

# The Inside Baseball of Sudden Stop Models

Bingbing Dong\*

Jieran Wu<sup>†</sup>

Eric Young<sup>‡</sup>

## Abstract

This paper examines the role of collateral in “sudden stop” models that feature occasionally-binding constraints and endogenous growth. We show how different assumptions regarding the nature and valuation of collateral alter the dynamics of crisis episodes and the welfare costs of pecuniary externalities. For example, in a model with land as collateral, we show that valuing collateral at the “expected future price” leads to substantially weaker Fisherian deflation effects compared to the case with collateral valued at the “current price.” However, the average size of sudden stops in the two economies are similar, because households endogenously avoid the region where large sudden stops would occur. The difference between different collateral valuations and the size of sudden stops are amplified when we abstract from endogenous growth. In another case, assuming collateral is income rather than land leads to smaller sudden stops as income is less volatile than asset prices. Finally, we show that the efficiency distortions also change in terms of nature and policy implications; using the expected future price case the competitive equilibrium is efficient, while other cases feature a small distortion.

## 1 Introduction

In models of sudden stops the collateral constraint is generally simply assumed into existence, rather than being derived explicitly from an underlying friction. As a result, there exist a number of plausible choices regarding what serves as collateral and how that collateral is to be valued. To be specific, suppose that the occupants of a small open economy can pledge land in return for international debt – examples in the literature suppose that either (i) individual land holdings

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\*School of Finance, Central University of Finance and Economics

<sup>†</sup>Academy of Financial Research and College of Economics, Zhejiang University

<sup>‡</sup>Department of Economics, University of Virginia

or (ii) aggregate land holdings serve as collateral, that this collateral is valued at either (i) the current price or (ii) the expected future price or (iii) the worst possible future price, and that either (i) current land or (ii) future land serves as collateral. All of these assumptions can be found in the literature – see Bianchi and Mendoza (2018), Devereux, Young, and Yu (2018), Jeanne and Korinek (2010a,b), Korinek and Mendoza (2014), Ma (2018), and Mendoza (2010). Another strand of literature views collateral as arising from current income, including Bianchi (2011) and Benigno et al. (2013, 2016, 2019), in which some type of terms of trade price appears.

In this paper we characterize how the behavior of a small open economy model with endogenous growth and collateral constraints (a “sudden stop” model) changes across different model specifications. Specifically, we consider alternative assumptions regarding (i) elastic vs. inelastic labor; (ii) endogenous vs. exogenous growth; (iii) collateral valuation at current vs. expected future prices; (iv) current vs. future land as collateral; (v) income vs. land as collateral; (vi) aggregate vs. individual collateral. The above assumptions correspond to various modeling choices in terms of the model environment (i, ii), collateral valuation (iii), and nature of collateral (iv,v,vi). We find that some of these choices substantially change the behavior of the model and the welfare losses associated with pecuniary externalities, while others make no quantitative difference. We concentrate here on presenting the key choices and the mechanisms that underlie the resulting differences, and relegate discussions of other choices to a short section.

The first set of key choices we investigate regards the valuation of collateral in a model with endogenous growth and inelastic labor. Let land  $n_t$  serve as collateral and be valued at either (i) the current price  $q_t$  or (ii) the expected future price  $E_t[q_{t+1}]$ . In the first case, the sudden stop displays a strong “Fisherian deflation” effect: a negative shock that causes the collateral constraint to bind forces consumption to fall, which in turn reduces asset values today (since land is productive) and then further tightens the constraint, necessitating further reductions in consumption. The result is a non-monotonic debt function; debt decreases rather than increases in the binding region. In contrast, the “feedback” mechanism is substantially weaker if land is valued at next period’s price – we find a “flat spot” in the debt accumulation rule in the binding region. The result is that a typical sudden stop will be smaller in the “future price” case. Since the collateral value is less vulnerable to bad shocks in the expected price case, agents reduce precautionary saving and accumulate higher debt on average, causing the crisis probability to

increase. Quantitatively, however, we find only small differences because agents endogenously avoid the regions of the state space where large sudden stops would occur.

The reduced severity of sudden stops is partially due to the presence of endogenous growth. Introducing endogenous growth affects the model dynamics through two channels. First, households have an extra tool to smooth consumption by changing the R&D investment, which generates endogenous variations in the stochastic discount factor that offsets the changes in the intertemporal marginal rate of substitution (IMRS). We term this effect as the “discount rate” channel, as it effectively changes the relative patience of agents. Second, the dividend stream gets augmented with an additional growth term, which in turn affects the collateral asset (land) value; we call this effect the “dividend channel”. Using a special analytical example with no “discount rate” channel, we illustrate that compared with the no-growth economy, the collateral value is higher and less sensitive to changes in productivity with endogenous growth. When a sudden stop happens, agents reduce the R&D investment and the growth rate. Compared to normal times, the decline in the growth rate causes agents to act more patient, increasing asset (land) demand and reducing borrowing. Although a drop in growth rate affects the land’s valuation through future dividend, its impact is transitory. Therefore, the drop in asset/collateral value during a crisis is mitigated, and this force is stronger if the collateral is valued at the future expected price.

The second set of key choices we investigate regards the nature of collateral with elastic labor. We consider both land and income as collateral, valued at current prices. In either case, we find smaller sudden stops. The reason is that, given the current state, the deflation spiral is weaker because agents can increase labor effort during a sudden stop; increasing labor effort both raises the value of land (due to complementarity in the production function) and raises income (since production is decreasing returns to scale in labor). As a result, the decision rule for debt does not “turn upward” but rather only falls at a slower rate (that is, agents always borrow more if their debt is higher).<sup>1</sup>

Finally, we consider the implications of our experiments for the measurement of pecuniary externalities; that is, how different would outcomes be if private agents internalized their effect on collateral values (as a constrained planner would do)? We define the “conditional efficiency”

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<sup>1</sup>We do not consider future price valuations here, because the presence of future controls in the current set of constraints raises time consistency issues.

following Benigno et al. (2012) and suppose the planner takes as given the collateral pricing function (that is, the price in a given state is fixed), but allow the planner to alter the set of states that are visited. We find that the “future price” case is efficient in this sense – the planner chooses to do nothing. The reason is that when constraint is binding, the planner can not change the value of collateral by adjusting the debt positions. In contrast, with collateral (land or income) valued at current prices, the planner can improve outcomes by policy interventions. Qualitatively, both the land collateral and the income based collateral lead to over-borrowing in the competitive equilibrium, even though the sources of efficiency distortions are not identical. Quantitatively, the welfare cost of efficiency distortions associated with pecuniary externalities are small. In terms of policy implications, the planner can implement a capital control to correct the externalities with revenues rebated lump-sum. If the labor is elastic and the current income serves as the collateral, however, planner would need a combination of labor income tax and a capital control to implement the conditionally efficient allocation.

## 2 Model

We consider a small open economy model with an occasionally-binding collateral constraint and endogenous growth. The basic structure of the economy follows Ma (2018), with two exceptions. First, we allow for elastic labor such that the labor income can serve as the self-insurance against sudden stops. Second, we consider different specifications in terms of the nature of collateral. The preferences of the representative household are represented by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(c_t - hZ_t)^{1-\gamma} (1 - l_t)^{\omega(1-\gamma)}}{1 - \gamma} \right],$$

where  $c_t$  is the consumption,  $l_t$  is labor, and  $Z_t h$  is a subsistence consumption level with  $h > 0$  and aggregate investment  $Z_t$ .<sup>2</sup> The parameter of the relative share of leisure in preference is defined as  $\omega \geq 0$ ;  $\omega = 0$  corresponds to the case of inelastic labor.

Output of the representative firm is produced using land and labor:

$$y_t = A_t n_t^\alpha l_t^\eta,$$

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<sup>2</sup>The introduction of  $Z_t h$  ensures balanced growth.

with  $\alpha > 0$  and  $\eta > 0$ . Productivity  $A_t$  is endogenous and depends on a random shock  $\theta_t$  and past investment  $z_t$ :

$$A_t = \theta_t z_t.$$

The productivity shock follows the AR(1) process,

$$\log(\theta_{t+1}) = (1 - \rho) \log(\theta) + \rho \log(\theta_t) + \varepsilon_{t+1},$$

where the *i.i.d* Gaussian innovations  $\varepsilon_t$  have mean 0 and variance  $\sigma_\varepsilon^2$ .

The representative household can borrow from the international bond market at an exogenous interest rate  $r$ , subject to a collateral constraint given by

$$b_{t+1} \geq -\phi X_t,$$

where  $b_{t+1}$  is the time  $t$  choice of bond holding and  $X_t$  is the collateral (to be specified shortly).  $\phi$  is a fixed parameter that captures the maximal loan-to-value ratio. The household budget constraint is

$$c_t + \Psi(z_{t+1}, z_t) + b_{t+1} + q_t n_{t+1} \leq \pi_t + w_t l_t + (q_t + d_t) n_t + (1 + r) b_t,$$

where  $q_t$  is the land price,  $w_t$  is the wage rate,  $d_t$  is the dividend generated by land,  $\pi_t$  is profit of the firm, and  $\Psi(z_{t+1}, z_t)$  is a quadratic adjustment cost of changing productivity from  $z_t$  to  $z_{t+1}$ :

$$\Psi(z_{t+1}, z_t) = \left( \left( \frac{z_{t+1}}{z_t} - \psi \right) + \kappa \left( \frac{z_{t+1}}{z_t} - \psi \right)^2 \right) z_t.$$

where  $\psi > 0$  is the minimum growth rate (which requires no adjustment cost) and  $\kappa > 0$  is the adjustment cost parameter. Land is in constant supply so that in equilibrium we must have

$$n_t = N_t \equiv 1, \quad \forall t$$

The equilibrium balance of payment equation is given by

$$C_t + \Psi(Z_{t+1}, Z_t) + B_{t+1} \leq \theta_t Z_t L_t^\eta + (1 + r) B_t.$$

The equilibrium consistency conditions are given by  $c_t = C_t$ ,  $b_t = B_t$ ,  $l_t = L_t$ , and  $z_t = Z_t$ , for all  $t$ .

Finally, we can obtain an exogenous growth economy  $Z_t = \psi$ ,  $\forall t$  as a special case by setting  $\kappa = \infty$ , and specifically a no-growth economy by additionally assuming  $\psi = 1$ .

### 3 Collateral

Choices of collateral can be highly flexible, and we allow  $X_t$ , the value of collateral, to vary along three main dimensions. The first dimension considers the change in timing of the valuation of the collateral. In this case we suppose that either (a.1) the current price  $q_t$  or (a.2) the expected future price  $E_t[q_{t+1}]$  is used to value the land. In addition, we also allow collateral to be based on (a.3) current land  $n_t$  or (a.4) future land  $n_{t+1}$ .<sup>3</sup> The second dimension considers the nature of the collateral. In this case we suppose that either (b.1) individual land holdings  $n$  or (b.2) aggregate land holdings  $N = 1$  serve as collateral. The difference across the two specifications is whether households know that the value of their collateral is under their control; of course, in equilibrium it is not possible for them to acquire more land, but the incentive to try causes price effects that differ across the two specifications. In addition, we suppose that either (b.3) land or (b.4) the flow of income served as collateral when labor is elastic. Finally, we also examine different model environment including (c.1) inelastic labor supply, (c.2) elastic labor supply, (c.3) growth economy, and (c.4) no-growth (level) economy.

Combinations of these cases yield a large number of scenarios that we need to analyze, but many of these cases generate sudden stop dynamics with little difference.<sup>4</sup> To illustrate the main message in our findings, therefore, we proceed with following steps. First, we analyze the model economy with inelastic labor supply and compare the results when collateral is valued at  $X_t = q_t n_{t+1}$  versus  $X_t = E_t q_{t+1} n_{t+1}$ , as the choice on the timing of collateral prices manifests the starkest distinction in model's positive and normative predictions.<sup>5</sup> Then we introduce the elastic labor supply and compare the results when either the land or the flow of income serve as the collateral. That is, we consider the case  $X_t = q_t n_{t+1}$  versus  $X_t = w_t l_t + d_t n_t$ . Then in Section 6 we discuss other possible scenarios selected from the combinations of (a.1)–(c.4). Finally, we discuss the conditional efficiency and policy implications in section 7. Table 1 provides an easy taxonomy for the various cases discussed below.

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<sup>3</sup>Note that if collateral is aggregate land then it does not matter if it is  $N_t$  or  $N_{t+1}$ .

<sup>4</sup>We exclude some combinations of the above conditions with no economic justification, e.g. collateral valued as  $q_{t+1} n_t$ .

<sup>5</sup>The choice of inelastic labor is purely pedagogical, and we relax this assumption in discussion later.

Table 1: Taxonomy of Economies

Case I	$X_t = E_t q_{t+1} n_{t+1}$	Inelastic Labor	Growth
Case II	$X_t = q_t n_{t+1}$	Inelastic Labor	Growth
Case III	$X_t = q_t n_{t+1}$	Elastic Labor	Growth
Case IV	$X_t = w_t l_t + d_t n_t$	Elastic Labor	Growth
Case V	Other possible combinations		

## 4 Parameterization

In the baseline environment with inelastic labor supply and positive economic growth, we calibrate the parameters following Ma (2018). Unless specified otherwise, we hold these parameters all fixed across specifications in terms of the collateral. The calibration is conducted using Case II. In particular, we target an average growth rate of 2.3 percent, a probability of a sudden stop equal to 5.5 percent, a debt/GDP ratio of 27.8 percent, a consumption/GDP ratio of 77.5 percent, and a correlation between GDP and the current account of  $-0.26$ .

In the case of elastic labor, we set  $\omega = 2$  and  $\eta = 0.6$  to make average share of leisure in instantaneous utility  $2/3$  and share of labor income 0.6, respectively.<sup>6</sup> To make output and value of collateral under inelastic and elastic labor cases comparable, average productivity  $\theta$  and collateral ratio  $\phi$  are adjusted accordingly. Finally, in the case of level economy with no growth, we set  $\psi = 1$  and  $\kappa = \infty$ . Table 2 summarizes the calibrated parameters used in our analysis.

The model economy is solved using a policy iteration algorithm (see Benigno et al. 2016), with unknown functions parameterized as piecewise-cubic Hermite splines. The Fortran code to solve the various models are available upon request.

<sup>6</sup>In the elastic labor case, the calibration implies an average growth rate is of 3.25 percent, a probability of a sudden stop equal to 12.6 percent, a debt/GDP ratio of 30.3 percent, a consumption/GDP ratio of 86.1 percent, and a correlation between GDP and the current account of  $-0.32$ .

Table 2: Calibration

Baseline: Inelastic Labor, Growth		
Share of Land	$\alpha = 0.2$	Jeanne and Korinek (2010)
Risk-Free Interest Rate	$r = 0.06$	Benigno et al. (2013)
Risk Aversion	$\gamma = 2$	Ma (2018)
Relative Share of Leisure in Preference	$\omega = 0$	Inelastic Labor
Variance of Productivity	$\sigma = 0.04$	Output growth in crisis = $-5.65\%$
Minimal Growth Level	$\psi = 0.95$	Output growth one year after crisis = $3.28\%$
Adjustment Cost for Investment	$\kappa = 26.29$	Consumption/GDP = $77.5\%$
Subsistence Level	$h = 0.51$	Average Growth Rate = $2.3\%$
Discount Factor	$\beta = 0.968$	Probability of Crisis = $5.5\%$
Autocorrelation of Productivity	$\rho = 0.83$	Correlation of GDP and Current Account = $-0.26$
Collateral Limit	$\phi = 0.0852$	NFA/GDP = $-27.8\%$
Elastic Labor		
Leisure in Preference	$\omega = 2$	Relative share of leisure in utility = $2/3$
Labor in Production	$\eta = 0.6$	Share of Labor income = $0.6$
No-Growth Economy		
Minimal Growth Level	$\psi = 1$	No Growth
Adjustment Cost for Investment	$\kappa = \infty$	No Adjustment of Investment for R&D

## 5 Sudden Stop Dynamics

### 5.1 Current and Future Price

We first consider Case I and Case II where we use either current or future price to value the land collateral. As in Bianchi (2011), a sudden stop is defined as a positive change in the net foreign asset position ( $B_{t+1} > B_t$ ) that is larger than one standard deviation ( $B_{t+1} - B_t > \sigma(B_{t+1} - B_t)$ ) and is associated with a binding collateral constraint ( $\mu_t > 0$ ). The last condition, which obviously cannot be observed in data, ensures that the change in assets is not due to consumption smoothing in the presence of a large increase in  $\theta_t$ , which implies that sudden stops are associated with recessions.

Figure 1 plots the equilibrium debt function  $b_{t+1} = g(b_t)$  for the two cases when low value of the shock  $\theta_t$  hits the economy. That is, state 1 refers to the low value of  $\theta_t$  that is two standard deviations away from the mean, while state 2 refers to the low value of  $\theta_t$  that is one standard deviations away from the mean. We also plot the intersection of the equilibrium debt function under the mean TFP shock and the 45 degree line (two dot points in the figure).<sup>7</sup> In both cases, agents increase borrowing when current  $b_t$  is high (i.e. debt level is low) and the collateral constraint is not binding, as the equilibrium debt functions lie below the 45 degree line. The differences between the two cases are not quantitatively significant in this region. **[Insert Figure 1 here]**

On the other hand, when the current debt is already high, the collateral constraint starts to bind and the equilibrium debt functions display kinks. Moreover, the equilibrium debt functions of the two cases are substantially different when the constraint starts to bind. When the collateral is valued at the current price, a deep contraction of debt results from a binding collateral constraint. In this case, our model features the “Fisherian deflation” discussed in Bianchi and Mendoza (2018). If a low productivity shock hits the economy, agents’ ability to borrow to prevent consumption declines is limited by the binding collateral constraint. The falling consumption in turn reduces output demand and therefore causes the value of the productive asset  $q_t$  to drop, which further tightens the constraint. Figure 2 illustrates this feedback loop that generates the sudden stop.

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<sup>7</sup>In the remainder of the paper, we only plot the equilibrium debt function under state 1, unless specified otherwise.

[Insert Figure 2 here]

If  $X_t = E_t [q_{t+1}n_{t+1}]$ , the equilibrium debt function display a flat region when the collateral constraint is binding, in contrast to the upward turn in the previous paragraph; as a result, the sudden stops in this region of the state space will be much smaller. To explore why these functions look so different, we first present some analytical results that shed light on the intuition.

### 5.1.1 Analytical Characterization

Consider the three Euler equations that governs the dynamic choices of endogenous productivity growth, land, and bond holdings for Case II,

$$\lambda_t \Psi_{t,1} = \beta E_t [\lambda_{t+1}(\theta_{t+1} - h - \Psi_{t+1,2})] \quad (1)$$

$$\lambda_t q_t = \mu_t \phi q_t + \beta E_t [\lambda_{t+1}(\alpha \theta_{t+1} Z_{t+1} + q_{t+1})] \quad (2)$$

$$\lambda_t = \mu_t + \beta E_t [\lambda_{t+1}(1 + r)] \quad (3)$$

where  $\lambda_t \equiv (c_t - hZ_t)^{-\gamma}$  is the Lagrangian multiplier associated with the budget constraint, which is equal to the marginal utility of consumption.  $\Psi_{t,1}$  and  $\Psi_{t,2}$  are the time  $t$  partial derivatives of the adjustment cost function with respect to its first and second arguments  $Z_{t+1}$ ,  $Z_t$ , respectively. Using the functional form on  $\Psi$ , we derive

$$\Psi_{t,1} = 1 + 2\kappa(g_t - \psi)$$

$$\Psi_{t,2} = -\psi + \kappa(g_t - \psi)^2 - 2\kappa(g_t - \psi)g_t$$

where  $g_t \equiv \frac{Z_{t+1}}{Z_t}$  is the endogenous productivity growth. Given  $Z_t$ , the quadratic adjustment cost implies that  $g_t \in [\psi, \infty), \forall t$ .  $\mu_t \geq 0$  is the Lagrangian multiplier associated with the collateral constraint. If collateral is valued at the expected future price, then equation (2) is given by

$$\lambda_t q_t = \mu_t \phi E_t q_{t+1} + \beta E_t [\lambda_{t+1}(\alpha \theta_{t+1} Z_{t+1} + q_{t+1})] \quad (4)$$

The non-linear dynamic system (1)-(3) is not analytically tractable with an occasionally binding constraint. As such, we leave aside the parameter calibration and impose the following assumption so that analytical characterizations are feasible.

**Assumption 1**  $\gamma = 0$ ,  $g_t = 1, \forall t$ , and  $1 - \beta(1 + r) > 0$

The linear utility without growth allows us to characterize the land price  $q_t$  in closed form.

**Proposition 1** *Suppose Assumption 1 holds, then the land prices in Case I and II are approximately linear in the shock  $\log \theta_t$ . If there exists no bubble and  $\frac{\beta}{1-\mu\phi} < 1$ , then the stationary equilibrium prices are given by*

$$q_t^I = \frac{\alpha\beta\rho}{1 - \mu\phi\rho - \beta\rho} \log \theta_t + C^I \quad (5)$$

$$q_t^{II} = \frac{\alpha\beta\rho}{1 - \mu\phi - \beta\rho} \log \theta_t + C^{II} \quad (6)$$

The constants are given by

$$C^I = \frac{\alpha\beta + m^I\beta + m^I\mu\phi}{1 - \mu\phi - \beta}$$

$$C^{II} = \frac{\alpha\beta + m^{II}\beta}{1 - \mu\phi - \beta}$$

with  $m^I = \frac{\alpha\beta\rho}{1-\mu\phi\rho-\beta\rho}$  and  $m^{II} = \frac{\alpha\beta\rho}{1-\mu\phi-\beta\rho}$ . The collateral constraint in Case I is less responsive to the shock than in Case II:

$$\frac{\partial E_t q_{t+1}^I}{\partial \log \theta_t} = \frac{\alpha\beta\rho^2}{1 - \mu\phi\rho - \beta\rho} < \frac{q_t^{II}}{\partial \log \theta_t} = \frac{\alpha\beta\rho}{1 - \mu\phi - \beta\rho}$$

If collateral is valued at the future expected price, the  $q_t$  process becomes smoother and less responsive to the exogenous shock  $\theta_t$ , which can be seen in equation (5) and (6). The future expected price  $E_t q_{t+1}^I$  is even smoother due to mean reversion in  $\theta_t$ , given the persistence parameter  $\rho < 1$ , as the expectation of a stationary random variable is inherently less volatile than the random variable itself. The result is that collateral values fluctuate less in Case I than Case II.

Although we use a special example to illustrate this mechanism, the dampening effect remains true in our general model in which we relax Assumption 1. In this case, both  $b_t$  and  $\theta_t$  serve as state variables. For given values of  $\theta_t$ , agents in Case I will always want to borrow more since the collateral constraint is less likely to be affected by a negative shock in  $\theta_t$ , as evident in Figure 1; that is, agents have weakened precautionary savings motives.

### 5.1.2 Quantitative Implications

In classical sudden stop models, the debt deflation spiral leads to deep debt contractions and shrink of asset-collateral values. However, our analytical results in the last section indicate that

this Fisherian debt deflation spiral is mitigated if the collateral is valued at the future expected price. Moreover, in Section 6.1 we show that introducing endogenous growth into the model lead to higher asset valuations than the no-growth economy. These modeling changes make the asset-collateral more valuable on one side and less responsive/elastic to change in exogenous shocks on the other side. To further explore the quantitative implications of our comparison, in Panel A of Figure 3 we plot the average sudden stop consumption paths for both Case I and Case II. As in Bianchi (2011), we define a sudden stop period as one in which the net debt position change is positive and larger than one standard deviation ( $B_{t+1} - B_t > \sigma(B_{t+1} - B_t)$ ) and the constraint is binding ( $\mu_t > 0$ ). When expected future prices are used to value the collateral, the size of sudden stop is a bit smaller as the debt contraction is now mitigated, but not by much. Figure 4 is the distribution function of consumption at the onset of a crisis, constructed from our simulation. While the mean values are very similar, the current price case contains a fat lower tail; the worst case sudden stop has a decline in consumption roughly 20 percent larger than the expected future price case. The reason we get very similar sudden stops on average is that most sudden stops occur as a transition from state 3 (the mean state) to state 2 (one standard deviation below the mean) and they start from the stationary debt level for state 3 for which the constraint is not binding; as noted already, the decision rules are very similar in this region, so the typical sudden stop does not look very different (see Figure 1). However, rarer sudden stops, such as those driven by the less-likely switch from state 3 to state 1 (two standard deviations below the mean), lead to significantly larger drops in the current price case. Figure 5 shows the ergodic distribution displays sharp spikes exactly at those debt levels that correspond to the fixed points of the debt function, with the largest spike at the fixed point from state 3, which justifies our explanation that most sudden stops originate there.

**[Insert Figure 3 - 5 here]**

The difference in the collateral constraints also affects the probability of sudden stops. Table 3 documents the summary of sudden stop crisis statistics using the simulated model time series. Using the current price to value the collateral reduces the probability of sudden stops by 22%. Note that this change reflects two forces: one, agents have different precautionary savings motives, as we have already noted, which lead them to move closer or farther away from the constraint; and two, the reference point for defining a sudden stop changes with each case (the volatility of

debt is not invariant to the form of collateral). To decompose this change, we note that if we view Case I using the volatility of debt of Case II, the sudden stop probability only declines by roughly 6 percent; the vast majority of the change is due only to the change in the volatility of debt. The intuition is straightforward; since the current land price is more sensitive/elastic to changes in  $\theta_t$ , agents take precautionary actions by saving more/borrowing less, reducing the frequency of sudden stops.

Table 3: Summary of Crisis

Categories	Scenario	Prob. of Crisis (%)	Ave Debt	Std of Debt
Inelastic Labor	$X_t = E_t q_{t+1} n_{t+1}$ (Case I)	7.072	-.2969	$7.47 \times 10^{-3}$
	$X_t = q_t n_{t+1}$ (Case II)	5.535	-.2720	$1.32 \times 10^{-2}$
	$X_t = q_t n_t$	4.885	-.2656	$1.38 \times 10^{-2}$
Elastic Labor	$X_t = q_t n_{t+1}$ (Case III)	11.29	-.2983	$2.16 \times 10^{-2}$
	$X_t = w_t l_t + d_t n_t$ (Case IV)	11.20	-.2725	$5.81 \times 10^{-3}$

## 5.2 Elastic Labor: Assets vs Income

In this section, we allow for elastic labor and compare Case III and Case IV in which the land asset value or wage and dividend income flow serve as collateral. When agents are able to adjust their labor effort, labor income serves as a buffering device that mitigates the effect of negative productivity shocks. The effect of this buffering mechanism is twofold. First, the collateral value will be less vulnerable to shocks in  $\theta_t$ , whether we use land or income as collateral. If we use land as collateral, adjusting labor increases the expected marginal return on land,

$$\lambda_t q_t = \mu_t \phi q_t + \beta E_t [\lambda_{t+1} (\alpha \theta_{t+1} Z_{t+1} L_{t+1}^\eta + q_{t+1})],$$

which raises the borrowing capacity of households in equilibrium. If instead income serves as collateral, its value may also increase when agents work more (although this result depends on the elasticity of the wage with respect to aggregate labor, in our model the total effect is positive).

Second, elastic labor income provides a direct insurance against negative productivity shocks that smooth consumption during crisis. Figure 6 and Panel B of Figure 3 plot the equilibrium debt functions and the sudden stops of consumptions for Case III and IV. In Figure 6 for comparison we also plot the inelastic labor case with collateral given by  $X_t = q_t n_{t+1}$ . Clearly, allowing for adjustable labor income reduce the debt contraction when the constraint is binding at large current debt level. In particular, the debt continues to increase in the binding region when agents can use labor income to expand their borrowing limit. When  $X_t = q_t n_{t+1}$ , the borrowing limit does shrink, but the contraction is much smaller in size compared with the inelastic labor case. The intuition is clear: agents does not have direct control over the value of collateral: the land price while the indirect link between labor and land price is weaker than the income case.

**[Insert Figure 6 here]**

Since the debt contraction is less severe in the elastic labor case, the declines of consumption during sudden stops are also much smaller in size, independent of the types of collateral used. Panel B of Figure 3 confirms this intuition, the percentage deviation in consumption is mild compared to Panel A. In Table 3 we report the crisis statistics for the elastic labor cases. Under both collateral valuations, the probabilities of crisis are higher than the case with inelastic labor, while the volatilities of debt are smaller. The intuition of higher incidence of debt drops is similar: the enhanced borrowing capacity at the bad state lead to over-borrowing in the good state where the borrowing is yet to be binding.<sup>8</sup>

## 6 Alternative Cases

In addition to the Case I–IV, there are many other possible setups on modeling collateral constraints. In this section, we discuss these alternatives and their implications of sudden stops.

### 6.1 Growth vs Level Economy

We now move to explain the role of endogenous growth in generating sudden stops. We write the normalized system of Euler equations that governs the dynamic choices of R&D investment, land

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<sup>8</sup>We also considered a collateral constraint in which agents can borrow against the maximum value of land and income; we found that almost every sudden stop involved land remaining more valuable than income and therefore this change made little quantitative difference. Details of these experiments are available upon request.

(asset) holdings, and international borrowing,

$$\tilde{\lambda}_t \Psi_{t,1} = \beta g_t^{-\gamma} E_t \left[ \tilde{\lambda}_{t+1} (\theta_{t+1} - h - \Psi_{t+1,2}) \right] \quad (g_t)$$

$$\tilde{\lambda}_t \tilde{q}_t = \tilde{\mu}_t \phi \tilde{q}_t + \beta g_t^{1-\gamma} E_t \left[ \tilde{\lambda}_{t+1} (\alpha \theta_{t+1} + \tilde{q}_{t+1}) \right] \quad (\text{land})$$

$$\tilde{\lambda}_t = \tilde{\mu}_t + \beta g_t^{-\gamma} E_t \left[ \tilde{\lambda}_{t+1} (1 + r) \right] \quad (\text{bond})$$

where labor is inelastic; we assume collateral is valued at the current price  $q_t$ , since it will turn out that Cases I and II are very similar qualitatively without growth. The first equation can be rewritten as

$$g_t^\gamma = \beta E_t \left[ \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \mathcal{H}(g_t, \theta_{t+1}) \right] \quad (7)$$

$$\mathcal{H}(g_t, \theta_{t+1}) = \frac{\theta_{t+1} - h - \Psi_{t+1,2}}{\Psi_{t,1}} \quad (8)$$

Equation (7) implies that everything else being equal,  $g_t$  and the IMRS move in the same way since  $\gamma \geq 0$ ,

$$g_t^\gamma \propto \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} = \frac{(C_t - hZ_t)^\gamma}{(C_{t+1} - hZ_{t+1})^\gamma}.$$

So when current consumption drops (due to a negative shock),  $g_t$  also decreases. Alternatively, two components of the effective SDF,

$$g_t^{-\gamma} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t},$$

move in the opposite direction. We term this effect the “discount rate” channel through which variations in growth affect the economy.

To see how the endogenous growth affects the land price (and therefore the collateral value), we can write

$$q_t = \frac{\tilde{\mu}_t \phi \tilde{q}_t}{\tilde{\lambda}_t} + \beta E_t \left[ \left\{ g_t^{-\gamma} \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \right\} g_t (\alpha \theta_{t+1} + \tilde{q}_{t+1}) \right]$$

In addition to the “discount channel” discussed above, endogenous growth also affects the value of land through future dividend. Higher growth increases the dividend and therefore the value of land. We term this effect the “dividend channel”. It is clear that  $q_t$  is affected via both channels; however,  $g_t$  affects the borrowing only through the discount channel, since the payoff of the bond is fixed. Note also that none of above derivations depend on whether  $\gamma$  is smaller or bigger than 1, but their relative strengths will depend on a number of parameters.

### 6.1.1 Analytical Characterization

We first shut down the discount rate channel by assuming that  $\gamma = 0$  and provide some analytical characterization on the role of endogenous growth through the dividend channel. The following proposition characterizes the equilibrium land prices and endogenous productivity growth (recall that we are valuing collateral at the current price).

**Proposition 2** *Suppose Assumption 1 holds except  $g_t$  is endogenous and time-varying and  $\frac{\beta}{1-\mu\phi} <$*

1. *If the equilibrium land price does not have a bubble, it is given by*

$$\tilde{q}_t = \alpha \sum_{k=1}^{\infty} E_t \left[ \left( \frac{\beta}{1-\mu\phi} \right)^k \left( \prod_{i=1}^k g_{t+i-1} \right) \theta_{t+k} \right]$$

with  $\tilde{q}_t \equiv \frac{q_t}{Z_t}$  is the normalized land price, and the transversality condition given by

$$\lim_{k \rightarrow \infty} E_t \left[ \left( \frac{\beta}{1-\mu\phi} \right)^k \left( \prod_{i=1}^k g_{t+i-1} \right) \tilde{q}_{t+k} \right] = 0$$

The endogenous growth rate  $g_t$  is characterized by the following first-order non-linear stochastic difference equation:

$$g_t = \psi + \frac{\beta \left[ E_t [\theta_{t+1}] - h + \psi \right] - 1}{2\kappa} - \frac{\beta\psi^2}{2} + \frac{\beta}{2} E_t g_{t+1}^2 \quad (9)$$

Proposition 2 indicates that the endogenous growth acts like a “discount rate” shock that affects the price of assets. The effect of  $\theta_t$  shock on  $g_t$  depends on the adjustment cost  $\kappa$ . Larger  $\kappa$  dampens the impact of  $\theta_t$  on the growth rate. If  $\kappa$  is large enough, the variance of the growth rate is extremely small (our numerical results confirm this fact). Therefore, instead of analyzing the global dynamics of (9), we perform a first-order approximation of this equation around the point where  $\theta_t \equiv 1$ . This approximation is accurate provided that the adjustment cost is sufficiently large. Using this linear approximation, there are two candidate solutions for the steady state growth rate:

$$g_- = \frac{1 - \sqrt{1 - \beta D}}{\beta} \quad g_+ = \frac{1 + \sqrt{1 - \beta D}}{\beta}$$

where  $D = 2\psi - \beta\psi^2 + \frac{\beta(1+\psi-h)-1}{\kappa}$ . However, the only plausible steady state equilibrium occurs when  $\bar{g} = g_-$ . To see why, note that the steady state level of endogenous growth should be

decreasing in  $\kappa$ , which penalizes attempts to raise growth above the minimum value. In particular, when deviating from the minimum growth level is infinitely costly ( $\kappa \rightarrow \infty$ ),

$$\lim_{\kappa \rightarrow \infty} g_- = \frac{1 - \sqrt{1 - \beta D}}{\beta} = \frac{1 - \sqrt{1 - \beta(2\psi - \beta\psi^2)}}{\beta} = \psi$$

Given this steady state, the log-linearized solution of the endogenous growth is given by

$$\tilde{g}_t = \frac{\beta\rho}{2\kappa\bar{g}(1 - \beta\bar{g})} \tilde{\theta}_t \quad (10)$$

with  $\bar{g} = g_-$ . We provide a numerical example of the local dynamics of the growth rate in this case. In the left panel of Figure 7, we plot the value of steady-state  $g$  as function of the adjustment cost  $\kappa$  where we use the parameter  $\psi = 0.95, \beta = 0.968, \rho = 0.83, h = 0.75$ .<sup>9</sup> If  $\kappa$  is not too large (roughly smaller than 28), the steady state of  $g_t$  exceeds 1 (in our quantitative model the growth rate will almost always be greater than one as well). In the right panel of Figure 7 we plot the impulse response of  $g_t$  with respect to a negative 1% shock in  $\theta_t$  for several different values of  $\kappa$ . The response of endogenous growth to  $\theta_t$  are quantitatively small due to the rigidity caused by the adjustment cost. The key message displayed here is that there exists a parameter region for  $\kappa$  such that the growth rate can be higher than 1 while remaining insensitive to changes in  $\theta_t$ .

[Insert Figure 7 here]

### 6.1.2 Quantitative Implications

We now compare the sudden stops in the level economy and in the growth economy. In the level economy, we set  $\psi = 1$  and  $\kappa = \infty$  so that  $z_t = 1, \forall t$ . We also adjust the parameter value of  $\phi$  and  $\beta$  in the level economy such that two economies have (i) same probability of sudden stops and (ii) same Debt/GDP ratio. Figure 8 plots the equilibrium debt functions for five cases – the benchmark endogenous growth Cases I and II, the same two cases with no growth, and a fifth case with endogenous growth and a higher value of  $\kappa = 35$ . It is clear that growth acts to mitigate the Fisherian deflation effects, and this mitigation is stronger when growth varies less. We use the insight of Proposition 2 to understand these differences.

[Insert Figure 8 here]

First, consider the endogenous growth cases – the difference between the two, as we mentioned earlier, is the strength of the Fisherian deflation, which is very strong in Case II and relatively weak

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<sup>9</sup>We adopt the parameterization to ensure that the steady state exists.

in Case I. The fifth curve shows that the deflation effects are even weaker when growth changes less, because of the dividend channel – if growth falls less during a sudden stop, dividends are expected to grow more and therefore asset prices decline less. The discount factor channel is weak, since it has forces operating in offsetting directions. In contrast, the no-growth economies both display much stronger deflation responses, because the overall level of growth is lower (remember that the endogenous growth rate is almost always above one, even during crises) and therefore asset price levels are lower; as before Case I generates a weaker deflation than Case II.

**[Insert Figure 9 here]**

Turning to the size of sudden stops, we now see a much larger difference between Cases I and II as illustrated in Panel C of Figure 3. That is, absence of endogenous growth amplifies the size of sudden stops and also increases the difference between the current and future collateral valuation cases. We can see why by looking at Figure 9, which is the no-growth version of Figure 1. In both cases, the debt contraction is more severe as agents can not mitigate the crisis through the discount channel. Finally, Panel D of Figure 3 plots comparison of the sudden stops of consumption between growth and level economy with land collateral valued at the expected future price; the no-growth case displays much larger consumption declines on average.

## 6.2 Individual vs Aggregate Land

We consider the case where the collateral is valued using individual land or aggregate land holdings, that is,  $X_t = q_t n_{t+1}$  and  $X_t = q_t N_{t+1}$ . When individual land is used to value the collateral, agents take into account the fact that their action affect their borrowing capacity, while in the aggregate land case the borrowing capacity is exogenous to individual agents which they take as given. Agents borrow more when they have (at least partial) control over their borrowing capacity, but the qualitative properties are similar and quantitative difference is small. Due to space limitations we omit the figure for the equilibrium debt function with both individual and aggregate land serving as the collateral, given the same level of shock  $\theta_t$ , but is available upon request.

Similar to the comparison between individual and aggregate land, the timing of land ownership in collateral valuation may also generate difference in sudden stop dynamics. We observe that the difference is even smaller in this comparison. Again, we set the labor choice to be inelastic. While

using the current land  $n_t$  implies that agents can not adjust their borrowing capacity immediately, they are free to do so over time.<sup>10</sup>

## 7 Conditional Efficiency

In this section, we consider how these different assumptions affect the size of efficiency distortions. Following Benigno et al. (2011, 2012), we discuss our model’s normative properties in terms of the notion of “conditional efficiency”. Conditionally-efficient allocations are defined as the optimal solution to the social planner’s problem in which the planner takes as given the equilibrium function that maps the state  $(B, \theta)$  into the collateral valuation. That is, in Case II the planner takes  $q_t$  as a function of  $B_t$  as given, which we denote  $Q(B, \theta)$ , while in Case I the planner takes  $E_t q_{t+1}$  as a function of  $B_t$  as given, which we denote  $\widehat{Q}(B, \theta)$ .<sup>11</sup> The allocation problem that the social planner solves can be formulated as

$$\max_{C_t, L_t, B_{t+1}, N_{t+1}, Z_{t+1}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(C_t - hZ_t)^{1-\gamma} (1 - L_t)^{\omega(1-\gamma)}}{1 - \gamma} \right], \quad (\text{SP})$$

subject to constraints

$$C_t + \Psi(Z_{t+1}, Z_t) + B_{t+1} \leq \theta_t Z_t L_t^\eta + (1 + r) B_t,$$

$$B_{t+1} \geq -\phi X_t,$$

$$\log(\theta_{t+1}) = \rho \log(\theta_t) + \varepsilon_{t+1}.$$

Compared with the social planner allocation, the competitive equilibrium features an inefficiency associated with the binding collateral constraint; the literature terms this distortion a “pecuniary externality” since it operates through inefficient pricing.<sup>12</sup> Agents fail to internalize the fact that their actions affect the asset price and the collateral constraint, leading to an overborrowing

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<sup>10</sup>The case where we use  $N_t$  or  $N_{t+1}$  turn out to be identical, as agents understand that the aggregate land is in constant supply.

<sup>11</sup>The alternative, constrained efficiency, is significantly more complicated due to the forward-looking nature of the asset price  $q_t$ . If distortions are significantly different across cases under conditional efficiency, they will also be significantly different under constrained efficiency; the opposite may not hold (see Benigno et al. 2012). We leave the investigation of constrained efficiency for future work.

<sup>12</sup>In the language of Davila and Korinek (2018), we have a “collateral externality” as opposed to a “distributive externality”, as our planner is not concerned with the welfare of the international creditors.

effect.<sup>13</sup> Since the pecuniary externality operates through prices and the collateral constraint, the size of efficiency distortions and corresponding policy implementations differ as well. We first analyze the size of efficiency distortions in different cases we considered before and then discuss implementation using capital controls.

## 7.1 Efficiency Distortions

From the planner's perspective, whether collateral is  $N_{t+1}$  or  $n_{t+1}$  or  $n_t$  is irrelevant, but the particular function  $q_t$  that faces the planner varies. As mentioned above, our focus is on conditional efficiency, i.e., to explore how the planner improves allocations by varying the socially optimal debt position. Figure 10 compares optimal debt functions under the competitive equilibrium and social planner for the four cases.

**[Insert Figure 10 here]**

Case I shows that when borrowing is using future value of land, the social planner allocation is identical to that of competitive equilibrium. The social planner will do nothing and the competitive equilibrium is conditionally efficient as the slope of the pricing function in the constrained region is zero, leaving no scope for the planner to act. Formally, we can write the Euler equations for optimal debt positions as

$$\begin{aligned} c_{CE,t}^{-\gamma} &= \beta g_{CE,t+1}^{-\gamma} (1+r) E_t c_{CE,t+1}^{-\gamma} + \mu_{CE,t} \\ c_{SP,t}^{-\gamma} &= \beta g_{SP,t+1}^{-\gamma} E_t [c_{SP,t+1}^{-\gamma} (1+r) + \mu_{SP,t+1} \phi D_1 X(B_{SP,t+1}, \theta_{t+1})] + \mu_{SP,t} \end{aligned}$$

where we refer CE as the competitive equilibrium and SP as social planner's allocations. We define  $D_1 X(B_{SP,t+1}, \theta_{t+1}) \equiv \frac{\partial X(B_{SP,t+1}, \theta_{t+1})}{\partial B_{SP,t+1}}$ . Under the conditional efficiency, social planner can improve welfare by internalizing the effect of their debt positions on the collateral constraint. Under Case I,

$$D_1 X(B_{SP,t+1}, \theta_{t+1}) \equiv \frac{\partial E_{t+1} q_{t+2}(B_{SP,t+1}, \theta_{t+1})}{\partial B_{SP,t+1}} = \frac{\partial \widehat{Q}}{\partial B} \quad (11)$$

Clearly, CE and SP allocations coincide when the constraint is not binding ( $\mu = 0$ ). When the collateral constraint starts to bind ( $\mu > 0$ ), the derivative in (11) is zero, as in Figure 1 we deduce

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<sup>13</sup>One must be careful to clarify exactly what allocation is the reference one; compared to the conditionally-efficient allocation agents are overborrowing, but compared to the first best they are severely underborrowing; see Benigno et al. (2016, 2019) for discussions of this distinction.

that in equilibrium

$$\frac{\partial b_{t+1}}{\partial b_t} = \frac{\partial E_t q_{t+1}}{\partial b_t} = \frac{\partial \widehat{Q}}{\partial B} = 0, \mu > 0$$

Therefore, social planner cannot affect the collateral valuation by changing the debt position tomorrow, and therefore the competitive outcome is efficient.

In contrast, there is room for policy intervention when the collateral is valued using the current price (whether it is  $n_t$ ,  $n_{t+1}$  or  $N_{t+1}$  does not matter). In particular, the social planner internalizes the pecuniary externality of individuals and instructs agents to borrow less when the collateral constraint is not currently binding, provided it may bind in the future. The planner also pushes up the debt limit for sudden stops, that is, the kink of the social planner's debt function lies to the left of that in competitive equilibrium. In other words, the social planner mitigates the over-borrowing problems of individuals. Panel C shows the elastic labor case with land collateral. The social planner again reduces overborrowing as in Case II. However, the quantitative differences between CE and SP allocations are negligible and the welfare costs under conditional efficiency are small. When constraint is binding, the planner cannot affect the collateral value as the current debt is already determined, but rather is only able to adjust the future collateral value by changing  $B_{t+1}$ .

Panel D plots the comparison between CE and SP allocations when we use current income (labor income plus dividend) as collateral in Case IV. The quantitative difference between the two cases is again quite small. Both the social planner and agents in the competitive equilibrium understand that by increasing the labor supply they are able to increase the value of collateral and the borrowing capacity. Nevertheless, agents in the competitive equilibrium do not internalize the effect of their labor supply choices on the wage and dividend. When the collateral constraint starts to bind, there is a pecuniary externality through wages such that agents work more and save more due to the precautionary motive, leading to under-borrowing in the equilibrium. On the other hand, there is also a second source of pecuniary externality through wages and dividend in the sense that agents do not take into account the effect of their decisions on the value of collateral, creating over-borrowing in equilibrium. In net, the second source of efficiency distortion is larger and the competitive equilibrium features over-borrowing.

## 7.2 Policy Implementations

We now construct time-consistent tax policies that can implement the conditionally-efficient allocations from the previous sector. While the implementations are not unique, we couch them in terms of capital controls, as this instrument has been the focus of much of the literature. We are cognizant of the arguments in Benigno et al (2016,2019) that constrained planning allocations may not identify the full benefits of various tax instruments.

We find that when land serves as collateral, a capital control is sufficient to implement the efficient allocations. Suppose a positive (negative)  $\tau_t$  is tax (subsidy) on debt, the budget constraint of individuals becomes,

$$c_t + \Psi(z_{t+1}, z_t) + b_{t+1}(1 - \tau_t) + q_t n_{t+1} \leq \pi_t + w_t l_t + (q_t + d_t) n_t + (1 + r) b_t + T_t,$$

where  $T_t \equiv -\tau_t b_{t+1}$  are lump-sum transfers to households. The new Euler equation for individuals in Case II is,

$$C_{CE,t}^{-\gamma}(1 - \tau_t) = \beta g_{CE,t+1}^{-\gamma}(1 + r) E_t C_{CE,t+1}^{-\gamma} + \mu_{CE,t} \quad (12)$$

The social planner's optimal condition for debt is given by,

$$C_{SP,t}^{-\gamma} = \beta g_{SP,t+1}^{-\gamma} E_t [C_{SP,t+1}^{-\gamma}(1 + r) + \mu_{SP,t+1} \phi D_1 q(B_{SP,t+1}, \theta_{t+1})] + \mu_{SP,t} \quad (13)$$

where  $D_1 q(B_{SP,t+1}, \theta_{t+1}) \equiv \frac{\partial q(B_{SP,t+1}, \theta_{t+1})}{\partial B_{SP,t+1}}$  is the derivative of  $q(B_{SP,t+1}, \theta_{t+1})$  with respect to its first argument  $B_{SP,t+1}$ . Equating (12) and (13) and solving for  $\tau_t$  yields,

$$\tau_t = 1 - \frac{g_{SP,t+1}}{g_{CE,t+1}} \left( \frac{C_{SP,t}}{C_{CE,t}} \right)^{-\gamma} \frac{g_{CE,t+1} \mu_{CE,t} + \beta g_{CE,t+1}^{1-\gamma} (1 + r) E_t [C_{CE,t+1}^{-\gamma}]}{g_{SP,t+1} \mu_{SP,t} + \beta g_{SP,t+1}^{1-\gamma} E_t [C_{SP,t+1}^{-\gamma} (1 + r) + \mu_{SP,t+1} \phi D_1 q(B_{SP,t+1}, \theta_{t+1})]}. \quad (14)$$

Note that for the capital control, the collateral constraint multipliers today and tomorrow both play a role; however, only the future multiplier determines whether the tax is positive or zero, the current multiplier only determines the size of the tax if the future multiplier  $\mu_{t+1}$  is positive in some state. This type of “ex ante” or “prudential” policy requires the planner to act before the crisis hits, because it cannot act during the crisis.

As noted above, with elastic labor choice and income-based collateral, the competitive equilibrium features externalities in both labor market and in the collateral constraint. Hence, to implement the conditionally efficient allocations, we need both capital control and a labor tax.

To correct the labor distortion, we introduce a labor income tax  $\tau_t^w$  in addition to the capital tax; the government budget constraint is

$$T_t = -\tau_t B_{t+1} + \tau_t^w L_t.$$

The equilibrium tradeoff between leisure and consumption satisfies

$$\omega C_{CE,t}^{1-\gamma} (1 - L_{CE,t})^{\omega(1-\gamma)-1} = \lambda_{CE,t} w_{CE,t} (1 - \tau_t^w) + \mu_{CE,t} \phi w_{CE,t}, \quad (15)$$

while the optimal choice for the planner satisfies

$$\omega C_{SP,t}^{1-\gamma} (1 - L_{SP,t})^{\omega(1-\gamma)-1} = \lambda_{SP,t} \theta_t Z_t \eta L_{SP,t}^{\eta-1} + \mu_{SP,t} \phi w_{CE,t}. \quad (16)$$

In each expression,  $\lambda_t$  is the Lagrange multiplier on the resource constraint, which equals the marginal utility of consumption. The optimal labor tax can be reconstructed by equating the above two equations as

$$\tau_t^w = 1 - \left( \frac{C_{CE,t}^{1-\gamma} (1 - L_{CE,t})^{\omega(1-\gamma)-1}}{C_{SP,t}^{1-\gamma} (1 - L_{SP,t})^{\omega(1-\gamma)-1}} (\lambda_{SP,t} \theta_t Z_t \eta L_{SP,t}^{\eta-1} + \mu_{SP,t} \phi w_{CE,t}) - \mu_{SP,t} \phi w_{CE,t} \right) \frac{1}{\lambda_{CE,t} w_{CE,t}} \quad (17)$$

The corrective capital control is now given by

$$\tau_t = 1 - \frac{g_{SP,t+1}}{g_{CE,t+1}} \left( \frac{\lambda_{SP,t}}{\lambda_{CE,t}} \right) \frac{g_{CE,t+1} \mu_{CE,t} + \beta g_{CE,t+1}^{1-\gamma} (1+r) E_t [\lambda_{CE,t+1}]}{g_{SP,t+1} \mu_{SP,t} + \beta g_{SP,t+1}^{1-\gamma} E [\lambda_{SP,t+1} (1+r) + \mu_{SP,t+1} \phi I_{t+1}]}.$$

where  $I_{t+1} = D_1 w(B_{SP,t+1}, \theta_{t+1}) L_{SP,t+1} + D_1 d(B_{SP,t+1}, \theta_{t+1})$ .  $D_1 w$  and  $D_1 d$  are the derivatives of wages and dividends with respect to their first argument  $B_{t+1}$ , respectively.

## 8 Conclusion

This paper may seem narrow, in that we study the effects of modeling choices in one specific model of sudden stops; indeed, the term “inside baseball” in the title of the paper refers to a discussion that is “appreciated by only a small group of insiders or aficionados”. However, since these models are being used to guide policy interventions in a number of asset markets, it is important to understand how these choices matter. While we find that some choices are not important, providing some cover for authors to use whichever formulation that is analytically or computationally convenient, there are also choices that do matter: different setups in the collateral

constraint and other model environment lead to distinct model predictions in terms of the size and frequency of sudden stops, the nature of efficiency, and the timing and implementation of optimal policy interventions.

Our results here also have implications for policymaking more generally. For example, Devreux, Young, and Yu (2015) show how the value of commitment changes significantly between models that value collateral at  $q_t$  versus  $E_t q_{t+1}$ . We leave such questions for future work.

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## Appendix

### A Proofs

**Proof of Proposition 1:** Under Assumption 1, Equation (3) implies that  $\mu = 1 - \beta(1 + r)$  is constant overtime. Equation (2) is now simplified to

$$q_t = \mu_t \phi q_t + \beta E_t [(\alpha \theta_{t+1} + q_{t+1})] \quad (\text{A.1})$$

$$q_t = \mu_t \phi E_t q_{t+1} + \beta E_t [(\alpha \theta_{t+1} + q_{t+1})] \quad (\text{A.2})$$

for Case I and II. Note we have used the equilibrium condition that  $n_t = N_t = 1$ . To compute the expectation, we observe that

$$\log(1 + \theta_t - 1) \approx \theta_t - 1$$

when  $\theta_t$  is close to 1. Therefore,  $\theta_t \approx 1 + \log(\theta_t)$ . Now conjecture that the asset price follows

$$q_t = m^i \log \theta_t + C^i, \quad i = I, II$$

Substituting this conjecture into (A.1) and (A.2) and matching coefficients yields the solution in Proposition 1. To ensure causal-stationarity, we need the condition  $\frac{\beta}{1-\mu\phi} < 1$ . Q.E.D

**Proof of Proposition 2:** First note that  $\mu = 1 - \beta(1 + r)$  is still constant overtime, and the linear utility assumption lead to the fact that  $\lambda_t = 1$  is also constant. Then we transform (2) in normalized form by dividing  $Z_t$  at both sides, leading to

$$\tilde{q}_t = \mu_t \phi \tilde{q}_t + \beta g_t E_t [\alpha \theta_{t+1} + \tilde{q}_{t+1}] \quad (\text{A.3})$$

Iterating forward (A.3) yields the expressions in Proposition 2. Derivation of (9) is trivial and hence omitted. Q.E.D

## B Figures

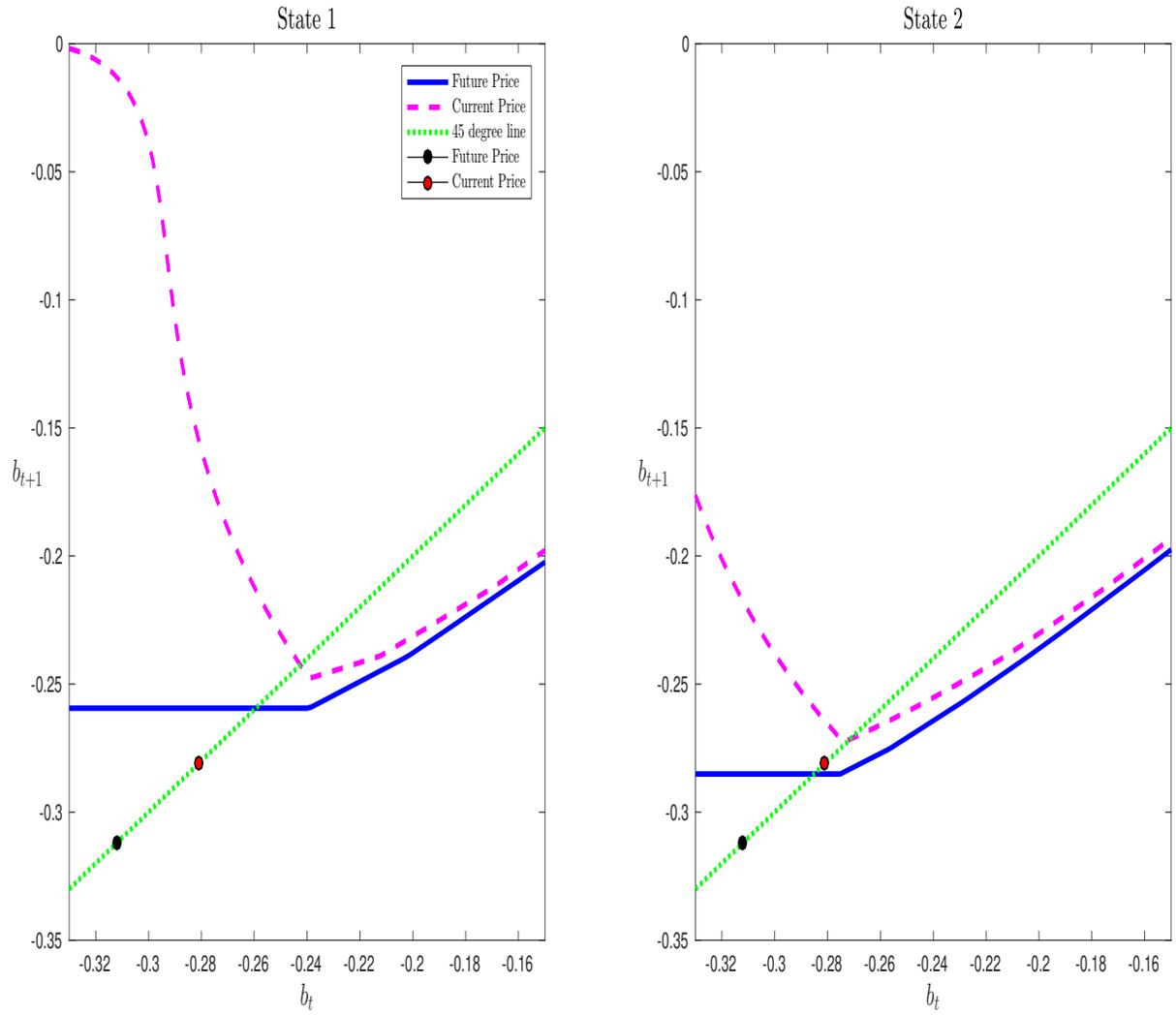


Figure 1: Equilibrium Debt Functions: Current vs Future Price

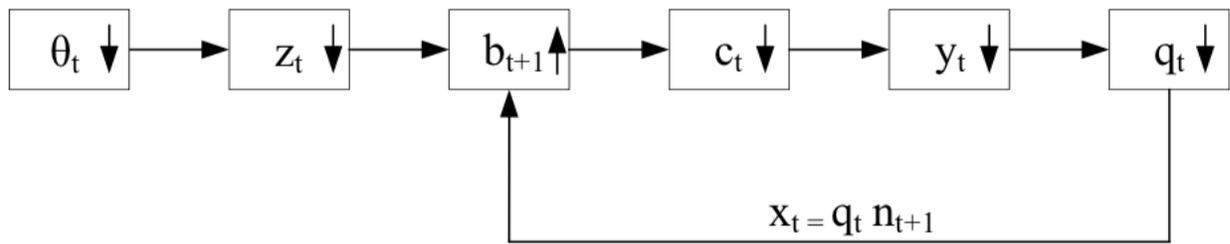


Figure 2: Standard Sudden Stop Mechanism

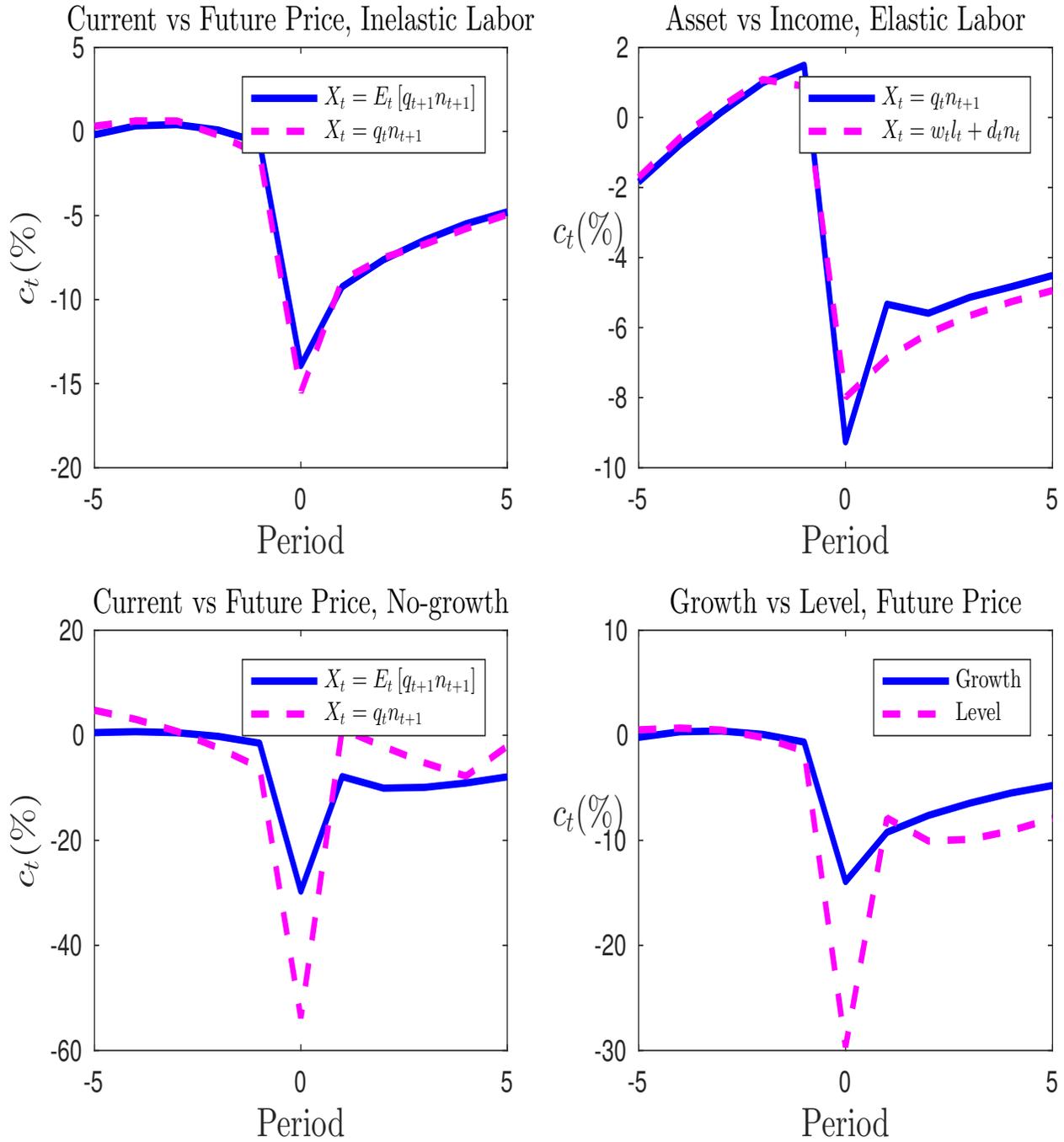


Figure 3: Sudden Stops of Consumption in Different Cases

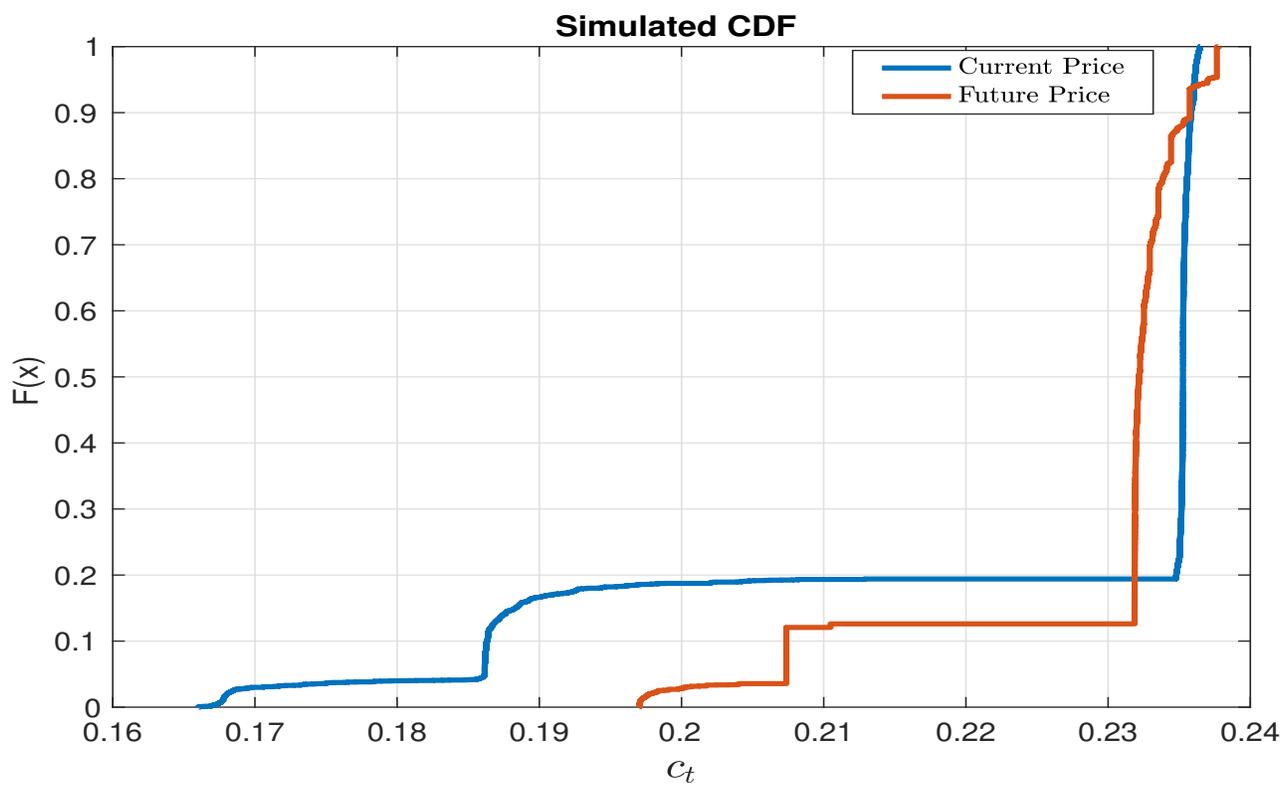


Figure 4: Simulated CDF of Consumption during Crisis

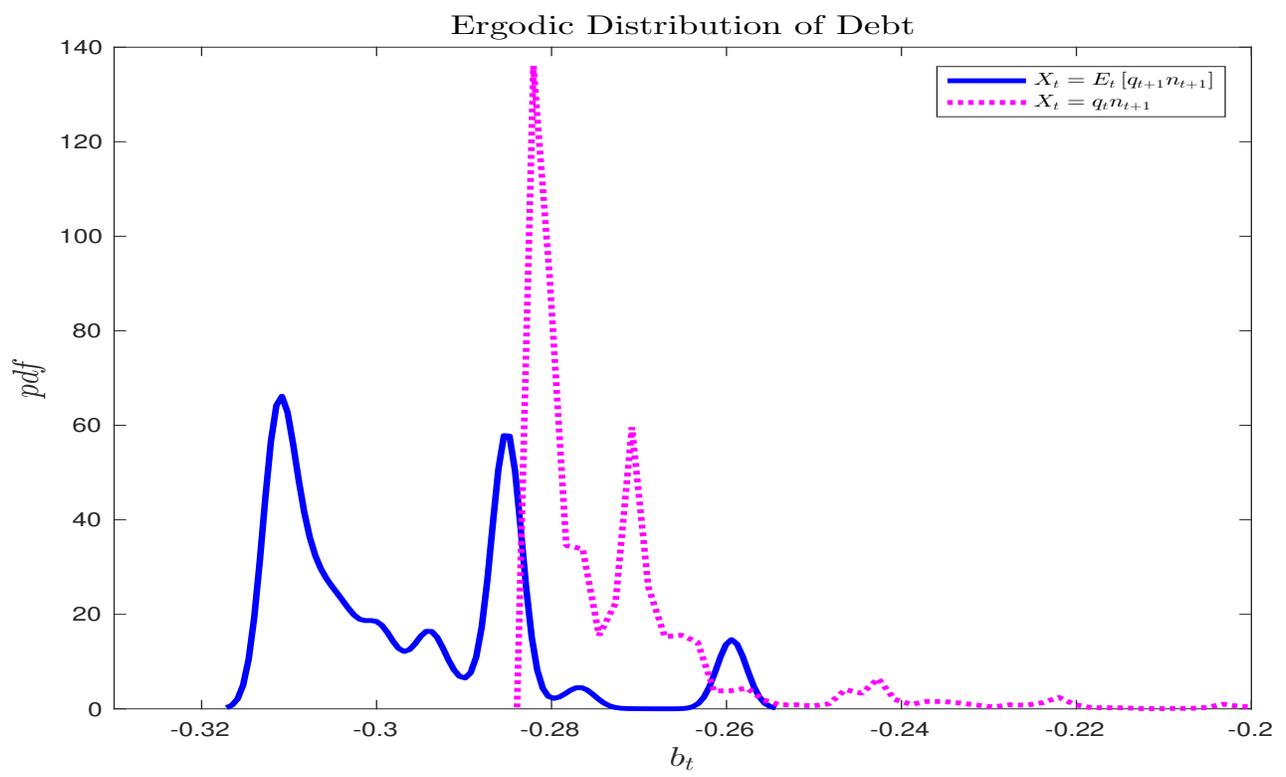


Figure 5: Ergodic Distribution of Debts: Current v.s Future Prices

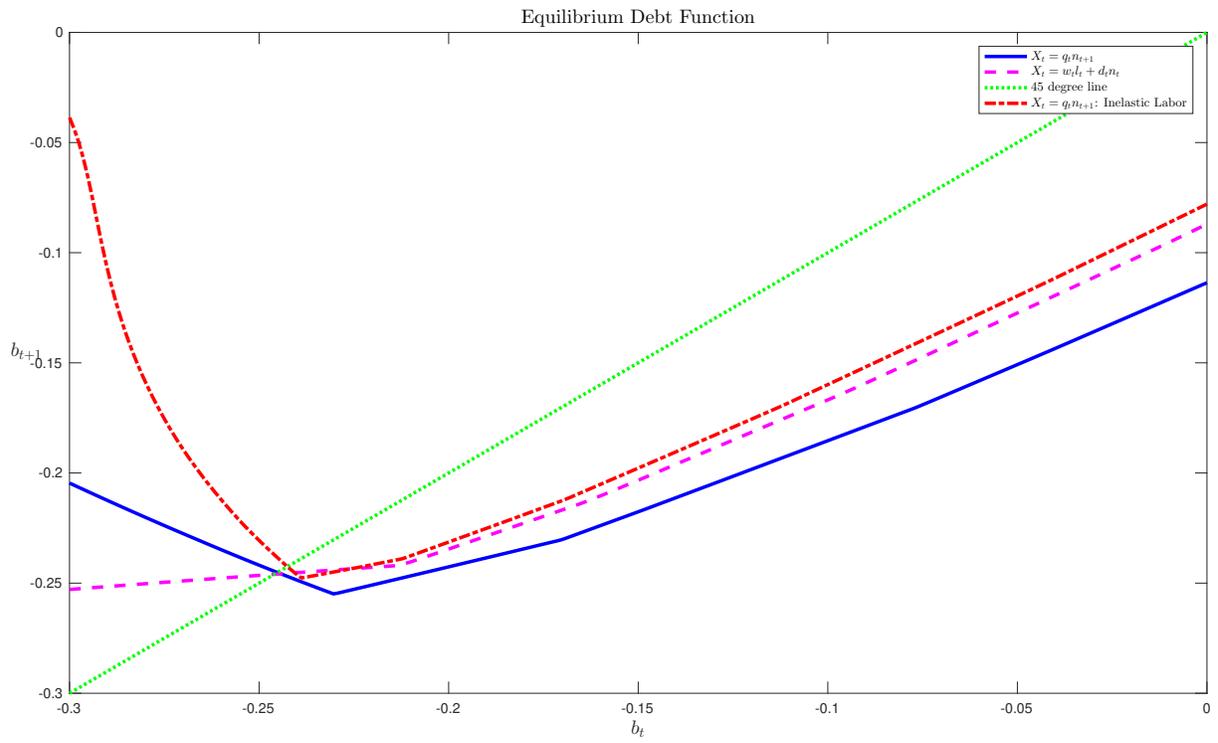


Figure 6: Equilibrium Debt Functions:: Elastic Labor

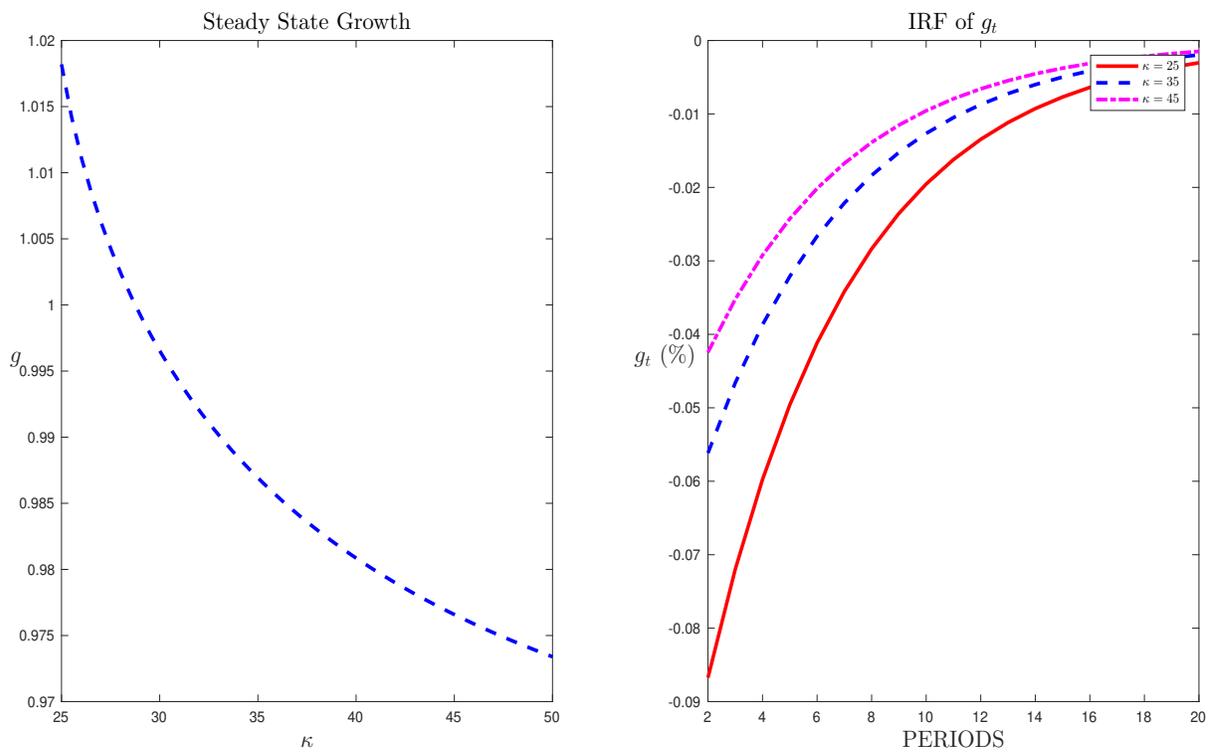


Figure 7: Local Dynamics of  $g_t$

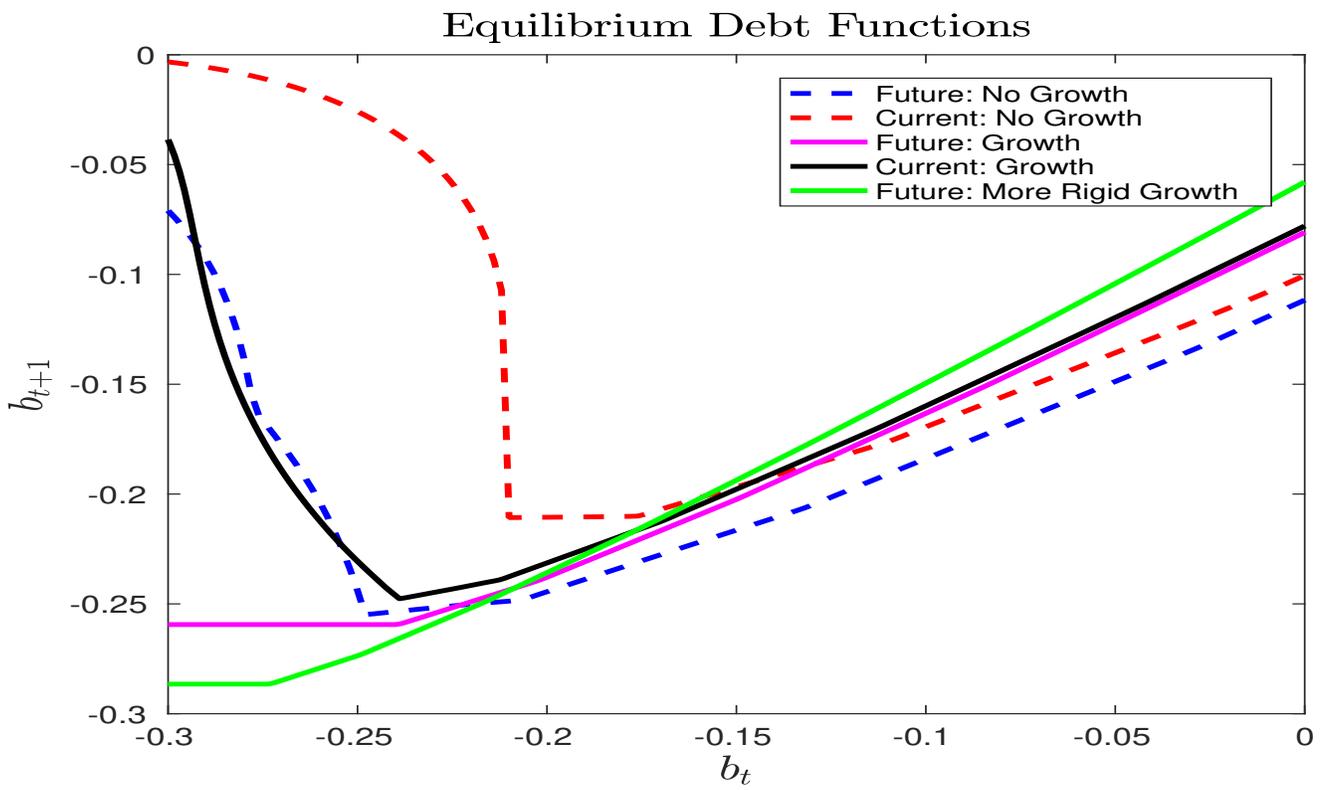


Figure 8: Equilibrium Debt Functions: 5 cases

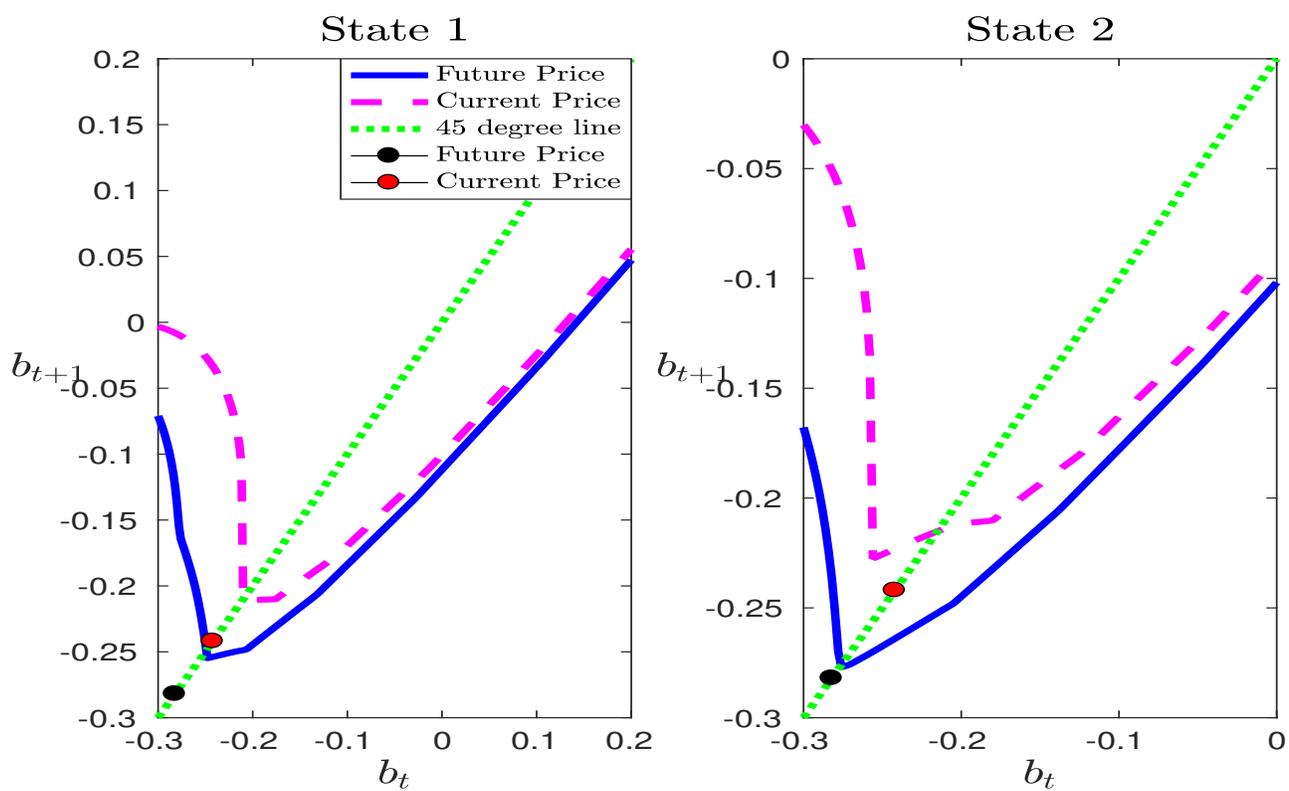


Figure 9: Equilibrium Debt Functions: Current v.s Future Prices with no growth

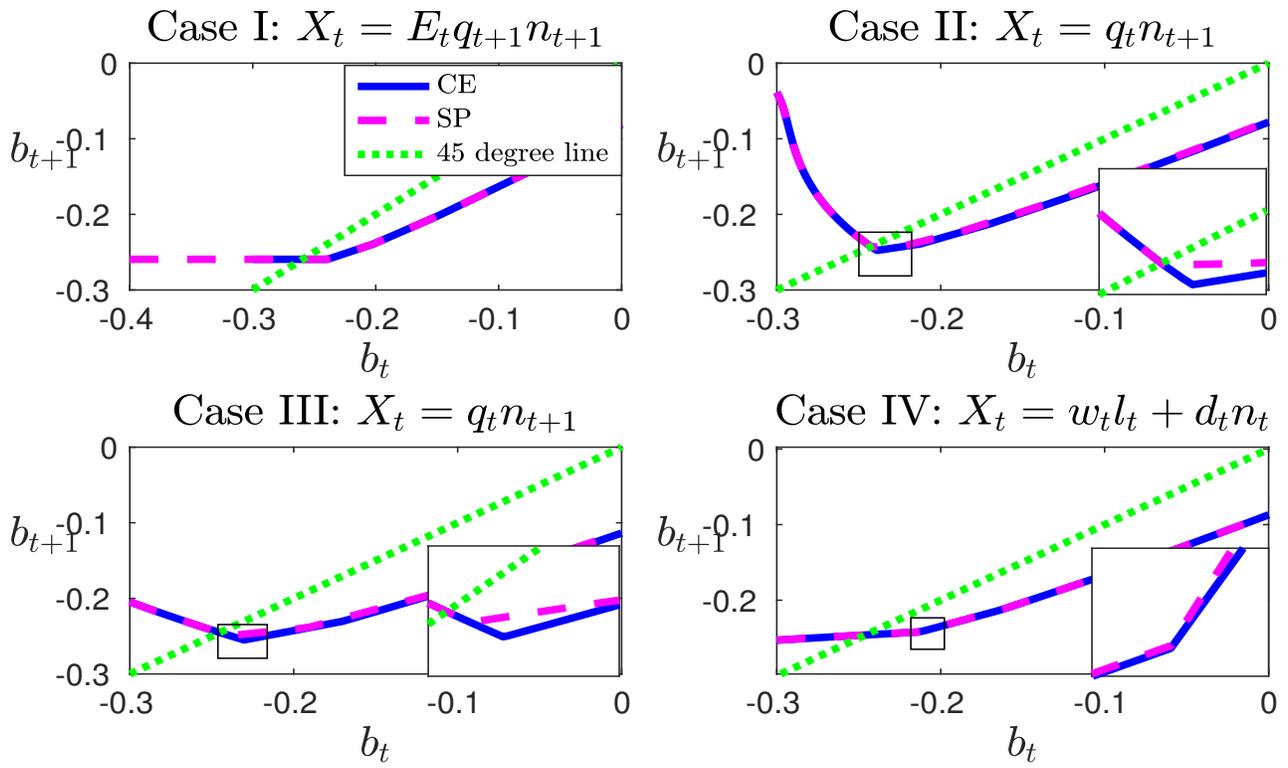


Figure 10: Equilibrium Debt Function: CE vs SP