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Money Creation, Reserve Requirements, and Seigniorage*

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In this paper, we examine the impact that changes in the rate of money creation and reserve requirements have on real seigniorage revenue. We consider two additional features that differ from previous analyses. First, the model economies grow endogenously, and that growth depends on the accumulation of intermediated capital. Second, agents have two means of financing; one is bank deposits against which reserves must be held, and the other is a nonbank intermediary. Thus, growth-rate effects and financing-substitution effects are both present, and one can assess the quantitiative importance of each factor. *Journal of Economic Literature* Classification Numbers: E6, H6.

1. INTRODUCTION

The purpose of this paper is to quantitatively assess the impact that changes in monetary policy—both money growth rates and reserve requirements—have on the present value of real seigniorage revenue col-

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lected by the government. There is a long tradition of studying the theoretical linkages between monetary policy actions and real seigniorage revenue.[1] In this paper, we specify a dynamic general equilibrium model, calibrate it, and apply standard numerical techniques to compute the present value of real seigniorage revenue collected under different monetary policy settings. A computational exercise complements the vast theoretical literature on monetary policy and seigniorage. To our knowledge, little has been done in terms of computational experiments.[2]

In this paper, two key features are drawn from the theoretical literature on monetary policy and seigniorage. First monetary policy is linked to growth through the intermediation process.[3] More specifically, capital accumulation is intermediated through either banks or nonbanks. Banks pool together small savers costlessly, but face a reserve requirement. Second, capital accumulation can also be financed through a nonbank intermediary.[4] The nonbank sector is not subject to a reserve requirement, but faces a resource cost to intermediate savings. Monetary policy actions operate on output growth through the real return on savings. Thus, faster money growth and/or higher reserve requirements, for example, reduce the return to deposits, thereby reducing the incentive to save and slowing the rate of capital accumulation. Thus, monetary policy affects the size of the seigniorage revenue tax base through two channels; one is the rate at which the tax base increases, and the other is through the relative size of the bank versus the nonbank.

The results presented in this paper are easily summarized. The computational experiments naturally lend themselves to the identification of a seigniorage-revenue maximum.[5] Our results show that if monetary policy

[1] For example, Bailey [2], Friedman [16], Brock [6], and Easterly et al. [12], to name just a few.

[2] One notable exception is a paper by Bullard and Russell [7]. In their analysis, Bullard and Russell look at how changes in the money growth rate affect the ratio of government revenue to output, focusing on the stationary equilibrium.

[3] Kormendi and Meguire [24], Fischer [14], and DeGregario [10], among others, provide empirical support for the notion that inflation and growth are related. In general, these studies find that countries with high inflation rates tend to grow more slowly than countries with low inflation rates. Boyd et al. [5] find that inflation is negatively correlated with financial market performance, suggesting that monetary policy may operate on capital accumulation through the intermediation process. Jones and Manuelli [22] and Chari et al. [9] explore this avenue in a veriety of model economies.

[4] See Bencivenga and Smith [3] for an analysis of model in which two types of intermediaries—a bank and an informal nonbank—coexist.

[5] The consequence of this focus is that we ignore welfare considerations. Indeed, in this model economy, the optimal policy settings are for money growth to follow the Friedman Rule or to set the reserve ratio to zero. See Freeman [15], Chari et al. [8], Mulligan and Sala-i-Martin [25], and Dotsey and Ireland [11] for analyses on the welfare costs of inflation in alternative economic environments.

affects the growth rate, revenue-maximizing values for both the inflation rate and reserve ratio are slightly below 10%. We find that, for baseline economy settings, the revenue-maximizing reserve ratio and inflation rate are well below the mean values we compute from a multicountry dataset. We also consider the robustness of these results to changes in the rate at which disintermediation (the switching from bank deposits to nonbank contracts) occurs. Not surprisingly, as disintermediation occurs more rapidly, both the revenue-maximizing inflation rate and reserve requirement ratio fall. In addition, we consider the quantitative effects in a less sophisticated intermediary structure, one in which nonbanks do not exist, finding that the revenue-maximizing reserve ratio and inflation rate are above 20%, well above the mean values we found from the cross-country evidence.

The paper is organized as follows. In Section 2, we describe the baseline model economy. Computational experiments are presented in Section 3. To assess the importance of the different effects, we modify the basic economy to eliminate the capital substitution effect in Section 4. We review the findings and suggest several extensions in Section 5.

2. THE BASELINE MODEL

The economy is populated by five types of decision makers: firms, households, banks, nonbank intermediaries, and the government. Firms rent capital from banks, producing units of the consumption good. Banks offer deposit contracts, maturing in one period, to households. The deposits are used to acquire capital or fiat money. Households receive the principal and interest from the deposit contracts and the (gross) rental payments from the capital plus any undepreciated capital to acquire consumption, capital, or deposits.

The bank holds fiat money to satisfy a reserve requirement. The seigniorage tax base, holding everything else constant, is positively related to the reserve requirement. In addition, the seigniorage tax rate, again holding everything else constant, is positively related to change in the inflation rate. However, output growth is inversely related to both the inflation rate and the reserve requirement. Thus, the growth-rate effect translates into a tax base that increases at a slower rate. Moreover, a higher inflation rate or reserve ratio, for instance, has a one-time reallocative effect, causing households to shift from the bank to the nonbank. With fewer deposits, the seigniorage tax base declines. Throughout this paper, we refer to changes in the means of financing as disintermediation, although strictly speaking, all capital is intermediated in this model economy. The computational experiments compute the change in the present value of real seigniorage revenue for different values of the inflation rate and the reserve ratio.

The nonbank intermediary (hereafter, nonbank) also offers one-period contracts to households. Each nonbank contract stipulates that the nonbank accepts one good from the household, promising to repay the household next period with R^n goods. The nonbank then uses these contracts to acquire capital, which is then rented to firms in a competitive market to produce the capital-consumption good. The gross return on nonbank contracts, unlike deposits, is determined by the quantity of good received by the nonbank. To this end, we assume that the nonbank faces a resource cost $f(k^n)$, where k^n stands for the number of contracts executed with the nonbank.

The government taxes capital income and makes lump-sum transfer payments to households. The government can finance a deficit in any period by issuing one-period bonds. The bond sells for b^g_{t+1} units of the consumption good and pays off $R_{t+1}b^g_{t+1}$ units one period later. For simplicity, we assume that government bonds and capital are perfect substitutes. Throughout the analysis, we assume that the quantity of government debt is small enough that bonds, deposits, and nonbank contracts will be held by households.

2.1. Model Specification

We assume that there are a large number of identical households that solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1 - \sigma} \right) \tag{P1}$$

$$\text{s.t.: } c_t + b^g_{t+1} + d_{t+1} + k^n_{t+1} \le R_t d_t + R^n_t\left(b^g_t + k^n_t\right) + G_t, \tag{2.1}$$

where d_{t+1} denotes the quantity of goods deposited with the bank at time t, k^n_t denotes the stock of nonbank contracts available at time t, R_t denotes the gross real return on deposits, R^n_t denotes the gross real return offered on nonbank contracts and government bonds, τ_t denotes the tax rate on both bank-financed and nonbank-financed capital, and G_t denotes the value of the government transfer.[6] We assume the subjective discount factor, β, lies in the open unit interval. Similarly, capital depreciates at a constant rate, with $0 \le \delta \le 1$. The constant elasticity of substitution pa-

[6] Income taxes will not play a crucial role in the experiments. In equilibrium, the rate of return on deposits will depend on the income tax rate. We also conducted the quantitative analysis with $\tau = 0$. The results for the no-income-tax case are not materially different from those reported here and are available from the authors upon request.

rameter, $1/\sigma$, is strictly positive. Finally, population is constant, so that there is no aggregation bias associated with treating per capita quantities as aggregate quantities.

Equation (2.1) is a fairly standard budget constraint. The household uses proceeds from bank deposits, government bonds, and nonbank contracts, plus the real value of government transfer payments to acquire units of the consumption good and storage. Goods can be stored for future consumption by acquiring deposits, government bonds, or nonbank contracts. Here we are assuming that government bonds and nonbank contracts are perfect substitutes.

Letting λ_t denote the Lagrange multiplier, the consumer's first-order conditions are

$$\beta^t c_t^{-\sigma} - \lambda_t = 0 \tag{2.2}$$

$$\beta^{t+1} c_{t+1}^{-\sigma} - \lambda_{t+1} = 0 \tag{2.3}$$

$$\lambda_{t+1} R_{t+1} - \lambda_t = 0 \tag{2.4}$$

$$\lambda_{t+1} R_{t+1}^n - \lambda t = 0 \tag{2.5}$$

$$R_t d_t + R_t^n \left(b_t^g + k_t^n \right) + G_t - c_t - d_{t+1} - k_{t+1}^n - b_{t+1}^g = 0, \tag{2.6}$$

where the first-order conditions, Eqs. (2.2)–(2.6), are taken with respect to c_t, c_{t+1}, d_{t+1}, b_{t+1}^g (and k_{t+1}^n), and λ_t, respectively. Equations (2.4) and (2.5) imply that deposits, government bonds, and nonbank contracts will offer the same rate of return in equilibrium. In addition, a transversality condition is necessary to ensure the existence of the households's present-value budget constraint. The household's terminal constraint is interpreted as a no-Ponzi condition in which the household cannot borrow against the sum of future deposits, nonbank contracts, and government bonds, at a rate greater than can be repaid. Formally, the transversality condition is represented as

$$\lim_{T \to \infty} \left[\frac{b_T^g + k_T^n + d_T}{\prod_{s=0}^{T-1} R_s} \right] = 0. \tag{2.7}$$

As such, the date-t budget constraint (2.1) can be combined into an infinite horizon, present-value budget constraint. We now describe the environment from the perspective of the other types of decision makers in the model economy. Throughout our analysis, we assume that units of the consumption good can be transformed into units of capital at a one-for-one rate.

The firm rents capital from either banks or nonbanks, using it to produce the capital-consumption good. Capital is perfectly substitutable in the production process, and the firm is a price-taker in the input market. The production technology is of the form

$$Y_t = A(k_t + k_t^n), \tag{2.8}$$

where k_t is the stock of intermediated capital rented from banks and k_t^n is the stock of capital rented from the nonbank. The laws of motion for bank-financed and nonbank-financed capital are

$$k_{t+1} = (1 - \delta)k_t + x_t, \tag{2.9}$$

$$k_{t+1}^n = (1 - \delta)k_t^n + x_t^n, \tag{2.10}$$

where x_t and x_t^n denote the amount of investment added in time t. The rental price of bank-financed and nonbank-financed capital, denoted q_t and q_t^n, are determined competitively.

Because the firm rents capital from two sources at a competitively determined price, profit maximization simplifies to a series of static problems. Formally, the firm's problem is written as

$$\max_{k_t, k_t^n} A(k_t + k_t^n) - q_t k_t - q_t^n k_t^n. \tag{P2}$$

The first-order conditions for the firm are $A = q = q^n$. (We drop the time subscripts since A is not dependent on time.)

Banks accept one-period deposits from the households, using the proceeds to acquire capital and fiat money. Capital is then rented to firms, and fiat money is held to satisfy a reserve requirement imposed by the government. The bank maximizes profits in a perfectly competitive environment. For simplicity, we assume that the bank costlessly provides intermediary services. Because the deposits are one-period contracts, the bank's infinite-horizon program reduces to a sequence of static problems. When deposits are liquidated, the bank transforms its assets into the consumption good. Hence, each unit of capital rented to firms returns $A + (1 - \delta)$ units of the consumption good.

Because fiat money is rate-of-return dominated by capital, the reserve requirement γ_t dictates how much fiat money the bank will hold. In short, rate-of-return dominance implies that the asset allocation constraint, $\gamma_t p_{t-1} d_t \leq m_t$, is binding at each date t, where m denotes the per-household quantity of fiat money balances. The bank's profit-maximization problem is

$$\max_{k_t, m_t, d_t} (A + 1 - \delta)k_t + \frac{p_{t-1} m_t}{p_t} - R_t d_t, \tag{P3}$$

subject to the reserve requirement constraint and a balance-sheet identity, which is represented as $k_t + m_t/p_t = d_t$. Thus, the bank's first-order conditions can be represented as

$$R_t = (1 - \gamma_t)(A + 1 - \delta) + \frac{\gamma_t}{\pi_t}, \tag{2.11}$$

where $\pi_t \equiv p_t/p_{t-1}$ denotes the rate of inflation. Equation (2.11) indicates that the rate of return on deposits is inversely related to the inflation rate and the reserve ratio.

Capital can also be acquired from the nonbank intermediary. The nonbank intermediary accepts goods with the promise to pay off the contract one period later. With one-period contracts, the nonbank maximizes a series of static problems. Formally, the nonbank's date-t profit maximization problem is given by

$$\max_{k_t^n} \, (A + 1 - \delta)k_t^n - f(k_t^n) - R_t^n k_t^n. \tag{P4}$$

The nonbank's first-order condition is

$$R_t^n = (A + 1 - \delta) - f'(k_t^n). \tag{2.12}$$

As we have already noted in the household's problem, the return to nonbank contract and bank deposits will be equal in equilibrium.

In the data, both banks and nonbanks are used to finance capital accumulation. For our model economy to match this observation, there must be some wedge between the return offered on nonbank contracts and the marginal product of capital. We introduce the resource cost, denoted $f(k_t^n)$, as a way to generate an equilibrium in which both bank deposits and nonbank contracts would coexist. Without the resource-cost function, nonbank contracts would rate-of-return dominate bank deposits. We assume that the resource-cost function has a positive marginal cost function; that is, $f'(\cdot) \geq 0$. Moreover, we assume that the resource-cost function is convex; that is, $f''(\cdot) \geq 0$. It is fairly straightforward to show that the arbitrage condition requires that $f''(\cdot) \geq 0$ for the model economy to exhibit disintermediation. Formally, $dk^n/d\gamma$ and $dk^n/d\pi$ are nonnegative as long as the resource-cost function is convex.

The nonbank's resource-cost function is structured so that some properties of the model economy match some observations in the actual data. For example, Goldsmith [18] finds that the ratio of bank assets to GNP followed an upward trend during the period 1869–1963. With $A < 1$, our model economy can account for Goldsmith's observation. There is a price for this setup. In our model economy, the ratio of investment financed through bank deposits to output is constant along the balanced-growth path. However, the ratio of investment financed through nonbank contracts to output approaches zero. Thus, all growth is financed with capital financed through the bank. This aspect of the model economy is not observed in the data. One way to get around this problem is to make all growth exogenous. Another way would be to introduce technological innovation into the nonbank sector, captured as changes in $f(\cdot)$ over time. A third way would be to explicitly model the dynamic contracting problem between the intermediaries and the firm as one of costly state verification, thus making the resource cost for both the nonbank and the bank endogenous.

Finally, the government commits to a sequence $\{G_t\}_{t=0}^{\infty}$ of transfers which are financed by a combination of taxes and seigniorage. The government's budget constraint is

$$R_t b_t^g + G_t = \frac{m_t - m_{t-1}}{p_t} + \tau_t A[k_t + k_t^n] + b_{t+1}^g.$$

The government has at its disposal two tools of monetary policy: the reserve requirement and the rate of money growth. We assume that money evolves according to the policy rule $m_t = \theta_t m_{t-1}$, where θ is the money growth rate. Moreover, the government's ability to issue debt is constrained, thus ensuring that the government's present-value budget constraint exists over an infinite horizon.[7]

2.2. Equilibrium and Balanced-Growth Equations

An equilibrium in this model economy is a sequence of prices $\{p_t, q_t, q_t^n, R_t, R_t^n\}_{t=0}^{\infty}$, real allocations $\{c_t, x_t, x_t^n, k_t, k_t^n\}_{t=0}^{\infty}$, stocks of financial assets $\{m_t, d_t\ b_t^g\}_{t=0}^{\infty}$, and policy variables $\{\gamma_t, \theta_t, \tau_t, G_t\}_{t=0}^{\infty}$, such that

(i) The real allocations and stocks of financial assets solve the household's maximization problem, (P1), given prices and policy variables.

(ii) The real allocations solve the firm's date-t profit maximization problem, (P2), given prices and policy variables.

(iii) The stock of financial assets solves the bank's date-t profit maximization problem, (P3), given prices and policy variables.

(iv) The stock of financial assets solves the nonbank's date-t profit maximization problem, (P4), given prices and policy variables.

[7] Insofar as capital and bank deposits are nonnegative, the present value of government bonds must zero in the limit. This follows from equation (2.7).

(v) The money market equilibrium condition $m_{t-1} = \gamma_t d_t p_{t-1}$ is satisfied, $\forall t \geq 0$.

(vi) The goods market equilibrium condition $c_t + k_{t+1} - (1-\delta)k_t + k^n_{t+1} - (1-\delta)k^n_t = A(k_t + k^n_t)$ is satisfied, $\forall t \geq 0$.

In this economy, eventually all capital will be financed through the bank. The household's first-order conditions imply that the gross real return on deposits and nonbank contracts will be identical. Hence, $R_t = R^n_t$. (Note that the household pays taxes on capital income. Consequently, the gross after-tax real return is $(1-\gamma_t)[(1-\tau_t)A + 1 - \delta] + \gamma_t \pi_t$.) This arbitrage condition is represented as

$$(1-\gamma_t)\left[(1-\tau_t)A + 1 - \delta\right] + \frac{\gamma_t}{\pi_t} = \left[(1-\tau_t)A + 1 - \delta\right] - f'(k^n_t). \tag{2.13}$$

Now all one needs to solve for the two types of capital is an initial condition stipulating the quantity of total capital. Throughout the analysis here, we assume that the date-0 total capital stock—$k^n + k$—equals one. From (2.13), the stock of nonbank-financed capital will be constant as long as the policy variables and the total factor productivity terms are constant. With $x^n_t = \delta k^n_t$ for all t, the ratio of nonbank-financed capital to total capital approaches zero as $t \to \infty$. With $0 < A < 1$, the bank's asset-to-output ratio will rise over time. Thus, the model economy's prediction for intermediated capital matches a stylized fact regarding banks' behavior.

The consumer's first-order conditions also imply that

$$\frac{c_{t+1}}{c_t} = (\beta R_t)^{\frac{1}{\sigma}}. \tag{2.14}$$

Balanced growth implies that bank-financed investment, output, deposits, and government spending will grow at the same rate as consumption. With deposits growing at the same rate as consumption, the money market clearing condition implies the following relationship between money growth and inflation:

$$\theta_t = (\beta R_t)^{\frac{1}{\sigma}} \pi_t. \tag{2.15}$$

Therefore, we can consider the government as directly controlling the inflation rate, rather than simply the rate of money creation.

As noted in King and Rebelo [23], the agent's utility is finite if and only if $\beta(\beta R_t)^{(1-\sigma)/\sigma} < 1$. The King–Rebelo condition holds for all experiments conducted in this paper. For the remainder of the paper, we consider only cases in which the policy variables are constant over time. We also choose to disregard certain portions of the parameter space. As Jones and Manuelli [21] show, part of the parameter space results in no growth. In our model, this implies that all capital will be financed through the nonbank. We therefore limit ourselves to the part of the parameter space for which the quantity of intermediated capital is strictly positive and endogenous growth occurs.

3. MONETARY POLICY EXPERIMENTS

In this section, we compute the present value of government revenues for various settings of the reserve ratio and the inflation rate. Following Ireland [20], we begin our investigation by setting the baseline values of G^0, γ^0, and π^0. We then ask whether it is possible to fund the same sequence of expenditures for different values of γ or π. In essence, we ask whether the growth effects and capital substitution effects induced by changes in monetary policy will result in revenue sufficient to cover the revenue losses due to a lower tax base (reserve requirements) or tax rate (inflation rate).

3.1. Calibration

To quantitatively assess this model economy, we must first parameter values. Table I presents the parameter settings used in the baseline computational experiments. Most are standard in the literature; accord-

TABLE I
Calibrating the Model's Parameters

Variable	Baseline value
β	0.95
σ	2
A	0.165
δ	0.10
τ	0.2
π^0	1.214
γ^0	0.173

ingly, we reserve more detailed discussion for selecting values of the model-specific parameters. The values for the inflation rate and reserve requirement were obtained from cross-country data. We obtained price, bank reserve, and bank deposit data for a sample of 82 countries, spanning the period 1975–1994. The (gross) inflation rate and reserve ratio presented in Table I are the sample averages for those 82 countries.

There is little guidance in calibrating the parameters for the nonbank contracts. One useful observation is the fraction of the capital stock that could be financed with bank deposits. Data on the stock of private capital are measured in current dollars, using end-of-year figures. Capital is defined as the net value of fixed private capital plus consumer durable goods. (These data are taken from *Fixed Reproducible Tangible Wealth in the United States, 1925–89*.) We use the Federal Reserve's definition of M2, subtracting currency held by the nonbank public to get an aggregate measure of deposits in the two classifications.[8] In the model economy, the bank's balance sheet identity is $d = m + k$. We next subtract the value of bank reserves. The result is a measure of the fraction of capital accumulation that would be financed by bank deposits, provided that the bank uses all available deposit proceeds to purchase capital. Flow-of-funds data give us observations on banks' holdings of government bonds, which need to be subtracted from $d - k$ to obtain a measure of private capital. For the period 1959–1989, the fraction of bank-financed capital fluctuates around 22%.

To pin down the fraction of bank-financed capital in the model economy, we need to specify a functional form for $f(k^n)$ and use Eq. (2.13). Above, we argued that the resource-cost function must be convex for disintermediation to occur in the event of either higher reserve ratios or higher inflation rates. Consequently, the functional form is chosen from the family of functions $f(k^n) = B(k^n)^{\omega}$, where $\omega > 1$. We use the parameter B to help pin down the fraction of capital financed by nonbank contracts. Clearly, both B and ω affect the equilibrium outcome. With so little to guide our selection of these two parameters, it seems essential that we consider several different combinations to determine whether a robust set of results emerges. We use four combinations for B and ω. For each setting, the combination yields a model economy's fraction of bank-financed capital that is roughly equal to 22% at $\pi = 1.04$ and $\gamma = 0.07$: (i) $B = 0.0042$ and $\omega = 1.5$; (ii) $B = 0.0031$ and $\omega = 5$; (iii) $B = 0.0053$ and $\omega = 10$; (iv) $B = 21.5$ and $\omega = 50$.

[8]After 1992, reserve requirements apply against checkable deposits but not time and savings accounts. We use M2 deposits, checkable deposits, and time and savings accounts, because reserve requirements were applied against both deposit types for most of the sample period and across most of the sample countries.

3.2. Computational Experiments

In this paper, the computational experiments involve the present value of the government budget constraint. Balanced growth simplifies the computations in the sense that real transfer payments grow at the same rate as output. Hence, the ratio of real government spending to output is constant.

We begin by characterizing the government's budget constraint. Consider a case in which the government is balancing its budget at each date t. Thus, with $b_t^g = 0$, $\forall t$, the constraint becomes

$$G_t = \frac{m_t - m_{t-1}}{p_t} + \tau A[k_t + k_t^n]. \tag{3.1}$$

With $\alpha = G_t/K_t$ a constant,[9] one can represent the value of government's date-t expenditures as

$$G_t = \alpha A\left\{\beta\left[(1-\gamma)\big((1-\tau_t)A + 1 - \delta\big) + \frac{\gamma}{\pi}\right]\right\}^{\frac{t}{\sigma}}. \tag{3.2}$$

Next, we set Eq. (3.1) equal to (3.2) and substitute the money supply rule, yielding

$$(\theta - 1)\frac{\gamma d_t}{\pi} + \tau A[k_t + k_t^n] = \alpha A\left\{\beta\left[(1-\gamma)\big((1-\tau_t)A + 1 - \delta\big) + \frac{\gamma}{\pi}\right]\right\}^{\frac{t}{\sigma}}. \tag{3.3}$$

Further substitution of the bank's asset allocation constraint yields the date-t budget constraint as a function of the reserve requirement, the inflation rate, the income tax rate, and the stocks of capital financed through banks and nonbanks. Formally,

$$\frac{(\theta - 1)\gamma}{(1-\gamma)\pi}k_t + \tau A[k_t + k_t^n] = \alpha A\left\{\beta\left[(1-\gamma)\big((1-\tau_t)A + 1 - \delta\big) + \frac{\gamma}{\pi}\right]\right\}^{\frac{t}{\sigma}}. \tag{3.4}$$

[9] A constant value for G/K is equivalent to the limiting condition that G/Y is constant. We use G/K and Eq. (3.2) because it closely parallels the derivation in Ireland [20].

The first term on the left-hand side of (3.4) is date-t real seigniorage revenue. Summing over all dates yields the present value government budget constraint (PVG),

$$PVG = \sum_{t=0}^{\infty} (R)^{-t}\left[\frac{(\theta - 1)\gamma}{(1-\gamma)\pi}k_t + \tau A(k_t + k_t^n) - G^0\right]. \tag{3.5}$$

where G^0 is the "baseline" present value of government spending. In other words, G^0 represents the present value associated with monetary policy parameters set at γ^0 and π^0 and constant τ. Note that PVG is measured in units of the consumption good.

Now we characterize the change in the present value government budget constraint, d(PVG), for a given change in monetary policy. Suppose, for example, that monetary policy parameters change to new values, denoted π^1 and γ^1. The experiment asks whether the same sequence of expenditures, with the possibility of short-term debt financing, can be financed while the present value of government expenditures is still in balance. The present value of the government's budget constraint under the new parameters is computed and compared with the initial setting. The government can fund the same sequence of transfers if and only if

$$d(\text{PVG}) = \frac{x^1k_t^1 + \tau Ak_t^{n1}}{R^1 - (\beta R^1)^{\frac{1}{\sigma}}} - \frac{x^0k_t^0 + \tau Ak_t^{n0}}{R^1 - (\beta R^0)^{\frac{1}{\sigma}}} \geq 0, \tag{3.6}$$

where $x^0 \equiv (\theta^0 - 1)\gamma^0/((1-\gamma^0)\pi^0) + \tau A$ and $x^1 \equiv (\theta^1 - 1)\gamma^1/((1-\gamma^1)\pi^1) + \tau A$. Equation (3.6) will be the basis for our computational experiments. Specifically, the experiments quantify differences in the present value of government expenditures for different settings of the monetary policy parameters. As such, the results compare two different economies with different anticipated, permanent values for the inflation rate and reserve requirement ratio.[10]

From Eq. (3.6), it is straightforward to account for the multiple channels through which movements in the monetary policy variables operate on the change in the present value of government spending. Consider, for instance, an increase in the reserve requirement. First, the term x merges

[10] Bhattacharya et al. [4] find that imposing a binding reserve requirement might improve welfare for cases in which government deficits are large. The upshot is that welfare considerations may justify the presence of a reserve requirement.

movements in the seigniorage tax base and tax rate. For an increase in the reserve requirement, x increases; the higher the reserve requirement, the more fiat money the bank is forced to hold. Second, as noted from Eq. (2.11), an increase in the reserve requirement lowers the return on deposits. This effect manifests itself through a decrease in intermediated capital. Ultimately, the substitution from bank deposits to nonbank contracts reduces the seigniorage tax base. In addition, the lower return on deposits means that the economy's growth rate falls, implying a permanent decrease in the path of government expenditures. Last, the discount factor in the denominator of (3.6) is inversely related to the return on deposits. Hence, an increase in the reserve requirement means that the lower path of government purchases is discounted less heavily over time.

Overall, the effect of the increase in reserve requirements on the present value of government expenditures is ambiguous. Similar ambiguities arise when one considers an increase in the inflation rate. Thus, the computational experiments quantify the effects that different monetary policy settings have on government spending. A rise or fall in the present value of spending in response to economies with lower (higher) inflation rates can also be used to identify whether a dynamic Laffer curve is present for inflationary finance.

Figure 1 plots $d(\mathrm{PVG})$ associated with a change in the reserve requirement ratio, starting with the initial value $\gamma^0 = 0.173$.[11] Each cell in Fig. 1 corresponds to a different setting for the nonbank's resource-cost function. The experiments take the stream of expenditures and the policy setting as given. The question is whether the present value of real seigniorage revenue increases, decreases, or stays the same when there is a permanent, anticipated change in a policy variable. What the four cases show is that the model economies are qualitatively very similar. In each case, $d(\mathrm{PVG}) > (<) 0$ for $\gamma < (>) \gamma^0$. The most striking feature is the consistency of the plots; that is, the profile is fairly flat, dropping off for reserve ratios above 17%. Closer inspection indicates that a revenue-maximizing reserve ratio is present for each resource-cost function.

The quantitative findings mirror the economics embodied in Eq. (3.6). Specifically, the model's data indicate that with a lower reserve ratio, faster growth together with agents substituting from nonbank contracts to bank deposits can result in a higher tax base upon which seigniorage revenue is

[11] We compute $d(\mathrm{PVG})$ for $\gamma \in [0.01, 0.25]$. Note that as $\gamma \to 0.35$, the graph is dominated by the huge spike which arises because R^1 approaches $(\beta R^0)^{1/\sigma}$ from above, causing the denominator of $d(\mathrm{PVG})$ to change from a positive number to a negative number. A similar effect arises in the inflation rate experiments. Graphs for these cases are available from the authors upon request.

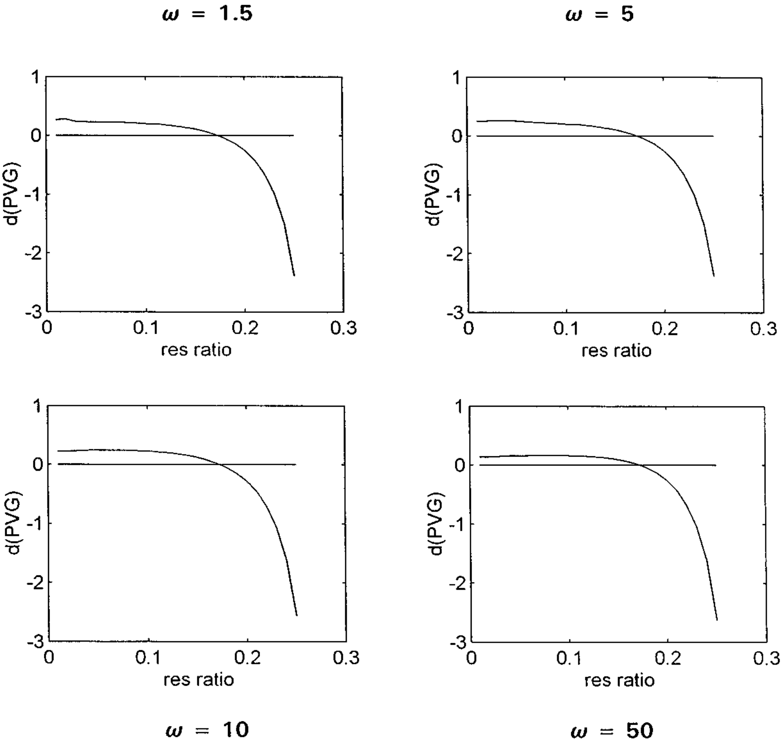

FIG. 1. Reserve requirements experiments—baseline model.

computed. Compared with the baseline policy setting, the increase in the tax base more than offsets the reduction in the tax base stemming from the lower reserve ratio in present-value terms.

Generally speaking, ω determines the speed of disintermediation; more specifically, a one percentage point increase in reserve ratios produces a larger decline in the fraction of date-1 capital financed via bank deposits when $\omega = 1.5$ than when $\omega = 50$. For seigniorage revenue, the different values of ω, therefore, imply that a given increase in the reserve ratio has a larger effect on the tax base when ω is smaller. In effect, smaller values of ω intensify disintermediation.[12] Changes in the resource-cost function

[12] To get a sense of the speed of disintermediation across the different cases, we calculated the fraction of next-period capital financed through the bank. With $\omega = 50$, the fraction of date-1 capital financed via banks falls from 23.3% at $\gamma = 0.01$ to 17.7% at $\gamma = 0.25$. With $\omega = 1.5$, the fraction of date-1 capital financed via banks falls from 88.1% at $\gamma = 0.01$ to 1.6% at $\gamma = 0.04$.

TABLE II
Revenue-Maximizing Policy Settings

A: Parameter settings	γ^{max}
$B = 0.0042$, $\omega = 1.5$	0.02
$B = 0.0031$, $\omega = 5$	0.03
$B = 0.0053$, $\omega = 10$	0.06
$B = 21.5$, $\omega = 50$	0.08
B: Parameter settings	π^{max}
$B = 0.0042$, $\omega = 1.5$	1.03
$B = 0.0031$, $\omega = 5$	1.01
$B = 0.0053$, $\omega = 10$	1.06
$B = 21.5$, $\omega = 50$	1.09

have substantial effects on the revenue-maximizing reserve requirement. Part A of Table II reports the reserve ratio that maximizes the present value of real seigniorage revenue for each of the four resource cost functions. In our experiments, the revenue-maximizing reserve ratio is 2% for $\omega = 1.5$ and 8% for $\omega = 50$.[13]

Figure 2 plots $d(\mathrm{PVG})$ in the inflation rate experiments. We compute the change in the present value of real seigniorage revenue for $\pi \in [1.0, 1.40]$. As with the reserve ratio experiments, we consider four sets of parameter settings for the nonbanks' resource cost function. The cells in Fig. 2 look similar to one another and similar to those in Fig. 1. Again, the values selected for the resource cost function are critical in determining the revenue-maximizing value. Part B of Table II reports the inflation rates that maximize the present value of real seigniorage revenue. Our findings indicate that a 1% inflation rate maximizes the present value of real seigniorage revenue when disintermediation is relatively fast, and that

[13]Another implication of Eq. (3.6) is that one can see that the effects of monetary policy settings are not independent of one another. Instead of using the sample mean from the cross-country dataset, we use the sample mean for the U.S. sample computed using data for the period 1959–1995; that is, $\pi = 1.042$. We then redid the reserve ratio experiments. The chief difference is that given a one percentage point increase in the reserve ratio, the decrease in the equilibrium rate of output growth is smaller when the inflation rate is lower. There is also less saving since the gross rate of return on deposits (and nonbank contracts) also falls. With $\omega = 50$, the revenue-maximizing reserve requirement is 8% when $\pi = 1.214$, but falls to 1% for the case in which $\pi = 1.042$.

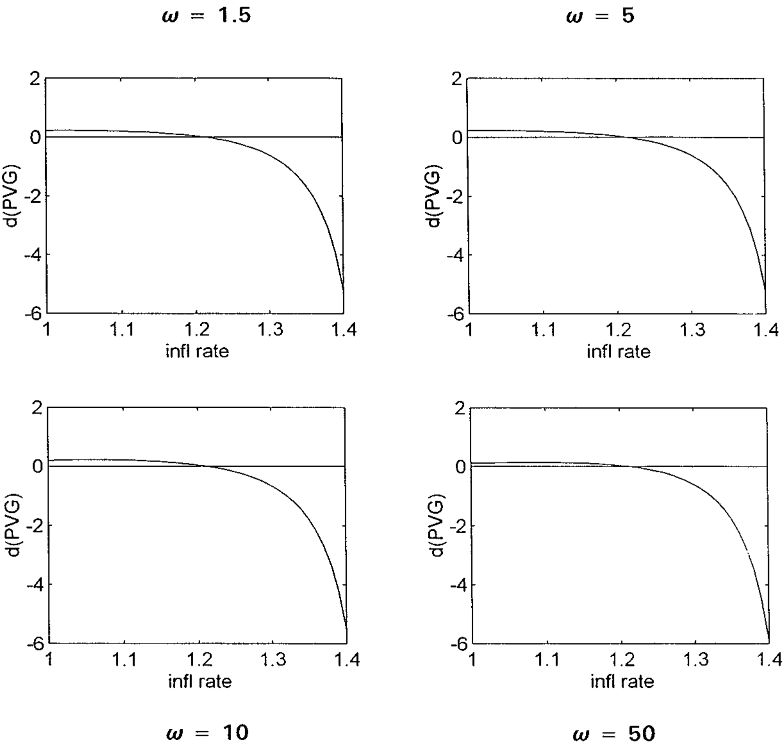

FIG. 2. Inflation rate experiments—baseline model.

a 90% inflation rate maximizes the revenue measure when disintermediation is slower.

Overall, our results show that a dynamic Laffer curve is present for reasonably calibrated economies. These findings reflect the fact that higher reserve ratios and inflation rates result in a slower growth rate and reduce the fraction of capital financed via bank deposits. The presence of the Laffer curve indicates that the growth-rate effects and disintermediation cause reductions in the quantity of real fiat money balances, more than offsetting the direct effects that higher reserve ratios and higher inflation rates have on real seigniorage revenue. Interestingly, the results of computational experiments suggest that the revenue-maximizing policy setting are well below the sample means taken from the cross-country data.

4. ASSESSING QUANTITATIVE IMPORTANCE

Next, we consider the case in which capital accumulation is financed completely through bank deposits.[14] In this model economy, the agent's budget constraint is given by

$$c_t + d_{t+1} + b_{t+1}^g \leq R_t(d_t + b_t^g) + G_t. \tag{4.1}$$

Note that the budget constraint eliminates nonbank contracts as a means of financing future consumption. (We also assume that government bonds and bank deposits are perfect substitutes as a means of storing for future consumption.) In the absence of the nonbank contracts, there is no capital substitution effect. With this modification, the present-value expression becomes

$$d(\text{PVG}) = \frac{x^1 k_t^1}{R^1 - (\beta R^1)^{1/\sigma}} - \frac{x^0 k_t^0}{R^1 - (\beta R^0)^{1/\sigma}} \geq 0. \tag{4.2}$$

The question we now consider is how $d(\text{PVG})$ responds to changes in the requirement or the inflation rate. Figures 3 and 4 provide quantitative answers to these questions, plotting the value of $d(\text{PVG})$ for different values of the reserve requirement and the inflation rate, respectively. As in previous figures, both show a $d(\text{PVG})$ curve that is hump-shaped. The chief difference is where the revenue-maximizing values of the reserve ratio and inflation rate occur.

In the absence of nonbank contracts, a higher reserve requirement lowers the return to bank deposits, which has two effects: the rate of growth declines and saving is less attractive. For real seigniorage revenue, deposits grow more slowly, resulting in a slower growth in the tax base. For this model economy, the present value of real seigniorage revenue is maximized when $\gamma = 0.23$.

Figure 4 plots $d(\text{PVG})$ for alternative values of the inflation rate. As with the experiments in which the reserve requirement changes, the $d(\text{PVG})$ curve is hump-shaped. The qualitative reasons are the same as those given for the reserve ratio experiments. For the inflation rate experiments, the present value of real seigniorage revenue reaches a maximum at $\pi = 1.34$.

[14] Espinosa and Yip [13] present a similar model economy in which reserve ratios affect the endogenous rate of capital accumulation. They show that a seigniorage-revenue-only policy is the preferred policy for cases in which the government purchases omit capital and when multiple equilibria are present.

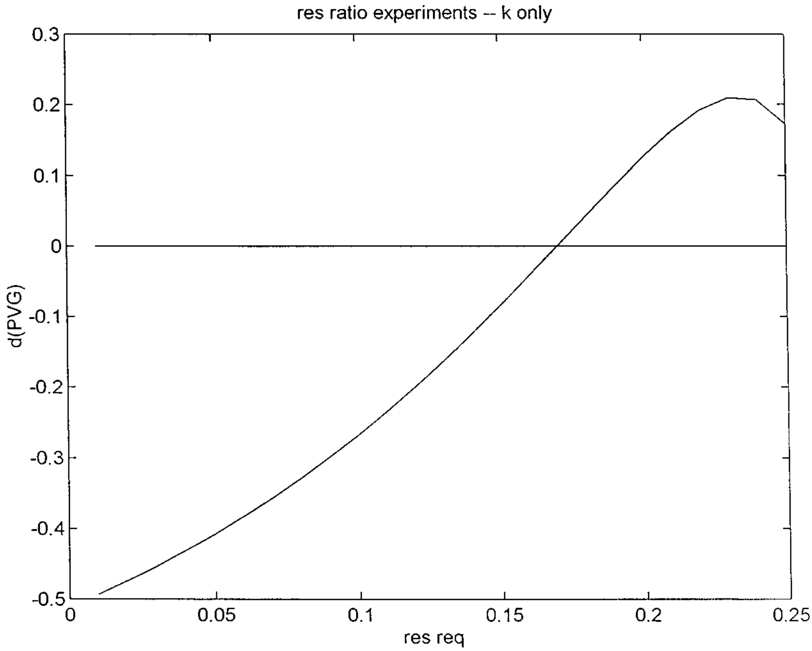

FIGURE 3

The experiments in this section offer some insight into how important disintermediation is for real seigniorage revenue. With disintermediation eliminated, the revenue-maximizing reserve ratio increases fro 8% to 23%, while the revenue-maximizing inflation rate increases from 9% to 34%.[15]

5. SUMMARY AND CONCLUSIONS

In this paper, we quantify the effect of two alternative monetary policy tools, the inflation rate and the reserve requirement, on the present value of government expenditures. Revenue in our model comes from a combination of income and inflation taxes. In this paper, we focus on the

[15] In most countries, currency in the hands of the public accounts for a majority of the inflation tax. It is possible to extend this setup to examine the revenue-maximizing policy settings when households have cash and credit as means of payment. In Haslag and Young (1998), we show that the revenue-maximizing policy settings depend on how close cash payment substitutes for credit payment.

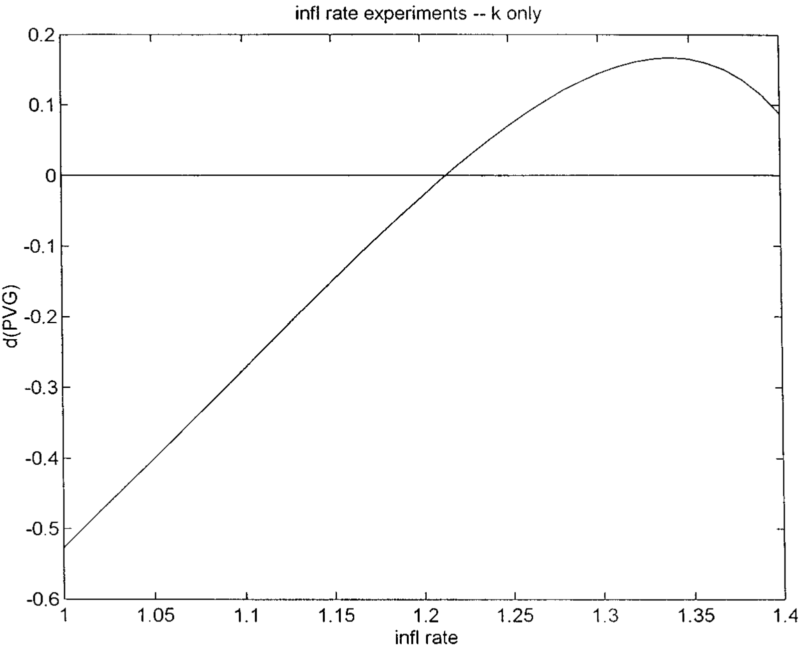

FIGURE 4

inflation tax. Indeed, the model economy possesses two distinct monetary policy experiments because the inflation tax base depends on the size of the reserve requirement ratio.

The chief contribution of this paper is to quantitatively assess the impact that changes in money growth and reserve requirements have on the present value of real seigniorage revenue. The model economy is rich enough to permit several different channels to operate simultaneously. Specifically, we consider two channels. One channel stresses the relationship between inflation and growth; in short, both faster money growth and higher reserve requirements, for example, retard the economy's growth rate. The other channel—disintermediation—permits households to avoid the inflation tax. Disintermediation occurs because there are two means of financing, only one of which is subject to monetary policy distortions. Inflation-tax avoidance, therefore, is permitted by simply depositing savings with the nonbank intermediary.

A dynamic Laffer curve is present in those model economies in which growth is endogenous. Hence, our monetary policy experiments are similar to the results found in Ireland [20] for fiscal policy. In an economy with a fairly sophisticated financial system, we find that the revenue-maximizing values for the monetary policy variables are well below the cross-country sample means observed in the data. The relationship between monetary policy and the rate of growth is the determining factor in finding a Laffer curve. The level of financial development, as measured by the resource cost associated with nonbank finance, also bears on the revenue-maximizing monetary policy settings, although its impact is relatively small when compared to the growth rate effect.

The specific quantitative results are as follows:

- When both the growth and disintermediation channels are operating, the revenue-maximizing reserve requirement is 8% and the revenue-maximizing inflation rate is 9%.
- For the same set of experiments, the revenue-maximizing reserve ratio and inflation rate fall to 2% and 1%, respectively, in economies with extremely fast disintermediation.
- If the disintermediation channel is eliminated, leaving only the growth-rate channel in effect, the revenue-maximizing reserve ratio and inflation rate are 23% and 34%, respectively.

Our main goal was to quantify the present value of real seignorage revenue across several different model economies. The economies are linked by systematically eliminating or adjusting specific channels that affect the seignorage tax base. Other authors have computed revenue-maximizing inflation rates. Fry [17], for example, finds that real seignorage revenue is maximized with inflation rates in excess of 50% for a stationary economy in which governments have monopoly power over both currency and deposits. Thus, our evidence bears on the quantitative importance of a supply-side channel and disintermediation for the present value of real seignorage revenue.

We have considered only revenue as a motivating factor for monetary policy. One potential extension would be to consider revenue issues at business cycle frequencies. In such models, it may be possible to build on the work of Auernheimer [1]. Another extension would be to focus on strategic issues between policymakers: when the monetary and fiscal authorities do not coordinate actions, what are the revenue implications? Such questions hark back to issues of decentralized policymaking common in the 1970's—would the fiscal authority try to use the inflation tax to covertly collect revenue in a model in which both fiscal and monetary authorities operate independently? It is likely that such considerations would lead to conclusions very different from the ones reached in this paper.

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