Flexibility and Frictions in Multisector Models*

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Abstract

We document two facts: *i*) there is substantial heterogeneity in the flexibility of production across US industries, defined as the elasticity of substitution between inputs, and *ii*) these elasticities are systematically correlated to sectoral bond spreads during the Great Recession. To explain our facts, we build a multisector model with CES sectoral technologies and occasionally-binding working capital constraints. Our model provides theoretical connections between sectoral elasticities and the severity of sectoral financing constraints. In the presence of financial constraints that are heterogeneous across inputs, higher flexibility in production reduces firms' exposure to financial frictions by facilitating substitutability from more constrained inputs to less constrained ones. Nevertheless, flexibility in production can increase firms' exposure to frictions when the constrained inputs become relatively cheaper.

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1 Introduction

The standard narrative of the Great Recession is one where financial frictions and interconnected sectors translated a small shock to a relatively unimportant sector – often argued to be an unexpectedly-large number of subprime mortgage defaults – into a large economy-wide decline in economic activity. Clearly, financial frictions and complementarities in production are critical for this story.[1] Nevertheless, it is a challenge to separately identify firms' production technologies from sectoral frictions.[2] Is the variation in firms' input shares we observe in recessions a result of optimal decisions or a result of frictions? In this paper, we seek to contribute to this literature by studying how fundamental features of firms' production technology – sectoral elasticities of substitution between inputs – relate to the severity of sectoral constraints during macroeconomic downturns.[3]

The literature on multisector models and frictions has assumed constant elasticity of substitution (CES) production functions that are homogeneous across sectors (Horvath (2000), Foerster et al. (2011), Atalay (2017)), and Bigio and La'O (2016)); that is, there is a common elasticity, generally equal to or below one, both between different types of intermediate inputs (denoted here as ϵ_M) and between intermediates and value-added (ϵ_Q).[4] Our first contribution in this paper is to reconsider this assumption – is there evidence that ϵ_Q and ϵ_M vary across sectors? We find substantial heterogeneity in both elasticities, with large differences in particular between manufacturing and service sectors. Our second contribution is to show that during the Great Recession, there is a systematic relationship between sectoral elasticities in production and sectoral bond spreads. Sectors with higher flexibility in production – higher ϵ_Q and ϵ_M– paid a lower spread. In addition, we find that high debt

[1]Recent work has shown the potential for sectoral productivity shocks to be amplified and propagated strongly when intermediate inputs and complementarities in production generate tight connections between sectors (Horvath (2000), Foerster et al. (2011), Atalay (2017), and Miranda-Pinto (2018)). Bigio and La'O (2016) show how financial shocks can be transmitted through the network in the presence of working capital financing constraints, leading to declines in measured aggregate TFP and aggregate labor wedges. Also, Jones (2011), Baqaee and Farhi (2017), and Osotimehin and Popov (2017) show the importance of sectoral linkages and sectoral elasticities in amplifying frictions in the use of inputs.

[2]Bigio and La'O (2016) identify sectoral frictions separate from production elasticities by assuming that production elasticities are unitary. Under that assumption, all the variation in firms' input shares are attributed to sectoral frictions.

[3]Several papers in the literature have followed Rajan and Zingales (1998) in defining sectors' external financial dependence as a proxy on how vulnerable are different sectors to tightening credit conditions. However, for the Great Recession, Kudlyak and Sánchez (2017) argues that firms with higher external financial dependence did not suffer large drops in sales, as expected in a financial crisis.

[4]We will not consider the elasticity of substitution between different inputs that produce value-added (capital and labor) here.

and high ϵ_Q service sectors paid higher premia, while high debt and high ϵ_Q manufacturing sectors paid lower premia.

We interpret these facts through the lens of a multisector model with sectoral linkages, heterogeneous elasticities, and heterogeneous working capital constraints. We focus on the role of the elasticity between labor-capital and intermediates (ϵ_Q). The main message from the model goes as follows. When firms face heterogeneous constraints in the financing of inputs, higher flexibility in production allows them to move away from the more constrained inputs to the less constrained ones. For example, suppose that firms pay to their workers after production takes place. On the other hand, firms need to borrow to pay all the cost of intermediates before production takes place. If the cost of intermediates relative to labor spikes – as observed during the Great Recession – flexible firms will demand relatively more labor and will reduce their external financing needs. However, firms with low flexibility in production will see an increase their external financing needs. Higher external financing needs imply that these firms will be more susceptible to become financially constrained if there are furhter increases in the intermediates cost or a tightening of credit conditions.

In our empirical analysis, we estimate sectoral elasticities in the US using panel fixed-effects (FE) and instrumental variables (IV). Our main estimates use data before the Great Recession. We restrict the sample motivated by our model's prediction that binding constraints in the use of inputs during the Great Recession induce significant bias in the estimation of elasticities.[5] Moreover, our instruments are stronger when we restrict our sample period. We find that manufacturing sectors have lower elasticities than service sectors in general, ϵ_Q is generally larger than ϵ_M, service sectors have an average ϵ_Q larger than one, and that manufacturing sectors have an average ϵ_Q less than one.[6] Our estimates for ϵ_M are generally less than one, as in Atalay (2017), except when we use IV and data before the Great Recession.[7]

We then study the empirical connections between sectoral elasticities and the spreads of firms' corporate bonds from Gilchrist and Zakrajsek (2012). First, spreads are countercyclical

[5]Given that sectoral spreads show little variation outside the Great Recession, we use sector fixed-effects and time fixed-effects to control for sectoral constraints that could be binding outside the Great Recession.

[6]The main results of the paper also apply when we use the estimated elasticities with the whole sample. However, we prefer the 1997-2007 sample as our instruments are stronger.

[7]Our paper focuses on the role of ϵ_Q. Most papers in the literature, except Atalay (2017), estimate ϵ_Q only for manufacturing firms. Atalay (2017) finds $\epsilon_Q = 1.2$ using OLS and $\epsilon_Q = 0.8$ using IV; however, the IV estimate is not statistically significantly different from one. Doraszelski and Jaumandreu estimate the elasticity between labor, capital, and intermediates, using data on Spanish manufacturing firms over the period 1990-2006. The authors find that manufacturing firms have technologies in which intermediates are complements. There is also important heterogeneity among the 10 sectors considered. Peter and Ruane (2017) estimate a long-run ϵ_M as find that is much larger than one.

for both manufacturing and service sectors. Second, sectoral bond spreads are systematically related to our estimated elasticities. During expansions, there is no evidence that either flexible (high elasticity) or inflexible (low elasticity) sectors pay higher premia, no matter which elasticity we consider. During recessions, however, sectors that have a relatively difficult time substituting between value added and intermediates (low ϵ_Q and low ϵ_M) will pay higher premia. This fact is particularly strong during the Great Recession in 2007-2009. For example, our results indicate that if auto makers ($\epsilon_Q = 0.9$) had the production flexibility of auto dealers ($\epsilon_Q = 5.7$), they would have paid a spread 1000 basis point lower during the Great Recession. Moreover, by focusing on the Great Recession period, we find that when corporate debt is high, service sector firms with high ϵ_Q pay higher premia, while manufacturing firms with higher ϵ_Q pay lower premia.

Motivated by these facts, we construct a multisector model with working capital constraints built on Bigio and La'O (2016). We allow for sectors to have heterogeneous values for ϵ_Q, and we explore different assumptions about which inputs are subject to the working capital requirement. To facilitate intuition, we start by studying a simple two-sector model. Sector one uses only labor to produce an intermediate good for use in sector two. Sector two combines intermediates from both sectors with labor to produce a good that can be consumed or used as an intermediate in sector two. We consider environments where the firms in sector two are required to finance labor or intermediate of inputs in advance of production via working capital loans, and these loans must be collateralized by sales.[8]

If the production function is Cobb-Douglas between intermediates ($\epsilon_M = 1$), we can analytically characterize the Lagrange multiplier as a function of sectoral productivity levels and sectors' ability to borrow, in the sense that we can sign the derivative with respect to ϵ_Q. The multiplier can be viewed as the shadow price of borrowing, which in turn therefore can be interpreted as a spread.[9] Our first result characterizes a key multiplicative "wedge" between the costs of labor and intermediates that depends on (i) what input is subject to the pay in advance constraint, (ii) the importance of labor in the production function, and (iii) the fraction of sales that can be credibly pledged as collateral. If this wedge exceeds one,

[8]Formally this arrangement is quite similar to 'Sudden Stop' models with flow constraints, as in Bianchi (2011) or Benigno et al. (2013). The assumption of sales being collateral for loans instead of the value of physical assets is consistent with the results in Li (2015), who finds that a model with heterogeneous firms and financial frictions matches firm dynamics facts of Japanese firms best if firms can pledge as collateral half of their one-year ahead earnings and one-fifth of their assets. The main reason is that earnings and sales fluctuate with firm productivity but current assets do not.

[9]In our Appendix C we show that our result between elasticities and the Lagrange multiplier of the collateral constraint is isomorphic to a relationship between elasticities and spreads, where the spread is defined by an upward sloping debt schedule that is activated when firms hit the constraint.

then the particular input is more costly.

Our first Proposition relates sectoral elasticities with the probability of a sector to become constrained, as a function of the multiplicative wedge. For the group of low elasticity sectors ($\epsilon_Q < 1$), the model can deliver countercyclical frictions when only intermediates have to be financed in advance and the wedge is smaller than 1. On the other hand, for the group of high elasticity sectors ($\epsilon_Q > 1$), the model delivers countercyclical frictions when only labor is financed in advance, and the wedge is smaller than 1. The wedge depends on the importance of labor in production, and the fraction of sales credible pledged as collateral. Using the observed steady state sectoral shares for labor-capital, our simple model implies that service sector and manufacturing sector firms can pledge at least a 50% of sales as collateral.

The same Proposition also accounts for the observed differential relationship between spreads and the interaction between sectors' leverage and ϵ_Q. We prove that for the group of low elasticity ($\epsilon_Q < 1$) intermediate-constrained sectors, increases in the relative cost of intermediates, increase leverage and the likelihood of reaching the borrowing limit. Leverage, and the likelihood of becoming constrained, increase more in relatively more inflexible sectors. For the group of high elasticity ($\epsilon_Q > 1$) labor-constrained sectors, increases in the relative cost of intermediates also increase leverage by increasing labor demand. As labor is constrained, more leveraged firms with relatively higher ϵ_Q will be more likely to be constrained. This prediction is supported by the observed increase in the relative cost of intermediates for manufacturing and service sectors, during the period 2002-2008.

Our second Proposition assumes that the constraint is binding and relates sectoral elasticities to the severity of the constraint, as measured by the Lagrange multiplier of the collateral constraint. We use this proposition to account for the observed negative correlation between elasticities and spreads during the Great Recession. The model is able to deliver the negative correlation between ϵ_Q and the severity of frictions, as long as manufacturing and service sector firms experienced a tightening of the borrowing limit. In this case, the constrained input has a high friction adjusted cost relative to the unconstrained input. Therefore, more flexible firms are relatively less constrained in the sense that they have higher flexibility to substitute the constrained input with the unconstrained one.

We provide additional evidence in favor of our mechanism by looking at the evolution of input quantities and prices. Before 2008, input quantities and prices moved consistent with the estimated elasticities. That is, for low elasticity manufacturing firm increases in the relative price of intermediates are accompanied with increases in the relative use of intermediates. For high elasticity service sector firms, increases in the relative price of intermediates are accompanied with declines in the relative importance of intermediates in production. However, during the Great Recession, relative quantities move exactly in the opposite direction from what predicted by the elasticity. This is evidence that service and manufacturing firms experienced a distortion in the use of inputs consistent with our models' predictions.

Finally, we perform a quantitative exercise for 62 U.S. sectors during the Great Recession. We measure the model implied wedges using BEA data on sectoral input and output prices and quantities. The results are consistent with the conclusions from our simple model. The model is able to deliver a negative correlation between elasticities (ϵ_Q) and frictions during the Great Recession if service sectors are constrained in the use of labor-capital while manufacturing sectors are constrained in the use of intermediates. In addition, the model implied wedges are negatively (positively) correlated with the interaction between ϵ_Q and leverage for the group of low (high) elasticity sectors, as in the data.

This paper makes two main contributions to the literature of financial frictions in multi-sector models. First, we provide evidence of substantial heterogeneity in sectoral elasticity of substitution. We are the first to point out that service industries are significantly different in their production flexibility. Although it is not the goal of this paper, our results are relevant to assess how sectoral shocks propagate along the production chain.[10]

Second, we present new evidence that relates firms' production technologies with the severity of sectoral constraints during macroeconomic downturns. We rationalize our evidence with a theoretical multisector model that yields clear connections between firms' working capital constraints and flexibility in production. This result has important implications for understanding what the most central industries in the economy are. With sectoral frictions, it is not simply the size of a sector that determines which sectors affect aggregate output the most, but also sectoral linkages and frictions (as in Bigio and La'O (2016)), and the sectoral flexibility in production.[11]

Our result linking sectoral technologies to sectoral external financial dependence also contributes to the literature following Rajan and Zingales (1998). Rajan and Zingales (1998)

[10] Horvath (2000), Atalay (2017), and Baqaee and Farhi (2017) show that low substitutability between inputs amplify business cycle fluctuations. On the other hand, Miranda-Pinto (2018) shows that in order for multisector models to make sense of the observed cross-country correlations between GDP volatility and input-output structure, it is important to account for sectoral heterogeneity in elasticities of substitution between inputs, as documented in this paper.

[11] Bigio and La'O (2016) assume Cobb-Douglas technologies and use their model to back out sectoral wedges in the use of inputs. Their identification relies on sectors optimally choosing a constant input share in production ($\epsilon_Q = 1$ for all sectors). We instead allow for optimal changes in input shares and identify sectoral wedges from the relationship between flexibility and sectoral bond spreads.

define sectors' external financial dependence, capital expenditures mines cash flows, as a proxy on how vulnerable are different sectors to tightening credit conditions. We provide an additional channel. Rather than looking at the equilibrium (observed) external financial dependence, which might not be directly related to financing frictions, we propose that production flexibility is important to determine how well can firms move away from input financing constraints.

The results in our paper can be used to conciliate the evidence in Kudlyak and Sánchez (2017). The authors use Rajan and Zingales (1998) approach to argue that firms with higher external financial dependence did not suffer larger declines in sales compared to lower external financial dependence firms, as expected in a financial crisis. Hence, the authors conclude that financial frictions might not be the best candidate to explain the sharp GDP decline during the Great Recession. Our results, on the other hand, do highlight the role of financial frictions during the Great Recession. The key is to change the way we think about firms' external financial dependence.

The paper is organized as follows. In section 2, we present the evidence on sectoral spreads, leverage, and estimated elasticities. In section 3, we estimate a panel fixed-effect regression to account for the relationship between spreads and elasticities, during the Great Recession. In section 4, we develop a theoretical two sector model that is able to explain the observed correlations. In section 5, we perform a quantitative exercise with the general model calibrated to the U.S in 2007. Finally, section 6 concludes.

2 Spreads and Elasticities

In this section we present our empirical evidence. We start by analyzing the evolution of sectoral bond spreads. We then present the framework to estimate our sectoral elasticities, ϵ_Q and ϵ_M.

2.1 Sectoral Bond Spreads and Leverage

We collect sectoral bond spread data from Gilchrist and Zakrajsek (2012).[12] The GZ credit spread measures for each non-financial firm the arithmetic average of the difference between firm i bond yield and a hypothetical Treasury security of the same maturity, for all the unsecured bonds issued by firm i at quarter t. The average maturity of the corporate bonds

[12]We are extremely grateful to Egon Zakrajšek for providing us with the data.

in Gilchrist and Zakrajsek (2012) is 13 years. However, because of the cash flows generated by coupon payments, the average duration of these bonds is considerably shorter. The sectoral bond spreads is defined as the median spread of all firms in sector j at time t.

Figure (1) plots the median spread among manufacturing and service firms for the period 1974-2016. Spreads are countercyclical. During macroeconomic downturns, credit supply tightens and/or firms' demand for external finance increases. During the Great Recession, firms in manufacturing industries paid up to 10% spread over the hypothetical Treasury security.

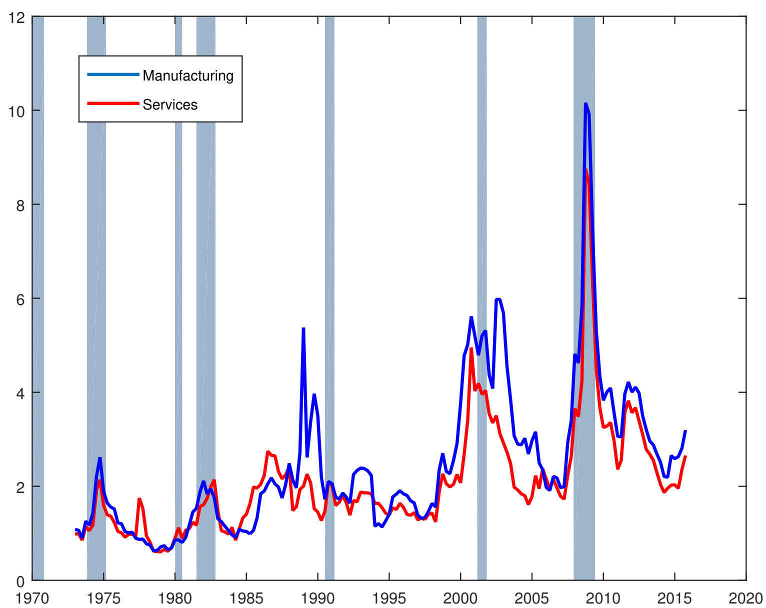

Figure 1
Sectoral Spreads

We also use COMPUSTAT firm level data to study firms' leverage. Figure (2) depicts manufacturing and service sector firms' debt to sales ratio, our measure of firms' leverage. We observe that leverage, in manufacturing and service sector firms, is also countercyclical. During downturns, firms' borrowing increases faster than firms' revenue.

We interpret this evidence as follows. The increase in borrowing costs and external finance needs during recessions are both a sign that, at the firm/sector level, binding financial constraints play a role in amplifying downturns. We now proceed to estimate sectoral elasticities of substitution in production, while accounting for the potential bias from binding financing constraints.

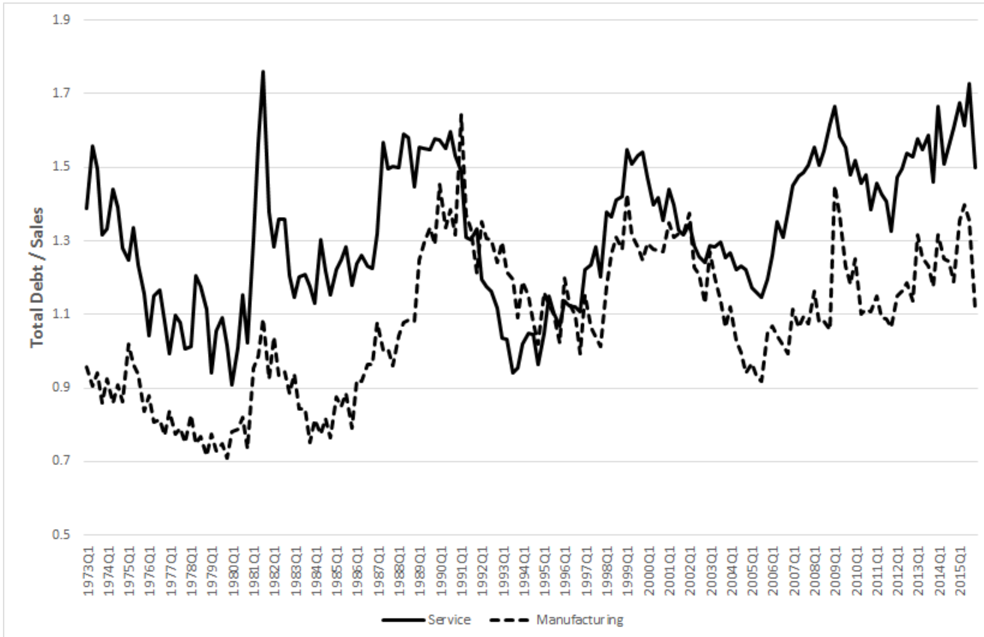

Figure 2
Sectoral Leverage

2.2 Estimation of Production Elasticities

Suppose that sectoral production uses an aggregate of capital and labor (value added V_j) and an aggregate of intermediates (material input M_j) to produce a final good Q_j:

$$Q_j = Z_j \left(a_j^{\frac{1}{\epsilon_{Q,j}}} V_j^{\frac{\epsilon_{Q,j}-1}{\epsilon_{Q,j}}} + (1-a_j)^{\frac{1}{\epsilon_{Q,j}}} M_j^{\frac{\epsilon_{Q_j}-1}{\epsilon_{Q,j}}} \right)^{\frac{\epsilon_{Q_j}}{\epsilon_{Q,j}-1}},$$

where $\epsilon_{Q,j}$ is the elasticity of substitution and is sector-specific. The sectoral total factor productivity is Z_j. The importance of labor in production is a_j. The material input bundle M_j is constructed using intermediates from other sectors:

$$M_j = \left(\sum_{i=1}^{J} \omega_{ij}^{\frac{1}{\epsilon_{M,j}}} M_{ij}^{\frac{\epsilon_{M,j}-1}{\epsilon_{M,j}}} \right)^{\frac{\epsilon_{M,j}}{\epsilon_{M,j}-1}},$$

where $\epsilon_{M,j}$ is the elasticity of substitution between different material inputs, and ω_{ij} represents how important are intermediate inputs from sector i in the total cost of intermediates of sector j.

In addition, suppose that firms are constrained in the financing of inputs. The working capital constraints are

$$\theta_j^w wL_j + \theta_j^m \left(\sum_{ij}^{N} P_i M_{ij} \right) \leq \eta_j P_j Q_j, \tag{1}$$

where θ_j^w and θ_j^m are the fraction of the labor input cost and intermediate input cost that must be paid in advance, respectively.[13]

The cost minimization conditions imply

$$\Delta \log \left(\frac{P_{jt}^M M_{jt}}{P_{jt} Q_{jt}} \right) = (1 - \epsilon_{Q_j}) \Delta \log \left(\frac{P_{jt}^M}{P_{jt}} \right) + (\epsilon_{Q_j} - 1) \Delta \log Z_{jt} + \epsilon_{Q_j} \Delta \overline{\mu}_{jt}. \tag{2}$$

and

$$\Delta \log \left(\frac{P_{it} M_{ijt}}{P_{jt}^M M_{jt}} \right) = (1 - \epsilon_{M_j}) \Delta \log \left(\frac{P_{it}}{P_{jt}^M} \right) + \epsilon_{Mj} \Delta \tilde{\mu}_{ijt}. \tag{3}$$

Here, P_{jt} is the price of output produced in sector j and P_{jt}^M is the price index for the bundle of intermediates used as inputs by sector j. The first regression (Eq. 2) identifies ϵ_{Q_j} by measuring the response of the share of intermediate expenditures to total revenue (which equals total expenditures due to the constant returns to scale) to a change in the relative prices, and the second regression (Eq. 3) identifies ϵ_{M_j} by measuring the response of the share of intermediates from sector i used in sector j (compared to the total expenditure by sector j) to a change in the relative prices.

The terms $\bar{\mu}_{jt}$ and $\tilde{\mu}_{ijt}$ are the sectoral wedges from the binding constraints in the bundle of intermediates and for a particular input (M_{ij}), respectively. These wedges are 1 when sectors are unconstrained and they are larger than 1 when sectors are constrained. The wedge is a function of the sectors' Lagrange multiplier of the collateral constraint, the fraction of sales to be pledged as collateral, and the fraction of inputs to be paid in advance.

Combining equations (3) and (4) we have the model's implied equation to estimate ϵ_M and ϵ_Q jointly

[13] The microfoundation for this constraint is detailed in Bigio and La'O (2016). Before production takes place, firms borrow from a competitive financial intermediary the amount of input expenses needed to produce. There is a limited commitment problem, since after sales firms can default on their debt without paying back to the intermediary. Therefore, firms are required to pledge a fraction of sales as collateral. If a firm does not repay, the financial intermediary seizes a fraction $1 - \eta_j$ of total sales. In an equilibrium without default, the incentive compatibility constraint implies that firms can externally borrow up to a fraction η_j of total sales.

$$\Delta\log\left(\frac{P_{it}M_{ijt}}{P_{jt}Q_{jt}}\right)=(1-\epsilon_{Mj})\Delta\log\left(\frac{P^M_{jt}}{P_{it}}\right)+(1-\epsilon_{Qj})\Delta\log\left(\frac{P_{jt}}{P^M_{jt}}\right)+(\epsilon_{Q_j}-1)\Delta\log Z_{jt}+\Delta\hat{\mu}_{ijt}, \tag{4}$$

where $\Delta\hat{\mu}_{ijt}$ summarizes how changes in sectoral wedges (on M_j or M_{ij}) affect input shares. Time variation in the unobserved wedge biases the estimation of sectoral elasticities. In general, tighter financial constraints are associated with increase in sectoral prices, which in turn generates an upward bias in the elasticities. As most of the variation in spreads is observed during the Great Recession, the level of sectoral wedges and small changes in wedges outside the Great Recession will be captured by including sectors' fixed-effects and time fixed-effects. To reduce the bias from large time variation in spreads during the Great Recession, our baseline specification only considers data before the Great Recession. The main results of the paper still apply when using the whole sample. However, as we will see later, the instruments that aim to correct for the endogeneoty in the estimation of the elasticities are weak when using data post 2007.[14]

There is an additional bias coming from unobserved sectoral productivities. Sectoral productivities are negatively correlated with sectoral prices which in turn implies a downward bias in the estimation of sectoral elasticities. To estimate the elasticities we follow Atalay (2017), but we allow for the elasticities to differ across sectors. We use the BEA annual Input-Output data for the period 1997-2007(2014). Originally, there are 71 sectors of the economy.[15] The empirical counterpart of equation (4) is:

$$\Delta\log\left(\frac{P_{it}M_{ijt}}{P_{jt}Q_{jt}}\right)=\alpha_j\Delta\log\left(\frac{P^M_{jt}}{P_{it}}\right)+\beta_j\Delta\log\left(\frac{P_{jt}}{P^M_{jt}}\right)+\nu_{ijt}, \tag{5}$$

where P_{it} and P_{jt} are sectoral output prices, and P^M_{jt} is the price of the sector j intermediate bundle. The error term is denoted by ν_{ijt}. We also include buyer-seller and time fixed effects. We can obtain the elasticities as

$$\begin{aligned}\epsilon_{Q,j}&=1+\beta_j\\ \epsilon_{M,j}&=1+\alpha_j.\end{aligned}$$

[14] While we could control for the Great Recession using time dummies, the fact that variation in sectoral wedges during the Great Recession can differ substantially across sectors, is still a worry.

[15] For each sector we keep the top 20 intermediate goods' supplier sectors.

2.2.1 Dealing with Endogeneity

As noted in Atalay (2017), there is an endogeneity problem in that relative prices will be correlated with unobserved sectoral productivity Z_{jt}, which is a component of ν_{ijt}. We consider the instrument used in Acemoglu et al. (2015) and Atalay (2017), namely sectoral military spending.[16] Higher military spending in sector j, or in sectors that use the output of sector j output intensively, increases the demand for sector j's output and therefore increases the price. The assumption is that military spending is orthogonal to changes in sectoral productivity, and that it only affects input shares through changes in the relative cost of inputs.

Following Atalay (2017) we define instruments for the output price of sector j (P_{jt}), the price of the intermediate input bundle of sector j (P^M_{jt}), and the price of the intermediate input from sector i (P_{it}) that is used in the production of sector j. To formally define the instrument, define S_{ji} as the share of sector j's output that is purchased by sector i. Our instruments are then

$$
\begin{aligned}
Military_{p_j,t} &= \sum_i (I-S)^{-1}_{ji} S_{i,military} \cdot \Delta \log(\text{Military Spending}_t), \\
Military_{p_i,t} &= \sum_j (I-S)^{-1}_{ij} S_{j,military} \cdot \Delta \log(\text{Military Spending}_t) \\
Military_{p^m_j,t} &= \sum_i \frac{P_{ijt} M_{ijt}}{P^M_{jt} M_{jt}} \cdot Military_{p_i,t}.
\end{aligned}
$$

The term $(I-S)^{-1}$ measures the sum of direct and indirect changes that occur due to network connections.[17] Changes in Military spending on sector i's output can have important indirect effects on sector j's output demand if military industries 1) purchase a large fraction of sector i's output (large $S_{i,military}$), ii) and sector i, directly and indirectly, purchases a large fraction of sector j's output (large $(I-S)^{-1}_{ji}$).

[16] Acemoglu et al. (2015) do not precisely use military spending as an instrument but rather as a demand shock. The authors study the propagation of different type shocks in economies with intermediate input linkages.

[17] Note that, unlike the well-known Leontief inverse matrix, this matrix does not account for the indirect upstream links – sectors supplier importance – but instead it measures the indirect downstream links. This is, it captures how important are other sectors in the demand of a given sector output.

2.2.2 Aggregated Sectors

Table 2.1 reports the panel FE estimation of regression (5). We define 3 broad sectors: Manufacturing sectors (sectors 7 to 26 in Table 6.1 of the Appendix B), Service Sectors (sectors 5, 6, and 27 to 66 in Table 6.1) and Primary sectors (sectors 1 to 4 in Table 6.1).[18] The results show that manufacturing sectors are the least flexible sectors. In terms of ϵ_M and ϵ_Q, service sectors and primary sectors have much larger elasticity estimates. This difference is even stronger if we exclude the Great Recession from our sample.

Table 2.1
Panel FE: Whole sample and Before 2008

VARIABLES	(1) 1997-2014	(2) 1997-2007
ϵ_M	-0.21***	0.22***
	(0.00)	(0.00)
ϵ_Q	0.33***	-0.62***
	(0.00)	(0.00)
$\epsilon_M \cdot service$	0.82***	0.54***
	(0.00)	(0.00)
$\epsilon_M \cdot primary$	0.95***	0.24
	(0.00)	(0.20)
$\epsilon_Q \cdot service$	0.33*	1.29***
	(0.07)	(0.00)
$\epsilon_Q \cdot primary$	0.98***	2.13***
	(0.00)	(0.00)
Observations	22,438	13,200
R-squared	0.044	0.021
Number of partner	1,320	1,320

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In Table 2.2 we report the IV results. The (not-reported) first stage results indicate, as expected, that $\frac{P_j^M}{P_i}$ is negatively correlated with the instrument for P_i ($Military_{p_i}$) and positively correlated with the instrument for P_j^M ($Military_{p_j^m}$). Similarly, as expected, $\frac{P_j}{P_j^M}$ is positively correlated with the instrument for P_j ($Military_{p_j}$) and negatively correlated with

[18]We drop government sectors (sectors 67-71 in table 6.1) from our analysis to make our OLS and IV estimates comparable.

the instrument for P_j^M ($Military_{p_j^m}$). The underidentification test rejects that the instruments and the endogenous variables are not correlated, the Hansen-J test of overidentifying restrictions does not reject orthogonality, but the weak instrument test does not reject weak instruments. However, we will see later that when we estimate more disaggregated elasticities, the hypothesis of weak instruments is generally rejected at a bias of IV that is at most 25 percent of the OLS bias.

One key result is that service sectors have a higher elasticity of substitution than manufacturing sector: for service sectors, ϵ_Q is significantly above one, regardless the sample used in the estimation.

Table 2.2
Panel FE - IV Military: Whole sample and Before 2008

VARIABLES	(1) 1997-2014	(2) 1997-2007
ϵ_M	-5.28**	5.33
	(0.03)	(0.19)
ϵ_Q	-4.78	4.32
	(0.11)	(0.43)
$\epsilon_M \cdot service$	12.21**	-5.71*
	(0.02)	(0.09)
$\epsilon_M \cdot primary$	9.18**	-1.85
	(0.01)	(0.52)
$\epsilon_Q \cdot service$	11.24**	12.49**
	(0.03)	(0.01)
$\epsilon_Q \cdot primary$	4.35	-2.92
	(0.37)	(0.44)
Observations	19,838	11,160
Number of partner	1,240	1,240
P-value Kleibergen-Paap LM	0.000	0.015
F Kleibergen-Paap	2.693	1.279
P-value Hansen test	0.336	0.733

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Tables 2.3 and 2.4 report the IV results for each subgroup of sectors separately, rather than pooling them and using dummy variables. Similar results apply. The elasticity of substitution for service sectors is consistently above one. As in Atalay (2017), we find that the

Table 2.3
IV Military: ALL, Manufacturing, Service, Primary 1997-2014

VARIABLES	(1) All	(2) Manuf.	(3) Service	(4) Primary
ϵ_M	-0.21	-4.47***	7.08***	4.03**
	(0.48)	(0.01)	(0.01)	(0.04)
ϵ_Q	1.16	4.97	5.76**	-2.18**
	(0.96)	(0.61)	(0.05)	(0.02)
Observations	21,118	6,398	12,160	1,280
Number of partner	1,320	400	760	80
P-value Kleibergen-Paap LM	0.00	0.08	0.00	0.00
F Kleibergen-Paap	4.075	1.834	5.921	4.572
P-value Hansen test	0.502	0.801	0.498	0.276

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

point estimates for ϵ_M are below one, when using the entire sample. However, the estimation of ϵ_M changes substantially when dropping the Great Recession from the sample, a result that can be due to small sample problems or the additional bias from binding constraints. The estimation of ϵ_Q, on the other hand, does not depend on the sample we use. The instruments used in IV though, tend to be stronger when dropping the Great Recession data.[19]

2.2.3 Disaggregated Sectors

We now proceed to estimate sectoral elasticities at a more disaggregated level. We use the sample before the Great Recession to avoid bias in the estimation of elasticities arising from frictions in the use of inputs during the crisis. We focus our analysis on the values of ϵ_Q.

We provide the estimated elasticities using panel FE and also IV approach. To improve

[19] In his Appendix D.2, Atalay (2017) estimates sectoral elasticity for primary sectors, manufacturing sectors and service sectors. His OLS and IV results for ϵ_Q show that is not possible to reject unitary elasticity of substitution for each of these sectors, except in the OLS regression for primary and manufacturing sectors where $\epsilon_Q > 1$. Regarding ϵ_M, all his estimates lie below 1 and service sectors present higher elasticities. The results in Atalay (2017) are different to the ones in this paper due to: i) Atalay (2017) aggregates the 71 industries in the BEA data to 30 industries to match KLEMS industry classification, while in this paper we use 66 sectors (we exclude the government sectors); ii) Atalay (2017) uses the top 10 intermediate input trade partners, while we use the top 20; and iv) his sample covers period 1997-2013 while we study subsamples as well.

Table 2.4
IV Military: ALL, Manufacturing, Service, Primary 1997-2007

VARIABLES	(1) All	(2) Manuf.	(3) Service	(4) Primary
ϵ_M	0.69	3.69	-0.42	-2.16
	(0.88)	(0.12)	(0.55)	(0.37)
ϵ_Q	6.76***	-2.16	15.18***	3.18*
	(0.01)	(0.59)	(0.00)	(0.07)
Observations	11,880	3,600	6,840	720
Number of partner	1,320	400	760	80
p-value Kleibergen-Paap LM	0.00	0.00	0.00	0.04
F Kleibergen-Paap	5.084	4.814	6.437	2.025
P-value Hansen test	0.0375	0.270	0.540	0.725

Robust pval in parentheses
*** p<0.01, ** p<0.05, * p<0.1

the precision in the estimation of elasticities we aggregate the 66 sectors into 30 sectors (for the panel fixed-effects) and 24 sectors (for the IV estimation). We make groups of several sectors based on the ranking of their point estimates using the 66 sectors and based on whether sectors are service sectors or not. In the IV estimation, the goal is also to increase the strength of the instruments. Therefore, we are forced to increase aggregation slightly, going from 30 sectors in the FE estimation to 24 sectors in the IV estimation.[20] The precision is greatly improved. In fact, for the panel FE estimation, all the sectoral elasticities are statistically different from the baseline sector. The ranking is also preserved. Thus, even though we aggregate sectors we gain sectoral heterogeneity compared to the case when we estimate sectoral elasticities for the 66 sectors. The only issue is the existence of several sectors – mostly non-service sectors – with elasticity estimates that are negative. As in Atalay (2017), we assume for future exercises that these sectors have strong complementarities in production and set their technologies to be almost Leontief (e.g., $\epsilon_Q = 0.1$).

The IV estimates are less precise (last column of Table 2.5). Nevertheless, we find a high correlation between the panel FE sectoral elasticities and the IV ones. The cross-sectoral correlation is 0.75 between the point estimates adjusted by significance and 0.81 between the

[20]The aggregation does not affect future results. The aggregation simply aims to increase the precision in the estimation of elasticities. After all, the estimated elasticities are generated regressors to be used in the next section.

estimates, adjusted by significance and non-negativity. Figure 3 depicts the estimates of ϵ_Q from the IV approach and from the panel FE approach.[21] We observe that they are highly correlated.[22]

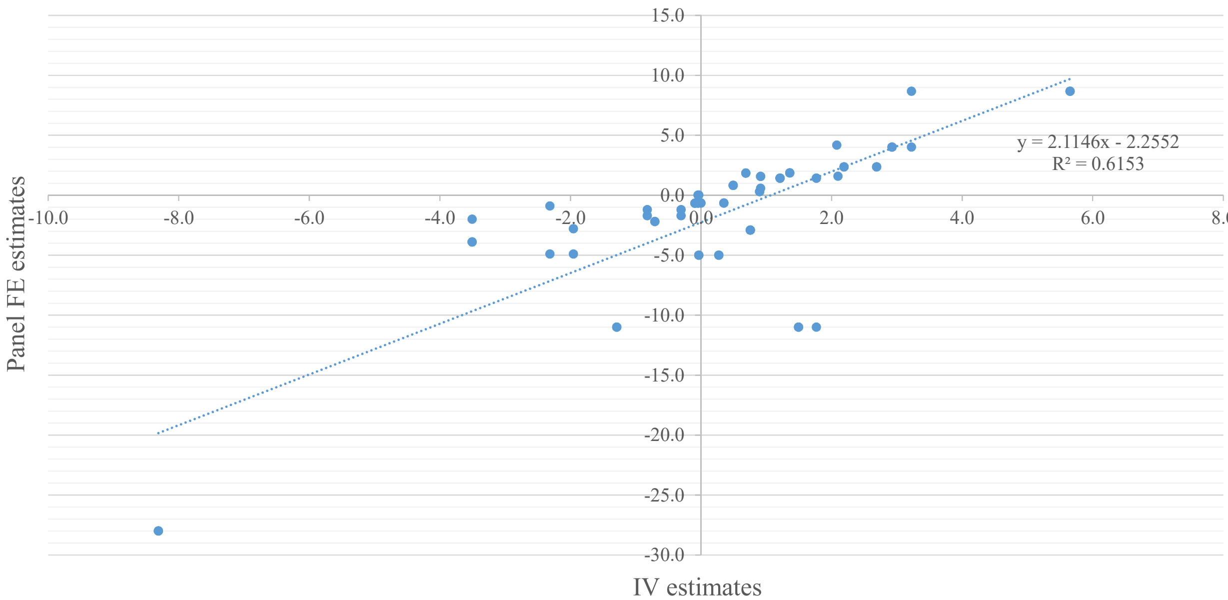

Figure 3
IV versus Panel FE estimates of ϵ_Q

In Table 2.5 we report the estimates for ϵ_Q and ϵ_M for the top 10 most flexible sectors in terms of ϵ_Q's point estimates. We report the adjusted point estimates that only use

[21] In general, the instruments are valid in the estimation of sectoral elasticities: the first stage relationships are strong and consistent with demand shifters and the overidentifying restrictions are satisfied.

[22] In Appendix B, we use the theoretical model in Section 3 as a guide to learn about bias in the estimation of sectoral elasticities. We argue that this bias might be important. However, consistent with our empirical results, when constraints are not binding in the model – using data before the Great Recession in our estimation – the ranking of sectoral elasticities is generally preserved.

statistically significant coefficients in (5). The elasticities that are statistically significant and below zero are set to be 0.1. Consistent with the more aggregated results, all the top 10 sectors in terms of ϵ_Q are service sectors, for both the panel FE and IV estimates.

Table 2.5
Sectoral Estimates of ϵ_Q and ϵ_M

			66 sectors		30 sectors		IV
Sector N	Sector Name	Service	ε_Q	ε_M	ε_Q	ε_M	ε_Q
28	Motor vehicle and parts dealers	1	5.8	0.3	5.7	0.1	8.7
27	Wholesale trade	1	5.6	0.1	5.7	0.1	8.7
62	Performing arts, spectator sports, museums	1	1.0	0.1	3.2	1.0	8.7
5	Support activities for mining	1	3.4	1.5	3.2	1.0	4.0
58	Ambulatory health care services	1	1.0	0.3	2.9	1.3	4.0
39	Warehousing and storage	1	3.2	2.8	2.9	1.3	4.0
66	Other services, except government	1	1.0	0.3	2.7	0.7	1
44	Federal Reserve banks, credit interm., and rel. act.	1	1.0	0.3	2.7	0.7	1
55	Administrative and support services	1	1.0	0.3	2.2	0.1	1
52	Computer systems design and related services	1	1.0	0.3	2.2	0.1	1

If we exclude the negative estimates or set them to be zero, the average ϵ_Q is larger than one for service sectors and is smaller than one for manufacturing sectors. One problem of estimating more disaggregated elasticities is that the precision of the estimates is low. In fact, about half of the top 10 estimates have an ϵ_Q that is statistically different from one. On the other hand, the estimates yield several sectors with an ϵ_Q that is statistically significant and negative. However, we still obtain enough heterogeneity in sectoral elasticities that will allow us in section 4 to lay out the relationship between flexibility in production and sectoral frictions.

In this section we have documented that not only is the assumption of common unitary elasticities across sectors ($\epsilon_Q = 1$) counterfactual (used in Bigio and La'O (2016)), but so is the assumption of a common $\epsilon_Q < 1$ across sectors (used in Baqaee and Farhi (2017)). In the next section, we explore the connection between sectoral flexibility in production and the severity of sectoral financing constraints, as measured by the spreads on corporate bonds.

3 Flexibility and Spreads in US Data

In this section, we proceed to study the relationship between sectoral elasticities and a proxy for the degree of financial frictions: the spread on corporate bonds over Treasury bills (corrected for duration). As pointed out by Gilchrist and Zakrajsek (2012), this measure contains information about aggregate credit conditions (supply) and firm level default risk (demand). In any case, when firms are financially constrained one would expect an upward sloping debt schedule.

To control for other firm level covariates – unconnected to the elasticity – that might cause a firm or sector to pay a higher premium at a given point in time, we use COMPUSTAT data. Our main covariates are sales, the value of tangible assets, the value of property and plants, inventories, leverage (total debt divided by sales), and working capital as a fraction of sales. Given that sectoral elasticities are assumed to be constant over time, our identification relies on interacting the elasticities with time-varying variables. In this case, we are interested in how sectoral spreads differ in recessions for firms with different elasticities of substitution, so we interact the elasticities with different recession dummies.

$$r_{jt} = \alpha_j + \beta_1 D_R + \beta_2 L_{jt} + \beta_3 \hat{\epsilon}_{Qj} D_{Rt} + \beta_4 \hat{\epsilon}_{Mj} D_{Rt} + \gamma X_{jt} + \nu_{jt}, \tag{6}$$

where r_{jt} is the median credit spread for sector j in quarter t, D_{Rt} is a recession dummy, L_{jt} is leverage measured by total debt divided by sales, and X_{jt} is the vector of controls from before.

All our specifications include year and sector fixed effects.[23] The estimated elasticities – $\hat{\epsilon}_{Qj}$ and $\hat{\epsilon}_{Mj}$ – correspond to the point estimates using the sample before the Great Recession. We still find that some sectors present negative elasticities of substitution. Our results include the negative elasticity sectors assuming they all have essentially Leontief technologies.[24]

We estimate (5) for the period 1974-2016. In the rest of the paper, we use the panel FE elasticities, since they are more precise and are highly correlated with the IV elasticities anyway.[25] To account for the generated regressors problem, we report the bootstrap standard errors in parentheses.[26] Our main identification lies in the cross sectional heterogeneity during macroeconomic downturns. The first set of results uses D_R based on the NBER definition of

[23] We include year instead of year-quarter fixed effect to better capture the effect of our crisis dummy D_R on spreads. Our results regarding the elasticities are the same with either time fixed effect dummies.

[24] We thus shut down additional heterogeneity within the group of negative elasticity sectors. However, we significantly enlarge our sample size, and still add additional heterogeneity compared to the sectors with positive elasticities.

[25] In Table 6.4 of our Appendix D we provide the estimated coefficients of equation (4) using the IV estimates of ϵ_Q. The results are very similar and even larger in magnitude than the panel FE estimates

[26] We perform a two step estimation. In the first step we estimate sectoral elasticities of substitution. Using the asymptotic distribution of our estimates, we then draw for each sector $M = 500$ realizations of sectoral elasticities. The second step uses these draws to estimate the corresponding M estimates of β_3 and β_4 in equation (5). The standard deviation of these M estimates is the bootstrap standard deviation we report in Table 4.1.

U.S recessions. The second set of results uses $D_R = 1$ for the Great Recession (2007q4-2009q2) and $D_R = 0$ outside the Great Recession.

The results in Table 3.1 when pooling all the sectors, column 1, show that during recessions sectors with higher flexibility in production – in terms of ϵ_Q and ϵ_M – paid a lower premia. The coefficient is only statistically significant for the interaction between D_R and ϵ_Q. When we split the sample between high elasticity service and low elasticity manufacturing sectors in columns 2 and 3, we also observe a negative correlation although it is not statistically different from zero.

Table 3.1
GZ Spreads and Panel FE Elasticities, 1974-2016 (NBER Recessions)

VARIABLES	(1) All	(2) Manufacturing	(3) Service
D_R	0.176***	0.119**	0.204***
	(0.0371)	(0.0511)	(0.0583)
L	0.142***	0.183***	0.0986***
	(0.0187)	(0.0328)	(0.0248)
$\hat{\epsilon}_Q \cdot \mathrm{D}_R$	-0.00671**	-0.00356	-0.00950
	(0.0036)	(0.00415)	(0.0068)
$\hat{\epsilon}_M \cdot \mathrm{D}_R$	-0.00588	-0.0252	-0.00915
	(0.0085)	(0.0101)	(0.0104)
Observations	6,603	2,830	3,352
R-squared	0.503	0.613	0.476
Number of sector	55	18	34
Controls	Yes	Yes	Yes
Time and Sector FE	Yes	Yes	Yes

Bootstrap standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

In Table 3.2 we change our recession dummy to only capture how sectoral flexibility and spreads relate during the Great Recession in 2007q4-2009q2. We observe strong negative correlations between sectoral spreads and both elasticities ϵ_M and ϵ_Q. The same result holds once we split the sample among high elasticity sectors and low elasticity sectors. However, these results are only statistically different from zero for the interaction between ϵ_Q and the Great Recession dummy.

The estimated coefficient for β_3 in column 1 implies that a 10 percent increase in the elasticity ϵ_Q is associated with a 0.20 percent decrease in the sectoral spread during the Great

Table 3.2
GZ Spreads and Panel FE Elasticities, 1974-2016 (The Great Recession)

VARIABLES	(1) All	(2) Manufacturing	(3) Service
D_R	0.169***	0.0844**	0.229***
	(0.0309)	(0.0390)	(0.0479)
L	0.140***	0.181***	0.0960***
	(0.0187)	(0.0326)	(0.0247)
$\hat{\epsilon}_Q \cdot \mathrm{D}_R$	-0.0198***	-0.0244***	-0.0177*
	(0.00381)	(0.011)	(0.0101)
$\hat{\epsilon}_M \cdot \mathrm{D}_R$	-0.0234***	-0.0365	-0.0182*
	(0.00671)	(0.058)	(0.0114)
Observations	6,603	2,830	3,352
R-squared	0.505	0.618	0.476
Number of sector	55	18	34
Controls	Yes	Yes	Yes
Time and Sector FE	Yes	Yes	Yes

Bootstrap standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Recession. To provide a sense of the magnitude of this elasticity, consider an example: if the motor vehicle sector ($\epsilon_Q = 0.9$) had the elasticity of the auto dealer sector ($\epsilon_Q = 5.7$), motor vehicle firms would have paid a spread about 1000 basis points lower during the Great Recession.

The previous results emphasize that financing frictions were more severe for less flexible firms, during the Great Recession. We also observe that during the Great Recession all firms paid higher spreads, and that more leveraged industries pay higher spreads. Nevertheless, we have not taken advantage of the fact that, for a given borrowing limit – e.g., collateral being 50% of total sales or value of plants–, highly leveraged and low elastcity industries might be more exposed to hit the constraint.

Therefore, we augment our regression, see Eq. 7, and add the interaction between firms' leverage – as measured by total debt over sales – and firms' elasticities. As the Great Recession is our main source of variation, to study the potential mechanisms behind our facts we estimate our equation (7) for the period 2002-2016. In Table 3.3 we observe that the coefficient of the interaction between leverage and elasticities offers interesting heterogeneity across service and manufacturing firms. High ϵ_Q manufacturing firms pay lower spreads when indebted, while high ϵ_Q service firms pay higher spreads when indebted. The interaction of ϵ_Q and the Great Recession dummy is still negative and statistically significant, as in previous tables.

Table 3.3
GZ Spreads and Panel FE Elasticities, 2002-2016 (The Great Recession)

VARIABLES	(1) All	(2) Manufacturing	(3) Service
D_R	0.389***	0.302***	0.432***
	(0.048)	(0.084)	(0.065)
L	0.351***	0.022	0.511***
	(0.055)	(0.133)	(0.081)
$\hat{\epsilon}_Q \cdot \mathrm{D}_R$	-0.013***	-0.018***	-0.010*
	(0.002)	(0.007)	(0.006)
$\hat{\epsilon}_M \cdot \mathrm{D}_R$	-0.010*	-0.021	-0.004
	(0.005)	(0.069)	(0.008)
$\hat{\epsilon}_Q \cdot \mathrm{L}$	0.010	-0.027*	0.028***
	(0.009)	(0.018)	(0.010)
$\hat{\epsilon}_M \cdot \mathrm{L}$	-0.010	-0.044	0.002
	(0.005)	(0.130)	(0.012)
Observations	2,493	989	1,376
R-squared	0.541	0.552	0.564
Number of sector	53	18	32
Controls	Yes	Yes	Yes
Time and Sector FE	Yes	Yes	Yes

Bootstrap standard errors are reported in parentheses.
*** p<0.01, ** p<0.05, * p<0.1

We have documented that during the Great Recession firms with higher flexibility in production paid lower spreads on corporate bonds. Additionally, we observe that highly leveraged and highly flexible sectors paid lower (higher) spreads within the group of manufacturing (service) sectors. In the next section, we construct a theoretical model that is able to explain our facts.

4 Theoretical Framework

The empirical evidence presented in the previous section suggests that higher flexibility in production allows firms to move away from financing frictions, during severe downturns. A simple example that is consistent with our results and the model presented in this section goes as follows. Two firms produce with two inputs, labor and energy. Workers are paid at the end of the month once the firms obtain their revenues. However, the firms have to paid all the cost of energy up front before production takes place. To pay for the energy bill firms need external funds. However, the external funds are limited up to fraction of total production. Due to the possibility of default, the financial contract implies that the lender can seize a fraction of current production (think of cars). Therefore, firms can only borrow up to a fraction of their total sales.

Now, suppose that firm 1 uses labor and energy as perfect substitutes, while firm two use them as perfect complements. An increase in the relative cost of energy reduces firm 1 financial needs as it will move completely to the use of labor input, the unconstrained input. Nevertheless, firm 2 really needs to produce using energy. Indeed, firm 1 financial needs increase with the cost of energy. Further increases in the cost of energy or tightening credit conditions in the economy will make firm 2 financially constrained.

4.1 The Model

We suppose there are only two sectors – the first sector produces using only labor, and the second sector produces using labor and intermediates from both sectors:

$$Q_1 = Z_1 L_1$$

$$Q_2 = Z_2 \left(a_2^{\frac{1}{\epsilon_{Q,2}}} L_2^{\frac{\epsilon_{Q,2}-1}{\epsilon_{Q,2}}} + (1-a_2)^{\frac{1}{\epsilon_{Q,2}}} \left(\omega_{12}^{\frac{1}{\epsilon_{M,2}}} M_{12}^{\frac{\epsilon_{M,2}-1}{\epsilon_{M,2}}} + \omega_{22}^{\frac{1}{\epsilon_{M,2}}} M_{22}^{\frac{\epsilon_{M,2}-1}{\epsilon_{M,2}}} \right)^{\frac{\epsilon_{M,2}}{\epsilon_{M,2}-1}\frac{\epsilon_{Q,2}-1}{\epsilon_{Q,2}}} \right)^{\frac{\epsilon_{Q,2}}{\epsilon_{Q,2}-1}}.$$

We suppose that each sector faces a collateral constraint on working capital:

$$\begin{aligned} \theta_1^w w L_1 &\leq \eta_1 p_1 Q_1 \\ \theta_2^w w L_2 + \theta_{12}^m M_{12} + \theta_{22}^m p_2 M_{22} &\leq \eta_2 p_2 Q_2. \end{aligned} \tag{7}$$

Firms in sector j need to externally finance a fraction θ_j^w of the wage bill wL_j, and a fraction θ_{ij}^m of the cost of intermediates purchased from sector i $p_i M_{ij}$. However, firms are limited in the amount of borrowing they can obtain. Different sectors can pledge a different fraction η_j of total sales as collateral. The variable μ_j denotes the Lagrange multiplier for the sectoral borrowing constraint in 7, which represents the firms' shadow cost of debt – that is, μ_j represents how much firms in sector j value a marginal increase in external funds that would allow them to produce closer to the optimal scale.[27] The model does not display capital. However, we can think of capital being fixed and equal to 1 for every sector.

The representative household maximizes

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma}$$

subject to the budget constraint

$$wL + \Pi \geq PC;$$

for ease of presentation we assume labor is inelastically supplied.

In equilibrium, labor market clearing requires

$$L = L_1 + L_2,$$

and goods market clearing requires

$$\begin{aligned} M_{12} &= Q_1 \\ C + M_{22} &= Q_2. \end{aligned}$$

Note that, for simplicity of the resulting algebra, the output of sector one is not consumed. Adding capital would not change our results if value added is produced using a Cobb-Douglas aggregate of capital and labor, so again for ease of presentation we simply ignore it.

We develop intuition through a series of special cases. We vary the values of the elasticities and examine the relationship between the Lagrange multiplier μ_2 on the collateral constraint for sector 2 and the elasticity of interest. We set $\sigma = 1$ and normalize $w = 1$. The total labor endowment $\bar{L}$ is normalized to 1. We set $\omega_{22} = 0$, $\theta_{22}^m = 0$, and $\omega_{12} = 1$. We assume that sector 1 is unconstrained. If we assume that sectoral production functions are Cobb-Douglas (as in Bigio and La'O (2016)) and constant returns the scale, then sectors are either constrained or

[27]The microfoundation for this constraint is detailed in Bigio and La'O (2016). Before production takes place, firms borrow from a competitive financial intermediary the amount of input expenses needed to produce. There is a limited commitment problem, since after sales firms can default on their debt without paying back to the intermediary. Therefore, firms are required to pledge a fraction of sales as collateral. If a firm does not repay, the financial intermediary seizes a fraction $1 - \eta_j$ of total sales. In an equilibrium without default, the incentive compatibility constraint implies that firms can externally borrow up to a fraction η_j of total sales.

unconstrained; for example, a sector is constrained if $\eta_2 < 1$ and $\theta_j^w + \sum \theta_{ij}^m = 1$ since the left-hand-side of the collateral constraint equals revenue at the unconstrained profit-maximizing point. To deal with this problem while maintaining Cobb-Douglas production functions, Bigio and La'O (2016) assume sector-specific decreasing returns to scale; their strategy for identifying the decreasing returns to scale parameter is indirect.

4.2 Flexibility and Frictions

Our evidence presented in section 2 supports abandoning Cobb-Douglas as an empirical description of sectoral production functions. In addition, the results in section 3 support the hypothesis that sectoral elasticities influence the severity of sectoral constraints during recessions. We now proceed to examine two moments from our model: (i) the frequency a given sector is constrained (extensive) and (ii) the extent a constrained sector is constrained (intensive); these moments correspond to the frequency with which $\mu_j > 0$ and the quantitative size of μ_j if $\mu_j > 0$. To simplify our analysis, we will focus on the empirically relevant cases where (i) constraints are countercyclical, and (ii) elasticities and frictions (spreads) are negatively correlated during downturns.[28]

The next proposition describes the extensive margin of sectoral frictions – how often unconstrained firms of different elasticities become constrained. In particular, we study how can the model deliver i) countercyclical frictions, and ii) a differential correlation between spreads and the interaction between ${}_Q$ and leverage for manufacturing ($\epsilon_Q < 1$) and service firms ($\epsilon_Q > 1$).

Proposition 1 *Let Z_1^* denote the threshold productivity in sector 1 that results in sector 2 being constrained. Then, if sector 2 only needs to externally finance the intermediates ($\theta_{12}^m = 1$ and $\theta_2^w = 0$), we have*

- *If labor and intermediates are complement inputs ($\epsilon_Q < 1$) and $\phi_m < 1$, leverage and the Lagrange multiplier of sector 2 are countercyclical (increases with lower Z_1), and a higher elasticity ϵ_Q reduces the likelihood of sector 2 becoming constrained, $\frac{\partial Z_1^*}{\partial \epsilon_Q} < 0$, where*

$$\phi_m = \frac{(1-\eta_2)(1-a_2)}{\eta_2 a_2}.$$

Also, if sector 2 only needs to finance the labor input ($\theta_{12}^m = 0$ and $\theta_2^w = 1$), we have

[28] A complete set of results, with all the possible combinations, are available in our Appendix.

- *If labor and intermediates are substitute inputs ($\epsilon_Q > 1$) and $\phi_w < 1$, leverage and the Lagrange multiplier are countercyclical, and a higher elasticity ϵ_Q increases the likelihood of sector 2 becoming constrained, $\frac{\partial Z_1^*}{\partial \epsilon_Q} > 0$, where*

$$\phi_w = \frac{(1-\eta_2)\,a_2}{(1-a_2)\,\eta_2}.$$

Proof.

First set $\theta_2^w = 0$ and $\theta_{12}^m = 1$. From the first order condition of sector 1, we have $p_1 = \frac{1}{Z_1}$. As the production of sector 1 is only supplied to sector 2, we have $M_{12} = Q_1$. Assuming the constraint is binding ($p_1 M_{12} = p_1 Q_1 = \eta_2 p_2 Q_2$), we find $Q_1 = Z_1 L_1 = Z_1 \eta_2$ and $L_2 = 1 - \eta_2$. Using the production function for sector 2 and the first-order condition for L_2 we obtain

$$\mu_2 = \left(\frac{(1-\eta_2)\,(1-a_2)}{a_2\eta_2}\right)^{1-\rho_Q} Z_1^{\rho_Q} - 1,$$

where $\rho_Q = \left(\epsilon_Q - 1\right)/\epsilon_Q$.

We define Z_1^* as the sector 1 productivity that results in sector 2 being exactly constrained, with $\mu_2 = 0$. Therefore,

$$Z_1^* = \phi_m^{\frac{1}{1-\epsilon_Q}},$$

so that

$$\frac{\partial Z_1^*}{\partial \epsilon_Q} = \phi_m^{\frac{1}{1-\epsilon_Q}} \frac{1}{\left(1-\epsilon_Q\right)^2} \ln\left(\phi_m\right).$$

The sign depends on whether ϕ_m is larger or smaller than 1. The interpretation depends on ϕ_m but also on whether ϵ_Q is smaller or larger than 1. When $\phi_m < 1$, within the group of firms with $\epsilon_Q < 1$, $\frac{\partial Z_1^*}{\partial \epsilon_Q} < 0$ means that more flexible sectors need a more negative shock to input suppliers in order to become constrained.

On the other hand, when if $\theta_{12}^m = 0$ and $\theta_2^w = 1$, we have

$$Z_1^* = \phi_w^{\frac{1}{\epsilon_Q - 1}},$$

so that

$$\frac{\partial Z_1^*}{\partial \epsilon_Q} = -\phi_w^{\frac{1}{\epsilon_Q-1}} \frac{1}{\left(\epsilon_Q - 1\right)^2} \ln\left(\phi_w\right).$$

The sign depends on whether ϕ_w is smaller or larger than 1. The interpretation depends on ϕ_w, and also depends on whether ϵ_Q is smaller or larger than 1. When $\phi_w < 1$, within the group of firms with $\epsilon_Q > 1$, $\frac{\partial Z_1^*}{\partial \epsilon_Q} > 0$ means that more flexible sectors need a smaller negative shock to input suppliers in order to become constrained. ■

Proposition 1 is important to identify what type of frictions were more likely to be binding during the Great Recession for high ($\epsilon_Q > 1$) and low elasticity firms ($\epsilon_Q < 1$). This proposition states that when the cost of intermediates relative to labor increases (lower Z_1 or lower η_1), for frictions to be countercyclical it has to be the case that high elasticity service firms faced frictions in the use of labor input and low elasticity manufacturing firms faced frictions in the use of intermediates. This prediction is supported by the fact that before and during the Great Recession, intermediate input did become relatively more expensive than labor-capital (see Figures 5 and 6).

Another fact from section 3 that supports this prediction is the sign of the correlation between sectors' leverage and sectors' elasticities. We observe that indebted service firms paid higher spreads, while indebted manufacturing firms paid lower spreads. This is precisely what the model predicts. In particular, the fact that declines in Z_1 affect the likelihood of the downstream sector to becoming constraint is due to the increased leverage in the face of more expensive intermediates. Take the case of flexible ($\epsilon_Q > 1$) labor-constrained service firms. With the increased relative cost of intermediates, these firms increased their relative demand for labor, which increased their leverage and the likelihood that these firms hit the borrowing-limit. The increase in financial external dependence – how close firms are to the borrowing limit – is larger the larger the elasticity (see Figure (7) of the Appendix). For the inflexible ($\epsilon_Q < 1$) intermediates-constrained manufacturing firms, the increased relative cost of intermediates increases their financial external dependence. The financial external dependence is higher the lower the elasticity (see Figure (6) of the Appendix).

Given that the data favors ϕ_m and ϕ_w below 1, and given the fact that we can measure a for service and manufacturing firms, our simple model also has implications for the values of η_2. In Table 6.1 we observe the steady state shares of labor-capital and intermediates. As a is on average smaller than 1/2 in manufacturing, for ϕ_m to be smaller than 1, it has to be the case that manufacturing firms can pledge anything higher than 50% of sales as collateral. For service sector firms, as $a > 1/2$, for $\phi_w < 1$, $\eta_2 > 1/2$ as well, meaning that at least a 50% of sales can be pledge as collateral.

The next proposition describes the intensive margin of sectoral frictions. Once sectors are constrained, how can the model deliver the negative correlation observed in the data (interaction dummy and elasticities).

Proposition 2 *Suppose sector 2 is constrained ($\mu_2 > 0$). Then, if sector 2 only needs to externally finance intermediate input expenses ($\theta_{12}^m = 1$ and $\theta_2^w = 0$), we have*

- *A higher elasticity ϵ_Q in sector 2 relaxes the constraint, $\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$, if the friction adjusted relative cost of intermediates is high, $\frac{\phi_m}{Z_1} > 1$,*

Also, if sector 2 only needs to externally finance the labor input ($\theta_{12}^m = 0$ and $\theta_2^w = 1$), we have

- *A higher elasticity ϵ_Q relaxes the constraint, $\frac{\partial \mu_2}{\partial \epsilon_Q} < 0$, if the friction adjusted relative cost of labor is high, $Z_1 \phi_w > 1$,*

Proof. First set $\theta_2^w = 0$ and $\theta_{12}^m = 1$, which implies $L_2 = 1 - \eta_2$ and $Q_1 = Z_1 \eta_2$. Using the production function for sector 2 and the first-order condition for L_2 we obtain

$$\mu_2 = \left(\frac{(1-\eta_2)(1-a_2)}{a_2 \eta_2} \right)^{1-\rho_Q} Z_1^{\rho_Q} - 1,$$

where $\rho_Q = \left(\epsilon_Q - 1\right)/\epsilon_Q$. Therefore,

$$\frac{\partial \mu_2}{\partial \epsilon_Q} = \frac{1}{\epsilon_Q^2} Z_1^{\rho_Q} \phi_m^{1-\rho_Q} \ln \left(\frac{Z_1}{\phi_m} \right).$$

If $\frac{Z_1}{\phi_m} = \frac{1}{p_1 \phi_m} < 1$ the derivative is negative, otherwise it is positive. Now set $\theta_2^w = 1$ and $\theta_{12}^m = 0$, which implies $L_2 = \eta_2$ and $Q_1 = Z_1 (1 - \eta_2)$. Again using the production function and the first-order condition for L_2 we obtain

$$\mu_2 = \left(\frac{(1-\eta_2) a_2}{\eta_2 (1-a_2)} \right)^{1-\rho_Q} Z_1^{-\rho_Q} - 1,$$

which implies

$$\frac{\partial \mu_2}{\partial \epsilon_Q} = -\frac{1}{\epsilon_Q^2} Z_1^{-\rho_Q} \phi_w^{1-\rho_Q} \ln (Z_1 \phi_w);$$

if $Z_1 \phi_w = \frac{1}{P_1} \phi_w > 1$ the derivative is negative, otherwise it is positive. ■

Given that in equilibrium $p_1 = \frac{1}{Z_1}$, the terms $\frac{\phi_m}{Z_1} = p_1 \phi_m$ and $Z_1 \phi_w = \phi_w / p_1$ can be interpreted as friction-adjusted relative prices of intermediates and labor, respectively. Therefore, Proposition 2 and Proposition 1 are jointly informative about what changes in Z_1 and η_2 –

that affected p_1, ϕ_w, ϕ_m – we should have observed during the Great Recession for our model to deliver the observed negative relationship between elasticities and constraints.

In particular, as Proposition 1 informed us about $\theta_{12}^m = 1$ and $\theta_2^w = 0$ (or more generally $\theta_{12}^m > \theta_2^w$) and $\phi_m < 1$ for low elasticity manufacturing firms ($\epsilon_Q < 1$), Proposition 2 implies that during the Great Recession manufacturing firms experienced an increase in intermediate input prices (lower Z_1, higher p_1) and/or a decline in the ability to borrow (lower η_2, higher ϕ_m), in a way that $\frac{1}{Z_1}\phi_m > 1$.

Regarding high elasticity service firms ($\epsilon_Q > 1$), Proposition 1 implied that for spreads to be countercyclical and for the model to match the differential relationship between leverage and elasticities, we need $\theta_{12}^m = 0$ and $\theta_2^w = 1$ (or more generally $\theta_{12}^m < \theta_2^w$) and $\phi_w < 1$. Therefore, Proposition 2 implies that during the Great Recession high elasticity service firms experienced a decline in the ability to borrow (lower η_2, higher ϕ_w) that was stronger than the increase in intermediate input prices (lower Z_1, lower $Z_1\phi_w$), in a way that $Z_1\phi_w > 1$.

4.3 Discussion: Model vs. Data

The empirical results in section 3 and the propositions from the model allow us to identify the connection between the elasticities in production (flexibility) and the multiplier on the working capital constraint, both in terms of the extensive and intensive margins. Based on Proposition 1 (extensive margin), for the constraint to be countercyclical (see Figure 1), high elasticity service sectors appear to have frictions in the use of labor-capital, while low elasticity manufacturing sectors appear to have frictions in the use of intermediate inputs.

The key mechanism in Proposition 1 that generates the aforementioned relationship is the increase in the relative cost of intermediate inputs due to lower productivity of suppliers (or tighter credit frictions on suppliers). When intermediates become relatively more expensive, labor-constrained high elasticity service sector firms increase their demand for labor, which increases their borrowing as a fraction of sales. On the other hand, with more expensive intermediates, intermediates-constrained low elasticity manufacturing firms increase their borrowing with respect to sales. Figure (4) and Figure (5) support this hypothesis. Between 2002 and 2008, the relative cost of intermediate inputs with respect to labor-capital increased 20% for manufacturing sector and 6% for service sectors.

Hence, our model predicts that between 2002 and 2008, service sector firms increased their leverage as they moved towards labor-capital (see Figure (2)). This substitution away from intermediates was stronger for relatively more flexible sectors (Proposition 1), which then accounts for the positive correlation between service sector spreads and the interaction between sectors' leverage and ϵ_Q in section 3. For low elasticity manufacturing sectors, the low substitutability between intermediates and labor-capital increased their leverage (see Figure (2)). The increase in leverage is stronger the more inflexible the sector (Proposition 1), which accounts for the negative correlation between manufacturing spreads and the interaction between leverage and ϵ_Q.

Proposition 2 predicts that for spreads and ϵ_Q to display a negative correlation during the Great Recession, it had to be the case that service sector firms received a tightening of credit conditions that was stronger than the increase in intermediate input prices. On the other hand, manufacturing firms had to receive an increase in the intermediate input price and/or a tightening of credit conditions. The fact that in Figure 4 and 5 we observe that intermediate inputs became much more expensive for manufacturing firms than for service sector firms suggests that our mechanism is plausible.

An additional piece of evidence in Table 4.2 suggests that before 2008 the ratios intermediates to value added (M/V) and the ratios cost of intermediates to cost of value added (P^M/V^p) co moved positively for manufacturing firms and negatively for service sectors (consistent with their elasticities). However, during 2008-2009 this relationship, captured by the interaction between a Great Recession dummy DR and relative prices, flips its sign, which suggests a disruption in the use of inputs. Absent of frictions outside the Great Recession, relative input quantities moved consistent with the evolution of relative input prices. However, binding constraints due to increased intermediate input prices and/or credit tightening (lower η_2) during the Great Recession, can generate the observed opposite relationship between relative input prices and effective input quantities. For example, when P^M/V^p increases, flexible service sector firms would decrease M/V. Nevertheless, due to distortions in the use of labor-capital (working capital constraints) during the Great Recession, service sector firms had to inefficiently increase M/V.

5 General Model: Flexibility and Implied Wedges

In this section, we perform a quantitative exercise to investigate whether a general version of our model is able to generate the correlation between sectoral elasticities and sectoral frictions (spreads) we observe in section 3. Rather than fully solving our non-linear occasionally-binding general equilibrium model, we use the firms' optimality conditions to back out the implied wedges. This approach is similar to Bigio and La'O (2016), except that we allow for elasticities and frictions to be heterogeneous across sectors. We use data for 62 U.S. sectors,

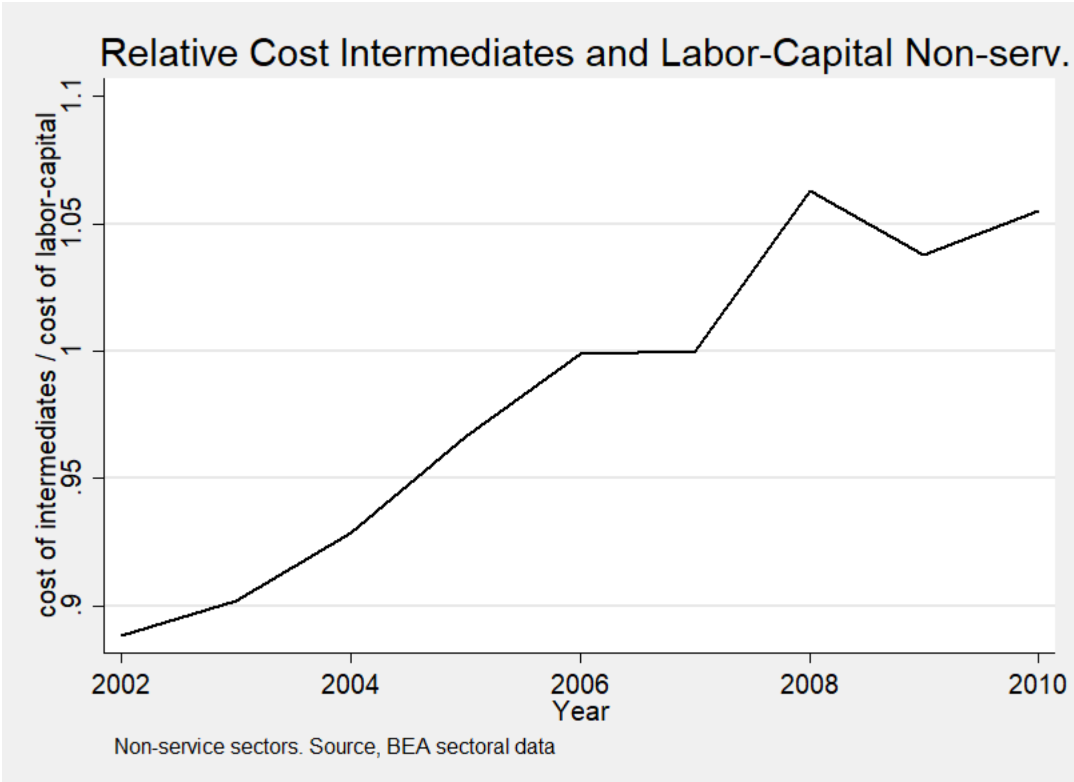

Figure 4
Relative Price Intermediates and Value Added: Manufacturing sectors

excluding government and FIRE sectors, from the BEA.[29] In this exercise we treat the labor and capital bundle (V_j) as a sole input. We focus on the role of ϵ_Q, so we assume unitary elasticity between labor and capital and a unitary elasticity between intermediates from different sectors.

From the sectoral production functions we back out sectoral productivity as

$$Z_j = \frac{Q_j}{\left(a_j^{1-\rho_j} V_j^{\rho_j} + (1-a_j)^{1-\rho_j} M_j^{\rho_j}\right)^{\frac{1}{\rho_j}}},$$

where

$$V_j = \left(\frac{K_{jt}}{\alpha_{jt}}\right)^{\alpha} \left(\frac{L_{jt}}{1-\alpha_{jt}}\right)^{1-\alpha}$$

is the labor-capital bundle. We use quantity-type index series for Q_j, V_j, and M_j from the BEA sectoral database. We choose the year 2007 as our baseline year. In this year we

[29] Same results hold if we add these sectors. We drop them because our theoretical model might not be the best way to describe these industries.

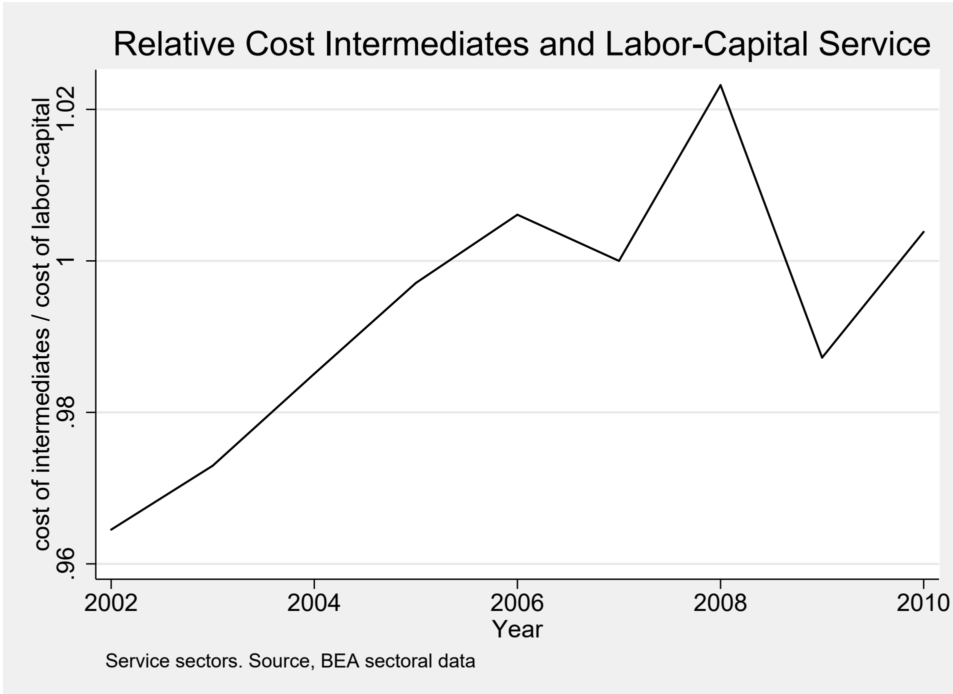

Figure 5
Relative Price Intermediates and Value Added: Service sectors

normalize sectoral productivities by assuming $Z_j = 1$ for all j. The parameter $\rho_j = \frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}$ is set according to our estimated sectoral elasticities. We calibrate a_j to our baseline year, implying that $1-a_j$ is equal to sectors' cost share of intermediates in gross output (see Table 6.1 in the appendix).

We proceed to measure sectoral wedges using the sectors' optimality conditions for labor-capital and intermediate inputs. For example, if sectors have working capital constraints in the use of intermediates, we have:

$$P_{jt}{Z_{jt}}^{\rho_j}\left(\frac{(1-a_j)Q_{jt}}{M_{jt}}\right)^{1-\rho_j} = P_j^M\frac{1+\mu_{jt}}{1+\eta_{jt}\mu_{jt}} = P_{jt}^M\vartheta_{jt}.$$

Given that η_j, the collateral constraint parameter, is smaller or equal than one, whenever the constraint binds there is a wedge ϑ_j, larger or equal than one, that increases the actual cost of intermediate inputs. In the baseline year, we have $P_j = P{j}^M = V^P = 1$ and $Z_j = 1$ for all j, where V^P is the chain-type price index for value added. Thus, in 2007 the sectoral wedges are equal to one and the constraints are not binding. We can then measure sectoral wedges using our proxy for sectoral productivity (Z_j), the observed input shares, our estimated sectoral elasticities, and the observed output and input price indices. The model implied wedges in

Table 4.1
Input Quantities and Prices

VARIABLES	(1) M/V service	(2) M/V non-service
P^M/V^p	-1.239***	0.506**
	(0.228)	(0.251)
$P^M/V^p \cdot DR$	1.387*	-1.156*
	(0.831)	(0.685)
DR	-1.426*	1.133
	(0.836)	(0.722)
Observations	169	117
R-squared	0.162	0.049

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

the use of intermediates and labor-capital are the following:

$$\vartheta_{jt}^{interm.} = \left(\frac{P_{jt}^{M} M_{jt}}{P_{jt} Q_{jt} (1-a_j)}\right)^{\rho_{Q_j}-1} \left(\frac{P_{jt} Z_{jt}}{P_{jt}^{M}}\right)^{\rho_{Q_j}},$$

and

$$\vartheta_{it}^{labor-capital} = \left(\frac{VA_{it}}{P_{it} Q_{it} \cdot a_i}\right)^{\rho_{Q_i}-1} \left(\frac{P_{it} Z_{it}}{VA_t^p}\right)^{\rho_{Q_i}}.$$

Note that changes in our sectoral wedges come from changes in μ_j and η_j. In our model, productivity shocks (Z_j) and financial shocks (η_j) are able to generate changes in the shadow cost of working capital μ_j. The goal of this paper is not to identify which shocks amplified sectoral frictions. Therefore, we simply test whether the implied wedges from our general calibrated model can match the observed facts in section 3.

Using the model implied wedges for the period 2002-2012, we estimate the same regression we estimated in section 3. That is

$$\vartheta_{jt} = \alpha_j + \beta_t + \gamma_1 \cdot D_R + \gamma_2 \cdot \epsilon_{Qj} \cdot D_{Rt} + \gamma_3 \cdot L_t + \gamma_4 \epsilon_{Qj} \cdot L_t + \nu_{jt}, \quad (8)$$

where DR is a dummy for the years 2008 and 2009, L_t is sectoral leverage, and α_j and β_t are sector and year fixed effects, respectively. The wedges, the elasticities, and leverage are in natural logs. Our calibration is inspired by the conclusion from our simple model. In this case, manufacturing firms are constrained in the use of intermediates and service sector firms are constrained in the use of labor and capital. In Table 5.1 column (1) we report the results for our calibration for all sectors together. In column (2) and (3) we split our sample between $\epsilon_Q < 1$ and $\epsilon_Q > 1$ sectors, respectively.

The results in column (1) confirm the predictions of our simple model (Proposition 2). During the Great Recession, sectors with lower flexibility in production had more severe distortions in the use of inputs, consistent with the empirical evidence on spreads and elasticities in section 3. In column (2), we also observe that within the group of low elasticity sectors, relatively flexible and leveraged sectors display lower wedges. On the other hand, in column (3) we observe that for the group of high elasticity sectors, relatively flexible and leveraged sectors display higher wedges. These two facts are also consistent with the predictions of the simple model (Proposition 1) and the evidence in section 3.

Table 5.1
Flexibility, Leverage, and Wedges

	(1)	(2)	(3)
VARIABLES	All	$\epsilon_Q < 1$	$\epsilon_Q > 1$
DR	0.282***	0.286	-0.00536
	(0.107)	(0.194)	(0.00899)
$\epsilon_Q \cdot DR$	-0.110***	-0.169**	0.00779
	(0.0423)	(0.0836)	(0.00941)
$L \cdot \epsilon_Q$		-1.692***	0.225***
		(0.364)	(0.0646)
Observations	682	462	220
R-squared	0.103	0.187	0.235
Number of sector	62	42	20

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

6 Conclusion

In this paper we have provided new empirical evidence that links sectoral technological characteristics – the elasticity of substitution between inputs – with the severity of financing constraints during the Great Recession. First, we find that in the US, service sectors are more flexible in production than manufacturing sectors. Within service and manufacturing sectors there is also important heterogeneity in flexibility. We then study the relationship between sectoral elasticities and sectoral spreads on corporate bonds. The results indicate that during the Great Recession, firms with higher substitutability in production paid lower spreads on corporate bonds. We also observe that highly leveraged service sectors with higher elasticity pay higher spreads, while highly leveraged manufacturing sectors pay lower spreads

We use this evidence to build a multisector model with occasionally binding working capital constraints in the use of labor or intermediate inputs. The model predicts clear connections between sectoral elasticities and the severity and the type of the working capital constraints. In particular, the model signs the relationship between sectoral elasticities and the Lagrange multiplier of the working capital constraint (shadow cost of debt). Similar connections between elasticities an spreads arise in a version of the model where firms face an upward sloping debt schedule (Appendix C). The sign of the relationship depends on: the relative cost of inputs, the importance of the constrained input in production, and how much collateral sectors can pledge.

We interpret our empirical correlations through the lens of our model and conclude that, during the Great Recession, manufacturing firms are mainly constrained in the use of intermediate input, while service sectors firms are mainly constrained in the use labor and capital. A quantitative exercise using the model implied wedges for 62 US sectors confirms the prediction of the simple model, and replicates the observed relationship between elasticities, leverage, and spreads.

There are several avenues for future research. One interesting question is how much of the business cycles fluctuations is driven by sectoral shocks, be they productivity or financial, as opposed to aggregate shocks (an extension of Foerster et al. (2011) and Atalay (2017)). Furthermore, this model displays network pecuniary externalities. Miranda-Pinto (2017) studies the optimal policy implications of this model.[30]

[30] See also Liu (2017).

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Appendix A: Tables and Figures

Table 6.1
U.S. Sectors 2014 (BEA)

Sector Number	Iocode	Sector Name	Capital	Labor	Intermediates	Sales Share
1	111CA	Farms	34%	7%	59%	1.41%
2	113FF	Forestry, fishing, and related activities	30%	42%	28%	0.17%
3	211	Oil and gas extraction	61%	9%	30%	1.39%
4	212	Mining, except oil and gas	48%	14%	38%	0.42%
5	213	Support activities for mining	26%	41%	33%	0.34%
6	22	Utilities	49%	18%	33%	1.35%
7	23	Construction	20%	35%	45%	3.89%
8	321	Wood products	10%	20%	71%	0.32%
9	327	Nonmetallic mineral products	18%	22%	60%	0.38%
10	331	Primary metals	10%	11%	79%	0.91%
11	332	Fabricated metal products	13%	25%	61%	1.22%
12	333	Machinery	14%	23%	63%	1.31%
13	334	Computer and electronic products	35%	34%	31%	1.25%
14	335	Electrical equipment, appliances, and components	16%	27%	57%	0.41%
15	3361MV	Motor vehicles, bodies and trailers, and parts	13%	11%	76%	1.92%
16	3364OT	Other transportation equipment	15%	22%	64%	1.12%
17	337	Furniture and related products	9%	26%	65%	0.23%
18	339	Miscellaneous manufacturing	19%	29%	52%	0.54%
19	311FT	Food and beverage and tobacco products	15%	10%	75%	3.13%
20	313TT	Textile mills and textile product mills	9%	22%	69%	0.18%
21	315AL	Apparel and leather and allied products	6%	21%	72%	0.13%
22	322	Paper products	13%	15%	71%	0.63%
23	323	Printing and related support activities	14%	30%	55%	0.28%
24	324	Petroleum and coal products	19%	2%	79%	2.64%
25	325	Chemical products	33%	12%	56%	2.62%
26	326	Plastics and rubber products	14%	18%	68%	0.75%
27	42	Wholesale trade	35%	31%	34%	5.09%
28	441	Motor vehicle and parts dealers	31%	41%	28%	0.81%
29	445	Food and beverage stores	28%	39%	32%	0.72%
30	452	General merchandise stores	26%	39%	35%	0.72%
31	4A0	Other retail	29%	31%	39%	2.76%
32	481	Air transportation	21%	23%	55%	0.61%
33	482	Rail transportation	27%	25%	48%	0.29%
34	483	Water transportation	18%	11%	71%	0.20%
35	484	Truck transportation	15%	26%	59%	1.07%
36	485	Transit and ground passenger transportation	25%	32%	42%	0.18%
37	486	Pipeline transportation	57%	19%	23%	0.11%
38	487OS	Other transportation and support activities	19%	33%	48%	0.70%
39	493	Warehousing and storage	15%	42%	43%	0.29%
40	511	Publishing industries, except internet	31%	32%	36%	1.07%
41	512	Motion picture and sound recording industries	54%	21%	25%	0.49%
42	513	Broadcasting and telecommunications	36%	14%	50%	2.65%
43	514	Data processing, internet pub., and other inf. servi	19%	24%	57%	0.67%
44	521CI	Federal Reserve banks, credit interm., and rel. act.	38%	32%	31%	2.28%
45	523	Securities, commodity contracts, and investments	4%	47%	49%	1.55%
46	524	Insurance carriers and related activities	26%	28%	47%	2.73%
47	525	Funds, trusts, and other financial vehicles	26%	1%	73%	0.49%
48	HS	Housing Services	90%	1%	9%	5.88%
49	ORE	Other Real Estate	33%	8%	59%	3.09%
50	532RL	Rental and leasing services and lessors of int. asse	46%	10%	44%	1.10%
51	5411	Legal services	33%	39%	28%	0.99%
52	5415	Computer systems design and related services	10%	60%	29%	1.14%
53	5412OP	Miscellaneous professional, scientific, and tech. S	17%	42%	42%	4.00%
54	55	Management of companies and enterprises	8%	48%	44%	1.93%
55	561	Administrative and support services	18%	47%	35%	2.41%
56	562	Waste management and remediation services	19%	28%	53%	0.30%
57	61	Educational services	7%	54%	40%	1.03%
58	621	Ambulatory health care services	13%	50%	37%	3.01%
59	622	Hospitals	6%	46%	49%	2.45%
60	623	Nursing and residential care facilities	7%	53%	39%	0.72%
61	624	Social assistance	9%	55%	36%	0.55%
62	711AS	Performing arts, spectator sports, museums	28%	32%	40%	0.50%
63	713	Amusements, gambling, and recreation industries	24%	33%	43%	0.45%
64	721	Accommodation	30%	32%	38%	0.73%
65	722	Food services and drinking places	16%	36%	48%	2.15%
66	81	Other services, except government	17%	43%	41%	2.07%
67	GFGD	Federal general government (defense)	26%	38%	36%	2.02%

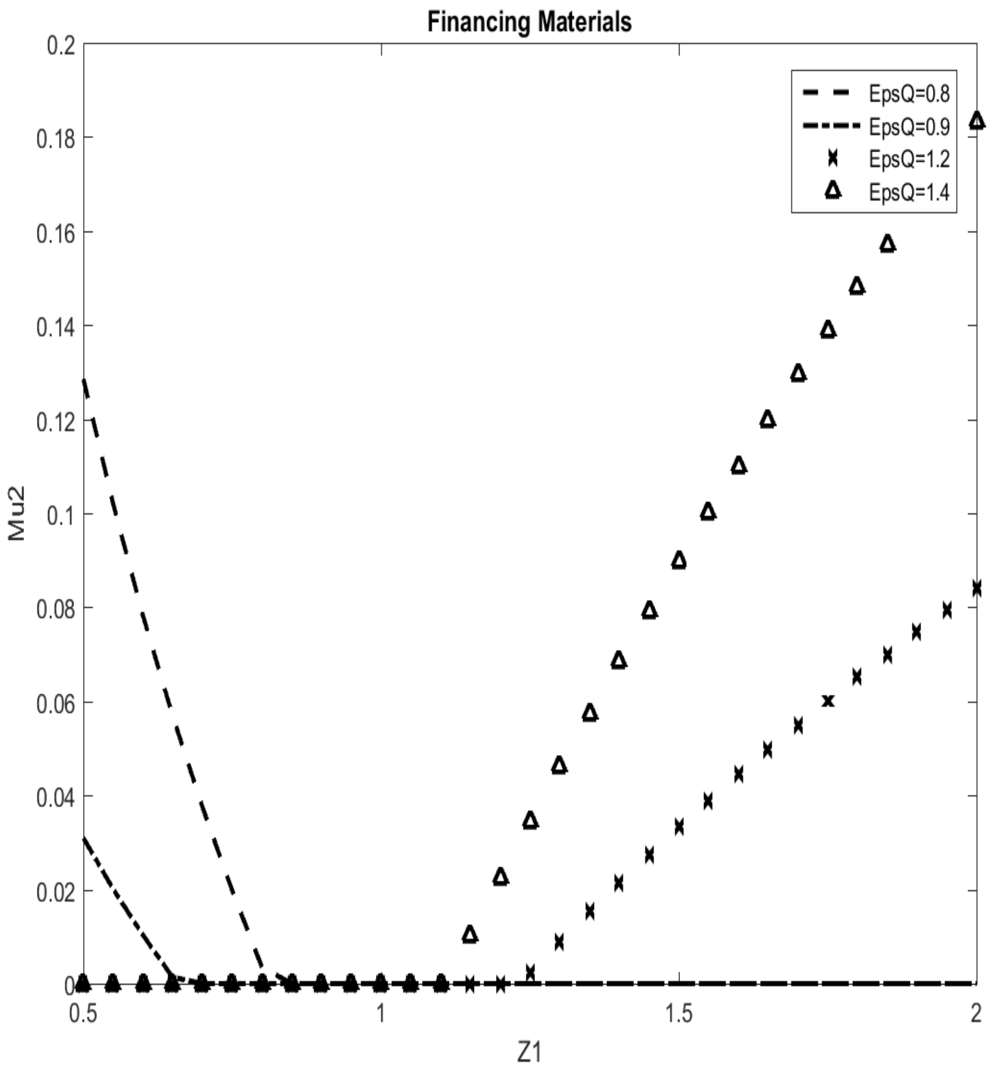

Figure 6
Lagrange Multiplier. Constraint on Intermediates $\phi_m < 1$

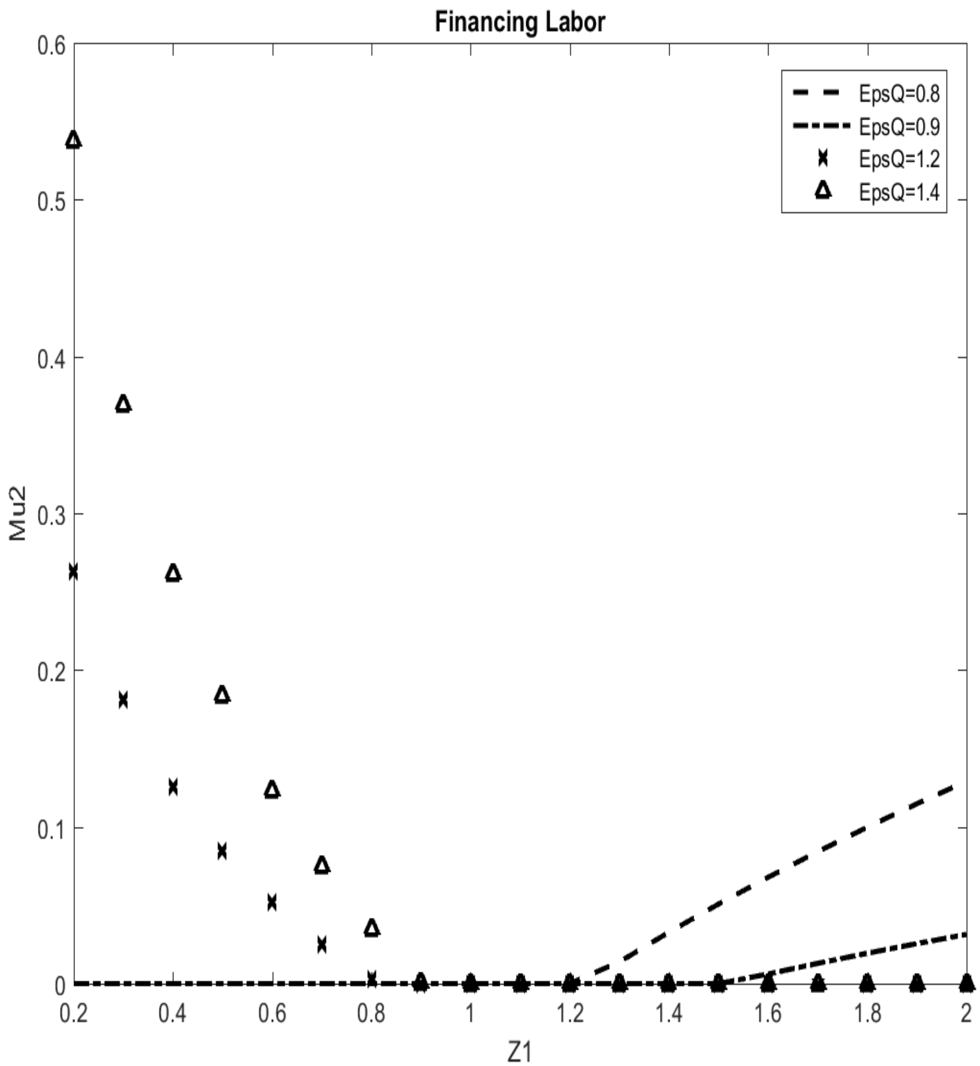

Figure 7
Lagrange Multiplier. Constraint on Labor $\phi_w < 1$

Appendix B: How Important is Endogeneity for the Estimation of ϵ_{Q_j}?

As discussed in the main body of the paper, the estimation of elasticities is biased due to unobserved productivity shocks that are correlated with prices and the input choice. An additional bias in the estimation of the elasticity is the presence of frictions in the use of inputs, e.g., working capital requirements. To evaluate how this bias can affect our results, we use the model as a guide. We simulate series of output, prices, and input demand to estimate the same OLS regressions as in section 2.3.[31]

The model used for the experiment is a more general version of the two sector model in section 2.3.

Firms in the intermediate good sector produce according to

$$Q_1 = Z_1 \left[a_1^{\frac{1}{\epsilon_{Q_1}}} L_1^{\frac{\epsilon_{Q_1}-1}{\epsilon_{Q_1}}} + (1-a_1)^{\frac{1}{\epsilon_{Q_1}}} M_1^{\frac{\epsilon_{Q_1}-1}{\epsilon_{Q_1}}} \right]^{\frac{\epsilon_{Q_1}}{\epsilon_{Q_1}-1}}, \tag{9}$$

where $M_1 = M_{11}^{\omega_{11}} M_{21}^{1-\omega_{11}}$.

Final good firms produce according to

$$Q_2 = Z_2 \left[a_2^{\frac{1}{\epsilon_{Q_2}}} L_2^{\frac{\epsilon_{Q_2}-1}{\epsilon_{Q_2}}} + (1-a_2)^{\frac{1}{\epsilon_{Q_2}}} M_2^{\frac{\epsilon_{Q_2}-1}{\epsilon_{Q_2}}} \right]^{\frac{\epsilon_{Q_2}}{\epsilon_{Q_2}-1}}, \tag{10}$$

where $M_2 = M_{22}^{\omega_{22}} M_{12}^{1-\omega_{22}}$.

The working capital constraints are

$$\theta_1^w w L_1 + \theta_1^m (P_1 M_{11} + P_2 M_{21}) \leq \eta_1 P_1 Q_1 \tag{11}$$

$$\theta_2^w w L_2 + \theta_2^m (P_1 M_{12} + P_2 M_{22}) \leq \eta_2 P_2 Q_2. \tag{12}$$

The market clearing conditions are

$$Q_1 = M_{11} + M_{12}, \tag{13}$$

$$Q_2 = C + M_{21} + M_{22}. \tag{14}$$

[31] Feenstra *et al.* (2014) perform a similar exercise to study how biased are the usual estimates of consumption elasticities that enter in the definition of trade elasticities.

Households solve the same problem as in Section 2.3.

To derive the model counterpart of Equation (5) we solve the cost minimization problem

$$\mathcal{L} = P_j^M M_j + wL_j + \lambda^1 \left(Q_j - Z_j \left[a_j^{\frac{1}{\epsilon_{Q_j}}} L_j^{\frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}} + (1-a_j)^{\frac{1}{\epsilon_{Q_j}}} M_j^{\frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}} \right]^{\frac{\epsilon_{Q_j}}{\epsilon_{Q_j}-1}} \right) +$$
$$\lambda^2 \left(M_j - M_{jj}^{\omega_{jj}} M_{ij}^{1-\omega_{jj}} \right) + \mu_j^C \left(\eta_j P_j Q_j - \theta_j^w wL_j - \theta_j^m \left(P_{we} M_{ij} + P_j M_{jj} \right) \right).$$

The first-order necessary and sufficient conditions for M_j are

$$P_j^M - \lambda \frac{\partial Q_j}{\partial M_j} + \mu_j^C \eta_j P_j \frac{\partial Q_j}{\partial M_j} - \mu_j^C \theta_j^m P_j^M = 0. \tag{15}$$

Rearranging and using the fact that in competitive markets the marginal cost of production in sector j (λ^1) is the price of good P_j, we have

$$P_j^M = Z_j^{\frac{\epsilon_{Q_j}-1}{\epsilon_{Q_j}}} \left(\frac{a_j Q_j}{M_j} \right)^{\frac{1}{\epsilon_{Q_j}}} P_j \frac{\left(1-\mu_j^C \eta_j\right)}{\left(1-\mu_j^C \theta_j\right)}. \tag{16}$$

Let $\overline{\mu}_j = \frac{1-\mu_j^C \eta_j}{1-\mu_j^C \theta_j}$. Raising the previous equation to the power of ϵ_{Q_j}, taking logs, and rearranging we obtain

$$\log\left(\frac{P_{jt}^M M_{jt}}{P_{jt} Q_{jt}} \right) = \log(a_j) + (1-\epsilon_{Q_j}) \log\left(\frac{P_{jt}^M}{P_{jt}} \right) + (\epsilon_{Q_j} - 1) \log Z_j + \epsilon_{Q_j} \overline{\mu}_j. \tag{17}$$

There are two unobserved variables for the econometrician in equation 17. The level of productivity of firms in sector j, Z_j, and the Lagrange multiplier of the working capital constraint in sector j, $\overline{\mu}_j$.

For the Monte-Carlo experiment, the TFP shocks in each sector are assumed to be either *iid* standard normal or having persistence by following an AR(1) process with persistence parameter 0.9. In the first simulation we assume $\epsilon_{Q_1} = 1$, $a_1 = 1$, $a_2 = 0.3$, and $\omega_{22} = 0$. Here we explore the effects of the bias – on what features of the environment does it depend?

In the next experiment we assume $\omega_{11} = \omega_{22} = 0.3$, $\eta_1 = \eta_2 = 1$, $a_1 = a_2 = 0.4$, $\theta_j^w = 0$, and $\theta_j^m = 1$. In this experiment one can study if the rank in terms of production flexibility is preserved. For example, for true pairs of elasticities like ($\epsilon_{Q_1} = 0.3, \epsilon_{Q_2} = 0.8$), is the OLS estimation still preserving the fact that sector 2 is more flexible?

We summarize our results for the bias as (we) there is no bias if the sector under

Table 6.2
OLS Bias

ϵ_Q	0.5	0.65	0.8	0.95	1.1	1.25	1.4
Only Z_1, iid							
OLS uncon	0.5	0.65	0.8	0.95	1.1	1.25	1.4
OLS con	1	1	1	1	1	1	1
(Z_1, Z_2), iid							
OLS uncon	0.93	0.95	0.97	0.99	1.01	1.03	1.05
OLS con	1	1	1	1	1	1	1
(Z_1, Z_2), persistence 0.9							
OLS	0.96	0.969	0.984	0.994	1.011	1.021	1.032
Binding Freq	0.58	0.57	0.57	0	0.42	0.42	0.42

investigation does not experience shocks (only the other sector does); (ii) when estimating only one elasticity, estimates are biased toward 1, even if constraints are not binding, and are exactly equal to 1 if constraints are always binding; (iii) estimates are biased downward when trying to estimate two elasticities if both sectors experience shocks. We conclude from these exercises that endogeneity may be an issue. However, the bias in the estimation does not alter the rank of sectors, in the sense that higher elasticity sectors are always identified relative to lower elasticity sectors. Since this cross-sector comparison is the key to our results, we believe that the endogeneity bias is not critical here.

Appendix C: Working Capital Constraints with Sectoral Spreads

In this section we show the mapping between the value of the Lagrange multiplier of sectoral collateral constraints and sectoral spreads. We assume that when sectors are unconstrained they can obtain intra-temporal working capital loans at the risk free interest rate, which we assume is $R = 0$. After sectors hit the collateral constraint they are able to borrow with an upward sloping debt schedule of the following form:

$$R = \frac{\text{Working capital loan} - \eta\text{Sales}}{Sales},$$

where η determines the ability to pledge sales as collateral for working capital loan at the risk free rate. In this environment, the insights from proposition 2 (extensive margin) are

Table 6.3
OLS Bias

ϵ_{Q_1}	0.5	0.6	0.5	0.8
ϵ_{Q_2}	1.2	1.2	1	1
Only Z_1, iid				
ϵ_{Q_1} OLS uncon	0.5	0.5	0.5	0.5
ϵ_{Q_2} OLS uncon	1.42	1.38	1.42	1.23
Only Z_2, iid				
ϵ_{Q_1} OLS uncon	0.5	0.6	0.5	0.8
ϵ_{Q_2} OLS uncon	1.42	1.36	1.42	1.29
(Z_1, Z_2), iid				
ϵ_{Q_1} OLS uncon	0.45	0.79	0.44	0.94
ϵ_{Q_2} OLS uncon	0.67	0.93	0.67	1.01

exactly the same. When sectors need to externally finance intermediates, and $\epsilon_Q < 1$ and $\phi_m < 1$, more inelastic sectors are more likely to hit the constraint and start paying a premium for their working capital loans. The predictions of proposition 1 will not follow directly. However, the insights are similar. Conditional on sectors being constrained, a negative shock on intermediate suppliers increases the spread more in sectors that are more inelastic. The intuition is the same, these inelastic sectors will have to bear the price increase and increase their working capital loans to finance now more expensive intermediates.

Figure 8 shows how the spread responds to productivity in the case where the constraint applies to intermediates:

$$P_j^M M_j \geq \eta_j P_j Q_j.$$

The higher ϵ_Q, the lower the spread, consistent with the main body of the paper for the case of $\epsilon_Q < 1$. Other cases deliver similar results and are available upon request.

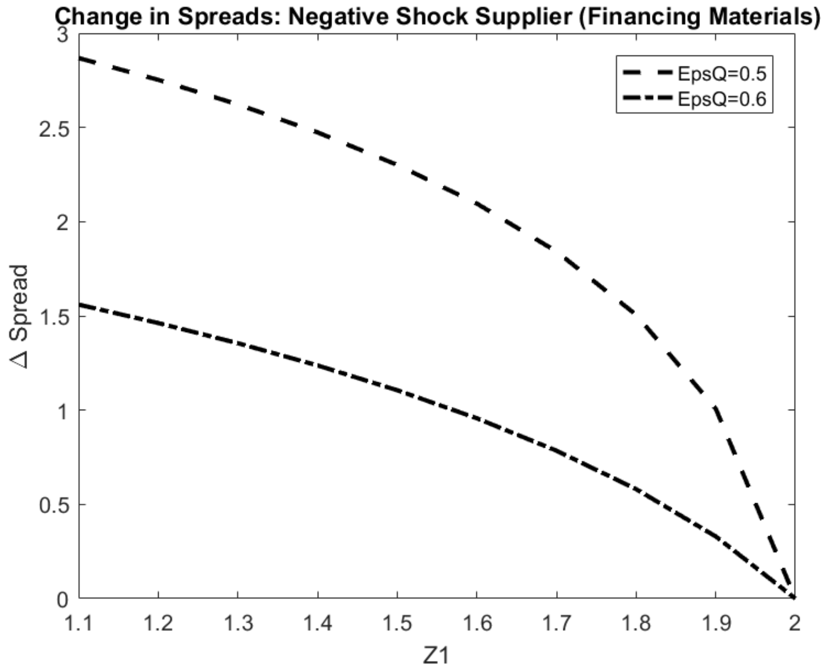

Figure 8
Spread and Elasticity

Appendix D: Additional Tables

Table 6.4
GZ Spreads and IV Grouped Elasticities, 2002-2016

VARIABLES	(1) All	(2) Manufacturing	(3) Service
D_R	0.431***	0.420***	0.431***
	(0.0597)	(0.0634)	(0.0704)
L	0.431***	0.334***	0.576***
	(0.0728)	(0.0912)	(0.0853)
$\hat{\epsilon}_Q \cdot \mathrm{D}_R$	-0.0172*	-0.0623***	-0.0137
	(0.00941)	(0.0162)	(0.0121)
$\hat{\epsilon}_Q \cdot \mathrm{L}$	0.0121	-0.0698***	0.0558***
	(0.0140)	(0.0260)	(0.0203)
Constant	0.914***	0.356	1.097***
	(0.185)	(0.322)	(0.289)
Observations	2,275	877	1,270
R-squared	0.533	0.553	0.555
Number of sector	48	16	29

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1