Homework #5

Mathematical Methods II

Spring 2009

1. Consider a variant of the growth model in which households value consumption and housing services. The utility function is

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \theta \log(s_t) \right]. \tag{0.1}$$

The resource constraint for the planner is

$$c_t + h_{t+1} + k_{t+1} = A k_t^{\alpha} + (1 - \delta_k) k_t + (1 - \delta_h) h_t, \tag{0.2}$$

where h_t is the stock of housing. Housing services are a multiple of the stock:

$$s_t = B h_t. \tag{0.3}$$

   1. Derive the equations that determine the steady state of this economy. Solve analytically for as much of the steady state as you can.
   2. Set $\delta_k = 0.025$, $\delta_h = 0.01$, $\beta = 0.99$, $\theta = 0.5$, $\alpha = 0.36$, and $A = 1$. Compute the values of (c, h, k) at the steady state. Use these values to compute a quadratic approximation to the utility function at those values, after substituting c using the resource constraint.
   3. Use the Ricatti equation to solve for the decision rules $k' = g_k(k, h)$ and $h' = g_h(k, h)$ and the value function $v(k, h)$. Is your value function concave? Plot the value function over a reasonable range.

2. The above economy was Pareto-efficient, so we could use the planning problem to compute the allocations. Imagine that we did not know this, and instead set out to solve for the recursive competitive equilibrium. The household budget constraint is

$$c_t + k_{t+1} + h_{t+1} \leq (r_t + 1 - \delta_k) k_t + w_t + (1 - \delta_h) h_t. \tag{0.4}$$

The firm rents capital and labor to produce output:

$$\max_{K_t, H_t} \left\{ AK_t^{\alpha} N_t^{1-\alpha} - r_t K_t - w_t N_t \right\}. \tag{0.5}$$

Markets clear if

$$\begin{aligned} k_t &= K_t \\ N_t &= 1 \\ C_t + K_{t+1} + H_{t+1} &= AK_t^{\alpha} + (1 - \delta_k) K_t + (1 - \delta_h) H_t. \end{aligned}$$

1. Write down the Bellman equation for the individual household.
2. Assume that the laws of motion for the aggregates are linear. Take a quadratic approximation to the utility function around the equilibrium steady state.
3. Use Kydland's method to obtain decision rules $k' = \widehat{g}_k (k, h, K, H)$ and $h' = \widehat{g}_h (k, h, K, H)$ and the value function $v (k, h, K, H)$. Verify that your decision rules aggregate to $K' = G_k (K, H)$ and $H' = G_h (K, H)$. Also verify that you obtain similar aggregate laws of motion as you obtained in Problem 1.