Robust Policymaking in the Face of Sudden Stops*

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Abstract

This paper considers policies to deal with Sudden Stops – declines in aggregate activity that are magnified by a binding collateral constraint – that occasionally occur in emerging market economies. Households and/or the government are assumed to face model uncertainty and desire robustness against alternative models. Welfare gains are small if the government trusts its model of household expectations, whether those expectations are altered by model uncertainty or not; in contrast, welfare losses are large if the government is uncertain about the household's probability model.

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1. Introduction

Emerging market economies frequently experience episodes – called Sudden Stops – in which output falls, the terms of trade deteriorate, and current accounts erode significantly. Figures (1) and (2) show one country – Mexico – that experienced a Sudden Stop in 1995 (the Tequila crisis). The ratio of the current account to GDP went from -5 percent to 4 percent in one quarter, and the trade balance went from -6 percent to 0. At the same time, output and consumption growth both collapsed – both were below -10 percent for the quarter (annualized). Mexico is far from alone in experiencing these kinds of large events, and the recent financial crisis has a similar flavor.

A recent debate has arisen over the nature of optimal policy in economies that face Sudden Stops. Contributions to this debate include Bianchi (2010), Bianchi and Mendoza (2011), Benigno *et al.* (2011a,b,c,d), and Jeanne and Korinek (2011a,b). There are two primary issues that the literature has raised. First, what are the appropriate tools to combat Sudden Stops? The papers cited above focus on two instruments: capital controls (Tobin taxes on new debt, see Yashiv 1997,1998) and exchange rate interventions (subsidies to nontradable sectors). Second, when should the government intervene? Here, the literature comes to two different conclusions; some papers (in particular Jeanne and Korinek 2011b) argue for "prudential" interventions that take place before the Sudden Stop, whereas others (Benigno *et al.* 2011c) argue for interventions only during the crisis ("mopping up" policies). The purpose of this paper is to examine this second question more closely.

Since Sudden Stops are contained in the ergodic set of the economy, policy must be specified for both normal and crisis periods. Naturally, the two are connected – policy in normal times will influence the probability and magnitude of a Sudden Stop, and policy during the Sudden Stop will influence the behavior of the economy in normal times.[1] Benigno *et al.* (2011b,c) studies this issue, computing optimal policy rules both in and out of a crisis state, in a simple model of an emerging economy subject to TFP shocks in the tradable goods sector, debt denominated in tradable goods, and a constraint that limits borrowing to a multiple of current income (which crucially depends endogenously on the price of nontraded output). The points made in those papers – which differ in the extent to which the government is constrained in the number of instruments – is that *ex post* interventions are far more important than *ex ante* ones in this class of models; indeed, the optimal

[1] Some papers in the literature consider Sudden Stops as zero probability events, such as Braggion, Christiano, and Roldos (2009), Hevia (2007), and Cúrdia (2009), and investigate the optimal policy once one is in a crisis. Obviously these papers have nothing to say about policies intended to prevent the crisis in the first place.

policy under restrictions is to do *absolutely nothing* unless the economy is currently in a crisis state.

The question taken up here is how policy interventions change in the presence of a fear of model misspecification (model uncertainty), particularly on the part of the government. I assume that the stochastic process generating the shocks to tradable TFP, trend productivity, and the interest rate are not completely trusted by the agents in the economy, and thus they twist their expectations in order to guard against the worst process contained within some specified set (as in Hansen and Sargent 2007); with multiple shocks, the households have to consider not only bad outcomes for each shock separately, but also bad outcomes that arrive *jointly* across uncorrelated processes.

Understanding the role of model uncertainty is important in models with Sudden Stops, since these events are rare by definition. Here, I will roughly calibrate the model to Mexican data, where the Tequila crisis noted in Figures (1) and (2) is the only Sudden Stop. Assuming agents know precisely (and trust completely) estimates of transition probabilities based on a single observation seems too demanding, as the error bounds on these estimates are large. In addition, combining all countries together to calculate a Sudden Stop probability, as in Calvo and Reinhart (2000) or the disaster literature (Barro and Ursúa 2011), carries some issues as well; for example, if crises have a significant country-specific (or regime-specific) aspect, the frequency estimated using pooled data could be highly misleading regarding the perceived probability of recurrence.

A desire for robust decision rules distorts the intertemporal relationship between the marginal utility of consumption today and tomorrow. As a result, the potential gains from intervening along this dimension – for example, by using a capital control aggressively before the constraint binds – could increase. However, robustness acts as a form of risk aversion (as noted in Tallarini 2000).[2] As a result, the economy may endogenously move away from the region where marginal gains are large, rendering welfare gains small again. The main result from these experiments is that there is still no scope for prudential intervention if both households and the government fear misspecification, but if only the government faces model uncertainty it intervenes aggressively in a prudential manner. The welfare gains from optimal policy are small if both agents want robust decision rules (and actually get smaller as the desire for robustness increases, reflecting the precautionary response of saving), but are large and negative if only the government does. The surprising part of the second result is not the sign, which is to be expected given that the government is not maximizing the subjective utility of the household, but rather the magnitude: if the government distrusts its

[2] In fact, Tallarini (2000) shows that robust decision-making implies that an agent has a restricted form of the preferences from Epstein and Zin (1989); the restriction is that the intertemporal elasticity of substitution is unity.

model of household expectations even by a modest amount, the resulting welfare losses are orders of magnitude larger than the gains that are obtained when the government trusts its model of household expectations.

There is by now an extensive literature on optimal fiscal policy under model uncertainty; representative examples include Karantounias, Hansen, and Sargent (2009), Carvalho (2005), and Svec (2011a,2011b), which consider Ramsey problems with commitment, and Luo, Nie, and Young (2012b) which investigates tax-smoothing in the Barro (1979) sense. There are also a number of papers on monetary policy with model uncertainty, including Adam and Woodford (2011) and Dennis (2010). Finally, the work here connects more tenuously to optimal policy with sovereign debt in open economies, both with model uncertainty (Costa 2009, Pouzo and Presno 2011) and without (Aguiar and Amador 2011a,b). While there are more examples than these, I refrain from a tedious listing and apologize to any authors whose work has not been referenced.

2. Model

2.1. Household Decision Problem

The model economy is populated by a continuum of identical households. Each household has identical preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\rho} \left(\left[\omega c_{T,t}^{\kappa} + (1-\omega) c_{N,t}^{\kappa} \right]^{\frac{1}{\kappa}} - \frac{\Gamma_{t-1}}{\delta} h_t^{\delta} \right)^{1-\rho} \tag{2.1}$$

where $\rho \geq 0$ is the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution), $\beta < 1$ is the pure time discount factor, $\frac{1}{1-\kappa}$ is the intratemporal elasticity of substitution between traded (c_T) and nontraded (c_N) consumption goods, $\omega \in (0,1)$ is a share parameter, h is labor effort, $\frac{1}{\delta-1} \geq 1$ is the Frisch elasticity of labor supply, and E_0 is an expectation operator to be defined below; for consistency with the numerical section I will assume κ is such that c_T and c_N are complements, and of course the utility function used here is known to have zero wealth effect on labor supply.[3] Γ is the trend component of TFP:

$$\Gamma_t = \exp(g_t) \Gamma_{t-1} \tag{2.2}$$

[3]See Greenwood, Hercowitz, and Huffman (1988).

for random variable g_t; because these preferences do not balance wealth and substitution effects caused by permanent wage increases, the household must "enjoy" leisure more as productivity grows. Normalizing by Γ_{t-1} yields an effective discount factor of

$$\beta^* (g_t) = \beta \exp (g_t (1 - \rho)) .$$

Households face a budget constraint of the form

$$(1 + \tau_{T,t}) c_{T,t} + (1 + \tau_{N,t}) p_{N,t} c_{N,t} + \frac{(1 + \tau_{B,t}) b_{t+1}}{1 + r_t} \leq b_t + w_t h_t + D_t + T_t \tag{2.3}$$

and a collateral constraint of the form

$$\frac{b_{t+1}}{1 + r_t} \geq -\varphi (w_t h_t + D_t) , \tag{2.4}$$

where b is the current net foreign asset position of the household, D is the dividend from owning the firms, w is the wage, p_N is the relative price of nontradable goods, r is the random real world interest rate, τ_i $(i = T, N, B)$ are flat taxes/subsidies on traded consumption, nontraded consumption spending and new debt respectively, and T is a lump-sum tax/transfer. Normalizing the constraints by Γ_{t-1} yields

$$c_{T,t} + (1 + \tau_{N,t}) p_{N,t} c_{N,t} + \frac{\exp (g_t) (1 + \tau_{B,t}) b_{t+1}}{1 + r_t} \leq b_t + w_t h_t + D_t + T_t \tag{2.5}$$

and

$$\frac{\exp (g_t) b_{t+1}}{1 + r_t} \geq -\varphi (w_t h_t + D_t) , \tag{2.6}$$

where all variables going forward are understood to be the normalized quantity. The fact that debt is denominated in tradable goods (foreign currency) implies that the economy faces a "liability dollarization" problem, as discussed in Krugman (1999) and Aghion, Bacchetta, and Banerjee (2004); it is rare that emerging economies can borrow externally in their own currency, so this assumption is consistent with empirical observations.

The collateral constraint is central to the results presented here, so I will explicitly discuss what happens when the constraint binds after presenting the rest of the model environment. The constraint implies that an individual household can post as collateral a security that entitles the lender to some part of current and future income (note there is no requirement that $\varphi < 1$, so future income is also serving as collateral). The constraint is not motivated explicitly from microfoundations, although it plays a role essentially identical to that in Kiyotaki and Moore (1997).

2.2. Firm Decision Problem

There are two stand-in firms in the domestic economy.[4] One firm produces tradable output, the other produces nontradable output, according to the production technologies

$$Y_{T,t} = \exp(z_t) H_{T,t}^{\alpha} \tag{2.7}$$

$$Y_{N,t} = A H_{N,t}^{\theta}. \tag{2.8}$$

z_t is a persistent but stationary shock to productivity in the tradable sector, $\alpha < 1$ and $\theta < 1$ are labor's share of income in each sector, and A is a normalization constant. The dividends for firm ownership paid to the households are

$$D_t = Y_{Tt} - w_t H_{Tt} + p_{Nt} Y_{Nt} - w_t H_{Nt}.$$

2.3. Competitive Equilibrium

Market clearing for the consumption goods requires

$$C_{T,t} = Y_{T,t} + B_t - \frac{\exp(g_t) B_{t+1}}{1 + r_t} \tag{2.9}$$

$$C_{N,t} = Y_{N,t}. \tag{2.10}$$

The efficient allocation of labor across sectors yields

$$w_t = \alpha \exp(z_t) H_{T,t}^{\alpha-1} = \theta p_{N,t} A H_{N,t}^{\theta-1}. \tag{2.11}$$

[4] It is equivalent to think of the model as populated by a single stand-in firm that produces two goods using different technologies.

In equilibrium it must be the case that aggregate and individual allocations coincide, requiring that

$$h_t = H_{T,t} + H_{N,t} \tag{2.12}$$

$$b_t = B_t \tag{2.13}$$

$$c_{T,t} = C_{T,t} \tag{2.14}$$

$$c_{N,t} = C_{N,t}, \tag{2.15}$$

hold, and the government budget constraint

$$T_t = \tau_{T,t} C_{T,t} + \tau_{N,t} p_{N,t} C_{N,t} + \tau_{B,t} \frac{\exp(g_t) B_{t+1}}{1 + r_t} \tag{2.16}$$

must also be satisfied.

2.4. Recursive Formulation with Fear of Misspecification

Following Hansen and Sargent (2007), households may not trust the probability model they use to predict the movements of the random variables in the model (z, g, and r). Here, I will represent this distrust using multiplier preferences; Strzalecki (2009) provides the axiomatic foundations for multiplier preferences and connects them to a wide range of other representations of uncertainty-averse preferences. A household who desires robustness solves the minmax problem

$$v(b, B, z, g, r) = \max_{b', c_T, c_N, h} \min_{m'} \left\{ \begin{array}{c} \frac{1}{1-\rho} \left(\left[\omega c_T^{\kappa} + (1-\omega) c_N^{\kappa} \right]^{\frac{1}{\kappa}} - \frac{1}{\delta} h^{\delta} \right)^{1-\rho} + \\ \beta \exp(g(1-\rho)) E_{\pi} \left[m' v(b', B', z', g', r') - \frac{1}{\varsigma} m' \log(m') \middle| z, g, r \right] \end{array} \right\}$$

where m' is the increment to the probability distortion M and

$$\begin{aligned} M' &= m' M \\ E[M | z, g, r] &= 1. \end{aligned}$$

Note that M is not a state variable in the recursive problem because the value function is homogeneous of degree 1 in M, meaning it can be dropped. The parameter $\varsigma \leq 0$ governs the strength of the demand for robustness and E_{π} is the standard expectation operator with respect to the empirical density π. As shown in the appendix (or in Backus, Routledge, and Zin 2005), solving the minimization yields the utility recursion

$$v(b,B,z,g,r) = \max_{b',c_T,c_N,h} \left\{ \begin{array}{c} \frac{1}{1-\rho}\left(\left[\omega c_T^{\kappa} + (1-\omega) c_N^{\kappa}\right]^{\frac{1}{\kappa}} - \frac{1}{\delta}h^{\delta}\right)^{1-\rho} + \\ \frac{\beta}{\varsigma}\exp\left(g\left(1-\rho\right)\right)\log\left(E_{\pi}\left[\exp\left(\varsigma v\left(b',B',z',g',r'\right)\right)\mid z,g,r\right]\right) \end{array} \right\}. \quad (2.17)$$

The essence of the robustness demand is that agents worry that their probability model of the shocks – taken here to be the true data-generating process – is misspecified in a way that is difficult to detect; specifically, they worry about alternative distributions that are absolutely continuous with respect to the true distribution over finite samples.[5] In this paper I assume the shocks follow finite-state Markov chains, so agents' fears will distort the transition probabilities but not the realizations of the shocks.

2.5. Recursive Competitive Equilibrium

Denote the state vector by $S \equiv (B,z,g,r)$. A recursive competitive equilibrium in this model is then characterized as a solution to the a system of functional equations:

$$F(S) = 0; \quad (2.18)$$

a complete statement of the equations is contained in the computational appendix, as well as the tricks involved in converting the complementary slackness conditions into equations.[6] The equilibrium value function is defined as $V(B,z,g,r) \equiv v(B,B,z,g,r)$.

I now highlight the critical equations for understanding the mechanisms in the model. Under model uncertainty, the expectation of future marginal utility is distorted by the ratio

$$\frac{\exp\left(\varsigma V\left(B',z',g',r'\right)\right)}{E_{\pi}\left[\exp\left(\varsigma V\left(B',z',g',r'\right)\right)\mid z,g,r\right]},$$

which means that states in which continuation utility is low are given more weight relative to those in which continuation utility is high. Defining

$$p\left(z',g',r'|z,g,r\right) \equiv \frac{\exp\left(\varsigma V\left(B',z',g',r'\right)\right)}{E_{\pi}\left[\exp\left(\varsigma V\left(B',z',g',r'\right)\right)\mid z,g,r\right]}\pi\left(z',g',r'|z,g,r\right)$$

[5] A measure ν is said to be **absolutely continuous** with respect to the measure μ if $\nu(A) = 0$ for each set A with $\mu(A) = 0$; that is, absolutely continuous pairs of measures have the same zero-probability events.

[6] Existence and uniqueness of solutions is an outstanding question. It seems reasonable to expect that the monotone-operator methods from Datta *et al.* (2005) could be adapted to this model, but it lies well beyond my goals here.

the Euler equation can be rewritten as

$$\frac{\lambda(B,z,g,r)}{1+r} = \beta E_p\left[\lambda\left(B'(B,z,g,r),z',g',r'\right)\middle| z,g,r\right] + \frac{\max\left\{\mu^*(B,z,g,r),0\right\}^2}{1+r}. \tag{2.19}$$

As $\varsigma \to 0$ $p(z',g',r'|z,g,r) \to \pi(z',g',r'|z,g,r)$, so that the agent's probability distribution is not distorted.[7] It is easy to understand which values of z and g the household views as unfavorable – low ones – but the values of r which are unfavorable depend critically on B'; if the household is currently indebted, high realizations of r are bad. In the parameterization considered here, the ergodic set will not place any mass on positive net foreign asset positions.

The other critical equation is

$$u_3(S) = w(S)\left(\lambda(S) + \varphi\max\left\{\mu^*(S),0\right\}^2\right), \tag{2.20}$$

where u_3 is the derivative of the utility function with respect to hours. This condition sets the marginal cost of labor supply equal to the marginal value of labor; a discussion of these two equations and how they illuminate the effects of a binding constraint are contained in the next subsection.

2.5.1. The Effect of a Binding Constraint

The crucial equations are (2.20) and (2.19), the intratemporal tradeoff between consumption and labor and the intertemporal Euler equation. Consider first the static condition (2.20); if the constraint is not binding ($\mu^* = 0$), then the marginal rate of substitution between tradable consumption and leisure is equated to the wage. If $\mu^* > 0$, however, the marginal utility from consumption will be too low, implying that total labor effort will be excessively high in this region. To see this, let

$$\widehat{\mu} \equiv \frac{\max\left\{\mu(S),0\right\}^2}{\left(\left[\omega C_T^{\kappa} + (1-\omega)C_N^{\kappa}\right]^{\frac{1}{\kappa}} - \frac{1}{\delta}(H_T+H_N)^{\delta}\right)^{-\rho}} \tag{2.21}$$

be the multiplier on the collateral constraint normalized by marginal utility. Then (2.21) can be rearranged to obtain

$$H_T + H_N = \left(\frac{\partial C}{\partial C_T} + \widehat{\mu}\varphi\right)^{\frac{1}{\delta-1}} w^{\frac{1}{\delta-1}}. \tag{2.22}$$

[7] As $\varsigma \to -\infty$, the recursion approaches the minmax utility functional from Epstein and Schneider (2003).

The right-hand-side is increasing in $\widehat{\mu}$, so that a binding constraint implies high labor effort; with the utility function employed here low leisure means low aggregate consumption.

Now consider the Euler equation (2.19). For notational simplicity ignore model uncertainty, so that the Euler equation (2.19) becomes

$$\frac{(1+\tau_B)\,\lambda(S)}{1+r} = \beta \exp(-g\rho)\, E_\pi\left[\lambda\left(S'(S)\right)\middle|\, S\right] + \frac{\max\{\mu(S),0\}^2}{1+r}. \tag{2.23}$$

A binding constraint implies that the marginal utility of tradable consumption today is too high relative to the unconstrained case, so that tradable consumption is low; with complementarity between C_T and C_N, nontradable consumption will also be low. The Euler equation also identifies the role of model uncertainty with respect to policy; model uncertainty alters the RHS of (2.23) by replacing E_π with E_p, increasing the RHS as it places more weight on states in which λ_{t+1} is large.

These two conditions identify the margins along which policy can improve allocations. In (2.21), the government can potentially counteract the effects of the constraint by altering the (after-tax) wage; one method for moving the wage is to alter the relative price of nontradables through τ_N (or τ_T). In (2.23) the government can alter the marginal utility of tradable goods today using τ_B (again, also τ_T).

There is a third important equation where policy can play a role, although it is an indirect one. In equilibrium, a household must be indifferent as to the sectoral allocation of her labor; that is, the marginal products of labor must be equated across sectors:

$$\alpha \exp(z)\, H_T^{\alpha-1} = \theta p_N A H_N^{\theta-1}. \tag{2.24}$$

When the constraint binds, tradable and nontradable consumption are not efficiently allocated; by extension, labor supply is not efficiently allocated either. The government can use τT and τ_N to correct this imbalance by shifting the relative price of nontradables.

2.6. Government Problem

The current policymaker solves the problem

$$W(B,z,g,r) = \max_{\psi_p,\psi_g} \left\{ \begin{array}{c} \frac{1}{1-\rho}\left(\left[\omega C_T^{\kappa} + (1-\omega)\, C_N^{\kappa}\right]^{\frac{1}{\kappa}} - \frac{1}{\delta}\left(H_T + H_N\right)^{\delta}\right)^{1-\rho} + \\ \frac{\beta}{\sigma}\exp(g(1-\rho))\log\left(E_\pi\left[\exp\left(\sigma W(B',z',g',r')\right)\middle|\, z,g,r\right]\right) \end{array} \right\} \tag{2.25}$$

subject to the conditions for a competitive equilibrium (2.18),where $\psi_g = (\tau_T, \tau_N, \tau_B)$ is the vector of policy tools and $\psi_p = (C_T, C_N, H_T, H_N, \lambda, \mu, B', V, p_N)$ is the vector of competitive equilibrium objects. Note that the government may have a different preference parameter for robustness (σ) than the private sector does (ς).[8]

In the government problem, if $\tau_N < 0$ then the government is subsidizing the consumption (and therefore also the production) of nontraded goods; of course, the same holds for τ_T. More subtly, if $\tau_B < 0$ the government is *taxing* new debt. To see why, consider a lending market in which the domestic household is issuing one new bond. In exchange for the new bond, which is a promise to repay one unit of tradable output tomorrow, the household receives $\frac{1+\tau_B}{1+r}$ units of tradable output today (ignoring the normalization); therefore, if $\tau_B < 0$ the household receives fewer resources today in exchange for the same amount of promised repayment. That is, the government is taxing the new issue.

Note that the Euler equation (2.23) contains future control variables (λ' and the continuation value function) which the current government must take as given (as a function of future states); a Markov perfect equilibrium results when the current government chooses the same functions today as are taken as given for tomorrow. Note also that the presence of the occasionally-binding constraint means that, in general, the policy functions are continuous but not differentiable; as a result, the smooth equilibrium of Klein, Krusell, and Ríos-Rull (2009) does not exist, so I look for equilibria in which the functions are only required to be continuous.[9] The appendix contains a discussion of the computational method used to solve the government problem.

A formal definition for the optimal policy equilibrium will now be given; for notational simplicity I use the "compact" definition defined in Krusell (2002). The "noncompact" definition involves indexing the competitive equilibrium by taxes today given tax functions for the future. Hopefully omitting a formal statement of this equilibrium does not leave the reader too confused.

Definition 1. *A* ***Markov-perfect equilibrium*** *for the government is (i) a government value function* $W : \mathcal{B} \times \mathcal{Z} \times \mathcal{G} \times \mathcal{R} \to \mathbf{R}$, *(ii) policy functions* $\psi_g : \mathcal{B} \times \mathcal{Z} \times \mathcal{G} \times \mathcal{R} \to \mathbf{R}^2$, *and (iii) equilibrium functions* $\psi_p : \mathcal{B} \times \mathcal{Z} \times \mathcal{G} \times \mathcal{R} \to \mathbf{R}^9$ *such that* $\left(W, \psi_g, \psi_p\right)$ *satisfy the recursion (2.25)*

[8] Klein, Krusell, and Ríos-Rull (2009) or Aiyagari *et al.* (2002).

[9] If the equilibrium for a finite horizon is unique, then my method selects the unique infinite horizon equilibrium that is the limit of these finite horizon equilibria. Other equilibria that cannot be reached as the limit of finite horizon equilibria may exist; see Klein, Krusell, and Ríos-Rull (2009) or Young (2006) for related examples of multiplicity. Jeanne and Korinek (2011a) discuss multiplicity of the finite horizon equilibria in a simple, but related, model. Multiple equilibria for a finite horizon can arise in this model if κ is close enough to 1 (meaning C_T and C_N are very close substitutes).

and the competitive equilibrium conditions (2.18).

Essentially, a Markov-perfect equilibrium requires that the government's choices for ψ_p be consistent with an equilibrium when policies are set using ψ_g; because the government takes the future functions $(\lambda, V)(B', z', g', r')$ as fixed, the Markov-perfect requirement bites by requiring that the government find it optimal *today* to use the same functions it takes as given for *tomorrow*.

If the government has a sufficiently-rich set of policy tools, the allocation chosen by the government satisfies a particular notion of constrained efficiency. Thus, to compute the model I follow Benigno *et al.* (2011b) and formulate a constrained-efficient social planning problem:

$$V^P(B, z, g, r) = \max_{\psi_p} \left\{ u(C_T, C_N, H_T + H_N) + \frac{\beta}{\varsigma} \exp(g(1-\rho)) \log \left(E\left[\exp\left(\varsigma V^P(B', z', g', r') \right) | z, g, r \right] \right) \right\} \tag{2.26}$$

subject to the resource constraints, the borrowing constraint, and the pricing equation

$$p_N = \frac{u_1}{u_2}, \tag{2.27}$$

where u_i is the derivative of the period utility function with respect to argument i. Following Bianchi (2010), I can construct the taxes that implement this allocation as a competitive equilibrium. When the set of policy tools is restricted, one gets a problem similar to that in Aiyagari *et al.* (2002); I will briefly describe the implications of restrictions on the set of taxes in a later section.

2.6.1. The Meaning of $\sigma < 0$

Because it plays an important role in the paper, I want to discuss explicitly how one should interpret $\sigma < 0$ when $\varsigma = 0$ and when $\varsigma < 0$. If $\varsigma = 0$, private agents do not face any model uncertainty; that is, they accept the objective probability distribution at face value. But if $\sigma < 0$, the government does not; in effect, the government distrusts its model *of the household's expectations.* In contrast, when I consider $\varsigma = \sigma < 0$, I am considering a case where the households distrust the model but the government trusts its model of household expectations. If both are negative but not equal, one would interpret the resulting environment as a combination of both forces – households distrust the probability model and the government distrusts its own model of the households' model.

2.7. Welfare

I calculate the welfare change of a given set of policy instruments relative to the competitive equilibrium with zero taxes. The percent increase in tradable consumption that makes an agent in the no-tax competitive equilibrium unwilling to move to the equilibrium of the policy game is given by the solution $\chi(B, z, g, r)$ to the equation

$$V(B, z, g, r) = V^{*}(B, z, g, r; \chi(B, z, g, r))$$

where

$$\begin{aligned} V^{*}(B, z, g, r; \chi) &= u((1+\chi) C_T, C_N, H) + \\ &\frac{\beta}{\varsigma} \exp(g(1-\rho)) \left[\log\left(E_{\pi}\left[\exp\left(\varsigma V^{*}\left(B', z', g', r'; \chi\right)\right) \middle| z, g, r\right]\right)\right]; \end{aligned} \tag{2.28}$$

the decision rules are evaluated using the competitive equilibrium functions with no taxes. This policy measure therefore takes into account any transient dynamics associated with moving toward the stationary distribution of the optimal tax allocation.

If $\sigma \neq \varsigma$ the equilibrium policies chosen by the government may not improve welfare, at least not as measured using the agent's subjective utility function; in addition, if the government does not have a full set of policy tools, even if $\sigma = \varsigma$ the allocation with policy may not dominate doing nothing (because doing nothing may not be an equilibrium). Average welfare changes are computed by integrating χ with respect to the stationary distribution from the competitive equilibrium.

3. Parametrization

I do not attempt to calibrate the robustness parameters. The detection error probability approach – advocated in Hansen and Sargent (2007) and used, for example, in Luo, Nie, and Young (2011,2012a) or Bidder and Smith (2011) – is too computationally costly given the nonlinearities inherent in the model. Therefore, I will roughly calibrate the other parameters of the model and examine how changes in ς and σ affect the allocations.

I set $\rho = 1$ (logarithmic preferences) and $\delta = 1.75$ (a Frisch elasticity of 1.5). $\alpha = \theta = 0.66$ is used to ensure that labor's share of income is consistent with Gollin (2002). $\kappa = -0.32$ ensures an elasticity of substitution consistent with Mendoza (2002). I assume

$$z_t = 0.690 z_{t-1} + 0.0083 \varepsilon_t^z$$
$$g_t = 0.524 g_{t-1} + 0.0138 \varepsilon_t^g$$
$$\varepsilon^z \sim N(0,1)$$
$$\varepsilon^g \sim N(0,1)$$

and approximate each process as a Markov chain with 7 and 9 states, respectively, using the procedure from Rouwenhorst (1995) and Kopecky and Suen (2010); these processes do a reasonable job matching the volatility of output and consumption growth (although consumption growth is a bit more volatile than in Mexican data).[10] The parameters for these processes, as well as $\beta = 0.965$, $\varphi = 1.6667$, and $\omega = 0.350$, generate a debt/income ratio of -36 percent, a frequency of Sudden Stops (defined as a period in which a reduction in debt exceeds two standard deviations and the constraint is strictly binding) of 2.1 percent, and a ratio of nontradable to tradable consumption of 1.66.

The interest rate follows a two-state Markov chain with realizations $\{r_1, r_2\}$ and transition matrix Π_r, where

$$r_1 = 1.0548$$
$$r_2 = 1.144$$

(both annualized) and

$$\Pi_r = \begin{bmatrix} 0.993 & 0.007 \\ 0.333 & 0.667 \end{bmatrix}.$$

Interest rate shocks therefore occur roughly 2 percent of the time in the stationary distribution, and last on average 1.5 quarters.[11] Table 1 summarizes the parameters for the benchmark model.

The model's predictions are insensitive to the parameters that are chosen; for example, I can vary the frequency of Sudden Stops, the size of the nontraded sector relative to the traded sector,

[10] I use more points for g because it generates more nonlinearity in the model. The answers are insensitive to increasing the number of points in each Markov process, but not to approximating the model using a different method, such as that advocated by Flodén (2009).

[11] These two values are the average interest rate in Mexico in "normal" times and in the four quarters surrounding the Tequila crisis.

the average level of debt (provided it remains everywhere negative), the elasticity of substitution between traded and nontraded goods, and the elasticity of labor supply significantly without changing them qualitatively (or quantitatively to any important degree).[12]

4. Results

The results section is divided into two main parts. First, I compare competitive equilibrium allocations with and without model uncertainty; that is, I compare $\varsigma = 0$ with $\varsigma < 0$. Second, I compare allocations with and without model uncertainty that solve the government problem (2.26); here I consider cases where $\varsigma = 0$ but $\sigma < 0$ and where $\varsigma = \sigma = 0$.

4.1. Competitive Equilibrium

4.1.1. Rational Expectations

I begin with the basic model where $\varsigma = 0$, so that private agents trust their model of the stochastic process. To give the reader some intuition about how the model works, Figure (3) shows the process of a Sudden Stop. The solid lines are the equilibrium functions for B' for fixed values of the exogenous states; specifically, for the highest and lowest values of z, given the mean value of g and the low value for r. The dashed lines are the collateral constraints for these shock values; where the dashed line disappears into the solid line is where the constraint begins to bind. Note first that both decision rules lie below the 45-degree line when the constraint is not binding; because $\beta(1 + r) < 1$, the agent would like to bring consumption forward in time and accumulate debt. At low values of z the household wants to borrow more than at high z values, for standard consumption-smoothing purposes. However, the collateral constraint for the low z state is tighter than for the high z state, implying that the household cannot do so indefinitely. Thus, a more indebted economy may actually have higher productivity.

To understand the dynamics of a Sudden Stop, suppose the economy has reached point A (it has received the highest z value for a very long time). Now suppose there is a negative shock; for the ease of presentation, this shock consists of a shift from the highest to the lowest value of z, but the intuition holds for smaller changes. The economy shifts to the 'Low z' decision rule, forcing debt to contract immediately to point B. If this contraction is large enough – that is, if the decision

[12] Provided the static equilibrium remains unique, at least; thus, I do not consider values of κ that imply c_T and c_N are close substitutes.

rules are sufficiently far apart at A – then the economy experiences a crisis event (remember that a sudden stop is defined as a reduction in debt that exceeds two standard deviations). If the economy remains in the low productivity state indefinitely, it will converge to point C.

To give the reader further insight into the effects of a binding constraint, Figure (4) plots the key endogenous variables (tradable and nontradable labor supply, tradable consumption, and the price of nontraded goods). The kinks correspond to the debt level (for the given shock values) at which the constraint begins to bind;. As the economy approaches a debt level at which the constraint will bind, labor in the nontraded sector is falling (due to a decline in p_N) and labor in the traded sector is rising. But when the constraint binds, labor effort in both sectors rise due to an incentive to shift the constraint outward; the result is a large drop in the relative price of nontraded goods. The drop in p_N enters into the collateral constraint and causes it to become tighter, setting off a debt-deflation mechanism (because debt is denominated in tradables, the drop in p_N actually raises the value of the existing debt).

4.1.2. Model Uncertainty

I now set $\varsigma < 0$, introducing model uncertainty on the part of households. To make the comparisons as transparent as possible, I consider two different parameter settings. First, I hold β fixed and simply decrease ς to -5; as noted elsewhere, in this model a decrease in ς is equivalent to an increase in risk aversion, so the debt distribution can shift significantly and complicate comparisons. With this effect in mind, I also set $\varsigma = -15$ and recalibrate $\beta = 0.9$ to equate the average debt-to-income ratios across the two allocations.

Figure (5) shows the ergodic sets from the three different settings. As is clear, the direct effect of decreasing ς is to increase precautionary saving – the ergodic set of debt levels shifts far to the right and becomes very symmetric. The symmetry arises because the constraint plays no role in this economy; the region where it might bind is far outside the ergodic set. As noted in Luo and Young (2011), increasing the demand for robustness is equivalent to increasing the discount factor.[13] Since the model economy with $\varsigma = -5$ does not display any important role for policy, I will concentrate the rest of this section on comparing the other two allocations. Even with the reduction in β the effect of $\varsigma = -15$ is still evident in the figure; the ergodic distribution is shifted a bit to the right and is noticeably more symmetric than the one with rational expectations. While

[13] This equivalence is exact for linear-quadratic economies; Tallarini (2000) shows a similar equivalence between increasing ς and increasing risk aversion.

not obvious from the figure, the debt-to-income ratios in the two economies are identical; what has happened is that labor supply has also changed, resulting in both less debt and less income and keeping the ratio constant.

Figure (6) compares the laws of motion for debt for these two economies. Relative to rational expectations, the robust environment has (for this particular constellation of shocks) a flatter decision rule for B' and a steeper constraint (the dashed lines); with RE this set of shocks has the constraint binding essentially at the 45^o line, while the robust control economy has the constraint binding past any debt level that would be chosen. The flatter decision rule is caused by a desire on the part of households to "make hay while the sun shines" (as noted in van der Ploeg 1993); that is, consumption (both today and in the future) responds more strongly to changes in income under robustness.

Figure (7) compares the ergodic set of debt levels in equilibrium to those in the *worst-case* scenario described by the probability distribution p; the calculation of the worst-case distribution is described in the Appendix. The worst-case distribution has several important differences from the objective distribution: it has a lower mean debt level (-1.38 vs. -1.35), a *lower* standard deviation (0.9 percent vs. 2.2 percent), and has significant left-skew. These differences arise because the agents in the economy distort the probabilities by adding correlation; specifically, the shocks in the worst-case scenario have positive correlation between z and g and negative correlation between r and both z and g. While the correlations are small, the effect on the distribution is noticeable; furthermore, a formal test of the stochastic processes along the lines suggested by Hansen and Sargent (2007) via detection error probabilities (used, for example, in Luo, Nie, and Young (2011,2012) would find it very difficult to distinguish the two distributions.

4.2. Constrained-Efficient Allocations

In this subsection I solve the government problem. As noted already, in this case I can solve the government problem by computing the solution to a constrained-efficient social planning problem and then construct the tax policies that implement this allocation as a competitive equilibrium, as in Bianchi (2010).

4.2.1. Rational Expectations

First, I present a proposition that describes the taxes that implement the solution to the government problem. The proof of this proposition is straightforward from a comparison of the optimality conditions for the competitive equilibrium and the social planner and is omitted.[14]

Proposition 1. *The tax on new debt τ_B is given by*

$$1+\tau_B = \frac{E\left[\lambda'_{CE}\right]+\mu_{CE}}{E\left[\lambda'_{T,OP}\right]+\mu_{OP}}\frac{\lambda_{T,OP}}{\lambda_{CE}} \tag{4.1}$$

where $\lambda_{T,OP}$ denotes the marginal utility of traded consumption (the multiplier on the traded resource constraint). The tax on nontraded consumption τ_N is given by

$$\tau_N = \frac{\mu_{OP}\varphi AH^{\theta}_{N,OP}\frac{1-\omega}{\omega}\left(\kappa-1\right)C^{\kappa-2}_{N,OP}C^{1-\kappa}_{T,OP}}{\lambda_{N,OP}p_{N,OP}} \tag{4.2}$$

where $\lambda_{N,OP}$ denotes the marginal utility of nontraded consumption (the multiplier on the nontraded resource constraint). Finally, the tax on traded consumption is given by

$$\tau_T = \frac{\mu_{OP}\varphi AH^{\alpha}_{T,OP}\frac{1-\omega}{\omega}\left(1-\kappa\right)C^{\kappa-1}_{N,OP}C^{-\kappa}_{T,OP}}{\lambda_{T,OP}}. \tag{4.3}$$

Benigno *et al.* (2011b) shows that in the constrained-efficient allocation the price of nontradables *rises* when the constraint binds, which is exactly opposite from that found in the competitive equilibrium. It is this feature that drives the behavior of the government here – it will strongly *subsidize* nontraded consumption via τ_N in order to eliminate the debt-deflation mechanism, and then use τ_B to eliminate the excessive borrowing that a high price for nontradables would otherwise encourage.

4.2.2. Model Uncertainty

Because the problem under unrestricted taxes is equivalent to a social planning problem, whether the household faces model uncertainty or not is relevant only to the welfare calculations of moving to the constrained-efficient allocation (and the taxes that are needed to implement it); unlike the restricted tax case I consider briefly below, the *allocation* itself will not be distorted. However, I will present the welfare effects of moving from the competitive equilibrium with and without model uncertainty.

It turns out that p_N behaves under model uncertainty ($\sigma = -15$) very similarly to the case where $\sigma = 0$ (recall that for allocations it does not matter whether the private sector shares the

[14] A related result can be found in Bianchi (2010), and the proof follows that approach.

model uncertainty of the government or not). Thus, the main intuition regarding the behavior of the government derived in that paper will go through unchanged. The reason that the presence of model uncertainty does not change the main distortion is that the distortion is primarily static – it is a misalignment of the current value of tradable vs. nontradable consumption and therefore p_N – and model uncertainty operates primarily through the dynamic relationship between consumption over time. However, indirectly the desire for robustness *could* have strong effects, since precautionary saving effects can operate through the labor supply channel.

Based on the previous two paragraphs, one might conclude that model uncertainty is largely irrelevant for the study of models of Sudden Stops. That conclusion would be premature, because the welfare effects of optimal policies will be significantly altered. Figure (8) presents the welfare functions generated by moving from the CE allocation to the OP allocation for two cases – one where $\sigma = -15$ but $\varsigma = 0$ and one where $\sigma = \varsigma = -15$; in the first case, the government is assumed to distrust its model of household expectations, whereas in the second the household distrusts its own model and the government is confident that it knows what the household thinks. What is remarkable is the difference in welfare costs – a government that does not trust that the private agent expectations are rational imposes a welfare loss of -16 percent on average! Even in the second case the difference is significant – the welfare gain from optimal policy with robustness can be over 3 times as large as under rational expectations, and the gap is increasing in the current stock of debt.

The welfare functions also have different slopes. For the two cases where the government and the private agents either both ignore model uncertainty or the government is certain what private sector agents believe, the more debt the economy has the higher the welfare gain from imposing the optimal tax system; this result is easy to understand, because the more debt the economy has the more likely a bad shock will result in a Sudden Stop due to a suddenly-binding constraint. For the case where the government is unsure about the private expectations but private agents fully trust their model, the welfare gains can be nonmonotonic in current debt, but generally slope upward – with more debt, the economy needs to accumulate more precautionary balances and this accumulation is costly in terms of current consumption.

Similarly, the situation regarding the values of the shocks differs across the two cases. When the government trusts its model of household expectations higher values of z generate smaller welfare gains, as again the key is the extent to which the economy is exposed to a potentially-binding constraint; with a high z the constraint is less likely to bind in the near future since z

is persistent and resources are abundant. In contrast, when the government is uncertain about private expectations the welfare loss for large debt levels is larger when z is high.

Given the high cost of having a robust decision rule for the government, it is useful to understand what the government fears about private expectations. As noted above, the worst-case distributions for this model involve a positive correlation between the productivity shocks and a negative correlation between those shocks and r. Figure (7) shows the ergodic distribution of debt for $\sigma = -15$ compared to the "worst-case" distribution; the worst-case distribution is the one that would result if the transition probabilities were given by p instead of π. The worst-case distribution has been shifted far to the left relative to the equilibrium one and is skewed left in addition, reflecting the fact that in the worst-case distribution the constraint would bind frequently.

5. Discussion

Here I want to make some comments on the robustness of the results that are discussed above. For brevity, and to avoid repeating myself, I do not formally present any of the results discussed here; some can be found in Benigno *et al.* (2011c) for the rational expectations case, and all are available upon request from the author.

5.1. Lump-Sum Taxation

First, the results are robust to eliminating the ability of the government to use lump-sum taxation; it turns out that the tax system balances revenues and expenditures, so that the absence of a lump-sum tax does not matter (formally, moving to a distortionary tax to balance the budget eliminates the constrained-optimality of the unrestricted tax case, but quantitatively it does nothing else). The intuition for this result is not clear, since the taxes used in decentralizing the constrained-efficient allocation depend on the strength of various distortions, and these distortions need not add up to zero, but the result is very robust numerically (in fact, it even holds when the government has only two instruments).

5.2. Denomination

Second, the results are also robust to the debt-denomination assumption. If I make the assumption that debt is denominated in nontradable goods, the budget constraint is changed to

$$(1+\tau_{T,t})\,c_{T,t} + (1+\tau_{N,t})\,p_{N,t}c_{N,t} + \frac{(1+\tau_{B,t})\,b_{t+1}}{1+r_t} \leq b_t + w_t h_t + D_t + T_t \tag{5.1}$$

and the collateral constraint becomes

$$\frac{b_{t+1}}{1+r_t} \geq -\varphi\,(w_t h_t + D_t)\,. \tag{5.2}$$

The results from the model are largely unchanged, except that the welfare gains are significantly reduced for all cases. The reason this reduction occurs is that there are large welfare gains of moving from tradable-denominated debt to nontradable-denominated debt (on the order of 1.2 percent for the benchmark rational expectations economy), and this change also reduces the Sudden Stop dynamics (as the nontraded price falls so does the face value of debt). Thus, there is less for the government to correct, and the resulting welfare gains are dramatically reduced.

5.3. Restrictions on Instruments

Suppose now the government cannot use all three instruments; as noted previously, the allocation will now no longer solve the constrained-efficient social planner problem, but is instead a nontrivial game between the current and future governments. To make the comparison concrete, I suppose the government cannot use τ_T, but the points here would hold as well if the government cannot use τ_N either; what matters is that the government *can* use τ_B.

The crucial difference between the unrestricted and restricted cases is that $\varsigma = 0$ affects the allocation; that is, it matters for outcomes, and not just welfare, that private households do not face model uncertainty. The reason this result arises is that the Euler equation (2.23) is now a relevant constraint, as the government can only implement allocations that respect how agents tradeoff consumption across time. From the government's perspective, private agents are not saving enough; effectively, the government is more risk averse than households, and wants additional precautionary balances. To achieve this goal, the government will use τ_B to tax debt *even if the constraint is not currently binding*; since τ_N is primarily useful for correcting the misallocation of labor supply, it is not used unless the constraint is binding. The resulting equilibrium has $\mu = 0$

in every state.

As a result, the ergodic set for B is shifted far to the right (debt is reduced), similar to the unrestricted case. Of course, this extra precautionary saving must come from consumption and leisure, so the economy will experience welfare losses as it transitions to the new ergodic set.[15] The average welfare loss is now 2.33 percent; as a comparison, the welfare gain from the optimal policy in the benchmark economy is only 0.09 percent. The lesson here is that restricting the government's options may be valuable if it cannot trust its own model of household expectations (remember the welfare loss from the unrestricted case was 16 percent); this result runs counter to that in Dennis (2010), where being robust improves performance in a standard New Keynesian model, and may arise from differences in the objective criteria.

6. Conclusion

This paper explored robust policymaking in models with Sudden Stops – declines in aggregate activity exacerbated by a binding collateral constraint – in an emerging market economy. Obviously, this paper is just a first step and is unsuited for making actual policy advice. A number of extensions would be natural, including the introduction of monetary factors (including nominal rigidities), physical capital, and sovereign default, although each poses significant computational challenges. Deriving the collateral constraint from formal microeconomic principles and endogenizing the interest rate also seem like important directions to proceed, perhaps by formally modeling the "rest of the world" lender.[16]

To conclude I will simply point out some challenges associated with these extensions. Benigno *et al.* (2011d) investigates the role of optimal monetary policy in a model similar to the one used here, but uses a three-period environment for tractability reasons. The primary obstacle for adding either nominal rigidities or capital is that the two-dimensional state space makes the optimal policy problem exceptionally difficult; preserving the shape of the value function is not easy to do in more than one dimension, and the algorithm relies critically on that shape (see Cai and Judd 2011 for a promising approach). Sovereign default introduces nonconvexities, rendering

[15] Note that τ_B is critical due to commitment issues; while it might seem that the government could use τ_T to replicate the effects of the capital control, that would require control over the tradable consumption tax *tomorrow*, which the current government does not have.

[16] Preliminary calculations using a patient, large country that simply imposes the collateral constraint (2.4) show counterfactual behavior of the interest rate when the constraint binds; specifically, the real interest rate collapses during a crisis. While that feature is consistent with recent events, it is not consistent with the behavior of interest rates during emerging market crises in the past.

the computation of the competitive equilibrium more challenging; with the exception of Mendoza and Yue (2008), most sovereign default models assume output is exogenous and thereby completely ignore the private sector's behavior.

Exploring the role of commitment is also important, although extremely difficult from a computational perspective. Benigno *et al.* (2011c) contains a brief discussion of how commitment matters in this context; here, I have avoided discussing the role of commitment by assuming that the government has enough tools to solve a standard dynamic programming problem – how to study commitment in the restricted policy case is unclear. Computationally, an extension using the multiplier method of Marcet and Marimon (2011) seems straightforward, but preliminary investigations have revealed that the domain of the multiplier state is difficult to characterize.

7. Appendix

7.1. The Utility Recursion

In this appendix I provide a proof that the utility recursion (2.17) represents the decision problem of an agent with multiplier preferences; this proof is nearly identical to that in Backus, Routledge, and Zin (2005) and is presented solely for the convenience of the reader. The continuation value of an agent who desires robust decision rules with respect to random variable z' is

$$\widehat{V}(z) = \min_{\{p(z')\}\in[0,1]^{\#Z}} \left\{ \sum_{z'} p(z') V(z') - \frac{1}{\varsigma}\left(\sum_{z'} p(z') \log\left(\frac{p(z')}{\pi(z'|z)}\right)\right) + \Omega\left(\sum_{z'} p(z') - 1\right)\right\};$$

note that the value depends on z through the conditional probability under the approximating model $\pi(z'|z)$. The problem is convex in $p(z')$, since $x\log(x)$ is a convex function and the other terms are linear; the constraint set is convex and compact as it is a finite product of intervals. The necessary and sufficient first-order condition with respect to $p(z')$ is

$$V(z') - \frac{1}{\varsigma}\log\left(\frac{p(z')}{\pi(z'|z)}\right) + \frac{1}{\varsigma} + \Omega = 0.$$

Now multiply by $p(z')$ and sum over z' to obtain

$$\sum_{z'} p(z') V(z') - \frac{1}{\varsigma}\left(\sum_{z'} p(z') \log\left(\frac{p(z')}{\pi(z'|z)}\right)\right) + \frac{1}{\varsigma} + \Omega = 0,$$

written more simply as

$$\widehat{V}(z) - \frac{1}{\varsigma} + \Omega = 0$$

so that

$$\Omega = -\widehat{V}(z) + \frac{1}{\varsigma}.$$

Substituting this expression into the first-order condition yields

$$p(z') = \pi(z'|z) \exp\left(-\frac{V(z') - \frac{1}{\varsigma} + \Omega}{\frac{1}{\varsigma}}\right)$$

or

$$p(z') = \exp\left(\varsigma \widehat{V}(z)\right) \pi(z'|z) \exp\left(\varsigma V(z')\right).$$

Now sum over z' to obtain

$$1 = \exp\left(-\varsigma \widehat{V}(z)\right) \sum_{z'} \pi(z'|z) \exp\left(\varsigma V(z')\right).$$

Taking logs and rearranging yields

$$\widehat{V}(z) = \frac{1}{\varsigma} \log\left(\sum_{z'} \pi(z'|z) \exp\left(\varsigma V(z')\right)\right).$$

7.2. Computational Method

Consider the system of functional equations that determine a recursive competitive equilibrium:

$$u_1(S) = \lambda(S) \tag{7.1}$$

$$u_2(S) = (1+\tau_N)\, p_N(S)\, \lambda(S) \tag{7.2}$$

$$u_3(S) = w(S)\left(\lambda(S) + \varphi \max\{\mu(S), 0\}^2\right) \tag{7.3}$$

$$\frac{(1+\tau_B)\,\lambda(S) - \max\{\mu(S), 0\}^2}{1+r(S)} = \beta \exp(-g(S)\,\rho)\, E_p\left[\lambda\left(S'(S)\right)\middle| S\right] \tag{7.4}$$

$$\max\{-\mu(S), 0\}^2 = \frac{B'(S)}{1+r(S)} + \varphi\left(w(S)\left(H_T(S) + H_N(S)\right) + D(S)\right)$$

$$w(S) = \alpha \exp(z(S))\, H_T(S)^{\alpha-1} \tag{7.5}$$

$$w(S) = \theta p_N(S)\, A H_N(S)^{\theta-1} \tag{7.6}$$

$$D(S) = (1-\alpha)\exp(z(S))\, H_T(S)^{\alpha} + (1-\theta)\, p_N(S)\, A H_N(S)^{1-\theta} \tag{7.7}$$

$$C_T(S) = B(S) + \exp(z(S))\, H_T(S)^{\alpha} - \frac{\exp(g(S))\, B'(S)}{1+r(S)} \tag{7.8}$$

$$C_N(S) = A H_N(S)^{\theta} \tag{7.9}$$

$$T(S) = \tau_N p_N(S)\, C_N(S) + \tau_B \frac{\exp(g(S))\, B'(S)}{1+r(S)}, \tag{7.10}$$

where E_p is the distorted expectations operator

$$E_p\left[f\left(S'\right)\right] = \frac{E_\pi\left[\exp\left(\varsigma V\left(S'\right)\right) f\left(S'\right)\right]}{E_\pi\left[\exp\left(\varsigma V\left(S'\right)\right)\right]}$$

and $S = (B, z, g, r)$. I solve these equations using an iterative approach, as in Coleman (1989). I fix a grid for B, with 300 points concentrated over a small interval at the lower end of the interval and 50 additional points more dispersed at the upper end; for stability it turns out to be important to include points where $B_i > 0$ even though they lie outside the ergodic set of debt levels.

I guess functions

$$\lambda^0\left(B', z, g, r\right) = E_p\left[\lambda\left(B', z', g', r'\right)\middle| z, g, r\right] \tag{7.11}$$

and

$$\mathcal{V}^0\left(B', z, g, r\right) = \log\left(E_\pi\left[\exp\left(\varsigma V\left(B', z', g', r'\right)\right)\middle| z, g, r\right]\right). \tag{7.12}$$

Using a hybrid Powell method (see Judd 1998), I solve the system of equations

$$u_1(S) = (1+\tau_T)\lambda(S) \tag{7.13}$$

$$u_2(S) = (1+\tau_N) p_N(S)\lambda(S) \tag{7.14}$$

$$u_3(S) = w(S)\left(\lambda(S) + \varphi \max\{\mu(S), 0\}^2\right) \tag{7.15}$$

$$\frac{(1+\tau_B)\lambda(S) - \max\{\mu(S), 0\}^2}{1+r(S)} = \beta \exp(-g(S)\rho)\lambda^0\left(B', z(S), g(S), r(S)\right) \tag{7.16}$$

$$\max\{-\mu(S), 0\}^2 = \frac{\exp(g(S)) B'(S)}{1+r(S)} + \varphi\left(w(S)\left(H_T(S) + H_N(S)\right) + D(S)\right) \tag{7.17}$$

$$w(S) = \alpha \exp(z(S)) H_T(S)^{\alpha-1} \tag{7.18}$$

$$w(S) = \theta p_N(S) A H_N(S)^{\theta-1} \tag{7.19}$$

$$D(S) = (1-\alpha)\exp(z(S)) H_T(S)^{\alpha} + (1-\theta) p_N(S) A H_N(S)^{1-\theta} \tag{7.20}$$

$$C_T(S) = B(S) + \exp(z(S)) H_T(S)^{\alpha} - \frac{\exp(g(S)) B'(S)}{1+r(S)} \tag{7.21}$$

$$C_N(S) = A H_N(S)^{\theta} \tag{7.22}$$

$$T(S) = \tau_T C_T \tau_N p_N(S) C_N(S) + \tau_B \frac{\exp(g(S)) B'(S)}{1+r(S)} \tag{7.23}$$

$$V(S) = u(S) + \beta \exp(g(S)(1-\rho))\mathcal{V}^0\left(B', z(S), g(S), r(S)\right) \tag{7.24}$$

for the functions $(C_T, C_N, H_T, H_N, B', \lambda, \mu, T, V, w, d, p_N)(S)$, and update my guesses; values for the functions $\lambda^0(B', z, g, r)$ and $\mathcal{V}^0(B', z, g, r)$ at points B' not on the grid are computed using cubic splines. This process is repeated until (if?) it converges; as noted already, existence and uniqueness have not been established for this model, but in no case did I detect any multiplicity. The true Lagrange multiplier can be recovered using the equation

$$\mu^*(S) = \max\{\mu(S), 0\}^2.$$

I then construct the stationary distribution and the ergodic set using the nonstochastic method from Young (2010), which converges more rapidly than simulation-based methods and deals better with rare events (like the interest rate shocks); this process uses an evenly-spaced grid of 1000 points for B.

The government problem is solved using standard dynamic programming methods, assuming that

$$\mathcal{W}^0\left(B',z,g,r\right) \equiv \log\left(E_\pi\left[\exp\left(\sigma W\left(B',z',g',r'\right)\right)\middle| z,g,r\right]\right) \tag{7.25}$$

is evaluated using cubic splines whenever necessary. The constrained maximization is conducted using a feasible sequential quadratic programming method (see Zhou, Tits, and Lawrence 1997) with the equilibrium conditions imposed as equality constraints for given functions $\lambda^0\left(B',z,g,r\right)$ and $\mathcal{V}^0\left(B',z,g,r\right)$. For numerical stability, I also impose a constraint that the argument of u be positive. All three guesses are updated and the process is iterated to convergence. Convergence does not always obtain here, particularly for cases where ς and σ are large in value or far apart; at least part of the reason is that when agents face a lot of model uncertainty, their precautionary savings motive pushes them into positive B regions where the worst-case interest rate shock changes identity. When I recalibrate β to match average debt levels in the $\varsigma = \sigma = 0$ case, I can explore higher values for these parameters without convergence becoming a problem.

The procedure actually used to solve the government problem is simpler than the one detailed above, which applies equally to the restricted tax case discussed briefly. In particular, because the allocation that solves the government problem satisfies constrained efficiency, I can solve it without including the optimality conditions of the household (that is, using only the resource constraints and the marginal condition for p_N); I then can back out the taxes needed to decentralize the allocation, again using cubic splines to interpolate the value of future marginal utility needed for τ_B.

A similar dynamic programming approach is used to calculate the welfare function $V^*\left(B,z,g,r,\chi\right)$ (I define a grid for χ and treat it as a state variable with a trivial transition function $\chi' = \chi$) and then Brent's method is used to solve the nonlinear equation

$$V^*\left(B,z,g,r,\chi\left(B,z,g,r\right)\right) = \mathcal{V}\left(B,z,g,r\right), \tag{7.26}$$

using cubic splines again to interpolate in the χ direction.

The worst-case stochastic process is governed by the probabilities

$$p\left(z',r',g'|B,z,r,g\right) = \frac{\pi\left(z',r',g'|z,r,g\right)\exp\left(\varsigma V\left(B'\left(B,z,r,g\right),z',r',g'\right)\right)}{\sum \pi\left(z',r',g'|z,r,g\right)\exp\left(\varsigma V\left(B'\left(B,z,r,g\right),z',r',g'\right)\right)}; \tag{7.27}$$

as noted previously by Bidder (2011), the worst-case process is not Markovian in the usual sense because the probabilities depend on B. I construct the ergodic distribution of the worst-case scenario using the non-stochastic approach from Young (2010). Because the worst-case distribution will generally involve correlation between the shocks, I construct a joint process for (z, r, g) involving a single state s:

$$p\left(s'|B,s\right) = \frac{\pi\left(z\left(s'\right), r\left(s'\right), g\left(s'\right)|z\left(s\right), r\left(s\right), g\left(s\right)\right)\exp\left(\varsigma V\left(B'\left(B, z\left(s\right), r\left(s\right), g\left(s\right)\right), z\left(s'\right), r\left(s'\right), g\left(s'\right)\right)\right)}{\sum \pi\left(z\left(s'\right), r\left(s'\right), g\left(s'\right)|z\left(s\right), r\left(s\right), g\left(s\right)\right)\exp\left(\varsigma V\left(B'\left(B, z\left(s\right), r\left(s\right), g\left(s\right)\right), z\left(s'\right), r\left(s'\right), g\left(s'\right)\right)\right)} \tag{7.28}$$

where $(z(s), r(s), g(s))$ is the unique trio of fundamental shocks associated with a particular s.

The computational procedure is not foolproof, however. It does not necessarily converge for reasonable parameter values, it will not converge if the grid for B is not carefully specified (particularly the lower bound), and it will fail if ς (or σ) is too large.

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Table 1

Parameters	
β	0.965
φ	1.667
ω	0.350
κ	-0.32
α	0.66
θ	0.66
δ	1.75
ρ	1
γ	0.05
ϖ	0.2
φ	0.4

Figure 1: Current Account Dynamics in Mexico

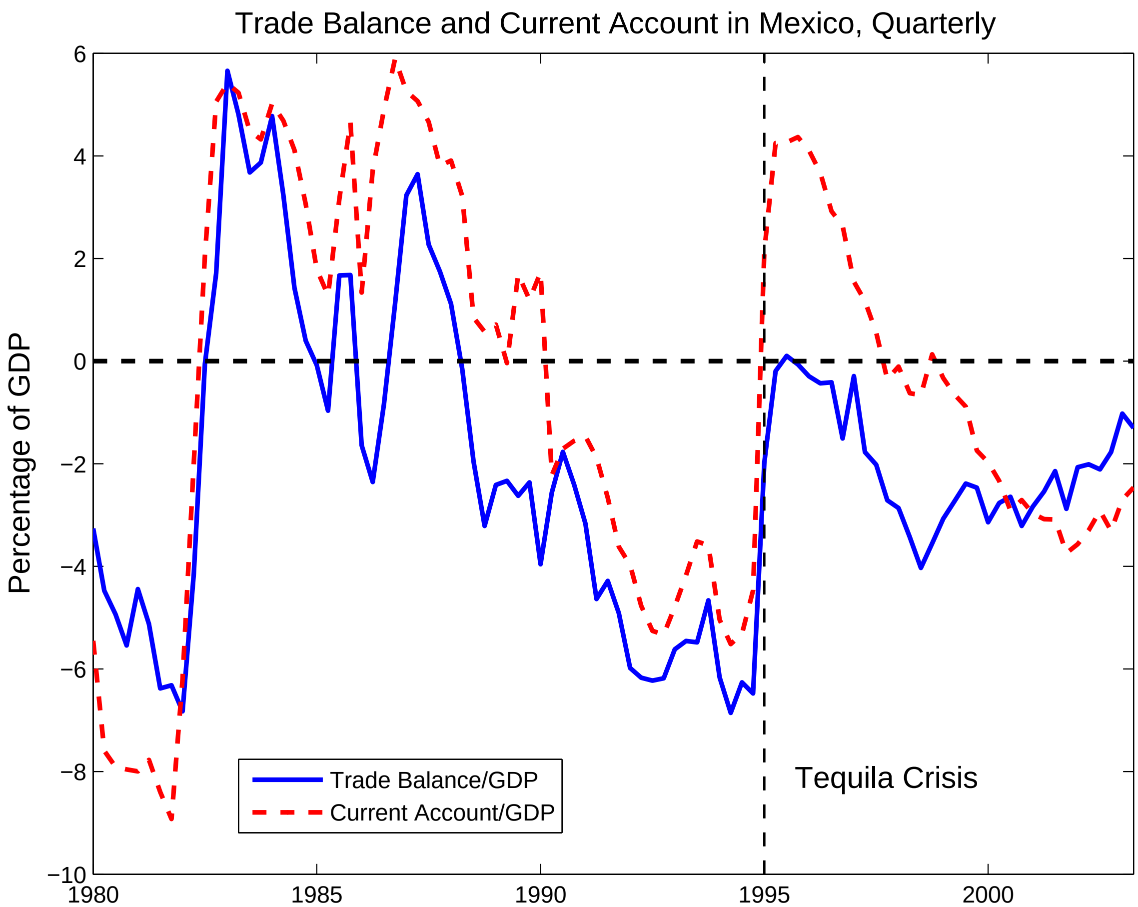

Figure 2: Consumption and Output Growth in Mexico

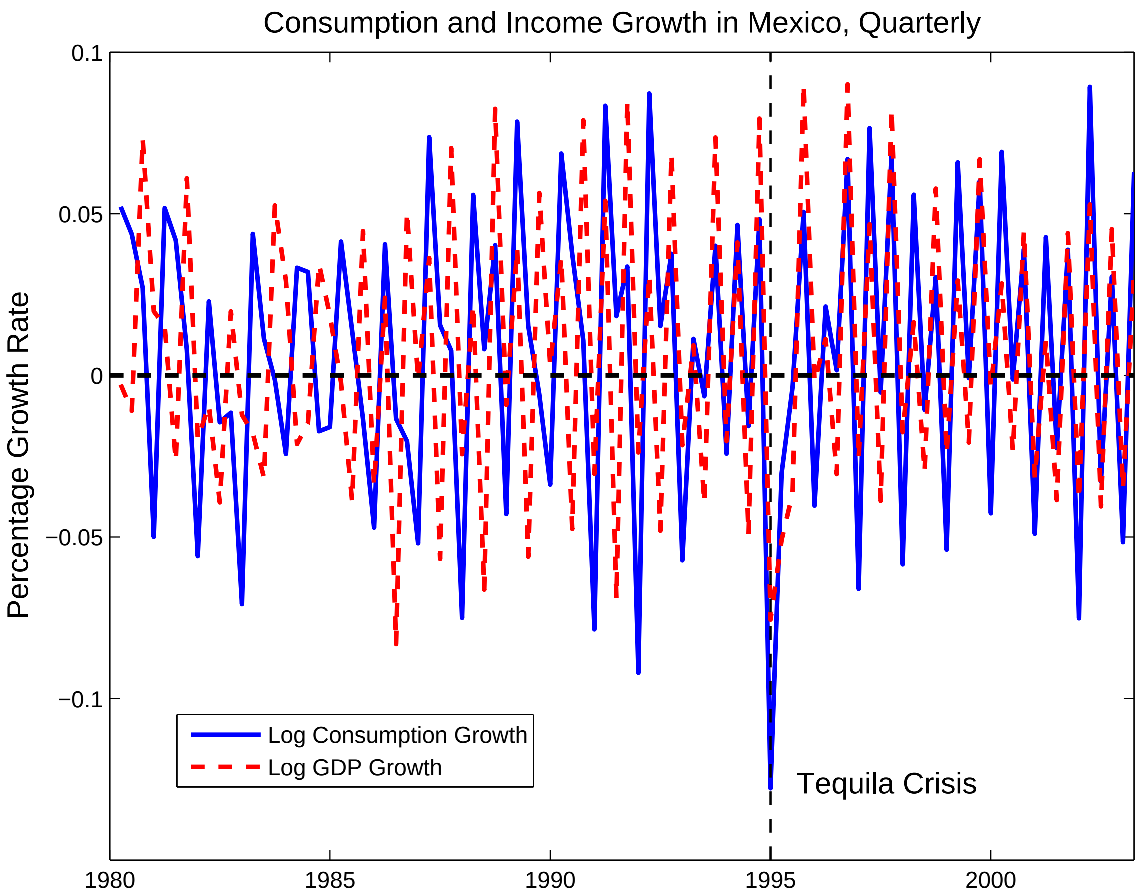

Figure 3: Sudden Stop

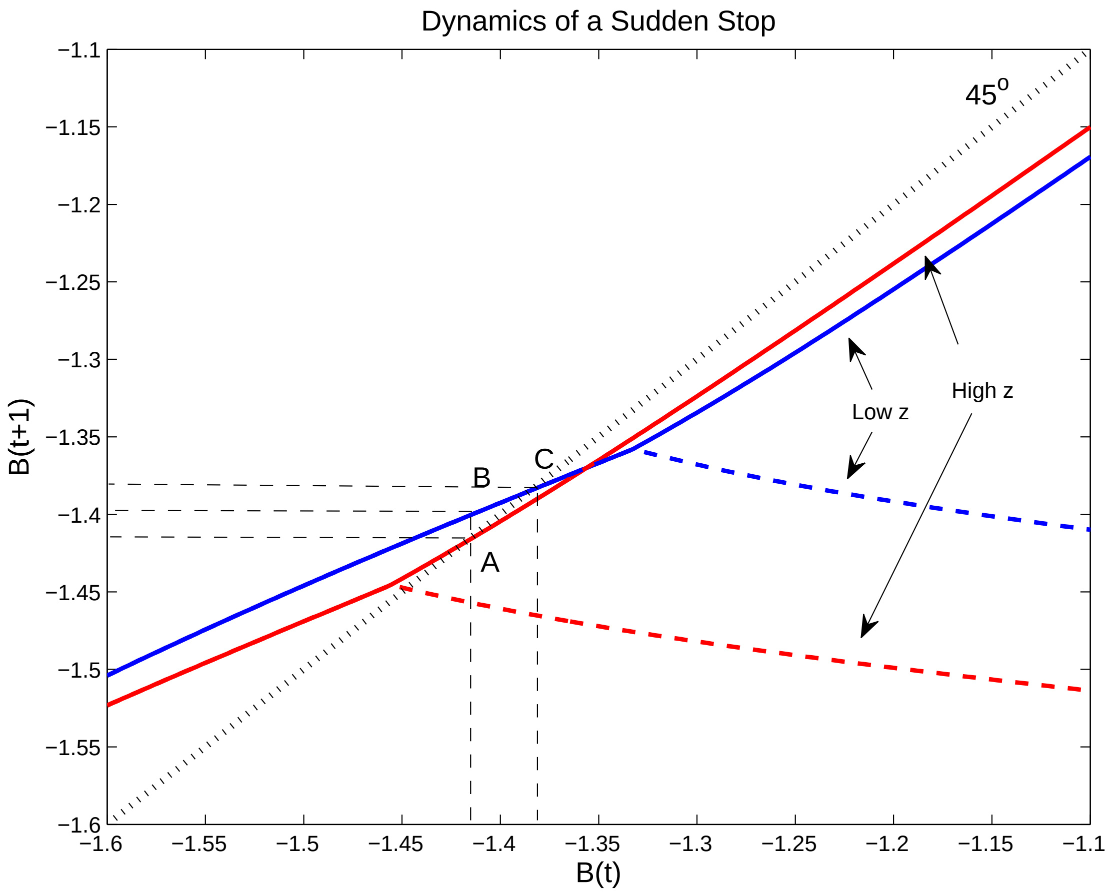

Figure 4: Endogenous Variables

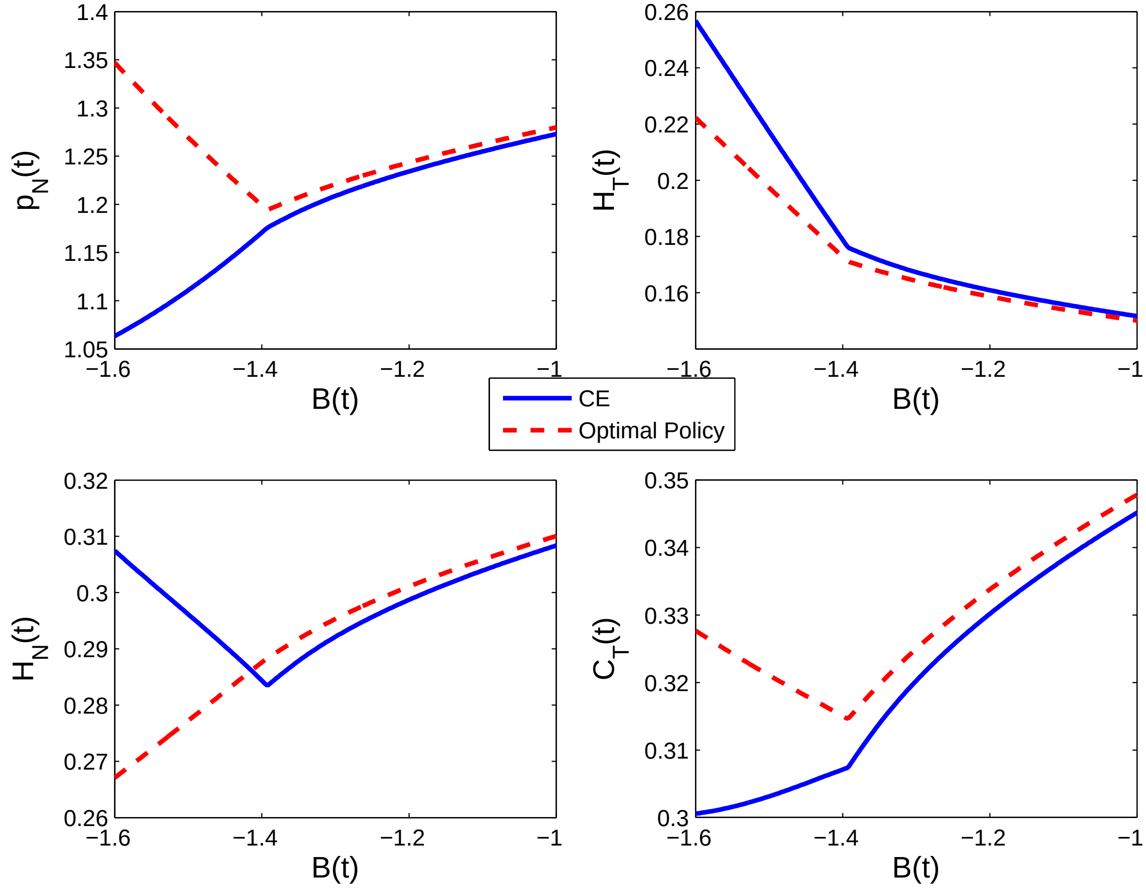

Figure 5: Ergodic Distribution

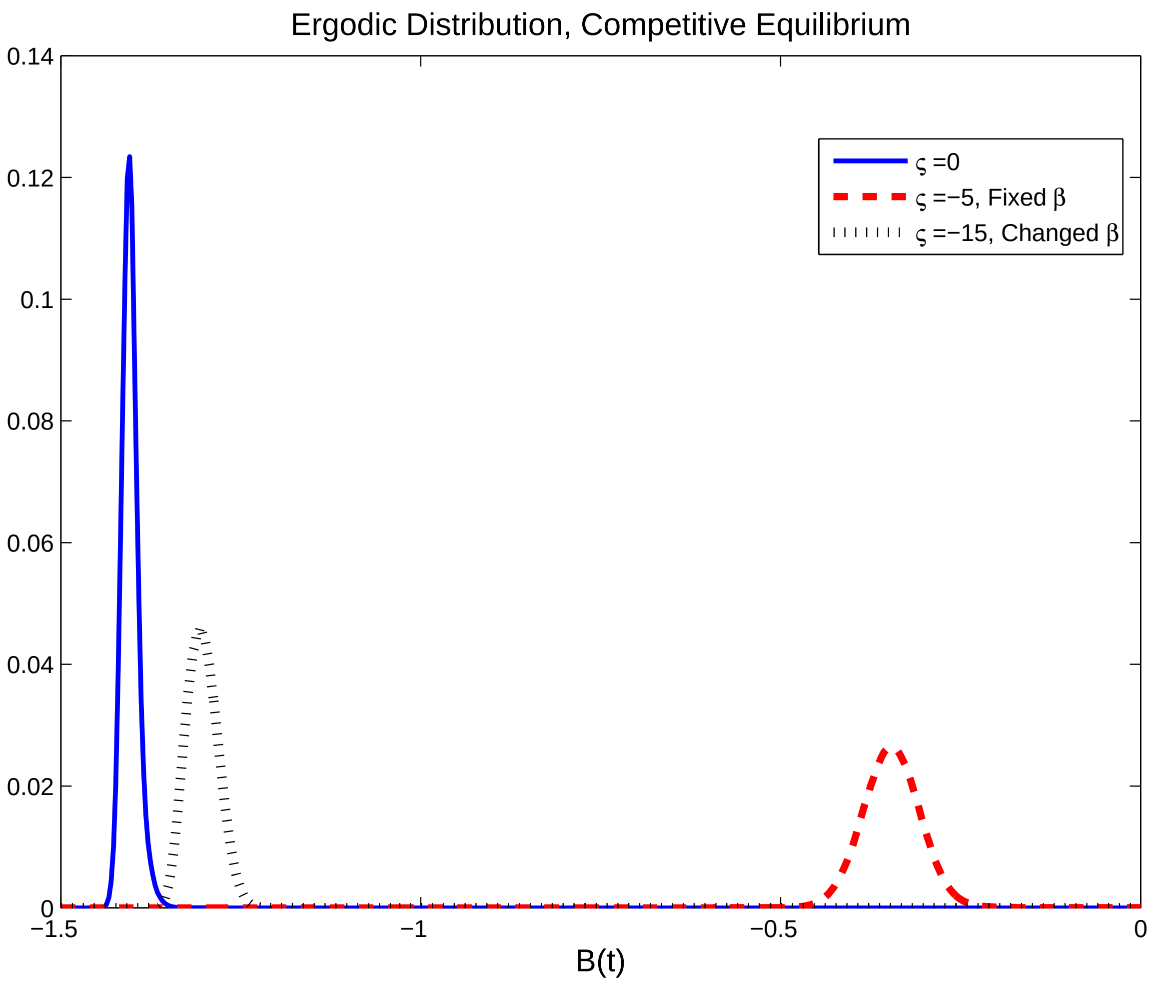

Figure 6: Law of Motion for Debt

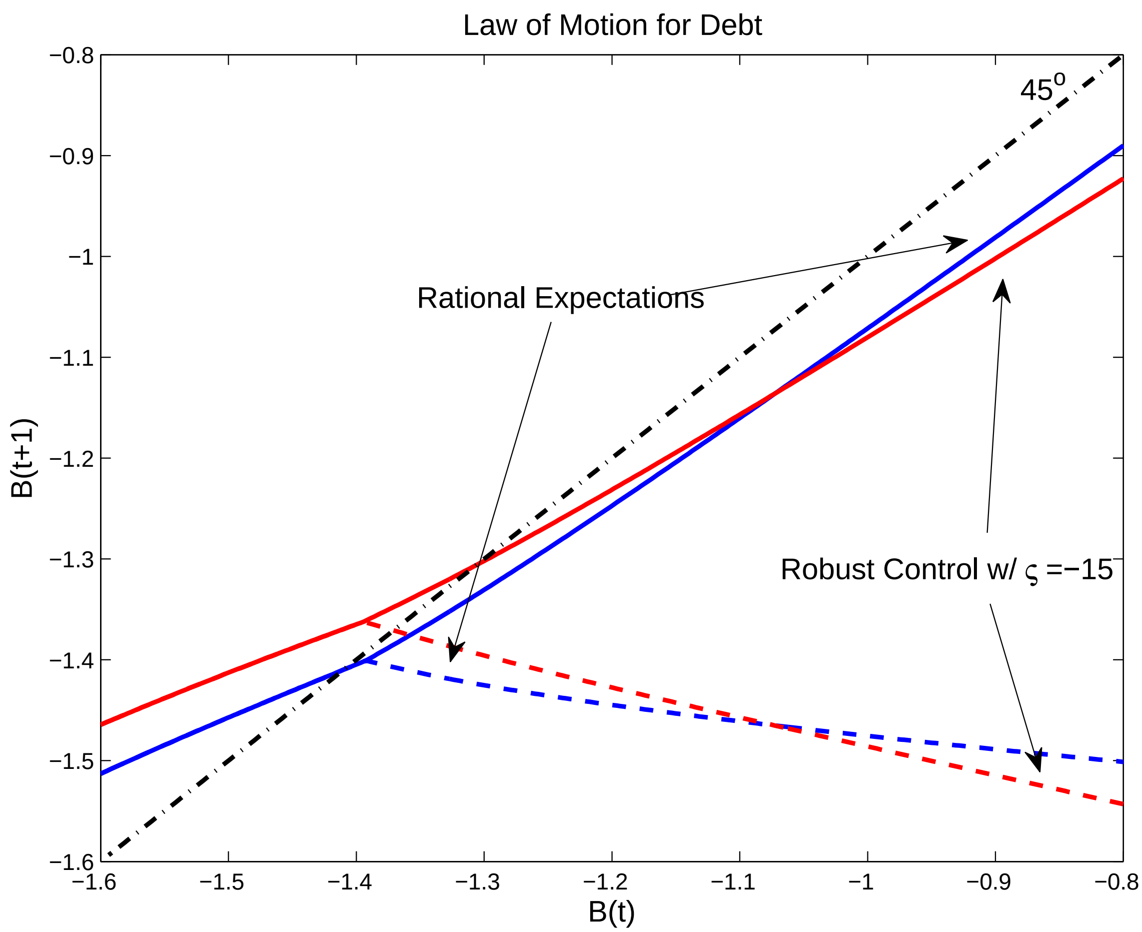

Figure 7: Worst-Case Distribution

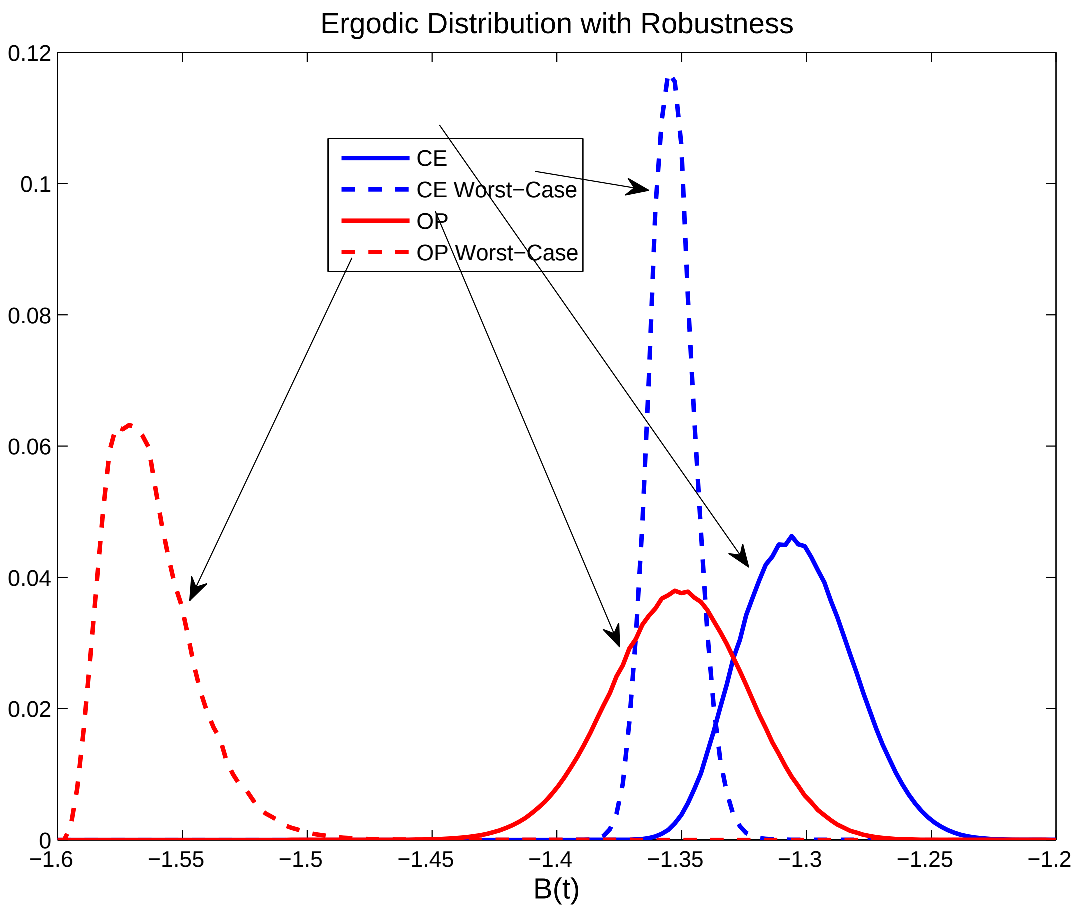

Figure 8: Welfare Cost Functions