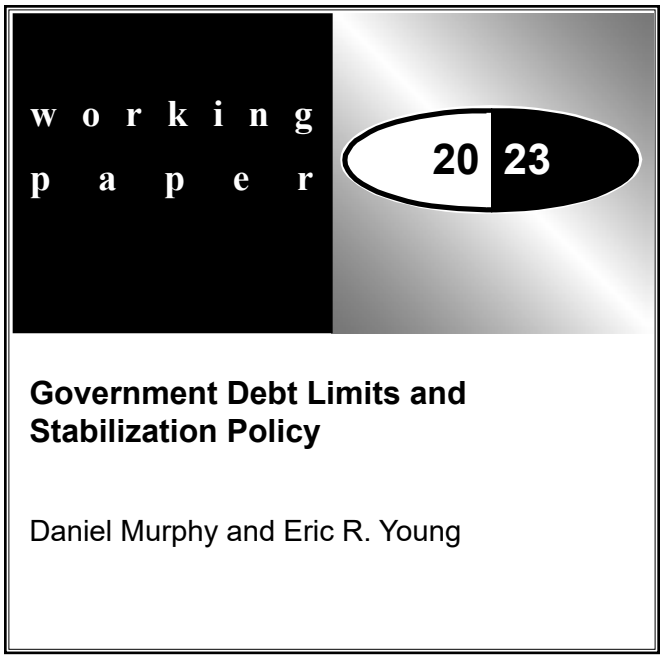

Government Debt Limits and Stabilization Policy

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Daniel Murphy and Eric R. Young

We evaluate alternative public debt management policies in light of constraints imposed by the effective lower bound on interest rates. Replacing the current limit on gross debt issued by the fiscal authority with a limit on consolidated debt of the government can ensure that output always reaches its potential, but it may permit excess government spending when the economy is away from the effective lower bound. The welfare-maximizing policy sets the gross debt limit to the level implied by Samuelson (1954), while the central bank finances government spending with money when the economy is at the effective lower bound.

Keywords: Government debt limits, zero lower bound, stabilization policy, inefficiency.

JEL Codes: E52, E58, E63.

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"The Federal government cannot afford [to spend more]. The national debt is now over $25 trillion." Scott Walker, former governor of Wisconsin, New York Times Op-Ed May 20, 2020.

1 Introduction

Public debt is a frequent preoccupation in popular discussions of the macroeconomy, as liabilities issued by the US Treasury have grown from approximately 50 percent of GDP in 2000 to over 100 percent of GDP in 2020. In response to concerns about rising public debt, Congress set a limit on Treasury debt with the Budget Control Act of 2011. The debt limit was reached in 2013, leading to severe contractions in government spending and a decline in GDP growth during the budget sequestration episode (e.g., Cashin et al. (2018)).[1]

The academic view of public debt has tended to be similarly cautionary. For example, there is the Reinhart and Rogoff (2009) book, which argues that sufficiently high debt leads to substantial negative effects on growth.[2] Other studies emphasize that high government debt is a reflection of politicians' incentives to overspend (e.g., Battaglini and Coate (2008), Yared (2010)) and prescribe debt limits as a solution to misaligned incentives between politicians and society. Yared (2019) surveys the literature on the causes and costs of rising public debt. Blanchard (2019) offers a more optimistic view, noting that interest rates below the growth rate of the US economy imply that the intertemporal solvency condition is not binding and the stock of debt is potentially irrelevant.

What is often overlooked in both the academic and the public discourse over debt is that an increasing share of debt issued by the Treasury is held by the Federal Reserve, itself a governmental organization. Indeed, the recent Treasury debt held by the Fed is the result of monetary expansion in response to the high-unemployment episodes associated with the Great Recession and, more recently, the COVID-19 recession. Netting out debt held by the central bank results in a debt-to-GDP ratio of only 75 percent, implying an increase since 2000 that is half as large as one would infer from gross debt. Despite the fact that the Fed is a governmental organization and the fact that the interest payments on the debt

[1] While Congress occasionally acts to lift these limits, they are nonetheless a constraint, even if only in the sense that they might not be lifted in the future.

[2] Herndon et al. (2014) offer an alternative interpretation of the evidence. See also Reinhart and Rogoff (2010).

are remitted back to the Treasury (net of Federal Reserve operating costs, which are quite small), current statutory restrictions on government debt in the US treat debt owned by the Fed as a liability that counts against the debt limit.[3]

In this paper we evaluate debt management policies in light of stabilization objectives alongside the objective of limiting excessive government spending. We consider an environment in which the central bank's balance sheet is at the forefront of the analysis. The central bank adjusts the money supply to stabilize output at its potential, while fiscal policy is controlled by politicians whose objective is to maximize deficits. The central bank exchanges money for government bonds. The ability of the central bank to achieve the socially optimal level of output is constrained by an effective lower bound (ZLB) on interest rates, while government spending is constrained by statutory limits on gross debt.[4]

In normal times (when interest rates exceed the lower bound and the central bank can maintain output at potential), optimal government debt is such that government spending is limited to the level prescribed by Samuelson (1954). At the Samuelson level, the marginal rate of substitution between government spending and private consumption equals their marginal rate of transformation. A limit on debt at this level prevents wasteful spending that would lower social welfare in normal times. However, when an adverse demand shock causes the natural interest rate to fall below the lower bound, the economy features excess capacity.[5] In this case, government spending stimulates employment and helps output to reach its potential. The debt limit prevents such spending and thus increases the likelihood of recessions. Therefore, optimal average government debt exceeds the limit specified by the Samuelson level.

We evaluate modifications to debt management that can remove this stabilization constraint while preserving limits to excessive government spending. One option is to simply raise the debt limit in light of the possibility that the economy will hit the effective lower bound, consistent with the prescription in Michaillat and Saez (2019) that optimal government spending should exceed the Samuelson level in the presence of idle resources. The downside of this policy is that it permits excessive government spending during episodes in which the economy is away from the lower bound.

[3] Hall and Sargent (2018) give a detailed history of debt limits; prior to 1939, limits were imposed security by security, rather than on the aggregate. Also, prior to 1922 the Federal Reserve did not actively buy and sell US government securities.

[4] We consider an effective lower bound of zero, without loss of generality, despite the recent move by the ECB to cut rates below zero. Rognlie (2016) studies the difference between zero and negative ELBs.

[5] We refer to excess capacity interchangeably with inefficient unemployment, slack, and/or idle resources. For formalizations of these concepts, see Michaillat and Saez (2015) and Murphy (2017).

An alternative policy is to replace the limit on gross Treasury debt with a limit on Treasury debt net of central bank holdings (net debt). In that case, government spending at the lower bound is financed by the central bank and does not violate the debt limit. Furthermore, this spending does not require an increase in future taxes, since payments on the debt owed by fiscal authorities to the central bank are remitted back to the fiscal authorities. Such a policy can in principle ensure that output always reaches its potential, thus overcoming the challenges posed by the ZLB. But it can also permit excessive government spending when the economy is away from the ZLB, because macroeconomic shocks may require adjustment in the money supply, and money supply adjustments affect net (consolidated) government debt. If the central bank must increase the money supply, it permits the government to spend above the socially optimal level when the economy is away from the ZLB. Therefore, this policy at least qualitatively suffers from a similar shortcoming of a policy of increasing the gross debt limit above that implied by the Samuelson level.

We document that a superior policy exists in which the gross debt limit is set to the Samuelson-implied level and the central bank has the authority to discretionarily finance government spending with money. If the central bank maximizes social welfare and the gross debt limit is set at the Samuelson level, the central bank will resort to money-financing of the fiscal authority only when the economy is at the ZLB. In that case, the economy can fully close the output gap when it is optimal to do so and ensure that government spending does not exceed the Samuelson level during periods of positive interest rates.

This policy achieves maximum expected utility when the policy tools are limited to monetary stimulus (subject to a ZLB) and government spending. An equivalent policy would be a state-dependent gross debt limit. However, the ability of politicians to follow such rules given political economy considerations is not certain. An independent central bank that is mandated to maintain output at potential can achieve the optimum without relegating decisions on debt limits to fiscal authorities that may have conflicting incentives.

Of course a potential concern with such a policy is that it may test the boundaries of central bank independence. Our view is that money-financed government spending is only one of many policies that push the boundaries of standard central bank operating procedures during severe recessions. As central banks reevaluate which policies are permissible, it is helpful to account for the benefits of different policies. We conjecture that if such a policy is established within the operating procedures of the Federal Reserve, then it can be insulated from political pressures to the same extent to which standard monetary policy adjustments are insulated.

A final concern with the welfare-maximizing policy is that it may threaten the solvency of the central bank, since the money used to finance the fiscal authority at the ZLB is a liability without a corresponding asset on the central bank's balance sheet. We consider such solvency requirements to be orthogonal to the central bank's raison-d'être, which is to stabilize output. While solvency indicators are very relevant for firms, for which solvency is an indicator of their ability to perform the core function of creating valuable goods and services, it is a less relevant metric for the central bank. Nonetheless, many central banks are subject to legal solvency requirements. Goncharov et al. (2017) argue that profits are a concern for the central banks themselves, and Berriel and Bhattarai (2009) and Hall and Reis (2015) show that such concerns may distort monetary policy.[6] We view our paper as demonstrating another potential cost of strict central bank solvency requirements.

Our paper is related to a literature on debt limits in the presence of political economy distortions (Battaglini and Coate (2008); Azzimonti et al. (2016); Yared (2010)); while we take these limits as given, it is comforting to know that they can arise from underlying frictions. Our paper is also related to a literature that measures fiscal limits (Davig et al. (2011); Trabandt and Uhlig (2011); Bi and Traum (2014)); note that these limits can be tighter than those required for intertemporal solvency and therefore may constitute an additional restriction on debt (for example, Bi and Traum (2014) estimate the debt levels that trigger sovereign defaults). We view our paper as guiding optimal debt restriction policy.

A separate strand of the literature considers the output effects of money-financed fiscal stimulus, including when the economy is away from the ZLB (e.g. Galí (2020)). Our contribution is to assess when such money financing is optimal and to consider how money financing interacts with debt management policies.

Other recent work re-evaluates the costs of government debt. Blanchard (2019) notes that interest rates below the growth rate of the US economy imply that the intertemporal solvency condition is not binding and therefore the stock of public debt is potentially irrelevant. Of course, such debt could become quite costly in the event of an increase in interest rates. We emphasize that increases in Treasury liabilities are potentially costless in a liquidity trap. In this sense, we offer a state-dependent perspective on when government debt is (more or less) costly.

Finally, we connect to the vast literature on the liquidity trap and optimal policy re-

[6]Another issue is that central bank solvency problems, in the sense of not having adequate income to "pay the bills" in a given period, might threaten central bank independence, which the literature shows is important for outcomes (Alesina and Summers (1993), Kokoszczński and Mackiewicz-Lyziak (2019), Lin (in press 2020)). We thank Ned Prescott for pointing out this concern.

sponses. Our modeling approach is similar to that in Eggertsson et al. (2019) in that there is an exogenously determined full-employment level of output (potential output) that the central bank targets subject to a ZLB constraint. Kiley and Rogers (2017) is among the papers that examine optimal policy responses to the ZLB. They explore a variety of policy options at the ZLB using the FRB/US model; the comments and discussion on their paper explore an even wider range of options. However, none of the suggestions consider problems related to the link between fiscal debt limits and central bank balance sheets, so our perspective adds something new to the debate.

2 Debt Limits and Stabilization Policy

We examine a setting in which the central bank's balance sheet is at the forefront of the analysis. The central bank adjusts interest rates to maintain full employment, subject to a zero lower bound on the interest rate. Interest rates are determined by a bond market clearing condition, and the central bank achieves its desired interest rate by swapping money (which it can create) for government bonds (which are issued by the fiscal authorities). Agents are willing to hold the money created by the central bank because of a cash-in-advance constraint.

The economy lasts for two periods – the initial period (0) and the future (1). Future output is a fixed endowment and serves as the numeraire, and the price level in the initial period is rigid. There is a potential (full-employment) level of output Y^F in the initial period, but due to rigid initial-period prices, output can fall below potential. Because the central bank faces an effective lower bound on interest rates, it may not be able to restore output to potential, so that our economy can feature inefficient "unemployment."

Fiscal authorities issue government debt in the initial period to finance government spending. This debt can be purchased by households and by the central bank and it is paid for by taxing households in the future. We assume that fiscal authorities' objective is to maximize spending, which is a reduced-form way of modeling the political economy forces that can lead to government spending that exceeds the socially optimal level (e.g., Battaglini and Coate (2008); Yared (2010); Krusell et al. (2006)). Government spending is constrained by an exogenously imposed debt limit, which represents the constraints that arise in political-economy models of government debt.

Our objective is to understand how alternative debt management policies, including alternative debt limit specifications, affect the ability of the central bank to achieve its

objective of maintaining output at potential in the face of the effective lower bound. The reason we find this exercise useful is that central banks are generally thought to be better insulated from political pressures, at least in the United States where the Federal Reserve is independent of the fiscal authority. But clearly the two policymakers cannot operate independently, since government debt is issued in nominal units of account.[7]

2.1 Model

Household: The representative household carries a stock of money into each period $t \in \{0, 1\}$. Agents exchange money and bonds at the beginning of the period in order to pay for their desired level of spending (which depends on a shock realization at the beginning of the period). Spending is subject to a cash-in-advance constraint. After obtaining and spending the desired level of money, households receive money income from the representative firm. The money income received by the household in the initial period is carried into the future period, and can be used to pay taxes.

Specifically, the household maximizes

$$U = \log(C_0) + v(G) + \beta E[\log(C_1)] \tag{1}$$

subject to

$$\begin{aligned} QB^H &= M_{-1} - M_0^H && (2)\\ M_0^H &= \theta P_0 C_0 && (3)\\ M_{0\to 1} &= P_0 Y_0 && (4)\\ P_1 C_1 + T &= M_{0\to 1} + Y_1 + B^H, && (5) \end{aligned}$$

where P_1 is the price index in the initial period, C is consumption, G is government spending in the first period, v is a concave and increasing function, and Y is income. Output in the future is the numeraire. The first two constraints state that the household can exchange money carried into the initial period M_{-1} and nominal bonds B^H (with price Q) to obtain the money M_0^H required to purchase consumption in the initial period. Each dollar permits θ units of private expenditure, which we interpret as the inverse of a money multiplier.[8] The

[7] Indeed, it is not clear at this point which actions are "fiscal policy" and which actions are "monetary policy," given that the Fed has begun buying many different assets and lending to a wide range of Special Purpose Vehicles.

[8] We can endogenize θ using a Baumol-Tobin style money management model, as in Freeman and Huffman (1991) and Freeman and Kydland (2000), in which households can replenish their money within a period at

third constraint states that households receive money income $M_{0\to 1}$ at the end of the period, which it carries into the future period. The last constraint is the household's future budget constraint. The household pays for future consumption and pays a tax T. The household pays for this tax and consumption using money it carries over into the future, future income, and the value of their bonds.

Government: The government purchases goods in the initial period and collects taxes in the future. We assume that government spending is controlled by politicians whose incentives are to maximize government spending. Their spending is limited by a debt constraint. The government's constraints are

$$\begin{aligned} M_0^G &= QB^G && (6)\\ M_0^G &\geq P_0 G && (7)\\ B^G &= T + T^{CB} && (8)\\ QB^G &\leq \overline{B}^G, && (9) \end{aligned}$$

where B^G is the amount of debt issued by the government. The second constraint is the government's cash-in-advance constraint, and the third constraint states that government deficits are paid for with future taxes T and remittances from the central bank T^{CB}. The last constraint states that the value of debt cannot exceed a limit $\overline{B}^G$.

Central Bank: The central bank creates money in the initial period and exchanges it for bonds. Its initial and future-period budget constraints are

$$\begin{aligned} M &= QB^{CB} && (10)\\ B^{CB} &= T^{CB}, \end{aligned}$$

where Q^{CB} is bonds held by the central bank and M is the amount of money created at the beginning of period 0. The central bank remits its future-period bond payments T^{CB} back to the fiscal authority.

The central bank's objective is to adjust the money supply to stabilize output at its potential level Y^F.

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a cost. For our purposes here this complication is not needed.

2.2 Equilibrium

The household's Euler equation is

$$C_0 = \frac{Q}{\beta P_0} E\left[C_1\right]. \tag{11}$$

Future-period output is endowed at $\overline{Y}$, and goods market clearing implies that

$$C_1 = \overline{Y}.$$

Substituting into the Euler equation and setting $P_0 = 1$ (due to fixed prices), we can rewrite the Euler equation as

$$C_0 = \frac{Q}{\beta} E\left[\overline{Y}\right]. \tag{12}$$

The equilibrium interest rate is determined by bond market clearing:

$$B^{CB} + B^H = B^G, \tag{13}$$

which can be written as

$$M + \left(M_{-1} - \theta P_0 C_0\left(Q\right)\right) = P_0 G. \tag{14}$$

Substituting in equation 12 and $P_0 = 1$ implies

$$M + M_{-1} = \theta \frac{Q}{\beta} E\left[\overline{Y}\right] + G, \tag{15}$$

This, along with the ZLB, implies

$$Q = \max\left\{\frac{\beta}{E\left[\overline{Y}\right]}\left(\frac{M - M_{-1} - G}{\theta}\right), 1\right\} \tag{16}$$

If $\frac{\beta}{E\left[\overline{Y}\right]}\left(\frac{M-M_{-1}-G}{\theta}\right) < 1$ then the bond market clears. Otherwise the ZLB binds and there is excess supply in the bond market.

Monetary Policy. The central bank adjusts the money supply to target

$$\frac{Q}{\beta} E\left[\overline{Y}\right] + G = Y^F. \tag{17}$$

Equilibrium output is

$$Y = \min\left\{Y^F, \frac{1}{\beta} E\left[\overline{Y}\right] + G\right\}, \tag{18}$$

and the government maximizes spending up to the debt limit:

$$G = \overline{B}^G. \tag{19}$$

Macroeconomic Shocks. We will consider two types of macroeconomic shocks. The first is a shock to $\frac{E[\overline{Y}]}{\beta}$ (a "demand shock") that can depress aggregate demand such that the central bank is constrained by the ZLB. The second is a shock to θ (a "money multiplier shock"). The effect of these shocks on the macroeconomy can be seen by examining 15. First consider the situation of a fixed money multiplier. In this case, the money supply pins down output when the economy is away from the ZLB and the central bank sets the money supply to hit its target level of output Y^F. A consumption demand shock implies that there is no need for further adjustment of the money supply, since Q simply adjusts to maintain consumption at the level required by Y^F. At the ZLB, consumption demand shocks translate into higher or lower consumption, and additional money stimulus has no effect on C.

Next consider the effect of shocks to θ. When the economy is away from the ZLB the central bank can offset the effects of θ by adjusting M. Therefore it can maintain consumption at the level necessary to hit Y^F without changes in Q. At the ZLB, there is excess supply in the bond market and shocks to θ do not affect Q (and hence C).

While shocks to θ do not affect the central bank's ability to manage consumption demand, they are relevant for the central bank's ability to stabilize the economy when government debt management policies are considered. As we explore below, changes in the money multiplier affect stabilization policy through its effect on the government's balance sheet and hence on G.

2.3 The Effects of Government Debt.

The fiscal authority's incentive to spend implies that government spending (and hence debt) in this model can exceed the socially efficient level. Samuelson (1954) demonstrates that government spending is socially efficient when the marginal rate of substitution between government spending and private consumption equals their marginal rate of transformation. In our context, this condition amounts to the marginal utility of consumption being equal to the marginal utility of government spending,

$$\frac{1}{C_0} = v'(G). \tag{20}$$

This condition, along with goods market clearing, implicitly defines optimal government spending G^{*F} when output is at potential. For any $G > G^{*F}$, the economy is allocating resources to public consumption that would be more valuable if reallocated toward private consumption.

Of course, output need not reach its potential. As demonstrated by Michaillat and Saez (2019), optimal government spending exceeds the Samuelson-implied level when the economy features inefficient unemployment. In our context, this condition implies that when the economy is at the ZLB, optimal government spending fills the gap between private consumption and potential output:

$$G^{*U} = Y^F - C_0(1). \tag{21}$$

If we parameterize $v(G) = \alpha \log(G)$, then we can write optimal government spending as

$$G^* = \begin{cases} \alpha C_0 & \text{if } \frac{E\left[\overline{Y}\right]}{\beta} \geq \frac{Y^F}{1-\alpha} \\ Y^F - C_0 & \text{if } \frac{E\left[\overline{Y}\right]}{\beta} < \frac{Y^F}{1-\alpha} \end{cases}, \tag{22}$$

where the economy hits the ZLB if

$$\frac{E\left[\overline{Y}\right]}{\beta} < \frac{Y^F}{1-\alpha}.$$

At an attendant increase in algebraic costs, we can consider utility functions with non-unitary elasticities, but the qualitative lessons would be the same.[9]

Are future tax liabilities costly? Government spending and debt are associated with taxes in the future. Unless these taxes are distortionary, there are no additional future costs of these taxes, since they do not affect the goods market clearing condition that future consumption equals the endowment. Therefore, the costs of government spending are based only on crowding out of consumption in the initial period. When the economy is at the ZLB, there are no crowding-out costs and therefore it is costless to increase G until output reaches potential.

However, it is possible that taxes have distortionary effects on future output, or at least that households perceive there to be costs of future taxes. Consider a situation in which

[9]The quantitative effects would be different, of course. Given that our goal here is qualitative in nature, and estimates of the elasticity of substitution between C and G are all over the place (see McGrattan et al. (1997), Bouakez and Rebei (2007), or Dawood and Francois (2018)), we do not regard the assumption as costly.

there is a (perceived or actual) adverse effect of T on $E\left[\overline{Y}\right]$ such that

$$E\left[\overline{Y}\right] = \Psi - \kappa T, \tag{23}$$

where Ψ is stochastic and κ measures the sensitivity of future output to taxes. Then the household's Euler equation becomes

$$C_0 = \frac{Q}{\beta}\left(\Psi - \kappa T\right).$$

If we substitute in $G - T^{CB}$ for T (using the fiscal authority's budget constraint), the Euler equation becomes

$$C_0 = \frac{Q}{\beta}\left(\Psi - \kappa\left[G - T^{CB}\right]\right). \tag{24}$$

In this case, there are two adverse effects of government spending (holding fixed the amount of remittances from the central bank T^{CB}) . First, the taxes lower current consumption, thus exacerbating the possibility of excess slack at the ZLB. Second, they can lower future output if they are indeed distortionary (and not just perceived to be so).

Crucial to understanding the potential tax costs of government spending is understanding how transfers from the central bank adjust with changes in G. If increases in G correspond one-to-one with increases in T^{CB}, then there is no additional tax burden for households. Below we demonstrate that the extent to which additional government spending is financed through taxes on households versus through central bank remittances depends on debt management policies.

3 Debt Management Policies

Consider a situation in which the debt limit is set so that government spending cannot exceed its socially efficient level when output is at potential:

$$\overline{B}^{*G} = G^{*F}. \tag{25}$$

The distribution of shocks in the initial period is such that the economy hits the ZLB with probability $\psi\left(\overline{B}^{G}\right)$ where $\psi' < 0$ and $0 < \psi < 1$. When the economy does hit the ZLB, realized output is $C^U + \overline{B}^{G} < Y^F$, where C^U indicates that consumption demand $(E[\overline{Y}]/\beta)$ is below what would be necessary to bring the economy to full employment at the current

debt limit. This policy is clearly inefficient at the ZLB even if it is optimal when the economy is away from the ZLB. At the ZLB, welfare could be improved by $\alpha \log(O)$, where $O \equiv Y^F - \left(C^U + \overline{B}^G\right)$ is the output gap, by increasing $\overline{B}^{*G}$ and expanding G. Expected welfare would rise by $\psi\alpha \log(O)$.

3.1 Alternative Policy 1: Increase the Gross Debt Limit

Increasing the gross debt limit above $\overline{B}^{*G}$ has the benefit of lowering the expected output gap. But it also causes government spending to deviate from the Samuelson level when the economy is at the full employment level. In general, the optimal gross debt level is higher than G^{*F}, with the extent to which the optimum exceeds G^{*F} depending on the distribution of shocks and the corresponding probability that the economy reaches the ZLB. Consider a situation in which $E[\overline{Y}/\beta] \equiv C^U$ has a probability distribution $f\left(C^U\right)$. Then the optimal gross debt limit $\overline{B}^{*\text{Gross}}$ (and hence the level of government spending) is the solution to the following problem:

$$\overline{B}^{*\text{Gross}} = \underset{\overline{B}}{\text{argmax}} \left\{ \int_{\underline{\psi}}^{Y^F - \overline{B}} \log\left(C^U\right) f\left(C^U\right) dC^U + \right. \tag{26}$$
$$\left. \int_{Y^F - \overline{B}}^{\bar{\psi}} \log\left(Y^F - \overline{B}\right) f\left(C^U\right) dC^U + \alpha \log \overline{B}, \right\}$$

where the demand shock $C^U \equiv \frac{E[\bar{y}]}{\beta}$ has the range $\left[\underline{\psi}, \overline{\psi}\right]$, $\underline{\psi} < Y^F < \overline{\psi}$. The first integral is the expected utility at the ZLB, where higher G is associated with a lower probability of hitting the ZLB. The second integral is expected utility away from the ZLB (where consumption equals Y^F net of G). Given that the gross debt limit pins down government spending, with probability 1 the household receives utility $\alpha \log(G)$. Note that θ is irrelevant for determining the optimal gross debt limit.

Figure 1 shows expected utility as a function of G if $E\left[\overline{Y}\right]/\beta$ is uniformly distributed on $\left[\underline{\psi}, \overline{\psi}\right]$. The vertical line is drawn at the Samuelson level of government spending (e.g., the optimal spending if the ZLB was not binding), and the top of the y-axis indicates the maximum expected utility that could possibly be achieved (e.g., if the government fully filled the output gap at the ZLB and was set to the Samuelson level above the ZLB). Optimal government spending is higher than the Samuelson level when there is a chance that the economy will hit the ZLB.

Result 1 *Consider a policy that imposes an ex ante limit on gross government debt. In the*

presence of an occasionally binding ZLB, the optimum gross debt limit exceeds the Samuelson-implied level.

This result is a corollary of the Michaillat and Saez (2019) observation that optimal government spending exceeds the Samuelson level in the presence of inefficient unemployment.[10] We will refer to this optimum implied by the presence of the ZLB as the ZLB-adjusted Samuelson level. The question that we address below is whether alternative debt limit policies can further improve welfare relative to an ex ante gross debt limit set at the ZLB-adjusted Samuelson level.

3.2 Alternative Policy 2: Replace the Gross Debt Limit with a Net Debt Limit

Turning attention to the consolidated government balance sheet, we can write net government debt as

$$B^{\text{Net}} \equiv B^G - B^{CB} = G - M. \tag{27}$$

Consider a policy that replaces a limit on gross debt with a limit on net debt ($\overline{B}^{\text{Net}}$). Suppose the economy hits the ZLB. In that case, money injections can relax the consolidated government balance sheet. Each additional dollar created by the central bank is used to purchase government debt. Since the debt limit now only applies to the consolidated balance sheet, there is no tightening of the government's debt constraint, and the fiscal authority can use the additional financing from the central bank to purchase goods. In principle, the central bank could inject exactly the amount of money needed so that the additional government spending brings the economy back to full employment.

While this policy can overcome the problem of the ZLB (and thus prevent output from falling below potential), whether it can guarantee that the level of government spending will be efficient when interest rates are above the ZLB depends on the nature of uncertainty in the economy.

If the only shock is to $E\left[\overline{Y}\right]/\beta$, then a net debt limit set at $B^{\text{Net}} = G^{*F}$ can ensure an efficient level of government spending when the economy is away from the ZLB. As $E\left[\overline{Y}\right]/\beta$ changes away from the ZLB, the central bank keeps the money supply fixed and the interest

[10] Our setting differs from that in Michaillat and Saez (2019) in that monetary policy also has a role to play. If the only stabilization tools are monetary stimulus (subject to a ZLB) and *ex ante* adjustments to the fiscal authority's gross debt limit, then the optimal debt limit will exceed the Samuelson level, as in Michaillat and Saez (2019)).

rate adjusts to maintain consumption demand at $Y^F - G^{*F}$. Since the money supply does not adjust, neither does G. Therefore, in the absence of shocks to θ, this policy achieves the maximum possible welfare:

$$E(U^{*Optimal}) = \int_{\underline{\psi}}^{Y^F - G^{*F}} \left[\log C^U + \alpha \log\left(Y^F - C^U\right)\right] f(C^U) dC^U + \int_{Y^F - G^F}^{\overline{\psi}} \left[\log\left(Y^F - G^F\right) + \alpha \log(G^F)\right] f(C^U) dC^U, \tag{28}$$

where the first integral is expected utility when the economy is at the ZLB (which occurs for $\underline{\psi} \leq C^U \leq Y^F - G^{*F}$). At the ZLB, the household receives utility from consumption C^U and government spending, which exactly fills the gap between potential output and consumption $(Y^F - C^U)$. Away from the ZLB (which occurs for $Y^F - G^{*F} \leq C^U \leq \overline{\psi}$), the household receives utility from consumption $(Y^F - G^F)$ and government spending, which is set to the optimal Samuelson level when output is at potential.

Result 2 *Consider a policy that imposes an ex ante limit on* net *government debt. If the money multiplier is constant, then the optimal net debt limit maximizes social welfare: government spending closes the output gap at the ZLB and is maintained at the Samuelson level away from the ZLB.*

Figure 2 compares welfare under the gross debt limit policy to welfare under the net debt limit policy, when θ is constant and C^U is uniformly distributed symmetrically around Y^F. The probability of hitting the ZLB is based on the variance of the uniform distribution. As the probability of hitting the ZLB (holding G fixed at G^{*F}) increases, welfare under the gross debt limit falls by more than does welfare under the net debt limit.

Implications of Money Multiplier Shocks. The welfare implications of the net debt policy are different if the money multiplier is stochastic. If the economy is away from the ZLB and θ increases, the central bank will respond by increasing M (to ensure that $M + M_{-1} = \theta Y^F + (1-\theta)G$, from 15). The increase in M implies a relaxation of the net debt constraint (27) and a corresponding increase in G such that $G > G^{*F}$. The higher government spending crowds out consumption and lowers welfare.

In general, welfare under the net debt policy if θ and C^U are stochastic is

$$\overline{B}^{\text{net}} = \arg\max_{\overline{B}} \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \left(\begin{array}{c} \int_{\underline{\psi}}^{\frac{\overline{B}-M_{-1}}{\theta}} \left(\log\left(C^U\right) + \alpha \log\left(Y^F - C^U\right)\right) f\left(C^U\right) dC^U + \\ \int_{\frac{\overline{B}-M_{-1}}{\theta}}^{\overline{\psi}} \left(\log\left(-\frac{\overline{B}-M_{-1}}{\theta}\right) + \alpha \log\left(\frac{\overline{B}-M_{-1}}{\theta} + Y^F\right)\right) f\left(C^U\right) dC^U \end{array} \right) g(\theta) d\theta \right\} \tag{29}$$

where $g(\theta)$ is the probability distribution function of the (inverse of) the money multiplier. Note that the ZLB binds for any $C^U \leq Y^F - G$ (and hence the upper limit of the first integral in the brackets is $Y^F - G$). Since $G = \frac{\overline{B}-M_{-1}}{\theta} + Y^F$ (by 15), this condition is equivalent to $C^U \leq -\frac{\overline{B}-M_{-1}}{\theta}$. Equation 29 resembles equation 28, with G^F replaced with $\frac{\overline{B}-M_{-1}}{\theta} + Y^F$ and the expression integrated over the range of θ.

Figure 3 shows expected welfare under the optimal gross debt limit and the optimal net debt limit when θ is distributed uniformly. C^U has the same distribution as in Figure 2. When θ is random, the net debt limit leads to lower welfare than the optimum in all instances. Higher variance of θ leads to lower welfare (for any level of the variance of C^U and hence any probability of hitting the ZLB), since government spending will be permitted to deviate more from the Samuelson level.

Furthermore, welfare under the optimal net debt policy is lower than welfare under the optimal gross debt policy when the probability of hitting the ZLB is sufficiently low. This is because the net debt policy can remove the output gap at the ZLB, but away from the ZLB, government spending will deviate from G^F as θ varies.

Therefore, limits on net debt cannot necessarily restrict government spending to its efficient level away from the ZLB. Whether this policy is preferable to one that fixes a limit on gross debt depends on the likelihood that the economy enters the ZLB. As ZLB episodes become more likely, this policy is relatively more appealing.

Result 3 *A policy that imposes an ex ante limit on* net *government debt does not achieve the social optimum in the presence of shocks to the money multiplier. Depending on the distributions of demand shocks and money multiplier shocks, the policy can be inferior to the limit on gross debt in Policy 1.*

3.3 Alternative Policy 3: Limit on Gross Debt, Central Bank Transfers Money to Fiscal Authority at ZLB

Finally, consider a policy whereby the fiscal authority is subject to a limit on gross debt set at the Samuelson-implied level but the central bank has the authority to monetize government spending at will. If the central bank maximizes social welfare, it will resort to money-financing of the fiscal authority only when the economy is at the ZLB. Away from the ZLB, government spending is fixed at the optimal level. This policy achieves maximum expected utility (equation 28) regardless of whether θ is stochastic. The maximum utility as a function of the variance of the demand shock (and hence the probability of hitting the ZLB) is depicted

by the top line in Figure 2 (and the top line of Figure 3). Since the limit is set on gross debt, G does not change with changes in θ. The economy can fully close the output gap when it is optimal to do so and ensure that government spending does not exceed the Samuelson level during periods of positive interest rates.

Additional Policy Options An equivalent policy would be a state-dependent gross debt limit. However, the ability of politicians to follow such rules given political economy considerations and a lack of commitment power is not certain. An independent central bank that is mandated to maintain output at potential can achieve the optimum without relegating decisions on debt limits to fiscal authorities that may have conflicting incentives.[11]

An alternative to central-bank-financed government spending at the ZLB is central-bank-financed transfers to households. In the context of our model, such transfers would not affect the household's Euler equation at the ZLB and therefore would not help close the output gap. However, if the transfers were designed such that they expire at the end of the initial period, then the household would have an incentive to spend the transfer in the initial period. This policy could improve welfare beyond money-financed government spending, since the marginal utility of consumption would exceed the marginal utility of government spending. Auerbach et al. (2020) propose additional fiscal transfer policies that are effective even when the economy is at the ZLB and households are on their Euler equation. We consider such transfer proposals - if financed by the central bank at the ZLB - to be categorized under the umbrella of Policy 3.

Is Central-Bank-Financed Spending at the ZLB a Free Lunch? Our model implies that money-financed government spending removes slack in the economy without any requisite increase in future taxes. In that sense, it is the definition of a free lunch. Stepping beyond our model, there are two potential costs of this policy to consider. The first is the potential for inflation. However, historically inflation has been below its target during ZLB episodes. The central bank could always pull back if inflation started to accelerate. A second potential cost is that the policy entails the central bank creating money (a liability) without a corresponding asset. Setting aside concerns about central bank solvency, there remains the possibility that the central bank would want to recall this money in the future but would not have the asset to trade for it. We consider this possbility to be unlikely. The central bank would only want to reduce the money supply if economic growth turned negative in the future and inflation were above target. Under any other cicrumstance the central bank can simply adjust money

[11] We do not attempt to solve the backward induction problem of why politicians would assign these abilities to the central bank and/or not revoke them. A related result on the value of limiting options in the absence of commitment is Devereux et al. (2019).

growth with growth in potential output. In the case of stagflation, history offers a guide as to whether the central bank would need to reduce the money supply: during the Volcker era, year-over-year growth in the monetary base was always positive (and has been since). A useful avenue for future work would be to examine a dynamic version of our model with aggregate supply shocks and endogenous inflation dynamics. This would help to quantify any potential costs of central bank liabilities without corresponding assets.

4 Conclusion

The pandemic cecession has reignited the debate over the relative merits of public debt expansions during a downturn. This debate is set against a backdrop of already high and rising public debt levels and monetary policy that is often constrained by an effective lower bound on interest rates. Much of the public discourse over debt is contentious and lacking a coherent framework to guide the debate.

We offer a state-contingent perspective on public debt management and stabilization policy. Our analysis incorporates insights from the public economy literature on debt limits with a balance-sheet treatment of monetary policy.

Much of the fiscal authority's debt is held by the central bank, itself a governmental organization. We address whether it would be optimal to replace limits on gross debt with limits on net government debt in order to remove slack when the economy is at the effective lower bound.

If there is a steady relationship between the monetary base and the full-employment level of output during normal times, (e.g., if the money multiplier is stable), then a net debt limit is the preferable policy. But if macroeconomic shocks disrupt the relationship between the monetary base and the full-employment level of output, then the central bank will have to adjust the monetary base during normal times, and this adjustment can relax the fiscal authority's net debt constraint. Whether the net debt limit or the gross debt limit is preferable depends on the frequency with which the economy hits the effective lower bound on interest rates and the composition of macroeconomic shocks.

An optimal policy that overcomes the limitations of pure gross debt limits and net debt limits is to maintain a gross debt limit at the Samuelson-implied level and for the central bank to directly pay for government spending when the economy is constrained by the effective lower bound on interest rates.

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Figures

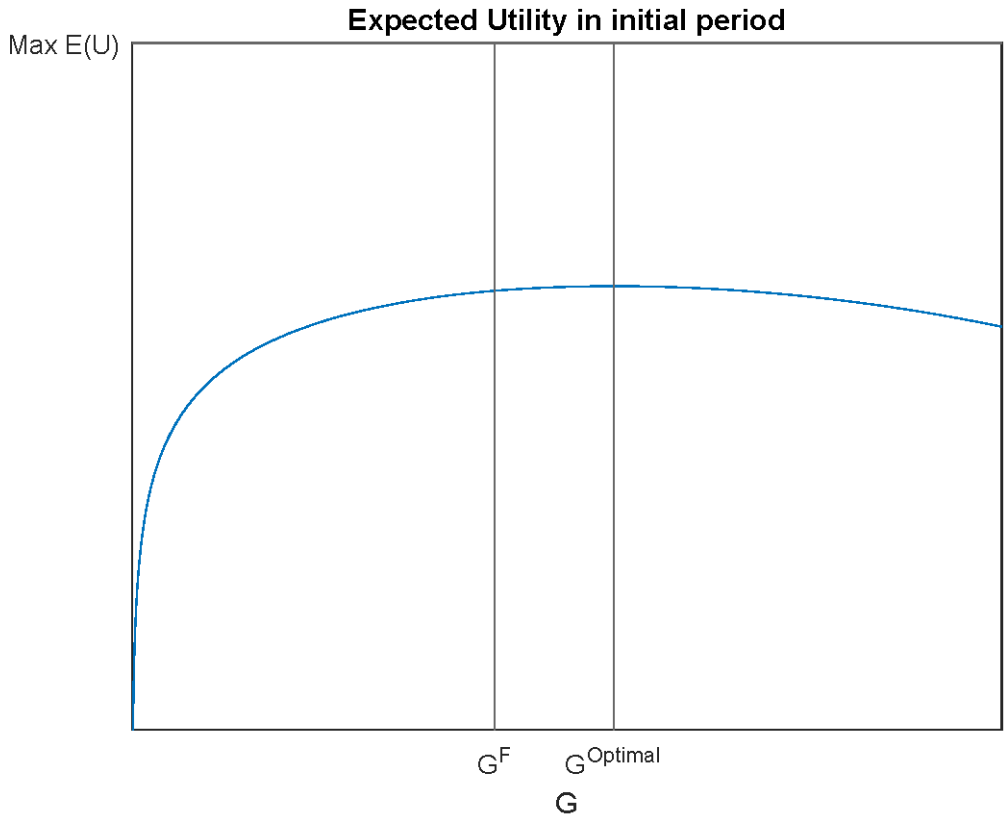
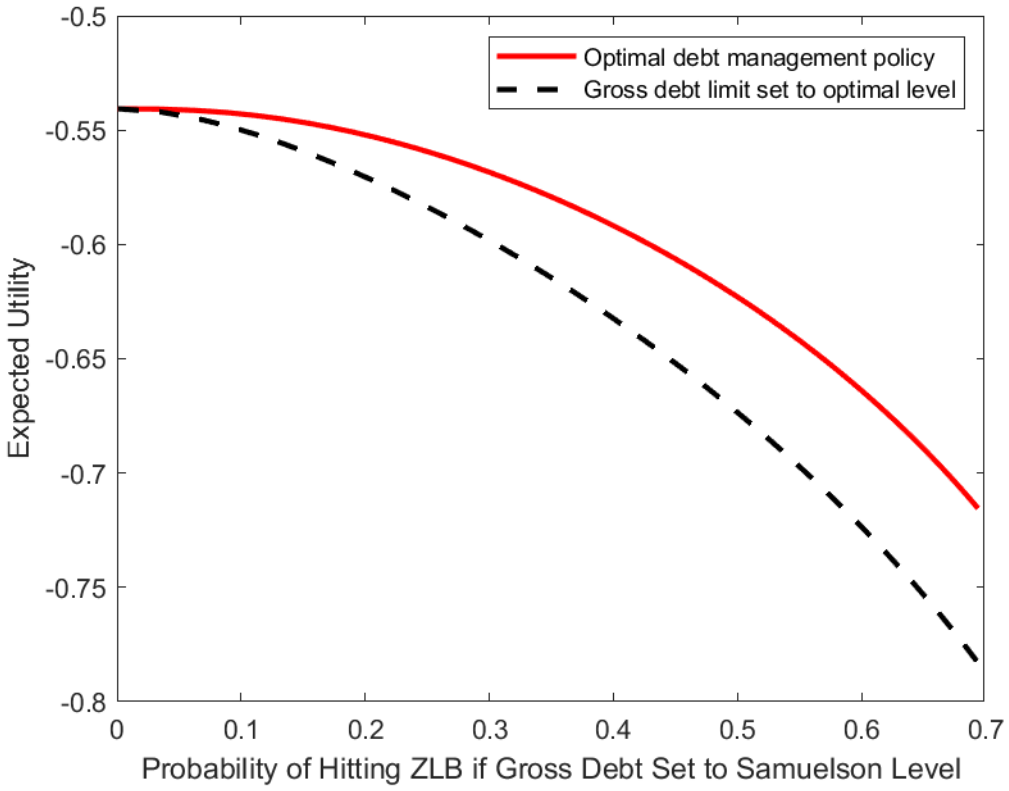


Figure 1


Optimal debt management policy
Gross debt limit set to optimal level
Expected Utility
-0.5
-0.55
-0.6
-0.65
-0.7
-0.75
-0.8
0
0.1
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0.3
0.4
0.5
0.6
0.7
Probability of Hitting ZLB if Gross Debt Set to Samuelson Level

Figure 2

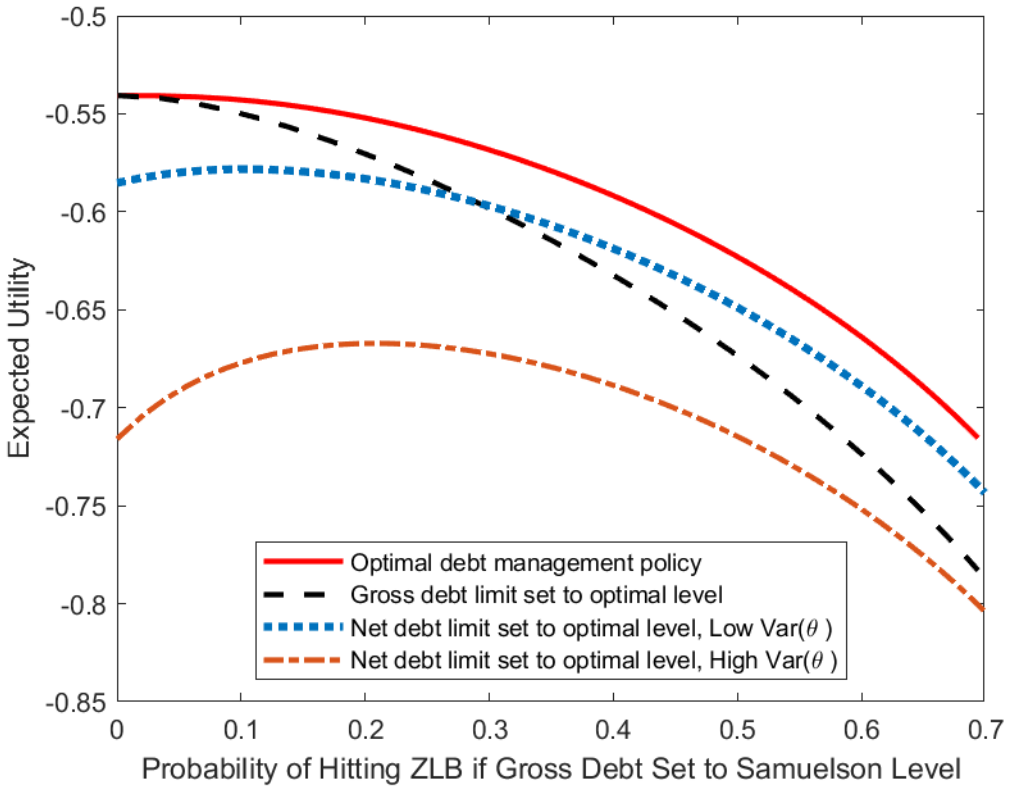
-0.5
-0.55
-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
Expected Utility
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Probability of Hitting ZLB if Gross Debt Set to Samuelson Level
Optimal debt management policy
Gross debt limit set to optimal level
Net debt limit set to optimal level, Low Var(θ)
Net debt limit set to optimal level, High Var(θ)

Figure 3