Fortifying the Banks

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Abstract

In this paper we study the costs and efficiencies associated with government guarantees in a network model of banking. We characterize a minimal set of banks such that every bank has a neighbor - defined using directed edges - in the set. We call this set a fortification. The government is permitted to transfer resources only to those banks in the fortification, similar to historical rules governing deposit insurance. We first explore what features of the banking network lead to small or large fortifications, as well as more costly or less costly fortifications and find that networks that are highly connected but are not concentrated around a few popular lenders are the easiest to fortify successfully. We also find that fortifications are more efficient than bailing out the banks considered too big to fail. Finally, we test the applicability of this method by finding the fortification of a real, historical financial network using data that describes interbank lending in the United States in 1867.

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1. Introduction

The lending relationships between banks form a directed network. Certain types of networks can propagate negative financial shocks and lead to cascading bank failures like those seen in the lead up to the 2008 financial crisis. [1] [13] [4] But what types of networks can be protected from such cascading failures? This paper proposes a method for identifying and stabilizing structurally important banks and characterizes the types of financial networks that can be most successfully protected.

The financial collapse of the 4th largest US investment bank, Lehman Brothers, in 2008 precipitated a global financial crisis and the worst economic downturn in the United States since the Great Depression. [14] One of the reasons this recession became so deep was that the collapse of Lehman Brothers sent a shock wave of cascading financial failures throughout the network of bank-to-bank lending. This network of banks is a critical component of the economy and can cause or protect against a global crisis. [14]

The recession that began in 2008 featured drops in GDP, employment, and household wealth from which countries and families are still recovering. The US real GDP fell by 4.3% between late 2007 and early 2009. [17] The unemployment rate rose above 10% for the first time since the Depression and did not return to pre-2008 levels until 2015. In addition to losing their jobs, households lost 27% of their retirement savings and $19.2 trillion in household wealth. After every economic indicator plummeted, it took an unprecedented amount of time for the economy to recover. [19]

To prevent the Great Recession from becoming the Great Depression, the US government passed the Emergency Economic Stabilization Act of 2008. This was an injection of $700 billion into the financial sector to bailout banks that were considered too big to fail. [10] The government lent the American International Group (AIG) $29 billion to buy Bear Stearns, which had also collapsed in 2008. AIG received a guarantee for further loans up to $85 billion later that year. Additionally, the government guaranteed up to $100 billion each to Fannie Mae and Freddie Mac, major government-backed mortgage lenders, to prevent their bankruptcy. While these bailouts were expensive and controversial, macroeconomists agree that it prevented further cascades of bank collapses, which would have lead to an even worse economic crisis. [10]

The lending relationships between banks form a directed network. In this network, the banks are the nodes, and the loans are the directed edges between them. The particular links that do exist and do not exist between individual banks can have an enormous effect on the macroeconomic outcomes in the financial sector. A change of one edge - one loan - can change the amount of loan dollars that are repaid to the lending banks by an order of magnitude. As a result, the particular network of lending must be taken into account when protecting against the prospect of cascading failures. In this paper, we propose a method for protecting against these cascading failures that is more efficient than bailing out banks considered too big to fail.

We define a fortification to be a set of nodes in a network such that every node in the network has an edge pointing to a node in the fortification. In the context of a financial network, a fortification corresponds to a set of banks such that every bank in the network is borrowing from a bank in the fortification. If the government, which is not a node in the network, can transfer resources to ensure that the banks in the fortification can repay their loans, it can prevent cascades of bank failures. Regardless of where a bank failure occurs in the network, it cannot travel along more than one edge of the network before encountering a stabilized bank. Such a targeted approach to identifying banks and protecting financial networks can improve outcomes as well as decrease costs. This concept exploits the amplification ability of networks: by saving only a small number of banks from failure, the entire network can see a large reduction in bank failures and loan defaults. We investigate which financial networks can be protected with the greatest success.

This paper contributes to the literature using financial networks to understand the strengths and weaknesses of the macroeconomy. It is well documented that financial networks can spread contagion, in the form of cascading delinquent loans, as the result of a negative enconomic shock. [12] [8] [2] But they may also protect against such contagion. Several papers have characterized the robust-yet-fragile nature of financial networks. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) characterize a level of network connectivity for which the network protects against shocks rather than propagating them. [1] The application of network theory to financial questions has advanced our understanding of the macroeconomy and has allowed us to characterize what network structures and characteristics play a role in propagating shocks. [13] [4] May et al. (2008), Caldarelli et al. (2013), and Schweitzer et al. (2009) all address network topology and its affect on aggregate outcomes. [15] [7] See Chinazzi and Fagiolo (2013) for an excellent summary of this body of work. [9] In this paper, we also explore how the structure of financial networks helps or hurts in the face of negative financial shocks but go further in suggesting and studying a method to improve stability.

Since 2008, there has been a great deal of work done in understanding the causes of the Great Recession and preventing its recurrence. In addition to cascading bank failures, there were many intertwined contributing factors. Brunnermeier (2009) analyzes the role that liquidity and credit constraints played. [5] Gorton (2008) gives an in-depth explanation of the subprime mortgage crisis. [11] Sapienza and Zingales (2009) discuss the amplifying role that the erosion of trust played in the worsening of the crisis after the collapse of Lehman Brothers and the bailout of AIG. [18] Swagel (2009) outlines the policy responses to the downturn. [16] Caballero et al. (2013) discusses the reforms of the financial sector going forward. [6] See the Federal Reserve Bank of New York's "Financial Turmoil Timeline," for a detailed description of the evolution of key events and policies. [20]

We use a network model of interbank lending, loan repayment, and protection to analyze the success of fortifications. We treat the network - including banks, loans, and interest rates - as exogenous and use the model of loan repayment described in Acemoglu et al. (2015). [1] The banks in the model borrow money from one another, which forms the directed edges of the network. They also invest in outside projects which have random returns and the success or failure of these projects contributes to the banks' ability to repay their loans. After loans and investments are made, equilibrium loan repayments and project liquidation decisions are determined. If a bank's project yields a low return, the bank may fail to repay its loans in full. This financial failure may lead to that bank's lenders failing in turn. This chain of failures to repay loans is how cascading failures occur.

We identify the smallest possible fortifications for a given financial network. When the banks in the fortification set receive resources to ensure that they repay the entirety of their loans, there are new equilibrium loan repayment amounts and liquidation decisions. A given network may have multiple minimal fortifications of the same size. When this is the case, we identify the fortification that is most successful and preventing delinquent loans. Our primary measure of fortification success is the difference between the dollar value of loans not repaid without the fortification and the dollar value of loans not repaid with the fortification. Depending on the particular network, the minimal fortifications may contain more or fewer banks, it may be more or less expensive to fortify, and it may be more or less successful at preventing delinquent loans. This paper identifies those networks that allow for the most successful, as well as the least costly, fortifications.

To characterize the types of financial networks that can be most successfully fortified, we analyze how the degree distributions, average path length, and clustering coefficient affect the number of banks needed to form a fortification of a given network and how successful that fortification is at preventing bank failures. The degree distributions describe the number of lenders and borrowers of each bank in the network. The path length measures how close one bank in the network is to any other; it describes how spread out the network is. The clustering coefficient is also a measure of network connectivity, but describes how concentrated the banks are around a few, very popular lenders or borrowers. We use the first measure of connectivity, path length, to capture how interconnected every part of the network is to every other part. If we wanted to walk along the links of the network from one bank to another, how many links would we be walking along, on average? We include the second measure of connectivity, the clustering coefficient, to measure the degree to which banks have separated into cliques. If we fortify a bank in a tightly grouped clique, is the benefit of that fortification going to get trapped in the the clique and never make it out to other banks in the network?

To understand fortification outcomes for many different financial networks, we simulate the model computationally by generating example networks and fortifying them. In each repetition of the simulation a random financial network is generated, including loans and loan amounts. Each financial network is *regular*, meaning that all of the loans are for the same amount and have the same interest rate. We do this to ensure that loan repayment outcomes are driven by network structure and not loan characteristics. For each network, we measure the degree distribution, average path length, and clustering coefficient. The unfortified repayments and liquidation decisions are determined. All possible minimal fortifications that contain the smallest number of banks for that particular network are found. For each of these fortifications, the associated new repayments and liquidation decisions are then computed.

The simulation results indicate that fortifications of most networks do not require many banks. Across all of the simulation repetitions, the average fortification size was only 3.26 banks. Perhaps counter-intuitively, networks with a few large banks that lend to many others, are the most difficult to protect and those without such hubs were the easiest to protect. Those with a shorter average path length and and smaller clustering coefficient had smaller fortifications. The networks that allowed for the smallest fortifications were those that were highly interconnected but not tightly clustered around a few large banks. That is, networks that have many connections that are distributed evenly amongst the banks, without separated clusters of banks, are the easiest to fortify with a small number of banks.

The simulations also showed that, in addition to requiring only a few banks, fortifications are worth the cost. Every fortification in the simulation saved more money in repaid loans than it cost to fortify the banks in it, and on average they saved over $2 for every dollar spent. Furthermore, it is the same types of networks that allow for small fortifications that can be fortified with the greatest success. Financial networks that are closely connected but not tightly clustered see the largest improvements in loan repayment. These results suggest that the interbank lending networks that are very interconnected but not concentrated around a small number of hub lenders are those that can be protected with the fewest fortified banks and with the greatest success.

To analyze the effectiveness of fortifications relative to other methods of stabilizing the financial network, we compare their performance to two alternative methods. The first alternative is to simply choose a random set of banks and cover their shortfalls. The financial process is the same as in a fortification but ignores the structure of the network in choosing the banks. Fortifications outperform these randomly chosen banks. On average, fortifications save more money - $670 million more on average - and prevent more banks from failing.

We also compare the performance of fortifications to a set of the most connected banks. These banks correspond to those that would be considered "too big to fail." Fortifications are more efficient than a set containing the most connected banks. They save more money in repaid loans per dollar spent. On average, fortifications are about 30% more efficient. By taking advantage of the entire network structure rather than simply considering which banks have the most lending partners, fortifications are a more frugal but still successful method of protecting the financial network from cascading failures.

We demonstrate the real world applicability of the fortification by finding the minimal fortification of a historical financial network. We use data on interbank lending relationships in Pennsylvania and New York in 1867 presented in Anderson, Paddrik, and Wang (2019). [3] This financial network is sparse; there are relatively few loans and the banks in the network are not very close to one another in terms of network distance. As a result, the fortification of this network is large. To fortify the 54 banks who borrow from other banks requires a fortification set that contains 37 nodes. This is consistent with the simulation results: networks that are spread out and not relatively interconnected require larger fortifications.

Fortifications work well on average but they can either work extremely well or extremely poorly depending on the particular financial network in question and the individual banks that are fortified. The same network can see an improvement in loan repayment as high as $5 billion or see no improvement at all, depending on the banks included in the fortification and on the structure of the network. Therefore it is critical to find and analyze all of the possible minimal fortifications and investigate which networks can be most successfully protected.

The remainder of this paper proceeds as follows. Section 2 describes the model of the interbank network, lending, repayment, and fortification. Section 3 describes simulations of this model and the results of these simulations, including comparisons to alternative methods of network stabilization. Section 4 presents a case study in which we find the fortification of a real, historical, financial network. Section 5 concludes.

2. Network Model

2.1. Model of Lending and Repayment

We use a model of interbank lending that describes the loans between banks using a directed network. The banks are represented by the nodes of the network and the directed edges represent loans from one bank to another.

A network G consists of a set J of nodes indexed $j = 1 \dots n$ and a set of directed edges between them. Let ij denote a directed edge from node j to node i. Figure 1 depicts a network with 10 nodes and 31 edges. We use $N_j^+(G)$ to describe the set of edges in G that point *to* node j and $N_j^-(G)$ to describe the set of edges in G that point *away from* node j.

$$N_j^+(G) = \{ji \in G\}$$

$$N_j^-(G) = \{ij \in G\}$$

Let $N_j(G)$ be the set of all neighbors of node j regardless of edge direction, $N_j(G) = N_j^+(G) \cup N_j^-(G)$. For example, in the network depicted in Figure 1, $N_2^+(G) = \{4, 8, 10\}$, $N_2^-(G) = \{4\}$, and thus $N_2(G) = \{4, 8, 10\}$.

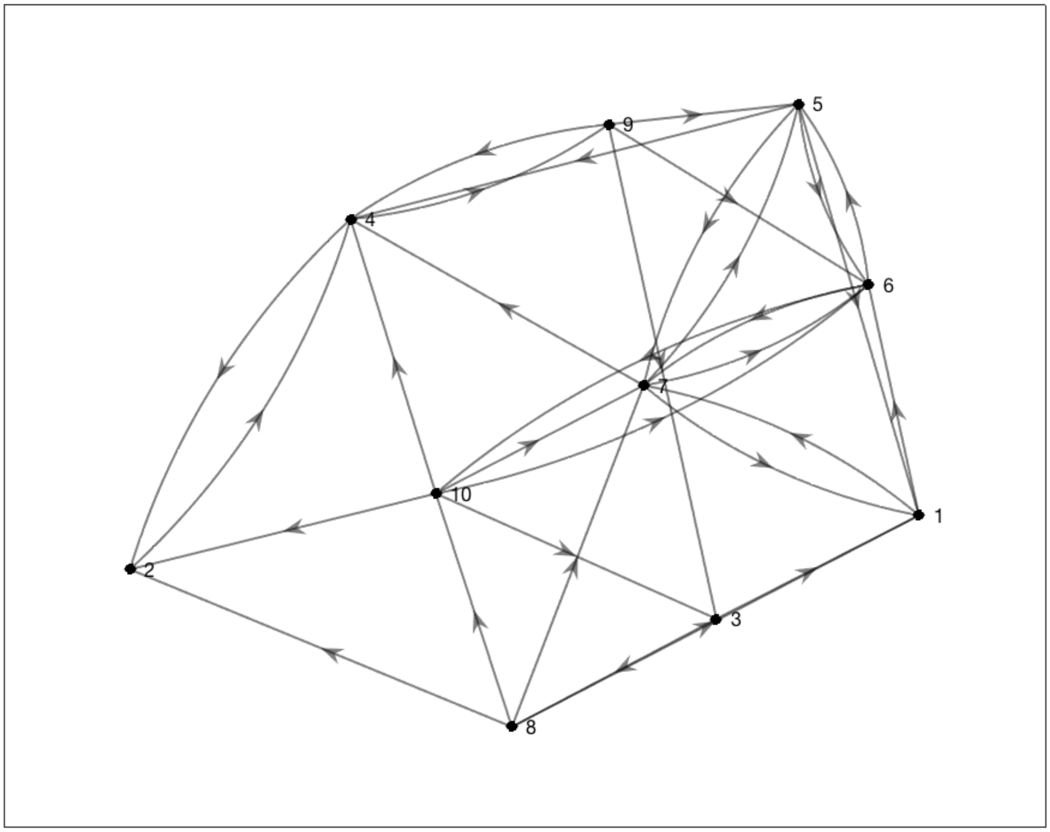


Figure 1: Network of Lending

The agents of the model are n banks, represented by the n nodes, which borrow money from one another and make liquidation decisions about investment projects based on their ability to repay their loans. A directed edge from bank j to bank i indicates that bank j borrowed from bank i and now must repay the loan with interest to bank i. That is, arrows indicate the flow of repayment. We use the model of loan repayment described in Acemoglu et al. (2015). [1] Each bank in the model borrows money from at least one other bank and has invested in an outside project. Each bank j is endowed with k_j dollars of capital that are allocated between loans and investment projects. Whatever capital is not loaned to other banks or invested in an outside project is held by the bank in cash. The projects yield random returns which determine how able each bank is to pay back its loans.

The network of loans is described using a matrix, $Y_{ij} = [y_{ij}]$, containing the face values of the loans, i.e., what must be repaid. Each element, y_{ij}, of this matrix describes the amount that bank j owes to bank i in repayment after borrowing a loan amount of l_{ij}. That is $y_{ij} = (1 + \rho_{ij})l_{ij}$, where ρ_{ij} is the interest rate on the loan from i to j. If bank j did not borrow from bank i then $y_{ij} = 0$. We do not allow for self loops so $y_{jj} = 0 \; \forall j \in J$. Let $y_j = \sum_i (y_{ij})$. This is the total amount that bank j owes in repayments to lenders.

Figure 2 depicts the same network as Figure 1, but with edge weights indicating loan amounts. Each loan is for \$100 and has an interest rate of 2.7%, thus each loan has a face value, $y_{ij} = 102.7$.

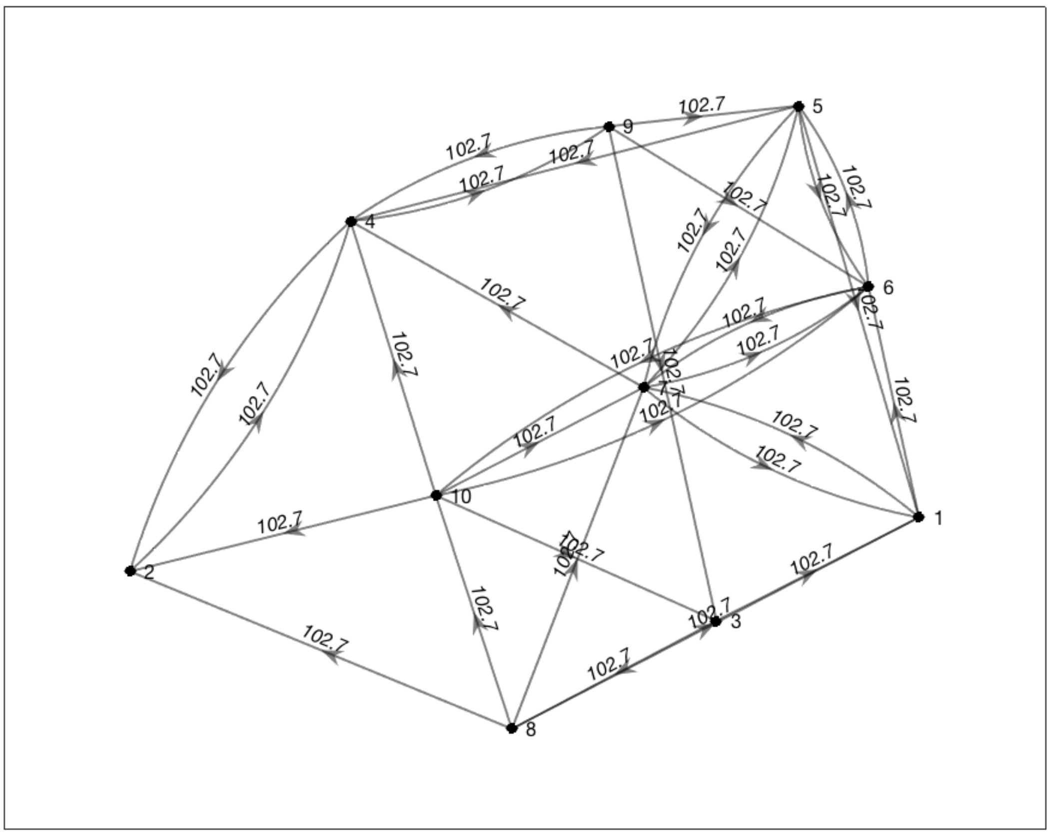


Figure 2: Network of Lending with Loan Amounts

Each bank, j, invests in an outside project and the success or failure of these projects determine the banks' abilities to repay their loans. The bank observes a preliminary random return, z_j, on the project and decides whether to liquidate or not. If they choose not to liquidate, they earn a fixed, non-pledgeable payout of A_j. If they choose to liquidate, they can recoup a fraction, ξ_j, of this yield.

Each bank holds an amount, c_j, of their funds in cash and has an outside obligation to their senior creditors, v_j, that they must pay before they repay their loans. This primary obligation can be interpreted as the bank's costs of operation, including wages, rent, etc. Label the bank's available resources at the time of repayment if they do not liquidate as $h_j = c_j + z_j + \sum_i r_{ji}$.

First loans and investments are made. The network of loans is treated as exogenous. Then, random returns are observed, liquidation decisions are made, and loans are repaid. Finally, any projects held to maturity yield their return, A_j. All repayment and liquidation decisions by all banks are made simultaneously. Banks take their future yields into account, but make repayment decisions before they are recieved. Following Acemoglu et al., the equilibrium repayment amounts are determined by

$$r_{ij} = \frac{y_{ij}}{y_j} \max\big\{ \min\{y_j, h_j + \xi_j L_j - v_j\}, 0\big\}$$

and the liquidation decisions are determined by

$$L_j = \max\big\{ \min\{\frac{1}{\xi_j}(v_j + y_j - h_j), A_j\}, 0\big\}.$$

There is an equilibrium repayment amount, $r_{ij} \in [0, y_{ij}]$, for each loan and a liquidation decision, $L_j \in [0, A_j]$, for each bank. If a bank has the resources to meet all of its liabilities at the time of the repayment, i.e. $h_j > v_j + \sum_i (y_{ij})$, then all loans are repaid in full, $r_{ij} = y_{ij}$, for each bank i to which bank j owes a repayment. If $h_j \leq v_j + \sum_i (y_{ij})$, the bank must either partially liquidate its project to cover the difference or liquidate entirely and pay back what it can. If a bank is able to meet its senior obligation, v_j, but not its loans, the repayments are made in proportion to their face values.

These equations describe how funds - or lack of funds - travel between banks. If bank j is unable to repay its loans in full, this shortfall propagates to bank j's lenders and they in turn may not be able to pay their loans in full, and so on throughout the network. Figure 3 depicts the equilibrium repayment amounts for the network of loans depicted in Figure 2 and the model paramterization used in Section 3. Many of the banks in this example are unable to repay their loans in full. For example, Bank 9 is only able to pay Bank 5 \$26.89 of the \$102.7 that they owed.

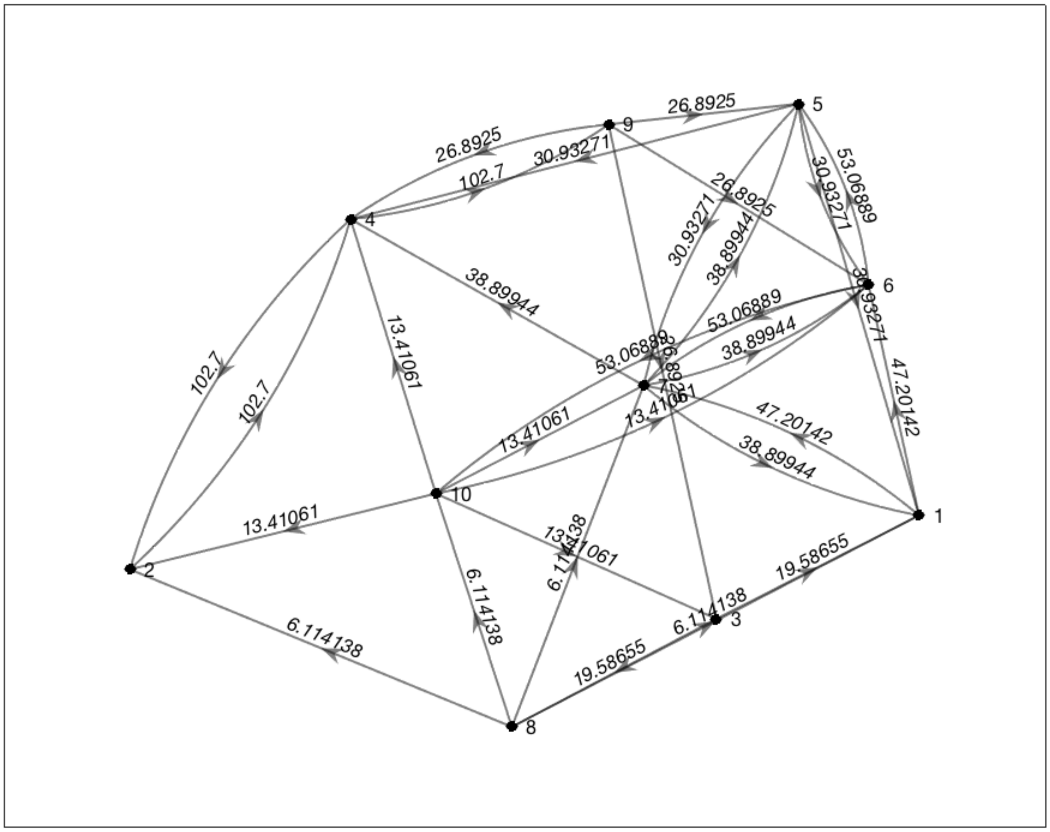


Figure 3: Repayment Equilibrium

A *bank failure* occurs when any bank is unable to repay its loans in full. Any such bank failure can lead to a cascade of failures as the lenders who are not repaid in full in turn are unable to repay their own loans in full. The repayments are determined simultaneously so these cascades do not occur sequentially in time, but rather result from the interrelated nature of the lending network. Let d denote the total dollars not repaid in the payment equilibrium for a given network, $d = \sum_{j=1}^{n} \left(\sum_{i \neq j} y_{ij} - r_{ij} \right)$. In the repayment equilibrium depicted in Figure 3,

$d = \sum_{j=1}^{10} \left(\sum_{i \neq j} 102.7 - r_{ij} \right) = 2,104.4$. Of the $102.7 \times 31 = 3,183.7$ dollars owed in loan repayment, $2,104.4$ of those dollars were not repaid in this example.

2.2. Fortifications

To limit these cascading bank failures, we delineate a set of nodes to bolster in the face of financial difficulty. We define a *fortification* to be a set, F, of nodes in a financial network such that every node in the network has an edge pointing from it to a node in F. This set is similar to the open-locating dominating (OLD) sets used in the graph-covering literature in mathematics. See Kincaid, Oldham and Xu for an example. Fortifications differ from OLD sets in that (1) they are defined for directed networks rather than undirected networks, and (2) they specify a direction (in-pointing) for the neighbor in the covering set. In the network depicted in Figure 1 the nodes 2 and 3 constitute a fortification.

The government transfers resources to the banks in the fortification to ensure that they pay their loans in full. Therefore, regardless of where a financial failure occurs, it can travel no further than one lending relationship before it encounters a fortified bank, thereby stemming the flow of financial failures from bank to bank throughout the network. Note that this does not ensure that a failure stops after one link of the network. The definition of a fortification merely ensures that the financial failure will encounter at least one fortified bank for every link it travels. Every bank has a fortified lending partner, so for every additional link along which the failure cascades, it is guaranteed to encounter at least one more fortified bank.

The *minimal fortification* is the smallest possible set of nodes that satisfies the definition of a fortification. There may be multiple minimal fortifications of the same size for any given network. For example, the network depicted in Figure 1 has four minimal fortifications, each containing four nodes: $\{4, 7, 8, 9\}$, $\{2, 4, 7, 8\}$, $\{1, 4, 7, 9\}$, and $\{1, 2, 4, 7\}$. The smallest possible fortification for any network must contain at least two nodes. This is because no bank lends to itself. So if a single bank (node) lends to every other bank - and therefore satisfies the definition of a fortification for (or *covers*) every other node in the network - the fortification will still need one more node to cover the original node.

The *cost* of a fortification is the total difference between what the fortified banks owe and their equilibrium repayment amounts.

$$cost^F = \sum_{f \in F} \Big(\sum_{j \neq f} (y_{jf} - r_{jf}) \Big)$$

The total cost of a fortification depends on the number of banks in the fortification as well as how far those banks are from repaying their loans in full. For a given network, two fortifications of the same size may have widely different costs. For example, the cost of the first fortification of the network depicted in Figure 1, $\{4, 7, 8, 9\}$, is \$944.78. Additionally, the cost of a fortification may be 0. This occurs when the particular banks in the fortification already pay their loans in full, regardless of whether other banks in the network are able to repay their loans in full.

There is a new payment equilibrium associated with any fortification. Denote the repayment and liquidation decisions associated with a particular fortification, F, be $\widetilde{r_{ij}}^F$ and $\widetilde{L_j}^F$, respectively. Figure 4 depicts the fortified repayment equilibrium using the first fortification, $\{4, 7, 8, 9\}$. The nodes in the fortification are designated with larger node markers and in red. Following the same notation, define $\widetilde{d}^F$ to be the total dollars not repaid in the fortified payment equilibrium. Before the fortification, \$2,104.4 were not repaid. After the fortification, only \$459.24 are not repaid. In the fortified payment equilibrium, no bank repays *less* than they did in the unfortified network. That is, $\widetilde{d}^F \leq d$.

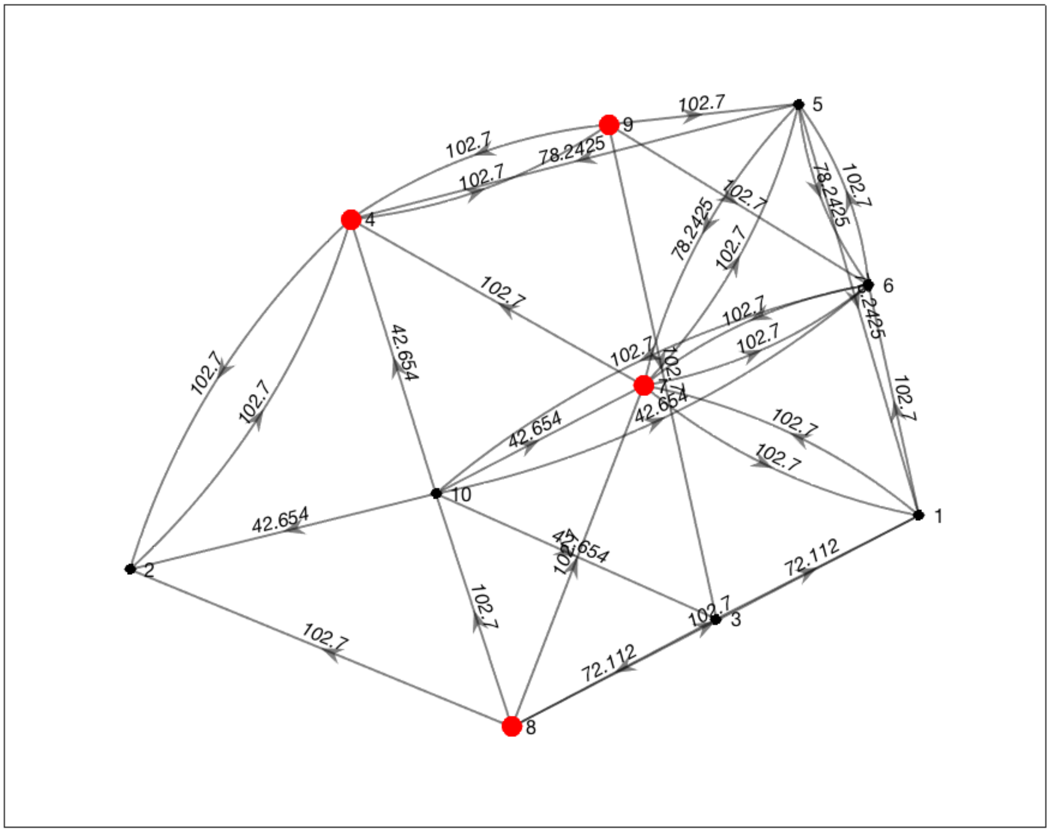


Figure 4: Fortified Repayment Equilibrium

How successful a fortification is at stymieing financial failures varies between different fortifications of a given network. Define the *success* of a fortification, s^F, to be the difference between the total dollars not repaid in the original payment equilibrium and the total dollars not repaid in the fortified payment equilibrium: $s^F = d - \widetilde{d}^F$. The success of the fortification used in the previous example is $s^F = d - \widetilde{d}^F = 2104.4 - 459.24 = 1,645.16$.

The ratio of fortification success to fortification cost describes the dollars saved per dollar spent on a fortification. Define the *efficiency* of a fortification to be the this ratio: $e^F = \frac{s^F}{cost^F}$. The most efficient fortifications are those that lead to a large amount of money being repaid that would not have been repaid without the fortifications while simultaneously costing the government as little as possible. The efficiency of the fortification used in the previous example is $e^F = \frac{s^F}{cost^F} = \frac{1645.16}{944.78} = 1.74$, so this fortification saved \$1.74 for each dollar spent.

2.3. Network Characteristics

The number of fortifications, the cost of each fortification, and the success of each fortification all depend on the structure of the financial network in question. We investigate how these outcomes depend on several different network characteristics: in-degree and out-degree distributions, path distance, and clustering coefficient.

The *in-degree* of a node is the number of edges pointing to that node: $in - deg_i = |N_i^+(G)|$. The *out-degree* of a node is the number of edges pointing from that node: $out - deg_i = |N_i^-(G)|$. The in-degree and out-degree distributions are the sets of in-degrees and out-degrees for every node in a network G, respectively. In a financial network, the mean of the in-degree distribution is the average number of borrowers each bank has, while the mean of the out-degree distribution is the average number of lenders each bank has.

In addition to the mean, we consider the variance and skewness of the degree distributions, as well. The variance describes how widely the number of lenders per bank and the number of borrowers per bank varies. A higher variance indicates that some banks have very few lenders/borrowers and some banks have many. The skewness describes how skewed the degree distribution is. A positive skewness indicates that the distribution is skewed left - many banks have a low number of lenders or borrowers, while a negative skewness indicates that the distribution is skewed right - many banks have a high number of lenders or borrowers.

The undirected[1] shortest path between two nodes in a network is the smallest sequential set of nodes and edges between those two nodes, regardless of the direction of the edges. We use the average of all of these shortest paths to measure how widely spread the nodes of the network are. The path distance of the network depicted in Figure 1 is 1.4286, so on average it takes between 1 and 2 links to get from one node in the network to another. A longer average path distance indicates that the network is more widely spread; it takes more edges, on average, to get from one node to another. In a financial context, a longer average path distance indicates that the network of loans is less interconnected in general.

The undirected local clustering coefficient describes how tightly grouped, or *cliqueish*, the nodes

[1] We use the undirected path distance and clustering coefficients because they are always defined for a connected directed network, while their directed counterparts may be infinite or undefined for such a network

of a network are. Specifically, for a given node in a network, the individual clustering coefficient computes the fraction of possible triangles that are actually present in the network. For a node i, suppose that there is an edge between i and j as well as between i and k (ignoring direction). The clustering coefficient of i computes the fraction of the time that there is an edge between j and k, as well. Let $\{ij\}$ be the undirected counterpart of an edge either from i to j or from j to i.

$$cc_i(G) = \frac{\#\{\{jk\} \in G : k \neq j, j \in N_i(G), k \in N_i(G)\}}{\#\{\{jk\} : k \neq j, j \in N_i(G), k \in N_i(G)\}}$$

The local clustering coefficient is then the average of these individual clustering coefficients across all nodes in the network, G.

$$cc(G) = \frac{\sum_{i \in J} cc_i(G)}{n}$$

The clustering coefficient for the network depicted in Figure 1 is 0.5921, so the third edge of a possible triangle occurs about half of the time. A larger clustering coefficient in a financial network indicates that if two banks have a lender or borrower in common, those two banks are more likely to lend or borrow from one another.

In the next section, we explore the connection between these network characteristics and the size, cost, and success of fortifications.

3. Simulation Results

We simulated the model of loan repayment and financial network fortification described in the previous section. We generated negative financial shocks and computed the network characteristics described in Section 2.3 to understand what role these characteristics play in the ease of fortifying the financial networks.

In each simulation repetition, we generated a random financial network with 25 banks. With no self-loops, a network of 25 nodes may have up to $25 \times 24 = 600$ edges. In our simulation, three different numbers of edges were used: in one third of the repetitions 25% of the possible edges were used, in one third 50% of the possible edges were used, and in one third 75% of the possible edges were used. We simulated 100 repetitions for each number of edges, for a total of 300 repetitions. Of the 600 possible edges, the appropriate number were selected randomly and uniformly without replacement. In 2 of the repetitions, the network that was generated was not fully connected and therefore those repetitions was dropped from the sample, for a total of 298 random financial network observations.

Each edge in the network represents a loan from one bank to another. Each loan was for \$100 million and the interest rate on each was 2.7%. Because each value of $[y_{ij}]$ is the same, the financial network is said to be *regular*. We used regular networks so that changes in outcomes were strictly driven by differences in network structure rather than by the loan values or the interest rates. The details of the model parameterization can be found in the Appendix.

In each of the repetitions, we found all of the minimal fortifications for the network and the cost and success of each fortification. We recorded the in-degree and out-degree distributions - including the mean, variance, and skewness of each distribution - as well as the average path length and clustering coefficient of each network. We investigate the relationship between these characteristics and the size and success of the fortifications.

3.1. Fortification Size

The average size of the minimal fortification across the 298 repetitions was 3.26, which represents 13% of the 25 banks in the network. The smallest possible size of a fortification, 2, was achieved in 35.91% of the repetitions. The percentage of fortifications containing 2 banks increases with the number of edges present in the network. None of the fortifications of the networks with 25% of the edges contained only 2 banks, 2.35% of the networks with 50% of the edges contained 2 banks, and 33.56% of the networks with 75% of the edges contained only 2 banks.

The cost of fortifications - the amount by which the fortified banks were short on their loans - varied widely across simulation repetitions as well as within individual networks. The mean cost across all fortifications in all repetitions was \$652.39 million, or about six times the loan size. The average cost of the *most* expensive fortification available for a given network was \$1.25 billion, or about ten times the loan amount, while the average cost of the *least* expensive fortification available for a given network was only \$269.30 million. It is common for the least expensive fortification to be zero-cost; the modal cost of the least expensive fortification available for a given network was 0.

On average, there were 31.88 different minimal fortifications for each network. There were as few as one single fortification for a particular network and as many as 276.

Below are the results of a linear regression of fortification size on the network characteristics described in the previous section and indicator variables for the number of edges used. The 50%

case is omitted as the base case, so the coefficient on the 25% edge indicator variable can be interpreted as the effect of having relatively few edges present in the network and the coefficient on the 75% edge indicator can be interpreted as the effect of having relatively many edges present. The mean of the degree distributions is omitted because it always takes on the same value for a given number of edges and is therefore perfectly collinear with the edge indicator variables.

Table 1: Fortification Size Regression Results

Network Characteristic	Coefficient
Edge % = 0.25	1.5284
Edge % = 0.75	−0.9976
In-Degree Dist. Std. Dev.	−0.2646
In-Degree Dist. Skewness < 0	−0.1019
Out-Degree Dist. Std. Dev.	0.0580
Out-Degree Dist. Skewness < 0	−0.1111
Average Path Length	2.1254
Clustering Coefficient	2.0205
Constant	−0.7118

As the number of edges in the financial network increases, the number of banks in the fortification falls. Compared to the base of half of the possible edges, a relatively small number of edges, 25%, leads to a larger fortification. On average, the fortification contains 1.53 more banks. In the other direction, when there are 75% of the possible edges, the fortification contains fewer banks, 1.0 banks on average. When there are more loans throughout the the network, it is easier to fortify with fewer banks.

The presence of many hub lenders and borrowers decreases the size of the minimal fortification. A negative skewness indicates that the degree distribution is skewed right. This corresponds to a larger number of banks lending to or borrowing from many other banks. A negative skew in the in-degree distribution decreases the fortification size by 0.10 banks, on average. Similarly, a negative skewness in the out-degree distribution decreases the number of banks in the fortification by 0.11 on average.

A financial network that is more connected but not tightly clustered allows for smaller fortifications. When the average path length increases by one link, the fortification grows by 2.13 banks on average. When it takes longer to get from one node in the network to another - when the network is more spread out - the fortification is larger and when the network is less widely spread, it allows for a smaller fortification. When the network is more tightly clustered around a small number of lenders or borrowers, the fortification is larger. A one unit increase in the clustering coefficient is associated with an increase in fortification size of 2.02 banks. These results indicate that a network that is relatively well connected but in a uniform way, without hubs, will allow for smaller fortification.

3.2. Fortification Success

The amount of money that a fortification saves in loans that are now able to be repaid that were not repaid before also varies widely across and within networks. A given network can be fortified very successfully or very unsuccessfully depending on the particular banks used in the fortification. On average, the most successful fortification for a given network leads to $3.05 billion being repaid that were not repaid without the fortification. The least successful fortifications, however, only saved $798 million on average and only saved a few cents in some cases. The standard deviation of the fortification success is on average $615 million; there is a wide spread in the success of a fortification.

Successful fortifications are often costly. The most successful fortifications for a given network cost an average of $1.24 billion. In 247 of the 298 repetitions, the most successful fortification was also the most expensive fortification. In only 27 repetitions was it the least expensive fortification. However, every fortification saved more money than it cost to fortify. Every fortification in the sample had an efficiency greater than 1 and, excluding zero-cost fortifications, the average efficiency was 2.50, meaning that on average a fortification saved over $2 in newly repaid loans for every dollar spent to fortify the banks in it.

Below are the results of a linear regression of the amount of money saved by the most successful fortification on the fortification size, the indicators for the number of edges, and the network characteristics. Because the dependent variable, fortification success, is measured in millions of dollars, coefficients can be interpreted as the increase or decrease in money saved in millions of dollars.

Table 2: Fortification Success Regression Results	
Network Characteristic	Coefficient
Fortification Size	1966.5
Edge % $= 0.25$	-4521.8
Edge % $= 0.75$	3424.1
In-Degree Dist. Std. Dev.	209.02
In-Degree Dist. Skewness < 0	52.63
Out-Degree Dist. Std. Dev.	965.22
Out-Degree Dist. Skewness < 0	-266.64
Average Path Length	-7919.5
Clustering Coefficient	-17942
Constant	15769

As the number of edges present in the financial network increases, so does the success of the fortifications of the network. A network with only 25% of the possible edges saves \$4.52 billion less relative to the base case of 50% of the edges, while a network with 75% of the possible edges saves \$3.42 billion more than a network with 50% of the edges. When there are more edges in the network to connect the banks, the fortification can save more money.

The presence of many hub lenders increases the success of fortifications while the presence of hub borrowers increases it. When a network features a negative skewed in-degree distribution - many banks that lend to many other banks - fortifications save \$52 million more relative to a positive skew or no skew. However, when a network features a negatively skewed out-degree distribution - many banks that borrow from many other banks - fortifications save \$267 million less.

The effect of the average path length and clustering coefficient on fortification success reflect the fortification size results. A more closely connected network that is not tightly clustered will be more successfully fortified. An increase in the average path length of one link is associated with an decrease in new repayment of \$7.92 billion, so a longer path length is associated with lower savings and a shorter path length is associated with higher savings. A one unit increase in the clustering coefficient is associated with a decrease in new repayment of \$17.94 billion. A network that is not widely spread out and not disproportionately clustered around a few key banks will have more successful fortifications. Such a network is likely to have a small fortification that saves a great deal of money in previously unpaid loans.

In general, if a network characteristic leads to smaller fortifications, it also leads to more successful fortifications. That is, if the coefficient is negative in the fortification size regression, it is positive in the fortification success regression. However, the out-degree distribution is an exception to this. More hub borrowers lead to smaller fortifications but they also lead to less successful fortifications.

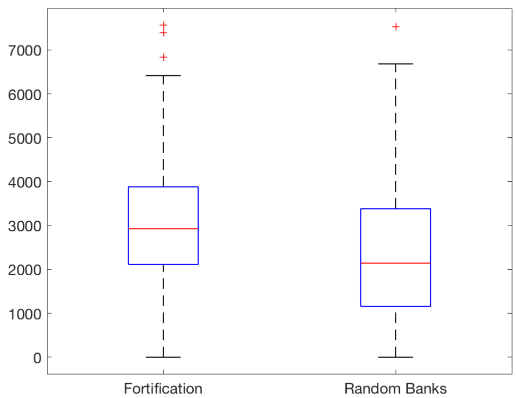
A fortification with more banks in it generally saves more money across the entire network. An increase in the fortification size by one bank leads to an increase in repayment of $1.97 billion on average. In this paper we only consider fortifications of the smallest number of banks allowed by the network. This result suggests that more research is needed to investigate whether the cost of fortifying more banks may be offset by increased savings.

3.3. Performance Comparisons

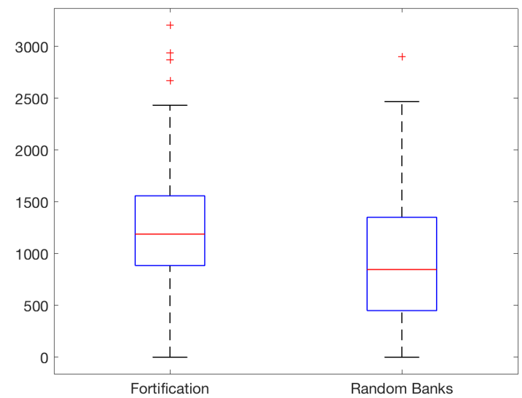
We compare the performance of fortifications to two alternatives. The first is a random set of banks of the same size. For each repetition of the simulation, a set of banks is chosen randomly and uniformly from the universe of banks and this set contains the same number of banks as the most successful fortification in the repetition. If any of these banks are unable to repay their loans in full, and if this shortfall is covered by the government as in a fortification, how does the performance of these two methods compare in terms of cost, efficiency, money saved in newly repaid loans, and banks saved from failure?

We also compare the performance of fortifications to that of a set containing the most connected banks in the network. Banks are ranked by their out-degree and banks are added to a set in order of highest out-degree until that set contains as many banks as the most successful fortification in the repetition. For example, if the fortification contains three banks, then this alternative set contains the top three most connected - in terms of out-degree - banks. As with fortifications and the set of random banks, any shortfall in loan repayment on the part of these most connected banks is covered and the ensuing increase in financial stability is compared to that of fortifications.

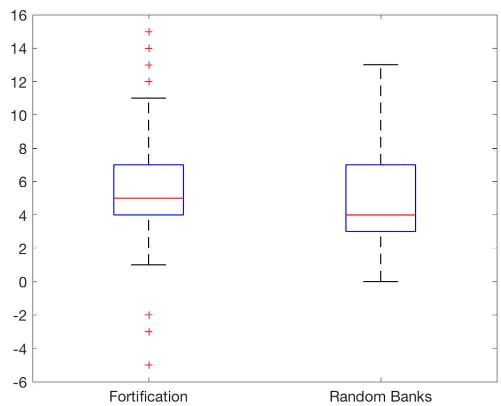
Figures 5 and 6 compare the measures of success for these two alternatives to those for the corresponding fortification. Figure 5 contains box plots that compare the money saved, cost, banks saved, and efficiency of fortifications and the random set of banks. Figure 6 contains box plots that compare the same measures between fortifications and the set of most connected banks. Tables 3 and 4 summarize the average money saved, cost, banks saved, and efficiency for each of these methods. All of the differences listed are statistically significantly different from zero. That is, the differences between fortification and the alternatives are statistically significant.



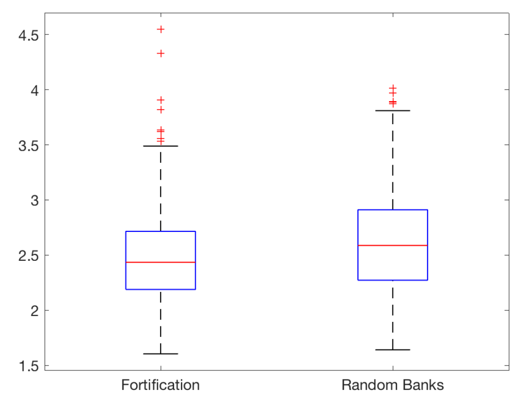
(a) Money Saved



(b) Cost

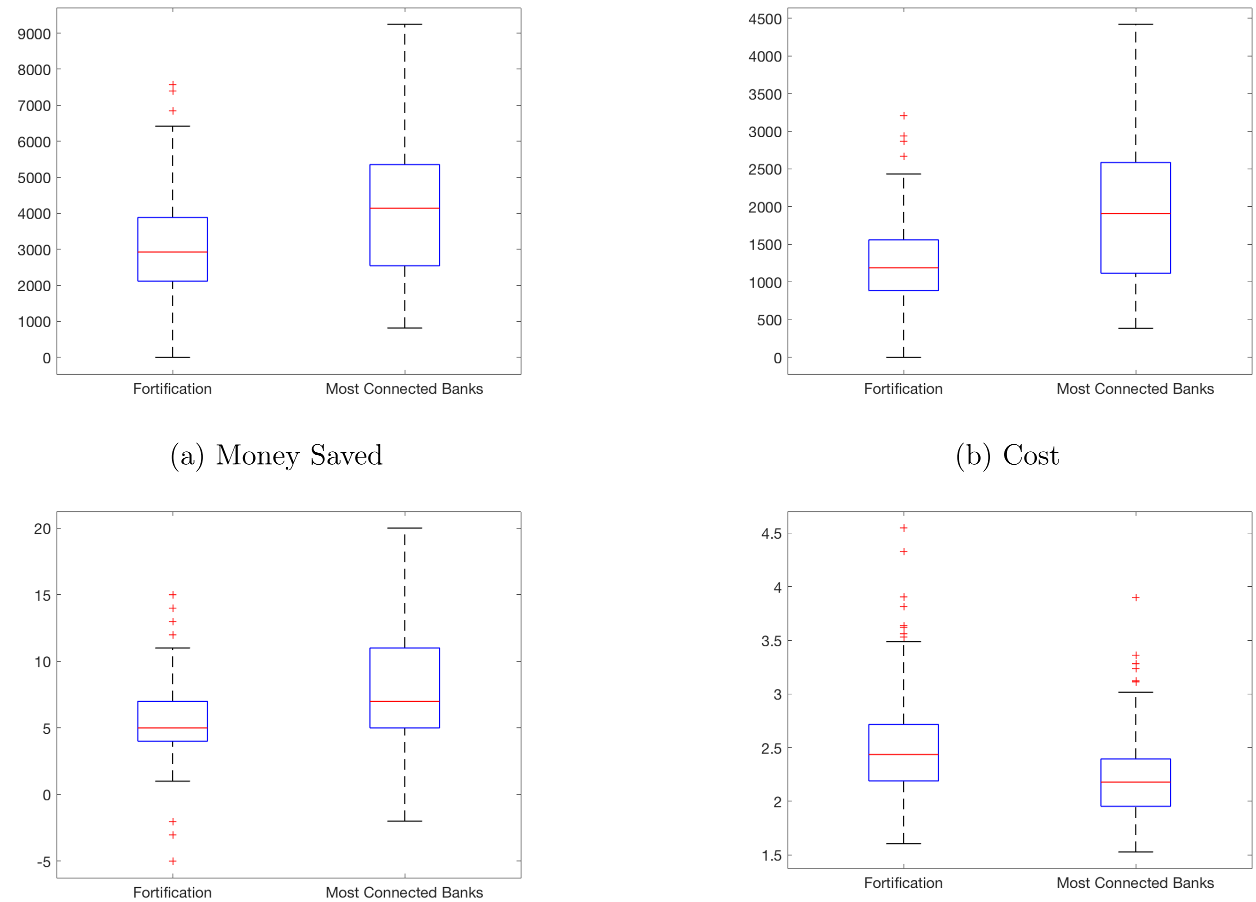


(c) Banks Saved



(d) Efficiency

Figure 5: Comparison of Fortifications vs. Random Banks



(a) Money Saved

(b) Cost

(c) Banks Saved

(d) Efficiency

Figure 6: Comparison of Fortifications vs. the Most Connected Banks

Compared to a set of the same size but containing randomly selected banks, fortifications save more money. On average, fortifications save \$670.62 million more than a random set of banks. This is because a random set of banks will not be connected by loan contracts to as many other banks as those in a fortification. Any funds that help a random set of banks will not have the same stabilizing influence as those used in a fortification because they will not propagate as far.

In addition to saving more money, fortifications save more banks from failure compared to a random set of banks. On average, fortifications save 0.69 more banks. As with money saved, this is driven by fortification-banks' centrality and influence on other banks' ability to repay loans.

Fortifications also cost more than a random set of banks. In general, fortifications cost \$305.43 million more. The reason fortifications are more successful in saving money and banks is the same reason that they tend to be more costly: banks in a fortification tend to be more central and connected to other banks both in lending and in borrowing. They tend to borrow money from more banks and therefore they have larger liabilities; however, covering those liabilities benefits more banks than covering the liabilities of randomly selected banks.

Perhaps counter-intuitively, on average, a random set of banks saves more money per dollar spent than a fortification. That is, the random set has a higher efficiency. However, this is simply the result of the low cost of covering the loan shortfall of random banks. They save a small amount of money and cost almost nothing. Fortifications save significantly more money and banks, while costing more to do so.

Table 3: Fortifications vs. Random Banks

	Fortification	Random Set	Difference
Money Saved (mill.)	$3,046.93$	$2,376,31$	670.62
Banks Saved	5.68	4.99	0.69
Cost (mill.)	$1,235.35$	929.92	305.43
Efficiency	2.50	2.63	-0.13

Table 4: Fortifications vs. Most Connected Banks

	Fortification	Most Connected	Difference
Money Saved (mill.)	$3,046.93$	$4,137.78$	$-1,090.85$
Banks Saved	5.68	7.60	-1.92
Cost (mill.)	$1,235.35$	$1,919.55$	-684.20
Efficiency	2.50	2.21	0.29

Compared to a set of the most connected banks, fortifications save less money and fewer banks. On average, a set of the most connected banks saves \$1.09 billion more than the corresponding fortification. Additionally, they save 1.92 more banks from failure. Because these banks are the most connected in the network, they have a direct and strong effect on the other banks in the network and on those banks' ability to repay their loans.

These most connected banks are also more costly to stabilize. To cover the shortfall of these highly connected banks costs on average \$684.20 million more than to pay for the fortification. They are costly for the same reason they save so much money and so many banks: they are very highly connected.

Finally, fortifications are more efficient than a set of the most connected banks. On average, fortifications save \$0.29 more for every dollar spent. Fortifications, while not saving as much money in repaid loans in total as compared to the most connected banks, still save \$3.05 billion and they do it at a lower cost. A dollar spent on a fortification goes farther in stabilizing the financial network.

Fortifications save more money in loan repayments and more banks from failure than a randomly selected set of banks and are more efficient than a more costly set of highly connected banks.

4. Case Study: US Financial Network in 1867

We use a historical data set describing interbank debts to analyze the fortification of a real financial network. Anderson, Paddrik, and Wang (2019) describe the network of lending between banks in Pennsylvania and New York City in 1867, following the National Banking Acts in 1863 and 1864. [3] We use network linkages constructed from the *Reports of the Several Banks and Savings Institutions of Pennsylvania (1863, 1868)* and the National Banks Examination Reports that resulted from the National Banking Acts. Figure 7 depicts this network consisting of 202 banks. We use this network to find a fortification of those banks engaged in interbank lending.

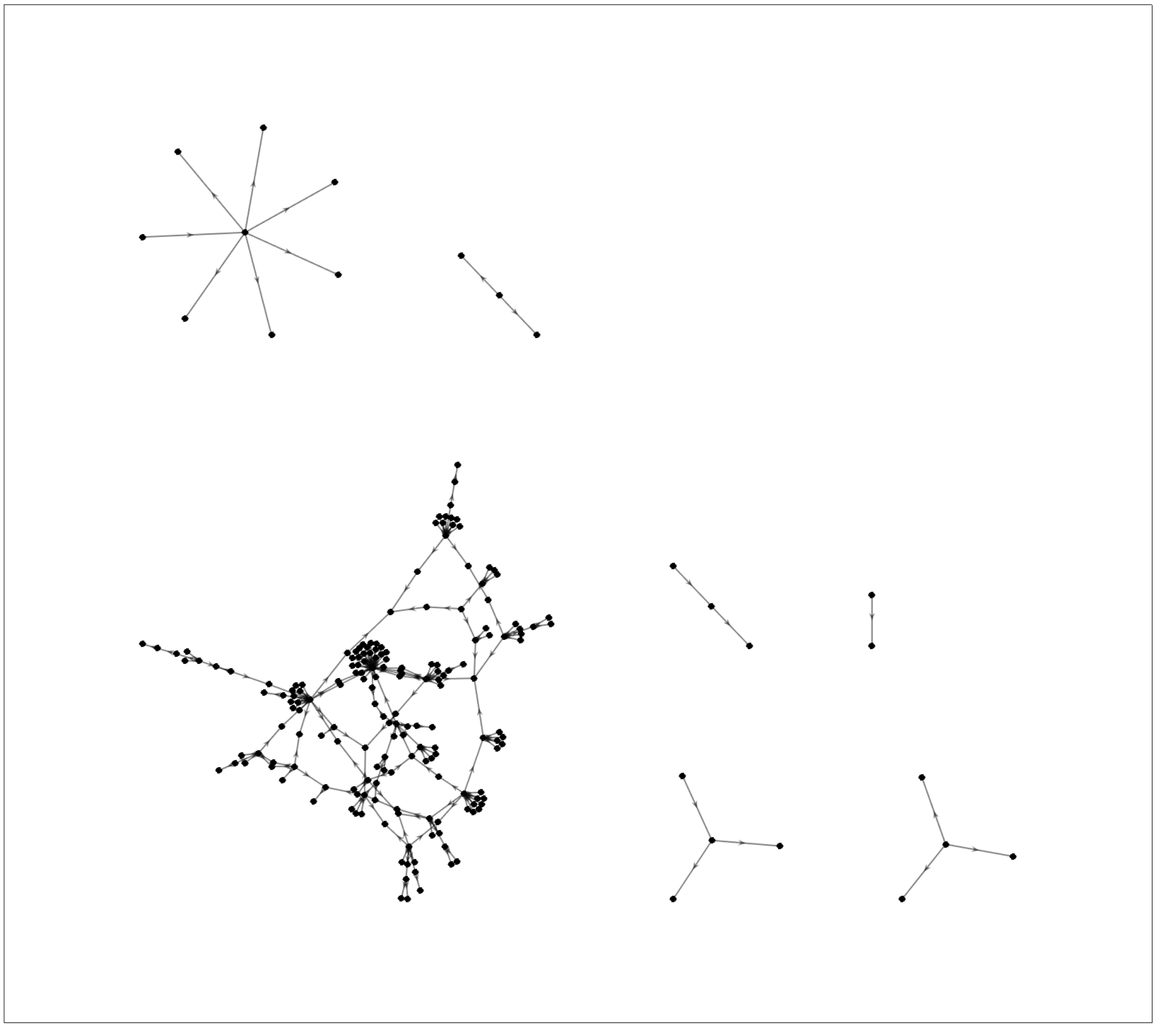


Figure 7: PA and NY Financial Network (1867)

There is little data available describing real financial networks. While the data we use in this paper is from 152 years ago, the presence of any data from Anderson, Paddrik, and Wang (2019) represents a significant improvement in financial network research. Additionally, the banks included in this sample cover a wide variety of bank types, ranging from local banks to large financial center banks. [3] This data allows us to demonstrate how the fortifications of real world networks can be constructed and that our simulation results are reasonable.

Not every bank in the network described by the data borrows money from another bank. As a result, not every bank in the network will have a neighbor in the fortification. Only banks who borrow from at least one other bank will have such a neighbor. Of the 202 banks in the network, only 54 of them borrow from at least one other bank. The average bank has only 1.07 debtor-banks. Figure 8 depicts the out-degree distribution of the network. That is, it depicts a histogram of the number of banks to which other banks owe loan repayments. The modal out-degree is 0, meaning the most common number of debtor-banks is none.

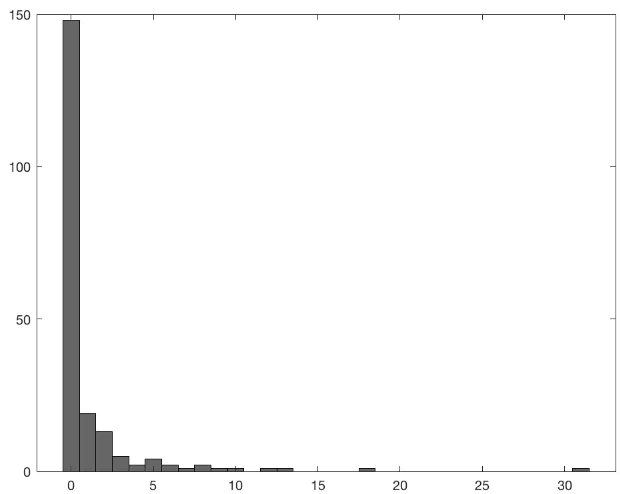


Figure 8: 1867 Financial Network Out-Degree Distribution

The minimal fortification of the 54 banks engaged in borrowing contains 38 nodes. In Figure 9, nodes in the fortification are highlighted with a larger node size and colored red. This large fortification is the result of the lack of connectivity in the network. Not only are there relatively few banks borrowing from one another but furthermore, very few borrow from the same bank. Of the 38 banks in the fortification, only 9 cover more than a single bank.

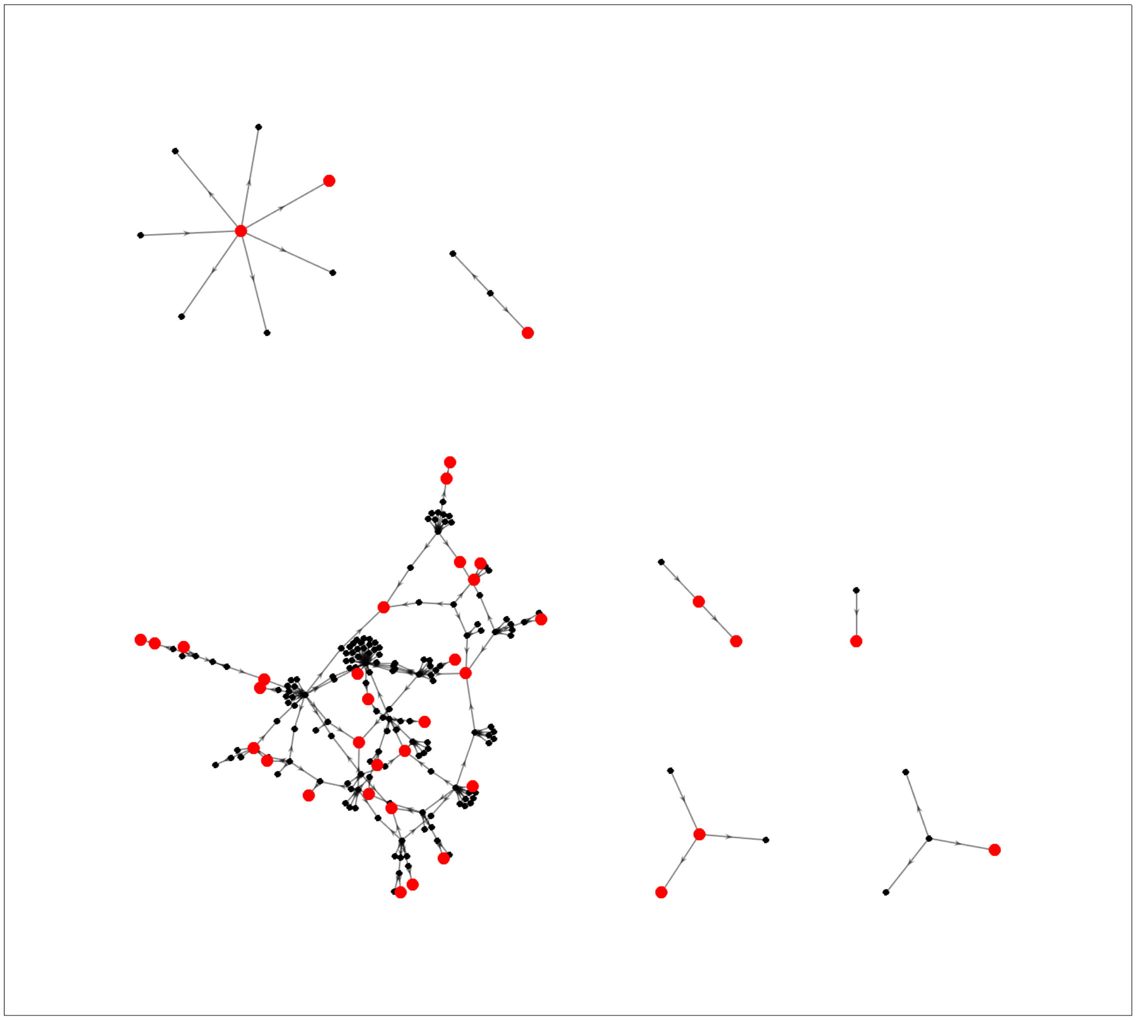


Figure 9: Fortification of 1867 Financial Network

These results are consistent with the simulation results discussed in the previous section. The number of edges (loans) present and the distance between banks in the network both play a large role in determining the fortification size. A network of 202 nodes with no self-loops can have up to $202 \times 201 = 40,602$ edges. This network has only 216, representing 0.5% of the possible edges. Additionally, because this network is not made up of a single connected component, a path does not exist from every node to every other node, and as a result the path distance is defined to be infinite. Both of these characteristics contribute to the large fortification relative to the number of borrowers. This is consistent with the simulation results regarding fortification size. As discussed in Section 3.1, as the number of edges present decreases, the size of the fortification increases. Similarly, as the path distance of a network grows, so does the fortification size.

5. Conclusion

In the face of recent research that identifies the financial networks that are most vulnerable to cascading financial failures, this paper identifies those networks that can be most successfully protected from such cascades. Networks that are globally interconnected but not overly cliqueish allow for the least costly and most successful stabilization. The method of stabilization presented in this paper performs well in comparison to alternative methods. Fortifications save more money and banks than a random selection of protected banks and fortifications are more efficient than protecting the banks with the most lending partners, that is, those that would be "too big to fail." Finally, the fortificaton of a real historical network is consistent with the simulation results; a network that is very disconnected requires a fortification containing many banks.

References

[1] Daron Acemoglu, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. "Systemic Risk and Stability in Financial Networks". In: *American Economic Review* 105.2 (Feb. 2015), pp. 564–608. ISSN: 0002-8282. DOI: 10.1257/aer.20130456. URL: https://www.aeaweb.org/articles?id=10.1257/aer.20130456 (visited on 06/03/2019).

[2] Franklin Allen and Ana Babus. *The Network Challenge (Chapter 21): Networks in Finance*. 1 edition. FT Press, May 19, 2009. 27 pp.

[3] Haelim Anderson, Mark Paddrik, and Jessie Jiaxu Wang. "Bank Networks and Systemic Risk: Evidence from the National Banking Acts". In: *American Economic Review* 109.9 (Sept. 2019), pp. 3125–3161. ISSN: 0002-8282. DOI: 10.1257/aer.20161661. URL: https://www.aeaweb.org/articles?id=10.1257/aer.20161661 (visited on 09/07/2019).

[4] Albert-László Barabási. *Linked: how everything is connected to everything else and what it means for business, science, and everyday life*. OCLC: ocn859186455. New York: Basic Books, 2014. 294 pp. ISBN: 978-0-465-08573-6.

[5] Markus K Brunnermeier. "Deciphering the Liquidity and Credit Crunch 2007fffdfffdfffd2008". In: (), p. 24.

[6] Ricardo J Caballero and Pablo Kurlat. "Public-Private Partnerships for Liquidity Provision". In: (), p. 13.

[7] Guido Caldarelli et al. "Reconstructing a credit network". In: *Nature Physics* 9 (Mar. 1, 2013), pp. 125–126. ISSN: 1745-2481. DOI: 10.1038/nphys2580. URL: https://www.nature.com/articles/nphys2580 (visited on 06/15/2019).

[8] Michele Catanzaro and Mark Buchanan. "Network opportunity". In: *Nature Physics* 9 (Mar. 1, 2013), pp. 121–123. ISSN: 1745-2481. DOI: 10.1038/nphys2570. URL: https://www.nature.com/articles/nphys2570 (visited on 06/15/2019).

[9] Matteo Chinazzi and Giorgio Fagiolo. "Systemic Risk, Contagion, and Financial Networks: A Survey". In: *SSRN Electronic Journal* (2013). ISSN: 1556-5068. DOI: 10.2139/ssrn.2243504. URL: http://www.ssrn.com/abstract=2243504 (visited on 06/13/2019).

[10] Marc Davis. *U.S. Government Financial Bailouts*. Investopedia. URL: https://www.investopedia.com/articles/economics/08/government-financial-bailout.asp (visited on 06/12/2019).

[11] Gary B Gorton. *The Panic of 2007*. Working Paper 14358. National Bureau of Economic Research, Sept. 2008. DOI: 10.3386/w14358. URL: http://www.nber.org/papers/w14358 (visited on 06/15/2019).

[12] Augusto Hasman. "A Critical Review of Contagion Risk in Banking". In: *Journal of Economic Surveys* 27.5 (2013), pp. 978–995. ISSN: 1467-6419. DOI: 10.1111/j.1467-6419.2012.00739.x. URL: https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-6419.2012.00739.x (visited on 06/15/2019).

[13] Matthew O. Jackson. *Social and economic networks*. OCLC: 254984264. Princeton, NJ: Princeton Univ. Press, 2010. 504 pp. ISBN: 978-0-691-14820-5 978-0-691-13440-6.

[14] Nick K. Lioudis. *The Collapse of Lehman Brothers: a Case Study*. URL: https://www.investopedia.com/articles/economics/09/lehman-brothers-collapse.asp (visited on 05/27/2019).

[15] Robert M. May, Simon A. Levin, and George Sugihara. "Complex systems: Ecology for bankers". In: *Nature* 451.7181 (Feb. 2008), pp. 893–895. ISSN: 1476-4687. DOI: 10.1038/451893a. URL: https://www.nature.com/articles/451893a (visited on 06/15/2019).

[16] Phillip Swagel. "The Financial Crisis: An Inside View". In: *Brookings Papers on Economic Activity* 2009.1 (2009), pp. 1–63. ISSN: 1533-4465. DOI: 10.1353/eca.0.0044. URL: http://muse.jhu.edu/content/crossref/journals/brookings_papers_on_economic_activity/v2009/2009.1.swagel.html (visited on 06/15/2019).

[17] Robert Rich. *The Great Recession | Federal Reserve History*. URL: https://www.federalreservehistory.org/essays/great_recession_of_200709 (visited on 06/13/2019).

[18] Paola Sapienza and Luigi Zingales. *The Results: Wave 1*. Financial Trust Index. 2009. URL: http://www.financialtrustindex.org/resultswave1.htm (visited on 06/15/2019).

[19] Caleb Silver. *10 Years Later, Lessons from the Financial Crisis*. URL: https://www.investopedia.com/news/10-years-later-lessons-financial-crisis/ (visited on 05/27/2019).

[20] *Timelines of Policy Responses to the Global Financial Crisis - FEDERAL RESERVE BANK of NEW YORK*. URL: https://www.newyorkfed.org/research/global_economy/policyresponses.html (visited on 06/15/2019).

6. Appendix

6.1. Model Parameterization in Simulations

Table 2: Model Parameterization

Parameter	Value
n	20
l_{ij}, in millions	100
$\rho_{ij} = \rho$	0.027
z_j	0.01
A_j, in millions	7
ξ_j	0.4
c_j, in millions	3
v_j, in millions	10

The data generated in the simulations are available from the author upon reasonable request.

Declarations of interest: none