

Homework #2  
 Mathematical Methods II  
 Spring 2009

1. Suppose that we have the quadratic dynamic programming problem

$$x^T P x = \max_a \{x^T Q x + a^T R a + x^T W a + a^T S x + \beta (x^T A^T + a^T B^T) P (A x + B a)\}$$

where  $x$  is an  $n \times 1$  vector (with 1 as the first element) and  $a$  is a  $k \times 1$  vector. The matrices are conformable to these vectors. Write a Matlab program that iterates on a functional equation in order to solve uniquely for  $P$  and the associated policy function  $a^* = Fx$ .

2. Let a firm choose a sequence of labor demands  $\{n_t\}_{t=0}^{\infty}$  to maximize the net present value of its stream of profits:

$$\max_{\{n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t \left[ f_0 n_t - \frac{1}{2} n_t^2 - w n_t - \frac{d}{2} (n_t - n_{t-1})^2 \right]$$

where  $n_{-1}$  is given. In the above problem,  $f_0 n_t - \frac{1}{2} n_t^2$  is the production of  $n_t$  workers,  $w$  is their wage, and parameters  $f_0$  and  $d$  are positive. The last term captures adjustment costs in the hiring and firing of workers.  $r$  is the interest rate at which firms discount the future and is positive.

- (a) Write down the Bellman equation for this model. What are the matrices  $Q$ ,  $R$ ,  $W$ ,  $A$ ,  $S$ , and  $B$ ?
- (b) Find the steady state level of employment  $n$ . How does this value depend on the parameters  $r$ ,  $w$ ,  $f_0$ , and  $d$ ?
- (c) Use your computer program to compute  $P$  given values for the parameters. Set  $w = 1$  and  $f_0 = 2$ . Solve the model for the decision rule  $n$  for each of the nine points

$$(r, d) \in \{\infty, 1, 0.1\} \times \{0, 1, 5\}.$$

Verify that the steady state level of employment agrees with your answer in the previous part for each parameter value. How do changes in  $r$  and  $d$  affect the coefficient on  $n_{-1}$  in your computed decision rules?