Homework #3
Mathematical Methods II
Spring 2009

1. Consider the deterministic growth model with elastic labor supply. The allocation for this economy is the one chosen by a social planner who solves the dynamic program

$$v(k) = \max_{c \geq 0, k' \geq 0, h \in [0,1]} \left\{ \frac{c^{1-\sigma}}{1-\sigma} + \nu \frac{(1-h)^{1-\gamma}}{1-\gamma} + \beta v(k') \right\}$$

subject to

$$c + k' \leq k^{\alpha} h^{1-\alpha} + (1-\delta) k.$$

Set $\alpha = 0.36$, $\beta = 0.99$, $\delta = 0.025$, $\nu = 2.0$, $\sigma = 2.0$, and $\gamma = 5.0$.

(a) Find the steady state values (k^*, h^*, c^*) using fsolve.

The steady state solves the system of equations

$$\begin{aligned} 1 &= \beta \left(\alpha (k^*)^{\alpha-1} (h^*)^{1-\alpha} + 1 - \delta \right) \\ c^* &= (k^*)^{\alpha} (h^*)^{1-\alpha} - \delta k^* \\ v (1-h^*)^{-\gamma} &= (1-\alpha)(k^*)^{\alpha} (h^*)^{-\alpha} (c^*)^{-\sigma}. \end{aligned}$$

For the parameter values given, the solution is

$$\begin{bmatrix} k^* \\ c^* \\ h^* \end{bmatrix} = \begin{bmatrix} 8.1859 \\ 0.5935 \\ 0.2155 \end{bmatrix}.$$

The function file that handles this question is **elastic_ss.m**.

(b) For each pair (k, k'), write a function file that can be used to compute (c, h).

Given (k, k'), the optimal pair (c, h) solve the system of equations

$$\begin{aligned} c &= k^{\alpha} h^{1-\alpha} + (1-\delta) k - k' \\ v (1-h)^{-\gamma} &= (1-\alpha) k^{\alpha} h^{-\alpha} c^{-\sigma}. \end{aligned}$$

Thus, we can solve for h by solving the unidimensional function

$$F(h) = v(1-h)^{-\gamma} - (1-\alpha) k^{\alpha} h^{-\alpha} \left(k^{\alpha} h^{1-\alpha} + (1-\delta) k - k' \right)^{-\sigma}$$

which is bracketed by $(0, 1)$. c is then obtained using the resource constraint. Note: for some (k, k') pairs $F(h)$ may not have a solution; you will need to handle those cases explicitly in part c) below. The function file that handles this question is **laborsupply.m.**

(c) Embed your programs from a) and b) into a discrete state space dynamic program that solves for $v(k)$ on a grid of 100 points that spans from 50 percent to 200 percent of the steady state.

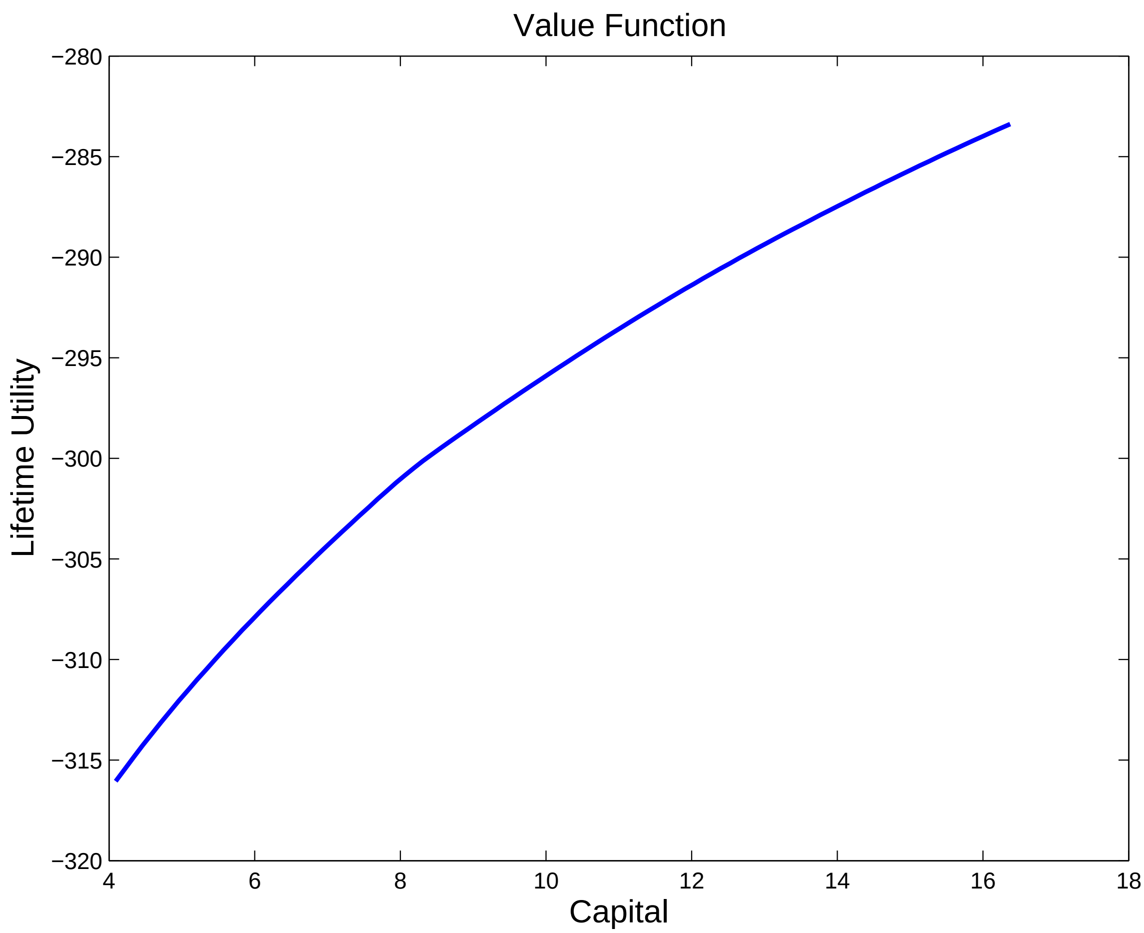
The most efficient approach is to solve for (c, h) at every pair (k, k') and then calculate

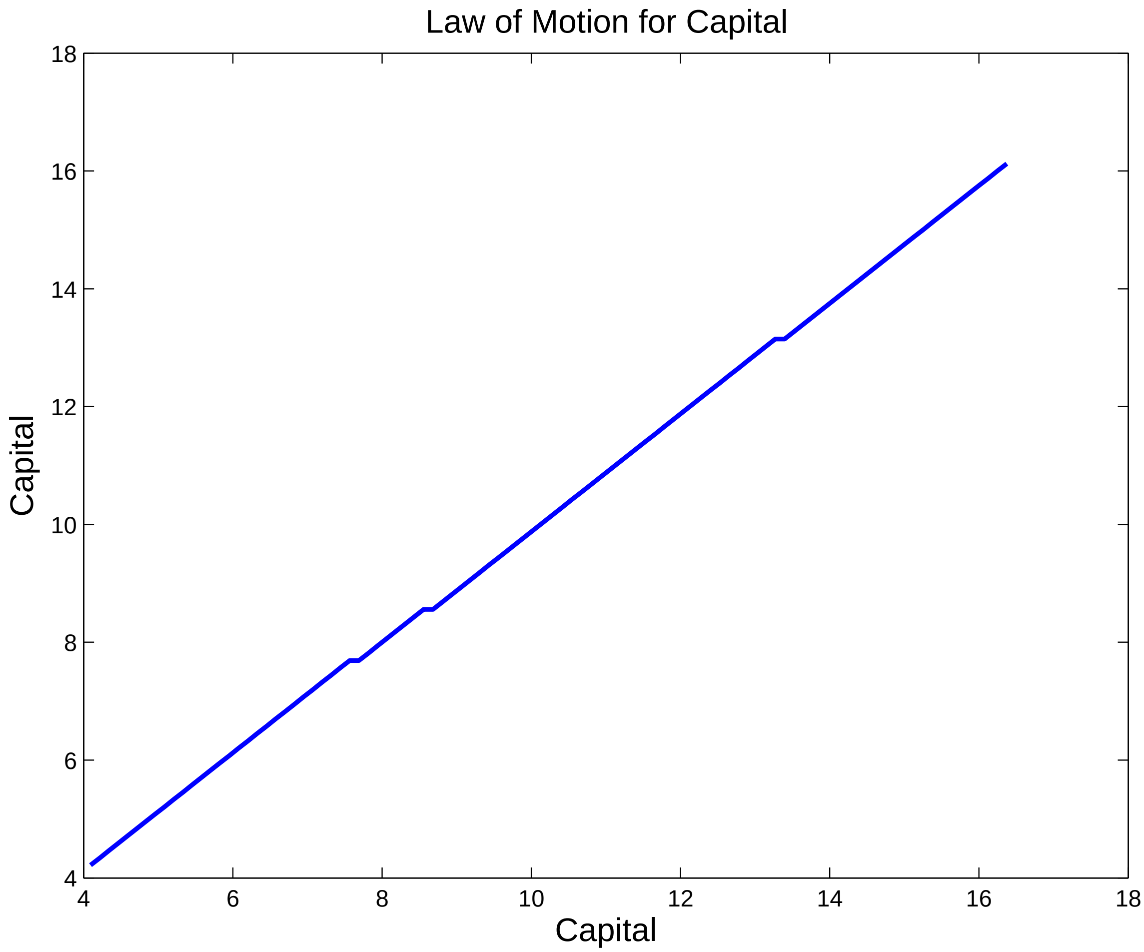
$$u_{i,j} = \frac{c_{i,j}^{1-\sigma}}{1-\sigma} + \nu \frac{(1-h_{i,j})^{1-\gamma}}{1-\gamma},$$

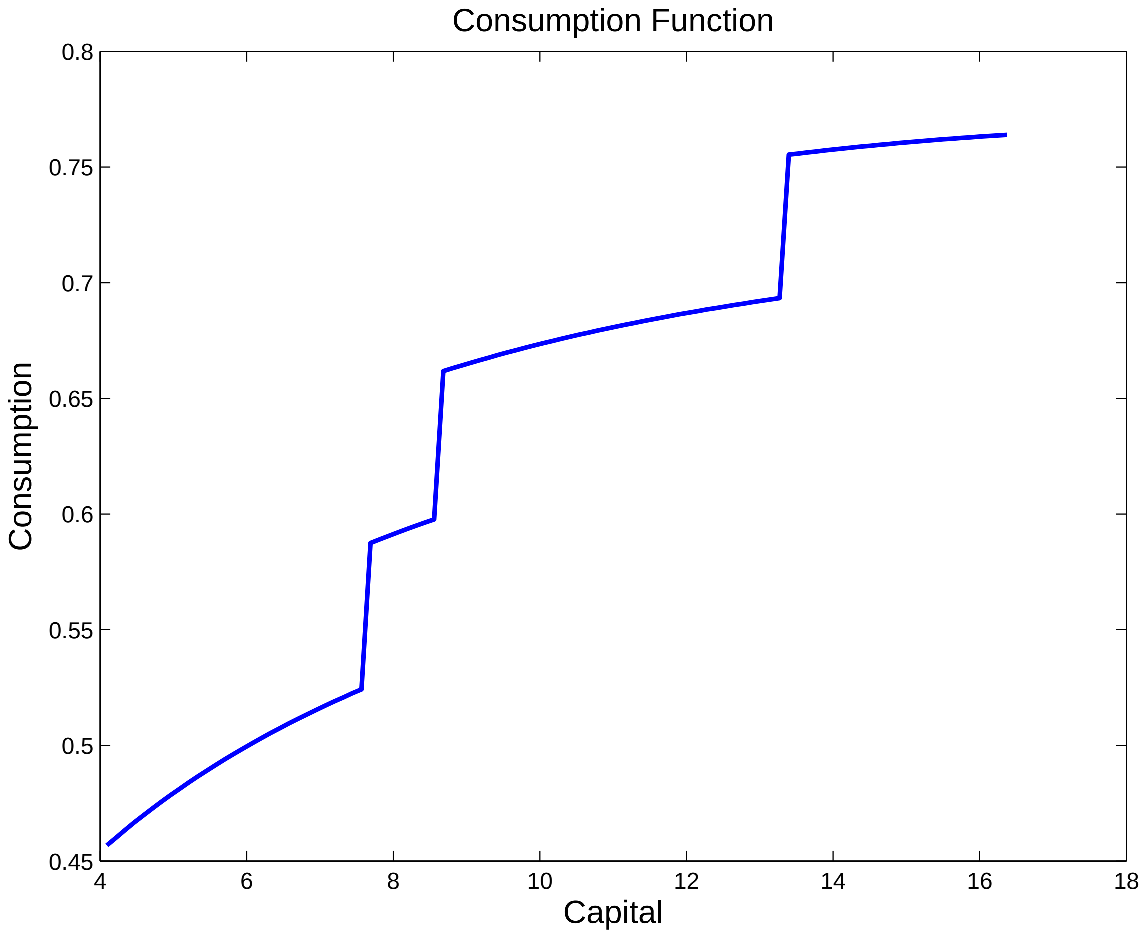
storing the results in a large 100×100 matrix. The value of any point where $c \leq 0$ or $h < 0$ is then $u_{i,j} = -\infty$ (-inf in Matlab). Then the optimization is done via simple finite grid search:

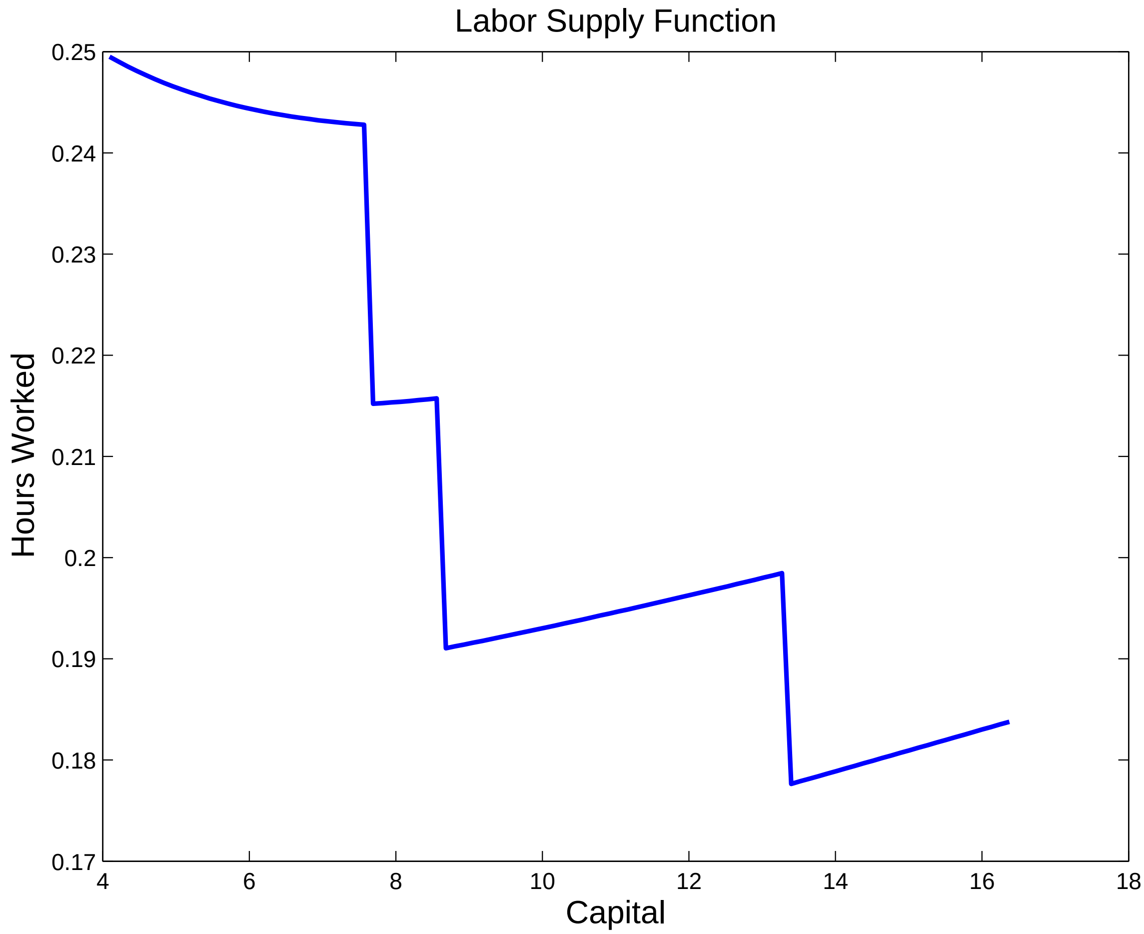
$$v^{n+1}(k_i) = \max_j \{u_{i,j} + \beta v^n(k_j)\}$$

from some initial guess $v^0(k)$. Figures 1 through 4 display the four functions – the value function, the law of motion for capital, the consumption function, and the hours function. The file that runs the entire program is called **discrete_elastic.m**.


Value Function
Lifetime Utility
Capital
−280
−285
−290
−295
−300
−305
−310
−315
−320
4
6
8
10
12
14
16
18


Law of Motion for Capital
Capital
Capital
4
6
8
10
12
14
16
18
4
6
8
10
12
14
16
18


Consumption Function
Consumption
Capital
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
4
6
8
10
12
14
16
18


Labor Supply Function
Hours Worked
Capital
0.25
0.24
0.23
0.22
0.21
0.2
0.19
0.18
0.17
4
6
8
10
12
14
16
18