boom managers will accumulate a substantial stock of negative information that they are able to hide effectively. As a result, stock prices become insensitive to upward changes in earnings, as most such reports will be rationally discounted as likely to be misrepresentations – that is, information is low during expansions.[10]

In downturns, however, stock prices are more sensitive to upward changes in idiosyncratic states, since the likelihood of a false report is lower. In addition, the more intensive investigations reveal more frauds, leading to larger downward movements in prices.

5.2.1 Countercyclical Crash Risk

Table 2 presents the model's implications for the behavior of crash risk. As noted previously, we consider two definitions — the Barro and Ursua (2009) definition and the Jin and Myers (2006) COUNT variable.The first column in Table 2 shows the average percentage of firms experiencing a stock crash, defined as cumulative real return of -0.25 or lower in Barro and Ursua (2009), in the good aggregate state and bad aggregate state respectively. The second and third column show the average value of COUNT at 0.1% and 0.01% frequencies in the good and bad aggregate state. Following Jin and Myers (2006), we calculate COUNT, as the frequency of crash, based on the number of residual returns exceeding k standard deviations above and below the mean, with k chosen to generate frequencies of 0.01% or 0.1% in the lognormal distribution. Following Jin and Myers (2006), we subtract the upside frequencies from the downside frequencies. A high value of COUNT indicates a high frequency of crashes. In the last column, we measure crash risk using the fourth moment of stock returns about the mean scaled by squared variance.

[10] This situation reverses that in van Nieuwenburgh and Veldkamp (2006), who have abundant information in booms as more firms in operation generate a higher signal-to-noise ratio for observed productivity. The information environment characterized in our model is motivated by the cyclical pattern of fraudulent financial reporting.

Table 2 Crash Risk				
	% Crash	COUNT(0.1%)	COUNT(0.01%)	Kurtosis
a=b	3.59	0.24	0.21	8.52
a=g	1.22	0.014	0.06	3.85

Our model delivers substantial countercyclical variation in crash risk; depending on the measure we get a drop in crash risk from 66 to 94 percent in good aggregate state. We also show that the model displays strong countercyclical movements in kurtosis. However, as Figure (10) shows, overall our model delivers considerably less variation in crash risk compared to the data – while it jumps up at the right time, it doesn't move enough. Part of this low volatility is likely due to the fact we only have two states in our Markov process for earnings; unfortunately the learning problem becomes rapidly intractable if we add more states.

5.2.2 Countercyclical Volatility and Negative Skewness

Table 3 shows how the cross-sectional volatility of stock returns varies with the aggregate state. While the model delivers less volatility than data overall, it matches almost perfectly that relative volatility in expansions and recessions.

Table 3 Higher-Order Moments of Returns				
	$\sigma(g)$	$\sigma(b)$	$\frac{\sigma(b)}{\sigma(g)}$	Skew
Model	0.03	0.04	1.53	-0.062
Data	0.1	0.14	1.39	-0.139

Gradual booms and sudden crashes in the financial markets have been well documented: stock prices declines are sharper than any boom of equal magnitude (see Veldkamp 2005). As a measure of this asymmetry between booms and crashes, the sknewness is negative both in our model and in the data. The size of sknewness is only half as large as found in the data; as with volatility, this shortfall is at least partially attributable to the two-state Markov process for earnings.

5.3 Time Irreversibility

In this section we consider more generally the asymmetric dynamics present in stock returns and how our model fares at reproducing it. Specifically, following Ramsey and Rothman (1996), we look at time reversibility, which we define here as in that paper.

Definition 1 *A random variable X is time-reversible if*

$$E\left[X_t^i \cdot X_{t-k}^j\right] = E\left[X_t^j \cdot X_{t-k}^i\right]$$

holds $\forall i, j, k$.

The superscripts denote powers and the subscripts denote time. We use $i = 2$, $j = 1$, and $k \in \{1, 2, 3, 4, 5\}$.

Table 4 shows the pattern of coefficients for the model and the data. The TR-test from Ramsey and Rothman (1996) strongly rejects the presence of time reversibility in both data and model time series for stock prices (the portmanteau test statistics are 13.767 and 18.686 in the data and the model, respectively, which are significant at 5 percent and 1 percent).

Table 4

Test of Time-Reversibility

k	1	2	3	4	5
Data	0.046	−0.606	−3.022	−1.981	0.586
Model	0.001	−2.030	−3.057	−2.042	−1.023

The model fits the general sign pattern well, although the magnitudes are off. The positive sign at $k = 1$ is another indication of "slow boom-sudden crash" dynamics (note that the coefficient in the model is in fact significant, because we have essentially unlimited data);

the meaning of the negative signs at higher values of k is not clear and we could not find any interpretation in the literature. These results are related to those found in Veldkamp (2005), who uses a related statistic from Hinich (1982) to characterize irreversibility.

5.4 Discussion: key drivers of model predictions

This subsection discusses which of the model's features are necessary for its key results. In substance, there are three essential features in the model. First and foremost, there is cyclical bias in regulatory intensity, which encourages frauds in booms and reveals them in recessions. Second, manipulation imposes adverse consequences on firms. These two elements deliver the asymmetries in stock returns: In bad times, strengthened regulation leads to waves of frauds to be revealed and penalized, causing stocks to plummet; while lack of information revelation in good times renders stock fluctuations mild. In addition, investors' Bayesian learning of manipulation also plays a role: rational investors are aware of but cannot perfectly see through manipulation, therefore the noise in financial information mitigates stock responses in booms and actual detection of frauds during downturns can lead to downward revisions in beliefs about firms' prospect, exacerbating the asymmetric pattern. We note that we abstract away from risk aversion to simplify our analysis, and it is natural to envision that with a state-dependent stochastic discount factor, the large monetary losses investors incur for fraud detection during downturns (exactly when investors' consumption growth is low) can translate into a higher equity premium demanded by investors. Now we discuss each of the three key elements in detail below.

Cyclical bias in regulation and frauds

Negative consequences of manipulation

In the model, revelation of manipulation imposes a large amount of financial losses on investors, which generates sharp drops in returns. This model feature is intended to capture the financial cost of manipulation that investors bear in practice, and it is also necessary in closing the model in a consistent manner. SEC enforcement actions and restatement announcements that are associated with drastic market reactions and negative impact on firms' future prospects (e.g. stock returns on average fall by about 10% around earnings restatements in the data). The Securities and Exchange Commission has collected over $20 billion penalties in fraud cases since 2002, and the amount of settlement fines has been growing over time. The loss of confidence in corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may force firms to forgo investment and accelerate layoffs. This financial cost also captures the adverse consequences of manipulation on firms' future productivity as manipulation is often done by taking suboptimal economic decisions: managers may engage in activities that boost current earnings at the expense of future benefits, such as forsaking profitable investment and postponing R&D and capital spending plans.[11]

Investors' Bayesian learning

One important and novel feature of the model is the Bayesian updating procedure that investors follow to extract information and price the assets. Whenever there is a new report from the manager, investors have to form their belief about historical performance (because penalties also depend on previous manipulation), make inferences about the current outcome, and predict future earnings. It is similar to Kalman smoothing problems, yet with Bayesian updating aspects. In the case with stochastic investigation, investors need to keep track of the entire history of reports and keep updating on everything that happened in the infinite past. This learning process creates an additional channel through which cyclical regulation dampens stock responses in good times (less learning) and generates large downward movements in bad times (belief revision).

6 Conclusion

We construct new indexes of financial regulatory intensity (FRI) and find patterns resembling a "regulatory sine curve": the intensity of financial regulatory intensity, gauged

[11]Graham et al. [2005] document that 78% of executives in their survey admit to sacrificing long-term value to maintain predictability in earnings.

through newspaper coverage, is negatively associated with the aggregate economic conditions. We then develop and calibrate a model that is meant to capture a simple piece of intuition about the effect of cyclical regulatory policies on stock markets: negative information hidden by corporate executives is more likely to be flushed out through investigations when the economy is falling, as opposed to rising. As we have argued, this mechanism can help shed light on a variety of stylized facts and in particular, countercyclical crash risk. We also provide empirical evidence for business cycle variations of crash risk in the data. Our model indicates that the dual role of regulatory activities in both deterring manipulation and revealing hidden negative information has considerable implications for asset pricing, and their timing is important for crash risk over the business cycle.

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Figure 1: Financial Regulation Index (Annual)

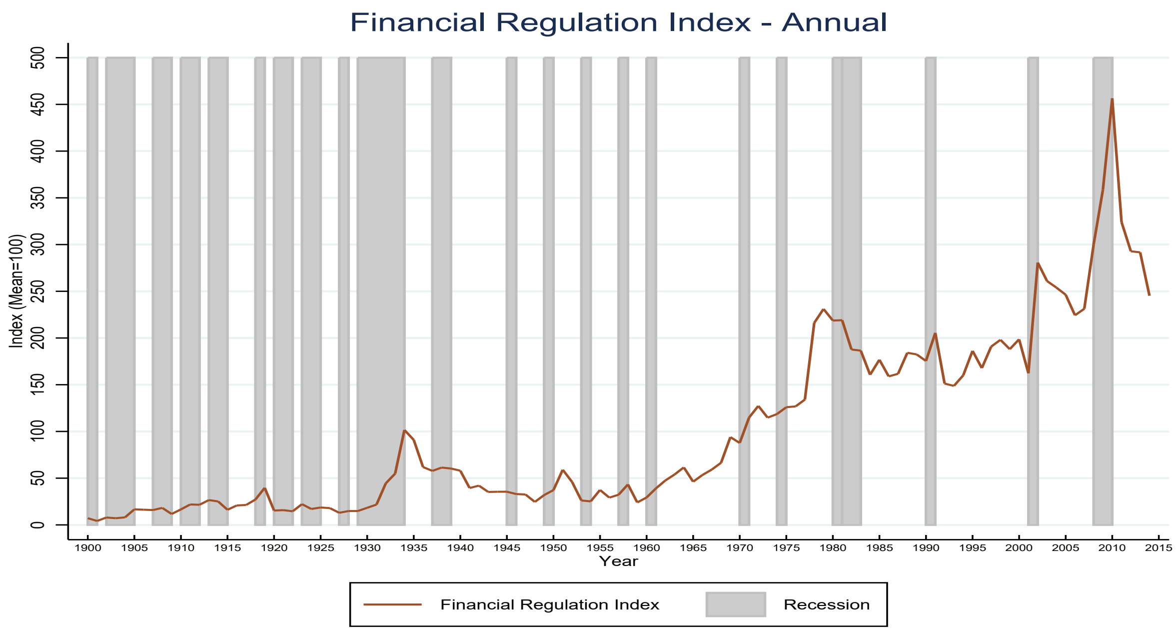


Figure 2: FRI post-1985 Pattern

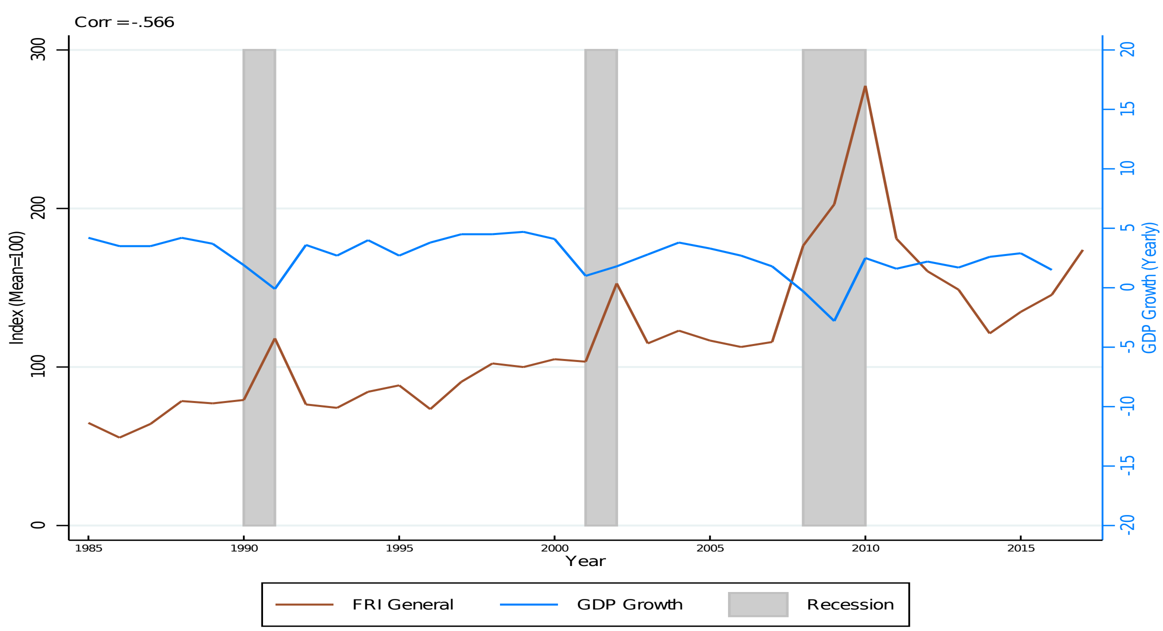


Figure 3: FRI subindex post-1985 Pattern

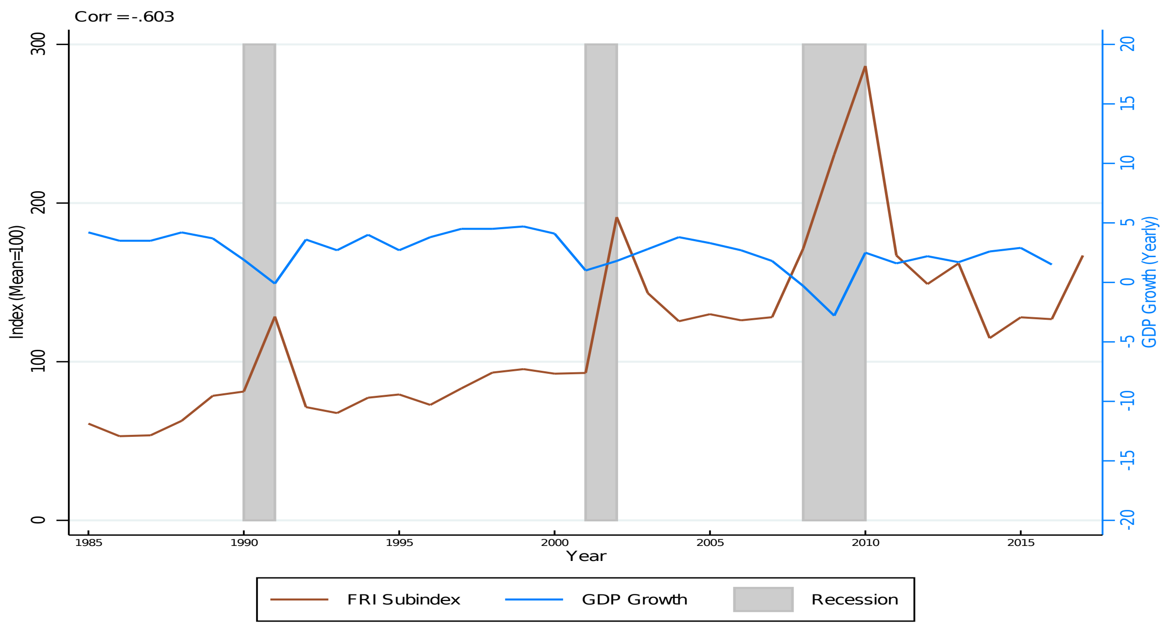


Figure 4: Barro-Ursua Crash Risk

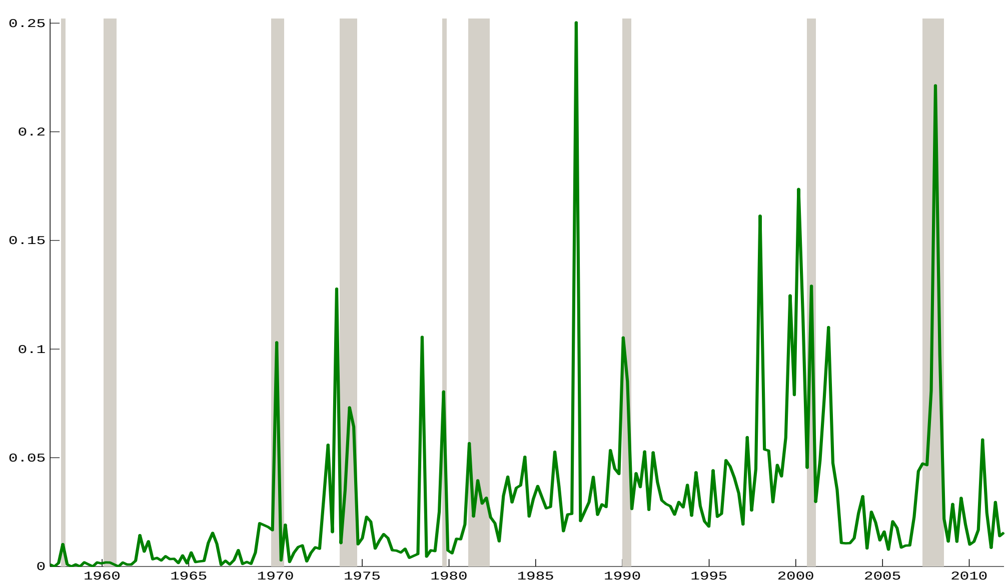


Figure 5: Jin-Myers Crash Risk

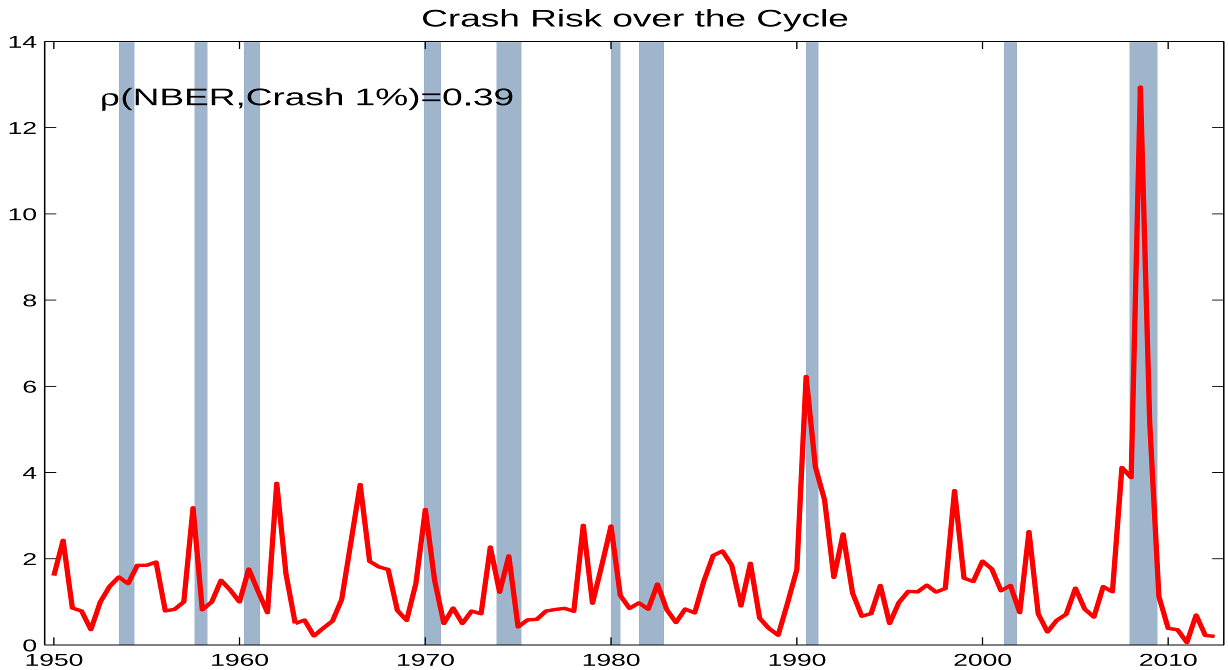


Figure 6: Expected Penalty Function

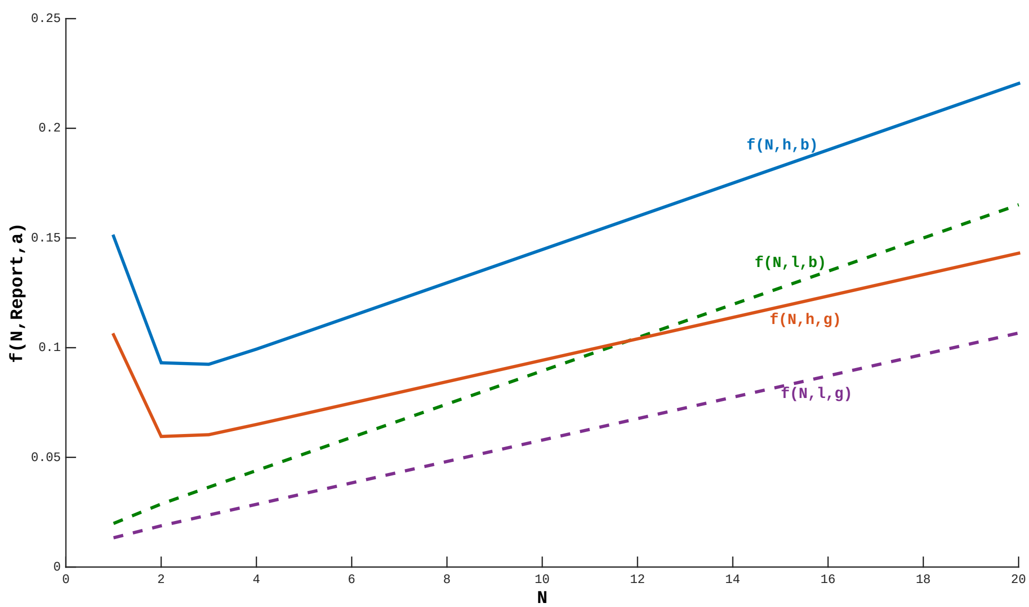


Figure 7: Pricing Function

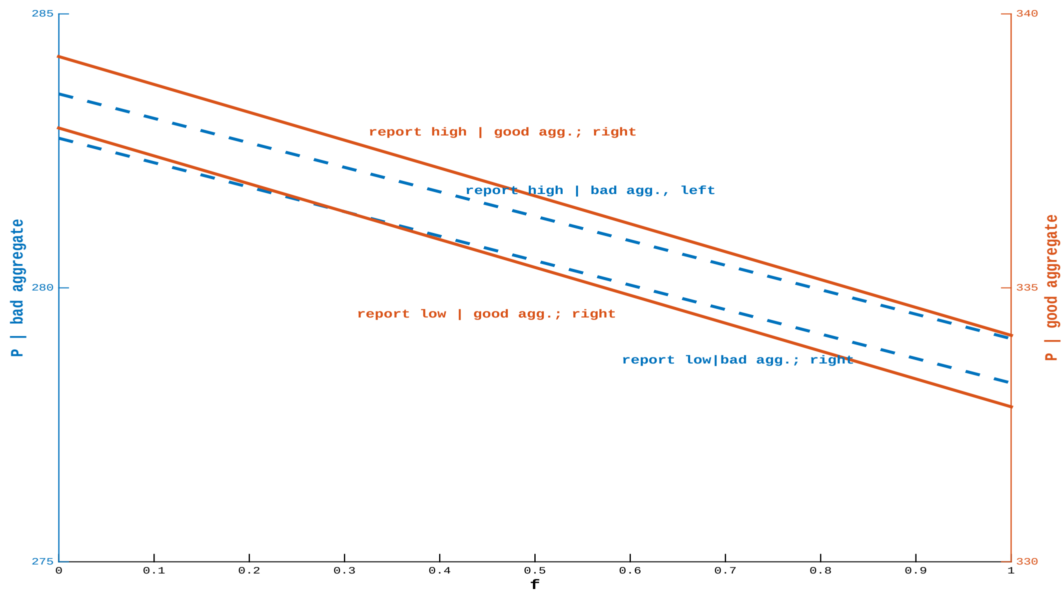


Figure 8: Pricing Function

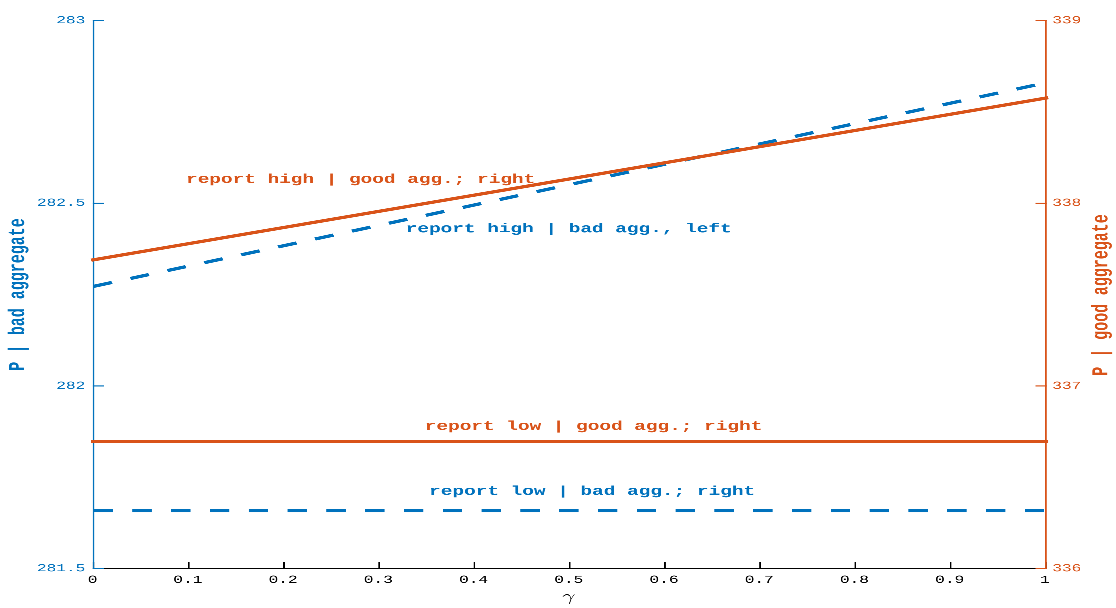


Figure 9: Pricing Function

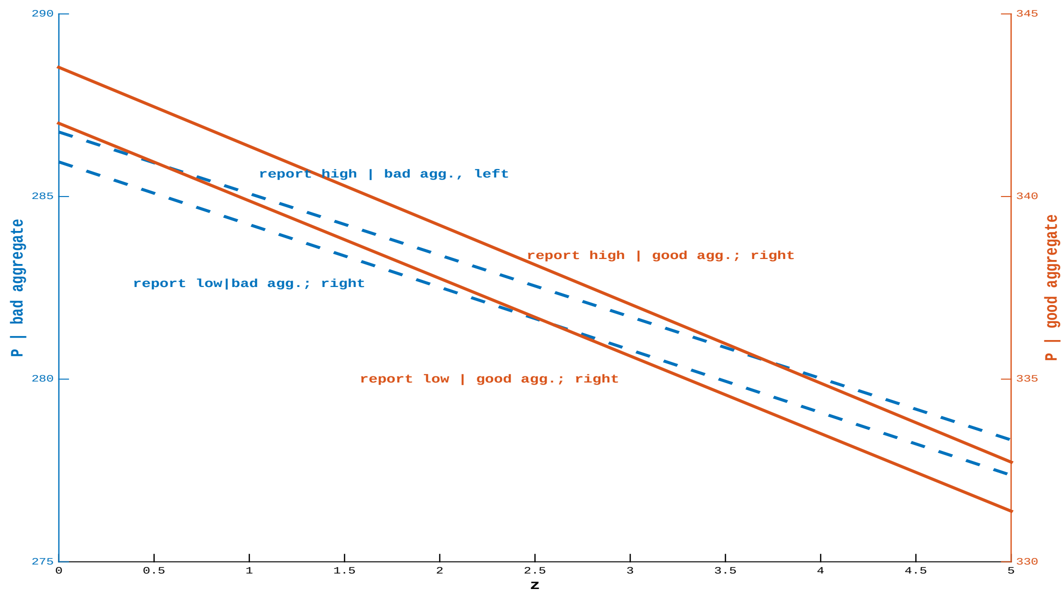
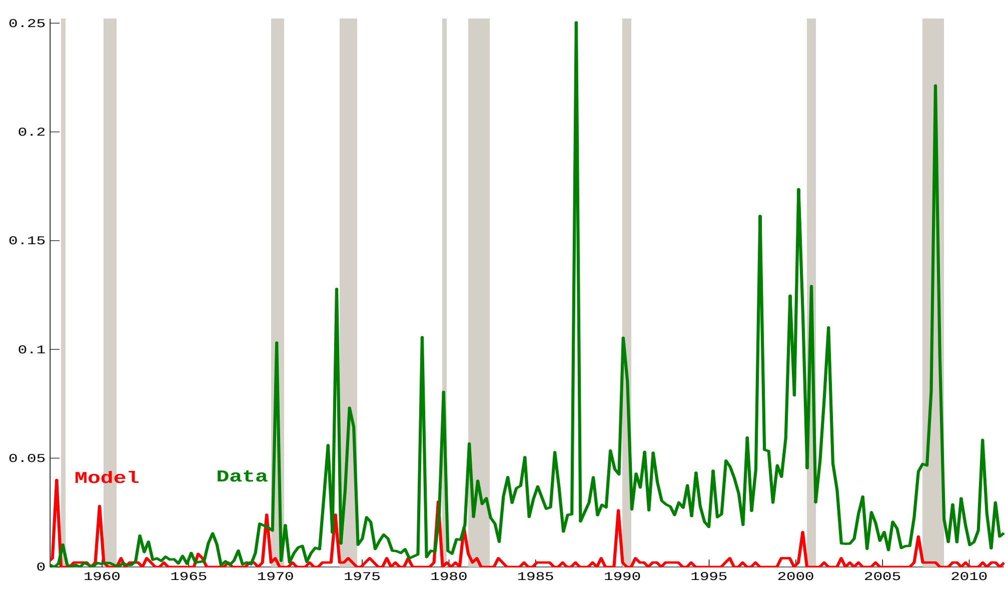


Figure 10: Barro-Ursua Crash Risk



Online Appendix

A Financial regulation index construction

The financial regulation index reflects automated text-search results for the newsstand edition of three major newspapers: New York Times, Wall Street Journal, and Washington Post. We use the ProQuest Newsstand database to search the electronic archives of each newspaper from January 1900 to January 2016 for terms related to monetary policy uncertainty. In particular, the search identifies articles containing the triple of ((i) "regulation" or "Regulatory", (ii) "financial" or "finance", and (iii) one or more of the following terms: "congress"; "federal reserve/fed"; "SEC/Security and Exchange Commission"; "OCC/Cffice of the Controller of the Currency"; "FDIC/Federal deposit insurance corporation," "Stock exchange". Based on these search criteria, we count in each newspaper how many articles contained the search terms above every day.

To deal with changing volume of newspapers over time, we normalize as follows. First, we divide, for each newspaper, in every inter-meeting period, the raw count of articles related to financial regulation (FR) by the total article count. For each newspaper i in period t, we calculate the share of articles containing financial regulation terms as

$$n(i,t) = \frac{\#\text{FR\_articles}(i,t)}{\#\text{total\_articles}(i,t)}.$$

We then normalize the share of articles so that, for each newspaper, the resulting series has a standard error of one over the sample period. This normalization controls for the possibility that different newspapers mention financial regulation with different frequency over time. That is, we denote the normalized share of articles using

$$nn(i,t) = \frac{n(i,t)}{stdev(n(i, 1985:2015))}.$$

Finally, we sum the $nn(i)$ series across newspapers and scale them so that the average value is 100 over the sample period. The scaling produces our financial regulation index (FRI):

$$FRI(t) = \left[\frac{\sum_i nn(t)}{avg(\sum_i nn(1985:2015))}\right] \times 100.$$

A human reading of a sample of the articles suggests that the news-based approach used to construct the index can provide a reasonable indicator of financial regulation. Newspapers typically cite financial regulation in one of the following cases:

- Newspaper articles describe regulatory structure and policy. For example, "Under the Treasury's proposal, the bank supervisory powers of the Fed and the Federal Deposit Insurance Corporation would be taken over by a single Federal Banking Commission. The commission would also take over the Treasury's two bank monitoring arms, the Office of the Comptroller of the Currency and the Office of Thrift Supervision." (25 Jan 1994, New York Times)
- Newspaper articles discuss a specific regulatory action. For example, "The OCC and FDIC, which regulate Citigroup's Citibank and Banamex USA subsidiaries, demanded the correction of deficiencies in its anti-money-laundering compliance." (27 Mar 2013, Wall Street Journal)
- Newspaper articles analyze current debate and potential movements in regulatory policy. For example, "In the White House's efforts to gain traction on its proposal to revamp financial-market supervision, a key flashpoint has been how much regulatory power should be centralized in the Fed and how much should be shared by a council of regulators". (02 Oct 2009, Wall Street Journal) "The Bush Administration is preparing to recommend the creation of a "super regulator" of banks, savings associations and other financial services, in a sweeping overhaul of the complex regulatory system." (07 Jan 1991, New York Times)
- Newspaper comment on issues with regulatory restrictions. For example, "Dodd-Frank restrictions on the Federal Reserve's powers to act as lender-of-last-resort, coupled with restrictions on federal guarantees for bank deposits and money-market funds, pose a threat to U.S. and global financial stability. In addition, the FDIC cannot expand guarantees to bank depositors without congressional approval, and the Treasury can't do the same to money-market funds without new legislative authority. These changes could make it difficult for the Fed and other regulatory bodies to act effectively in the next crisis." (02 Mar 2015, Wall Street Journal)

B Additional evidence on cyclical regulatory bias

In this section we gather additional evidence on the time-series pattern of regulatory behavior specifically targeted at managerial manipulation and show that the intensity of regulatory actions peaked during NBER recessions over the recent business cycles. There does not exist a long time series, besides our new indexes, that we can exploit here, so we assemble a large number of sources that tell us something about how regulators have responded to downturns. The evidence gathered in this section sheds some additional, albeit anecdotal, light on the plausibility of the cyclical bias in financial regulatory behavior.

Our first piece of evidence is the percentage of firms that received a Securities and Exchange Commission (SEC) comment letter. The SEC issues comment letters to registrants if the staff has questions or concerns related to a disclosure filing or if the staff believes the filing is incomplete or requires improvement. These comment letters may involve requests for additional information, revised disclosures, or additional disclosures of firm information. If the concerns in the comment letters are not resolved, formal investigations and enforcement actions will follow. As shown in Figure (11), the number of comment letters issued peaked during the Great Recession; unfortunately, since the SEC only began releasing these information (via the EDGAR database) in 2005, we cannot examine whether this particular measure similarly peaked during other recessions.

Another piece of evidence we use is the number of litigation cases related to financial information manipulation, using Audit Analytic's Litigation Database on all federal securities class action claims, SEC actions, and material federal civil legislation. Figure (12) shows that the percentage of firms subject to litigation related to performance manipulation rose during the recession of 2001, remained high (possibly due to the Sarbanes-Oxley Act of 2002), and then peaked again during the Great Recession; the correlation with an indicator of NBER recessions is 0.40, showing that recession periods are associated with more regulatory actions. Similarly, in the plot we include the S&P500 index price; the correlation with this index is -0.51.

Figure (13) is borrowed from Bertomeu and Magee (2011). The Financial Accounting Standards Board (FASB) is a private standards-setting organization that chooses the rules for accounting by nongovernmental entities; the decisions of FASB are officially recognized as authoritative by the SEC. The figure documents that the number of Statements of Financial Accounting Standards (SFAS) is significantly higher during NBER recessions compared to expansions; this pattern also holds for the length of these statements (an intensive margin), and is somewhat stronger in fact.

We also obtained data from the General Accounting Office (GAO) that documents the number of enforcement actions undertaken by the SEC during the period 1977-1984. Figure (14) plots the number of actions per staff member, adjusted for the length of time that a typical action lags behind the beginning of an investigation (1.64 years). It is easy to see that the "activity" of SEC staff members bottomed out at the onset of the 1979 recession and rose during the subsequent long downturn.

Our evidence on how regulatory actions related to managerial manipulation varies over the business cycle complements the existing studies on cyclical financial regulation in law and economics literature (see McDonnell 2013 for a review), and suggests that the general patterns in financial regulatory actions also apply to those specifically targeted at information manipulation.

Figure 11: Comment Letters from SEC

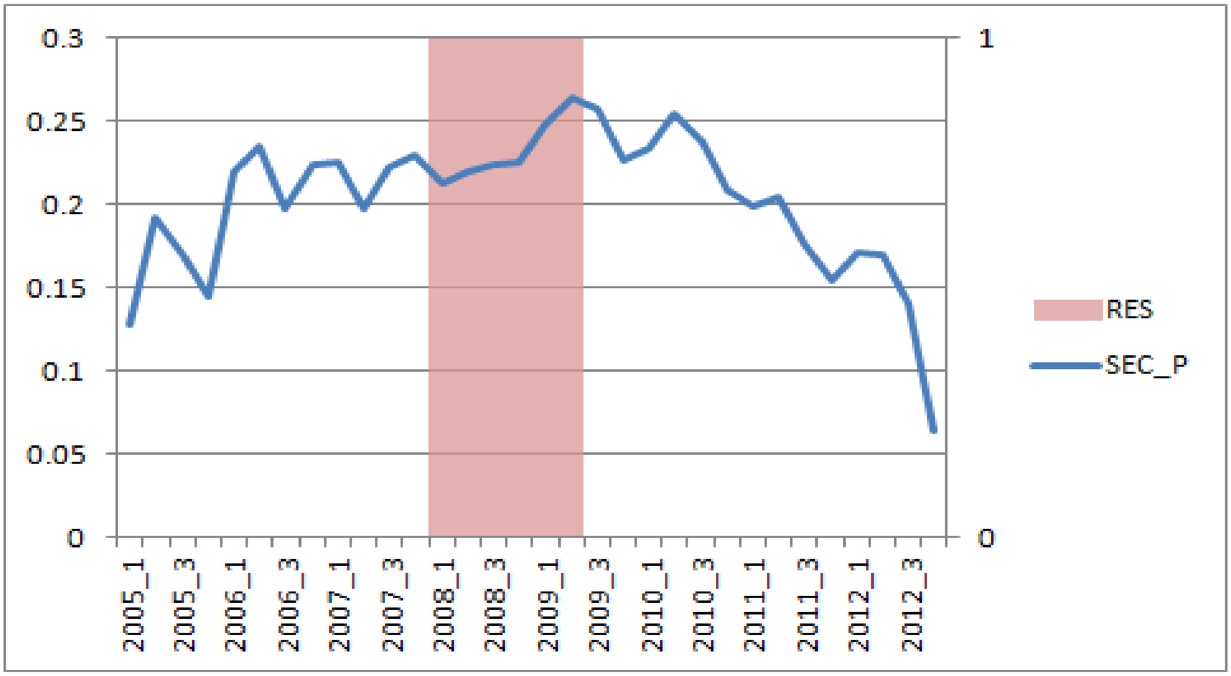


Figure 12: Legal Actions by the SEC

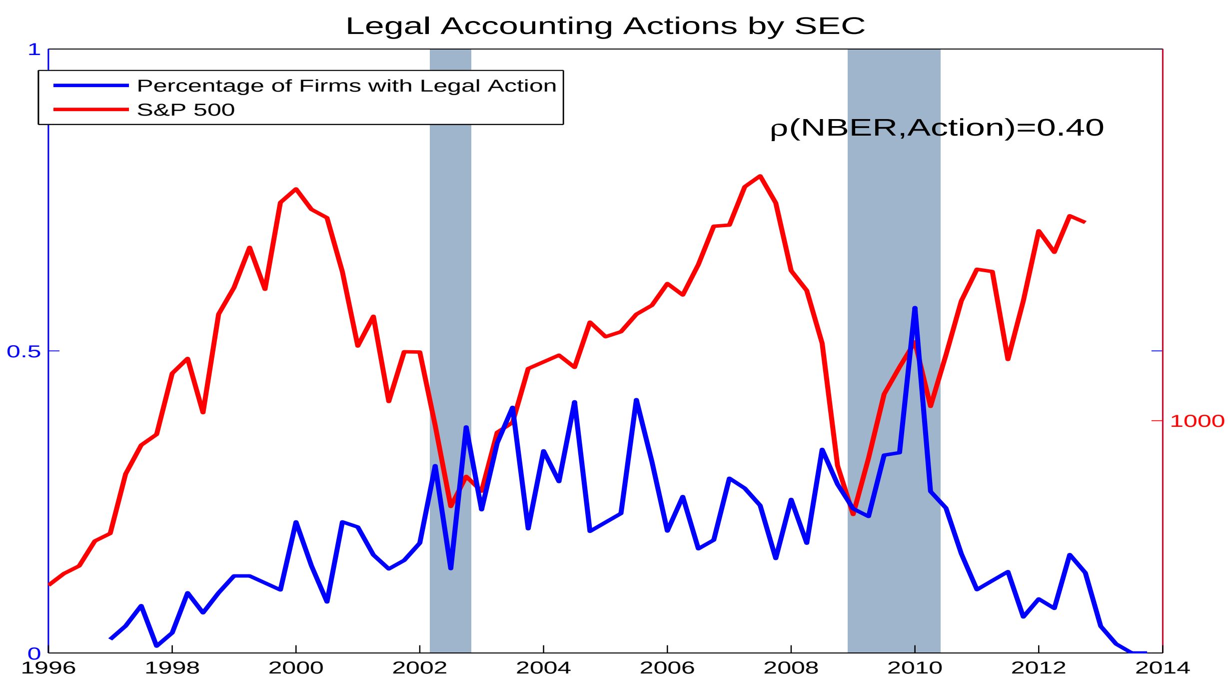


Figure 13: FASB Actions

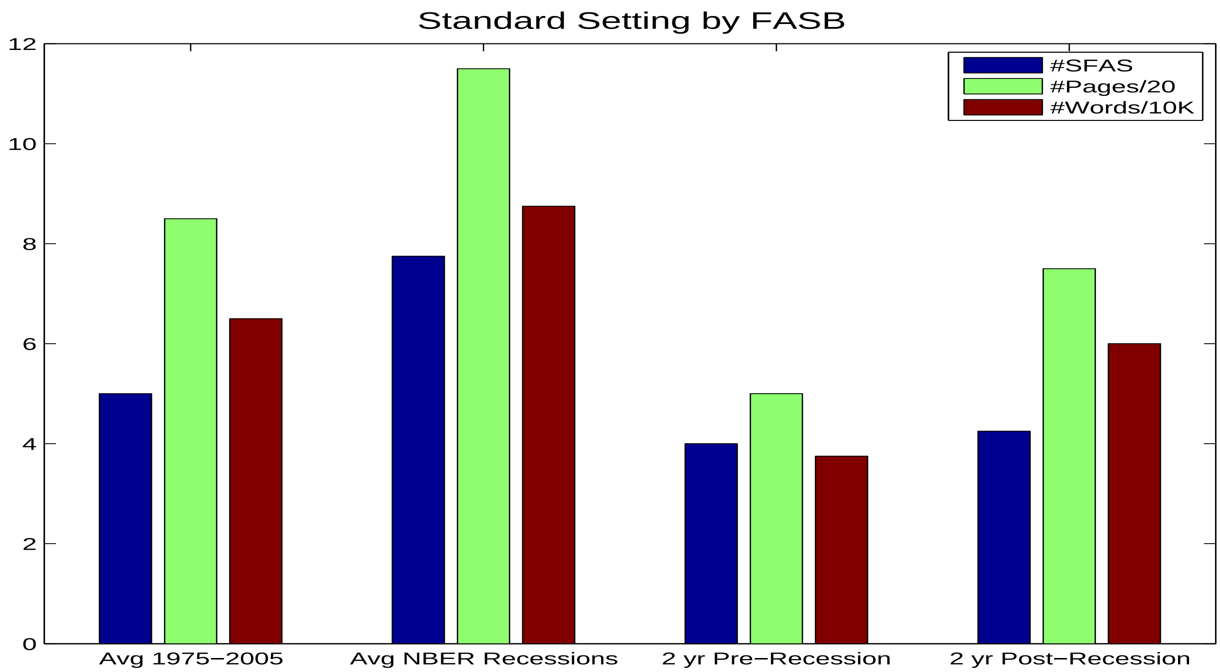
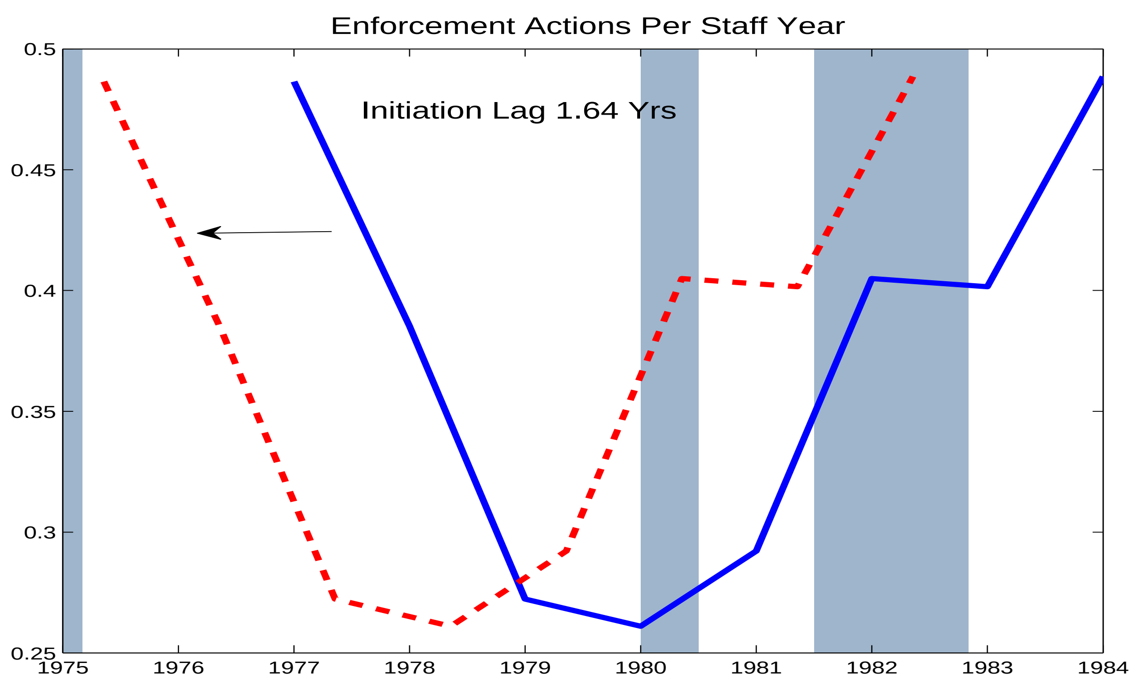


Figure 14: Enforcement Activity over the Cycle



C Regulator-Manager Model

C.1 Environment

Firms' true earnings are jointly determined by an aggregate and an idiosyncratic state in each period. There are two values for the aggregate state $a \in \{g, b\}$ with $g > b$ ("good" and "bad"), and two values for the idiosyncratic state $y \in \{h, l\}$ with $h > l$ ("high" and "low"). Each firm's earnings are then ay; we assume a law of large numbers holds for the idiosyncratic state conditional on the aggregate state. The aggregate state is perfectly and costlessly observed by both the regulator and the firm manager, whereas the idiosyncratic state is the private information of the manager; manipulation occurs when the manager reports a high value of y when the true value is low.[12]

The first agent in this model is a regulator; we suppose the objective of the regulator is to maximize the prevalence of truthful reporting, which is consistent with the SEC's mission of "facilitating capital information"[13] and the operating charter of the SEC:

> Companies publicly offering securities for investment dollars must tell the public the truth about their businesses, the securities they are selling, and the risks involved in investing.

The regulator chooses the frequency of investigations τ; the cost of investigation is given by the following quadratic form

$$C(\tau) = C\frac{\tau^2}{2},$$

where $C \geq 0$.[14]

The other agents in the model are a continuum of managers who operate individual firms. As noted above, managers choose a report $r \in \{h, l\}$. If a manager produces an inflated report, the manager will be fined in the event of a successful detection; the size of the fine is denoted F_m. The probability of a fine being levered given a false report is therefore $\tau p(a)$, where $p = 1$ in recessions and $p = \varepsilon$ in expansions.

Managers differ according to the private utility they receive from reported earnings — a report of r delivers $\theta a r$ utils to the mangers, where $\theta \sim UNI[0, 1]$. We view this

[12] Feroz, Park, and Pastena (1991) document that most SEC enforcement actions are aimed at overstatements, the average amount of restatements is negative, and over 75 percent of restatements are negative. Our model thus focuses on upward manipulation.

[13] See the mission of the SEC at http://www.sec.gov/about/whatwedo.shtml.

[14] The quadratic form facilitates a closed-form solution, but our model can accommodate any continuous, strictly increasing, and strictly convex function.

assumption as a simple shorthand for a wide variety of reasons that managers may have different valuations of manipulating earnings, including variation in pay-performance sensitivity, preferences, and the complexity of business operations. The value of θ is the private information of the manager.

Definition 2 *A Bayesian Nash equilibrium for this model is (i) a regulatory investigation policy* $\mathbf{T} : R(y) \rightarrow \tau$ *that maximizes the objective of the regulator given the reporting strategies of the managers; (ii) a reporting strategy* $\mathbf{R} : \tau \rightarrow R(y)$ *that maximizes the manager's utility, given the regulatory policy; and (iii) beliefs that are consistent with the actions of other players.*

C.2 Optimal Reporting

We first derive the optimal reporting strategy for the managers. Given (a, y, θ, τ), each manager chooses the report $r \in \{h, l\}$ to solve

$$\max_{r \in \{h,l\}} \{\theta a r - \phi(a, y, \tau, r)\},$$

where

$$\phi(a, y, \tau, r) = \begin{cases} 0 & \text{if } r = y \\ F_m & \text{if } r \neq y \text{ and manipulation is successfully detected} \\ 0 & \text{if } r \neq y \text{ and manipulation is not investigated or not detected successfully} \end{cases}.$$

The optimal reporting strategy is

$$r = \begin{cases} h & \text{if } y = h \\ h & \text{if } y = l \text{ and } \theta a h - \tau p(a) F_m \geq \theta a l \\ l & \text{if } y = l \text{ and } \theta a h - \tau p(a) F_m < \theta a l \end{cases}.$$

Thus, there exists a threshold θ, given by $\widetilde{\theta} = \frac{\tau p(a) F_m}{a(h-l)}$, above which managers that get low earnings will manipulate. Clearly, this threshold depends positively on the regulator's investigation choice τ, the success rate for investigations $p(a)$, and the fine amount F_m, and negatively on the size of misreported earnings $a(h-l)$. From this threshold and the distribution of θ, we can derive the fraction of misreported earnings in the population:

$$x(a, \tau) = 1 - \frac{\tau p(a) F_m}{a(h-l)}.$$

C.3 Optimal Regulation

Given the policy for reporting, the regulator chooses τ to solve

$$\max_{\tau} \left\{ \alpha \left[1 - x\left(a, \tau\right) \right] - \frac{C}{2} \tau^2 \right\}.$$

$\alpha > 0$ is a preference parameter that measures the effects of "outside pressure" on the regulator to act (we could also have absorbed this parameter into C); there are stories one could tell that would lead to α varying systematically with the aggregate state (a).[15] Consistent with these stories, Yu and Yu (2011) find that firms engaging in financial fraud spend more on lobbying, and corporate lobbying lowers the likelihood of fraud detection. In our benchmark analysis in Section 3, we assume that $alpha = 1$ regardless of the aggregate state for clarity.

C.4 Equilibrium

We prove that the equilibrium takes the following form.

Proposition 3 *The Bayesian Nash equilibrium in given by the regulatory actions*

$$\tau\left(a\right) = \frac{\alpha \varepsilon F_m}{Ca\left(h - l\right)},$$

the prevalence of fraud is

$$x\left(a\right) = 1 - \frac{\alpha p\left(a\right)^2 F_m^2}{Ca^2 \left(h - l\right)^2}.$$

These functions satisfy

$$\tau\left(g\right) < \tau\left(b\right)$$

$$x\left(g\right) > x\left(b\right).$$

Managerial incentives to misreport vary with the aggregate state (a) for several reasons. First, the variation in τ and $p(a)$ implies that detection is lower in good times; then there will be a strong incentive to inflate earnings when a is high. Second, the private benefit from manipulation is increasing in a. Third, if α moves countercyclically then that has an additional effect; during expansions regulators are "not interested" in investigation.

[15]Stigler (1971) and Peltzman (1976) emphasize the political economy of interest groups in regulatory decisions, in particular regulatory capture by industries. These effects would appear as changes in α.

These implications are consistent with the empirical literature. Wang, Winton, and Yu (2007) find that the effect of industry investment on fraud propensity is strongly positive. Cohen and Zarowin (2012) find that the tendency of firms to manipulate earnings upward to beat benchmarks is positively correlated with market-wide conditions. Both facts imply that, when the economy is booming, it is likely that more firms are misstating their earnings upward.

Due to frictions present in detecting frauds, our model generates the following features relevant for asset pricing. First, state-varying detection difficulties give rise to cyclical tendencies in financial regulation. Second, cyclical patterns in managerial manipulation emerge in response to asymmetric regulatory intensities over the business cycle. Third, rational investors who are informed about the regulator-managers interaction are uncertain about whether a particular report has been inflated. That is, investors can perfectly infer x given the equilibrium regulatory policy, but they cannot correctly gauge firms' idiosyncratic state. We show in the next section that the relationship between investigation intensity and manipulation frequency — together with the unrevealing financial reporting caused by manipulation — has considerable implications for the dynamics of financial markets over business cycles.

D Examples of state variables

As the monetary penalties upon investigation depends on the number of restated financial statements, the expected number of periods in which the manager inflates earnings since the most recent realization up to now is necessary in characterizing the prices. If there are N consecutive high reports and no low reports after the most recent investigation, a function of $f(N;\bar{y})$ determines the expected number of periods involving earnings management until the last period. If there is any low report after the last investigation, the sum of Z and $f(N;\bar{y})$ summarizes the history. In addition, γ and r incorporate the information regarding the current true state conveyed by the current report.

To be clear on what each variable represents, a set of clarifying examples is provided in the following. Now let today be $t = 10$ and let the last investigation happen at the beginning of $t = 5$. Suppose that the true state of $t = 4$ is revealed to be y_4.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 7; $N = 1$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 8 is known to be low (recall that all the low reports are honest reports).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$; and $r = \tilde{l}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports. Note that $\gamma = 0$ at $t = 10$, because the current low report is an honest one.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 8; $N = 0$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (all the low reports are honest reports). Note that in the case of $N = 0$, $\bar{y}$ is set to be y_{t-1} ($N = 0$ occurs only when the report at $(t-1)$ is low or the investigation happens at the beginning of t).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, 7, and 8; $N = 0$; and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (Again, all the low reports are honest reports).

Let today be $t = 5$ and let the investigation happen at the beginning of $t = 5$.

- If $r_5 = \tilde{h}$, then $Z = 0$, $N = 0$, $r = \tilde{h}$, and $\bar{y} = y_4$.

- If $r_5 = \tilde{l}$, then $Z = 0$, $N = 0$, $r = \tilde{l}$, and $\bar{y} = y_4$.

E Calculation of $f(N; \bar{y})$ in the model with stochastic investigation

Let the information set $\mathcal{R}_N^{\bar{y}} \equiv \{\bar{y}, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$. y_n represents the true earnings in period n, $\forall n \in \{1, 2, \cdots, N\}$. Thus $f(N; \bar{y})$ can be written as

$$f(N; \bar{y}) = \Pr[y_1 = l | \mathcal{R}_N^{\bar{y}}] + \Pr[y_2 = l | \mathcal{R}_N^{\bar{y}}] + \cdots + \Pr[y_n = l | \mathcal{R}_N^{\bar{y}}] + \cdots + \Pr[y_N = l | \mathcal{R}_N^{\bar{y}}]$$

The problem of deriving $f(N; \bar{y})$ in a recursive way is transformed into an equivalent problem, that is, to recursively derive

$$\Pr[y_n = l | \mathcal{R}_N^{\bar{y}}] = 1 - \Pr[y_n = h | \mathcal{R}_N^{\bar{y}}], \qquad \forall n \in \{1, 2, \cdots, N\}.$$

Note that

$$\mathcal{R}_N^h \equiv \{h, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$$

$$\mathcal{R}_N^l \equiv \{l, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$$

The proof includes two steps. In step 1, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are calculated. In step 2, I show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ and $\Pr[y_n = h|\mathcal{R}_{N+1}^h]$, $\forall n \in \{1, 2, \cdots, N+1\}$, can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l]$ and $\Pr[y_n = h|\mathcal{R}_N^h]$, $\forall n \in \{1, 2, \cdots, N\}$.

As the first step, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are derived as follows.

$$\begin{aligned}
\Pr[y_1 = h|\mathcal{R}_1^l] &= \Pr[y_1 = h|\bar{y} = l, r_1 = \tilde{h}] \\
&= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = l]}{\Pr[r_1 = \tilde{h}|\bar{y} = l]} \\
&= \frac{\pi_{lh}}{\pi_{lh} + (1 - \pi_{lh})x_a}, \\
\Pr[y_1 = h|\mathcal{R}_1^h] &= \Pr[y_1 = h|\bar{y} = h, r_1 = \tilde{h}] \\
&= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = h]}{\Pr[r_1 = \tilde{h}|\bar{y} = h]} \\
&= \frac{\pi_{hh}}{\pi_{hh} + (1 - \pi_{hh})x_a},
\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period 1.

In step 2, I first show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ can be calculated if $\Pr[y_n = h|\mathcal{R}_N^l]$ is known. For $n \in \{1, 2, \cdots, N+1\}$,

$$\Pr[y_n = h|\mathcal{R}_N^l, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l]}. \tag{2}$$

The denominator in (2), $\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l]$, is derived as the following.

$$\begin{aligned}
\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l] =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|\mathcal{R}_N^l] + \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|\mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|\mathcal{R}_N^l] \\
&+ \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l|\mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h|\mathcal{R}_N^l] + x_a \left[1 - \Pr[y_{N+1} = h|\mathcal{R}_N^l]\right],
\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N+1$, and

$$\begin{aligned}
\Pr[y_{N+1} = h|\mathcal{R}_N^l] =& \Pr[y_{N+1} = h, y_N = h|\mathcal{R}_N^l] + \Pr[y_{N+1} = h, y_N = l|\mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h|y_N = h, \mathcal{R}_N^l] \times \Pr[y_N = h|\mathcal{R}_N^l] \\
&+ \Pr[y_{N+1} = h|y_N = l, \mathcal{R}_N^l] \times \Pr[y_N = l|\mathcal{R}_N^l] \\
=& \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^l] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^l]\right]. \qquad (3)
\end{aligned}$$

As $\Pr[y_N = h|\mathcal{R}_N^l]$ is known from the supposition, this can be calculated. The denominator is obtained

$$\begin{aligned}
\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^l] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^l]\right] \\
&+ x_a\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^l] - \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^l]\right]\}, \qquad (4)
\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N+1$.

Now let us consider the numerator in (2). For $n = N+1$, $\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l]$ can be rewritten as

$$\begin{aligned}
\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l] &= \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|\mathcal{R}_N^l] \\
&= \Pr[y_{N+1} = h|\mathcal{R}_N^l],
\end{aligned}$$

where $\Pr[y_{N+1} = h|\mathcal{R}_N^l]$ is derived in (3).

For $n \in \{1, 2, \cdots, N\}$, the numerator $\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l]$ can be rewritten as

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l] = \Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] \times \Pr[y_n = h|\mathcal{R}_N^l].$$

Here, $\Pr[y_n = h|\mathcal{R}_N^l]$ is known from the supposition. Now we only need to check if $\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l]$ can be calculated. I rewrite

$$\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] = \Theta + \Lambda,$$

where

$$\begin{aligned}
\Theta =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& 1 \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l], \qquad (5)
\end{aligned}$$

$$
\begin{aligned}
\Lambda =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l | y_n = h, \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h} | y_{N+1} = l, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l | y_n = h, \mathcal{R}_N^l] \\
=& x_a \left[1 - \Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l]\right] \\
=& x_a[1 - \Theta],
\end{aligned}
\tag{6}
$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

If $n = N$, it is straightforward to determine that

$$
\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] = \pi_{hh}.
$$

Now let us consider $\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l]$ if $n < N$. Because actual earnings y follow a Markov process, all the past information is fully summarized in the most recent realization, and the prior realizations are informationally irrelevant. Thus,

$$
\begin{aligned}
\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] &= \Pr[y_{N+1} = h | y_n = h, \bar{y} = l, r_1 = \tilde{h}, \cdots, r_N = \tilde{h}], \\
&= \Pr[y_{N+1} = h | \bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}]
\end{aligned}
$$

and

$$
\Pr[y_{N+1} = h | \bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}] = \Pr[y_{N-n+1} | \bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}].
$$

Recall that $\mathcal{R}_{N-n}^h \equiv \{\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}\}$. Therefore,

$$
\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] = \begin{cases} \Pr[y_{N-n+1} = h | \mathcal{R}_{N-n}^h] & \text{if } n < N, \\ \pi_{hh} & \text{if } n = N. \end{cases}
\tag{7}
$$

and

$$
\begin{aligned}
\Pr[y_{N-n+1} = h | \mathcal{R}_{N-n}^h] =& \Pr[y_{N-n+1} = h, y_{N-n} = h | \mathcal{R}_{N-n}^h] + \Pr[y_{N-n+1} = h, y_{N-n} = l | \mathcal{R}_{N-n}^h] \\
=& \Pr[y_{N-n+1} = h | y_{N-n} = h, \mathcal{R}_{N-n}^h] \times \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] \\
&+ \Pr[y_{N-n+1} = h | y_{N-n} = l, \mathcal{R}_{N-n}^h] \times \Pr[y_{N-n} = l | \mathcal{R}_{N-n}^h] \\
=& \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] + \pi_{lh} \left[1 - \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]\right],
\end{aligned}
$$

where $\Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]$ is known from the supposition, since $N - n < N$. Therefore, Θ and Λ can be both calculated. Hence, the numerator in (2) can be derived following this procedure. The numerator is obtained

$$
\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l] =
\begin{cases}
\pi_{hh}\Pr[y_N = h|\mathcal{R}_N^l] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}_N^l]\right] & \text{if } n = N+1, \\
\Pr[y_N = h|\mathcal{R}_N^l]\left[\pi_{hh} + x_a(1-\pi_{hh})\right] & \text{if } n = N, \\
\Pr[y_n = h|\mathcal{R}_N^l]\Big\{\pi_{hh}\Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] + & \text{if } n < N. \\
\pi_{lh}\left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right] & \\
+x_a\{1 - \pi_{hh}\Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] - \pi_{lh}\left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right]\}\Big\}, &
\end{cases}
\tag{8}
$$

where $a \in \{b, g\}$ represents the aggregate state in period $N+1$.

Now combining the expressions (4) and (8), it has been shown that $\Pr[y_n = h|\mathcal{R}_N^l, r_{N+1} = \tilde{h}]$ can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l, r_N = \tilde{h}]$. The same procedure can be repeated for $\Pr[y_n = h|\mathcal{R}_N^h, r_{N+1} = \tilde{h}]$ as follows.

$$
\Pr[y_n = h|\mathcal{R}_N^h, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^h]}.
$$

where the denominator is

$$
\begin{aligned}
\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =& \pi_{hh}\Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] \\
&+ x\{1 - \pi_{hh}\Pr[y_N = h|\mathcal{R}_N^h] - \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right]\}.
\end{aligned}
$$

and the numerator is

$$
\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =
\begin{cases}
\pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] & \text{if } n = N+1, \\
\\
\Pr[y_N = h|\mathcal{R}_N^h] \left[\pi_{hh} + x_a(1 - \pi_{hh})\right] & \text{if } n = N, \\
\\
\Pr[y_n = h|\mathcal{R}_N^h] \Big\{ \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] + & \text{if } n < N. \\
\pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right] & \\
+ x_a \{1 - \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right]\} \Big\}, &
\end{cases}
$$

where $a \in \{b, g\}$ represents the aggregate state in period $N+1$.

Online Appendix

A Financial regulation index construction

The financial regulation index reflects automated text-search results for the newsstand edition of three major newspapers: New York Times, Wall Street Journal, and Washington Post. We use the ProQuest Newsstand database to search the electronic archives of each newspaper from January 1900 to January 2016 for terms related to monetary policy uncertainty. In particular, the search identifies articles containing the triple of ((i) "regulation" or "Regulatory", (ii) "financial" or "finance", and (iii) one or more of the following terms: "congress"; "federal reserve/fed"; "SEC/Security and Exchange Commission"; "OCC/Cffice of the Controller of the Currency"; "FDIC/Federal deposit insurance corporation," "Stock exchange". Based on these search criteria, we count in each newspaper how many articles contained the search terms above every day.

To deal with changing volume of newspapers over time, we normalize as follows. First, we divide, for each newspaper, in every inter-meeting period, the raw count of articles related to financial regulation (FR) by the total article count. For each newspaper i in period t, we calculate the share of articles containing financial regulation terms as

$$n(i,t) = \frac{\#\text{FR\_articles}(i,t)}{\#\text{total\_articles}(i,t)}.$$

We then normalize the share of articles so that, for each newspaper, the resulting series has a standard error of one over the sample period. This normalization controls for the possibility that different newspapers mention financial regulation with different frequency over time. That is, we denote the normalized share of articles using

$$nn(i,t) = \frac{n(i,t)}{stdev(n(i,1985:2015))}.$$

Finally, we sum the $nn(i)$ series across newspapers and scale them so that the average value is 100 over the sample period. The scaling produces our financial regulation index (FRI):

$$FRI(t) = \left[\frac{\sum_i nn(t)}{avg(\sum_i nn(1985:2015))}\right] \times 100.$$

A human reading of a sample of the articles suggests that the news-based approach used to construct the index can provide a reasonable indicator of financial regulation. Newspapers typically cite financial regulation in one of the following cases:

- Newspaper articles describe regulatory structure and policy. For example, "Under the Treasury's proposal, the bank supervisory powers of the Fed and the Federal Deposit Insurance Corporation would be taken over by a single Federal Banking Commission. The commission would also take over the Treasury's two bank monitoring arms, the Office of the Comptroller of the Currency and the Office of Thrift Supervision." (25 Jan 1994, New York Times)

- Newspaper articles discuss a specific regulatory action. For example, "The OCC and FDIC, which regulate Citigroup's Citibank and Banamex USA subsidiaries, demanded the correction of deficiencies in its anti-money-laundering compliance." (27 Mar 2013, Wall Street Journal)

- Newspaper articles analyze current debate and potential movements in regulatory policy. For example, "In the White House's efforts to gain traction on its proposal to revamp financial-market supervision, a key flashpoint has been how much regulatory power should be centralized in the Fed and how much should be shared by a council of regulators". (02 Oct 2009, Wall Street Journal) "The Bush Administration is preparing to recommend the creation of a "super regulator" of banks, savings associations and other financial services, in a sweeping overhaul of the complex regulatory system." (07 Jan 1991, New York Times)

- Newspaper comment on issues with regulatory restrictions. For example, "Dodd-Frank restrictions on the Federal Reserve's powers to act as lender-of-last-resort, coupled with restrictions on federal guarantees for bank deposits and money-market funds, pose a threat to U.S. and global financial stability. In addition, the FDIC cannot expand guarantees to bank depositors without congressional approval, and the Treasury can't do the same to money-market funds without new legislative authority. These changes could make it difficult for the Fed and other regulatory bodies to act effectively in the next crisis." (02 Mar 2015, Wall Street Journal)

B Examples of state variables

As the monetary penalties upon investigation depends on the number of restated financial statements, the expected number of periods in which the manager inflates earnings since the most recent realization up to now is necessary in characterizing the prices. If there are N consecutive high reports and no low reports after the most recent investigation, a function of $f(N;\bar{y})$ determines the expected number of periods involving earnings management until the last period. If there is any low report after the last investigation, the sum of Z and $f(N;\bar{y})$ summarizes the history. In addition, γ and r incorporate the information regarding the current true state conveyed by the current report.

To be clear on what each variable represents, a set of clarifying examples is provided in the following. Now let today be $t = 10$ and let the last investigation happen at the beginning of $t = 5$. Suppose that the true state of $t = 4$ is revealed to be y_4.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 7; $N = 1$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 8 is known to be low (recall that all the low reports are honest reports).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$; and $r = \tilde{l}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports. Note that $\gamma = 0$ at $t = 10$, because the current low report is an honest one.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 8; $N = 0$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (all the low reports are honest reports). Note that in the case of $N = 0$, $\bar{y}$ is set to be y_{t-1} ($N = 0$ occurs only when the report at $(t-1)$ is low or the investigation happens at the beginning of t).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, 7, and 8; $N = 0$; and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (Again, all the low reports are honest reports).

Let today be $t = 5$ and let the investigation happen at the beginning of $t = 5$.

- If $r_5 = \tilde{h}$, then $Z = 0$, $N = 0$, $r = \tilde{h}$, and $\bar{y} = y_4$.

- If $r_5 = \tilde{l}$, then $Z = 0$, $N = 0$, $r = \tilde{l}$, and $\bar{y} = y_4$.

C Calculation of $f(N; \bar{y})$ in the model with stochastic investigation

Let the information set $\mathcal{R}_N^{\bar{y}} \equiv \{\bar{y}, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$. y_n represents the true earnings in period n, $\forall n \in \{1, 2, \cdots, N\}$. Thus $f(N; \bar{y})$ can be written as

$$\begin{aligned} f(N; \bar{y}) = &\Pr[y_1 = l | \mathcal{R}_N^{\bar{y}}] + \Pr[y_2 = l | \mathcal{R}_N^{\bar{y}}] + \cdots \\ &+ \Pr[y_n = l | \mathcal{R}_N^{\bar{y}}] + \cdots + \Pr[y_N = l | \mathcal{R}_N^{\bar{y}}] \end{aligned}$$

The problem of deriving $f(N;\bar{y})$ in a recursive way is transformed into an equivalent problem, that is, to recursively derive

$$\Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] = 1 - \Pr[y_n = h|\mathcal{R}_N^{\bar{y}}], \qquad \forall n \in \{1, 2, \cdots, N\}.$$

Note that

$$\begin{aligned} \mathcal{R}_N^h &\equiv \{h, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\} \\ \mathcal{R}_N^l &\equiv \{l, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\} \end{aligned}$$

The proof includes two steps. In step 1, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are calculated. In step 2, I show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ and $\Pr[y_n = h|\mathcal{R}_{N+1}^h]$, $\forall n \in \{1, 2, \cdots, N+1\}$, can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l]$ and $\Pr[y_n = h|\mathcal{R}_N^h]$, $\forall n \in \{1, 2, \cdots, N\}$.

As the first step, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are derived as follows.

$$\begin{aligned} \Pr[y_1 = h|\mathcal{R}_1^l] &= \Pr[y_1 = h|\bar{y} = l, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = l]}{\Pr[r_1 = \tilde{h}|\bar{y} = l]} \\ &= \frac{\pi_{lh}}{\pi_{lh} + (1 - \pi_{lh})x_a}, \\ \Pr[y_1 = h|\mathcal{R}_1^h] &= \Pr[y_1 = h|\bar{y} = h, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = h]}{\Pr[r_1 = \tilde{h}|\bar{y} = h]} \\ &= \frac{\pi_{hh}}{\pi_{hh} + (1 - \pi_{hh})x_a}, \end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period 1.

In step 2, I first show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ can be calculated if $\Pr[y_n = h|\mathcal{R}_N^l]$ is known. For $n \in \{1, 2, \cdots, N+1\}$,

$$\Pr[y_n = h|\mathcal{R}_N^l, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l]}. \tag{1}$$

The denominator in (1), $\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$, is derived as the following.

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N] =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|\mathcal{R}^l_N] + \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|\mathcal{R}^l_N] \\ =& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}^l_N] \times \Pr[y_{N+1} = h|\mathcal{R}^l_N] \\ &+ \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, \mathcal{R}^l_N] \times \Pr[y_{N+1} = l|\mathcal{R}^l_N] \\ =& \Pr[y_{N+1} = h|\mathcal{R}^l_N] + x_a \left[1 - \Pr[y_{N+1} = h|\mathcal{R}^l_N]\right],\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$, and

$$\begin{aligned}\Pr[y_{N+1} = h|\mathcal{R}^l_N] =& \Pr[y_{N+1} = h, y_N = h|\mathcal{R}^l_N] + \Pr[y_{N+1} = h, y_N = l|\mathcal{R}^l_N] \\ =& \Pr[y_{N+1} = h|y_N = h, \mathcal{R}^l_N] \times \Pr[y_N = h|\mathcal{R}^l_N] \\ &+ \Pr[y_{N+1} = h|y_N = l, \mathcal{R}^l_N] \times \Pr[y_N = l|\mathcal{R}^l_N] \\ =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right]. \qquad (2)\end{aligned}$$

As $\Pr[y_N = h|\mathcal{R}^l_N]$ is known from the supposition, this can be calculated. The denominator is obtained

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right] \\ &+ x_a\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] - \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right]\}, \qquad (3)\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

Now let us consider the numerator in (1). For $n = N + 1$, $\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$ can be rewritten as

$$\begin{aligned}\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] &= \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}^l_N] \times \Pr[y_{N+1} = h|\mathcal{R}^l_N] \\ &= \Pr[y_{N+1} = h|\mathcal{R}^l_N],\end{aligned}$$

where $\Pr[y_{N+1} = h|\mathcal{R}^l_N]$ is derived in (2).

For $n \in \{1, 2, \cdots, N\}$, the numerator $\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$ can be rewritten as

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] = \Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}^l_N] \times \Pr[y_n = h|\mathcal{R}^l_N].$$

Here, $\Pr[y_n = h|\mathcal{R}_N^l]$ is known from the supposition. Now we only need to check if $\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l]$ can be calculated. I rewrite

$$\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] = \Theta + \Lambda,$$

where

$$\begin{aligned}
\Theta =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& 1 \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l], \qquad (4)
\end{aligned}$$

$$\begin{aligned}
\Lambda =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l|y_n = h, \mathcal{R}_N^l] \\
=& x_a \left[1 - \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l]\right] \\
=& x_a[1 - \Theta], \qquad (5)
\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

If $n = N$, it is straightforward to determine that

$$\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] = \pi_{hh}.$$

Now let us consider $\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l]$ if $n < N$. Because actual earnings y follow a Markov process, all the past information is fully summarized in the most recent realization, and the prior realizations are informationally irrelevant. Thus,

$$\begin{aligned}
\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] &= \Pr[y_{N+1} = h|y_n = h, \bar{y} = l, r_1 = \tilde{h}, \cdots, r_N = \tilde{h}], \\
&= \Pr[y_{N+1} = h|\bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}]
\end{aligned}$$

and

$$\Pr[y_{N+1} = h|\bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}] = \Pr[y_{N-n+1}|\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}].$$

Recall that $\mathcal{R}^h_{N-n} \equiv \{\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}\}$. Therefore,

$$\Pr[y_{N+1} = h | y_n = h, \mathcal{R}^l_N] = \begin{cases} \Pr[y_{N-n+1} = h | \mathcal{R}^h_{N-n}] & \text{if } n < N, \\ \pi_{hh} & \text{if } n = N. \end{cases} \tag{6}$$

and

$$\begin{aligned} \Pr[y_{N-n+1} = h | \mathcal{R}^h_{N-n}] =& \Pr[y_{N-n+1} = h, y_{N-n} = h | \mathcal{R}^h_{N-n}] + \Pr[y_{N-n+1} = h, y_{N-n} = l | \mathcal{R}^h_{N-n}] \\ =& \Pr[y_{N-n+1} = h | y_{N-n} = h, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}] \\ & + \Pr[y_{N-n+1} = h | y_{N-n} = l, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = l | \mathcal{R}^h_{N-n}] \\ =& \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}] + \pi_{lh} \left[1 - \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}]\right], \end{aligned}$$

where $\Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}]$ is known from the supposition, since $N - n < N$. Therefore, Θ and Λ can be both calculated. Hence, the numerator in (1) can be derived following this procedure. The numerator is obtained

$$\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}^l_N] =$$

$$\begin{cases} \pi_{hh} \Pr[y_N = h | \mathcal{R}^l_N] + \pi_{lh} \left[1 - \Pr[y_N = h | \mathcal{R}^l_N]\right] & \text{if } n = N+1, \\ \Pr[y_N = h | \mathcal{R}^l_N] \left[\pi_{hh} + x_a(1 - \pi_{hh})\right] & \text{if } n = N, \\ \Pr[y_n = h | \mathcal{R}^l_N] \Big\{ \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}] + & \text{if } n < N. \\ \pi_{lh} \left[1 - \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}]\right] & \\ + x_a \{1 - \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h | \mathcal{R}^h_{N-n}]\right]\} \Big\}, & \end{cases} \tag{7}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

Now combining the expressions (3) and (7), it has been shown that $\Pr[y_n = h | \mathcal{R}^l_N, r_{N+1} = \tilde{h}]$ can be calculated using $\Pr[y_n = h | \mathcal{R}^l_N, r_N = \tilde{h}]$. The same procedure can be repeated for $\Pr[y_n = h | \mathcal{R}^h_N, r_{N+1} = \tilde{h}]$ as follows.

$$\Pr[y_n = h | \mathcal{R}^h_N, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}^h_N]}{\Pr[r_{N+1} = \tilde{h} | \mathcal{R}^h_N]}.$$

where the denominator is

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] \\ &+ x\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] - \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right]\}.\end{aligned}$$

and the numerator is

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =$$

$$\begin{cases} \pi_{hh} \Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] & \text{if } n = N+1, \\ \\ \Pr[y_N = h|\mathcal{R}_N^h] \left[\pi_{hh} + x_a(1-\pi_{hh})\right] & \text{if } n = N, \\ \\ \Pr[y_n = h|\mathcal{R}_N^h] \Big\{ \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] + & \text{if } n < N. \\ \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right] & \\ +x_a\{1 - \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right]\} \Big\}, & \end{cases}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N+1$.