The long run effects of changes in tax progressivity
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A B S T R A C T
This paper compares the steady-state outcomes of revenue-neutral changes to the progressivity of the tax schedule. Our economy features heterogeneous households who differ in their preferences and permanent labor productivities, but it does not have idiosyncratic risk. We find that increases in the progressivity of the tax schedule are associated with long-run distributions with greater aggregate income, wealth, and labor input. Average hours generally declines as the tax schedule becomes more progressive implying that the economy substitutes away from less-productive workers toward more-productive workers. Finally, as progressivity increases, income inequality is reduced and wealth inequality rises. Many of these results are qualitatively different than those found in models with idiosyncratic risk, and therefore suggest closer attention should be paid to modeling the insurance opportunities of households.

1. Introduction
The purpose of this paper is to study the effects of flattening progressive tax functions when there is no risk. The literature on the gains from flattening the tax code is large — some recent quantitative examples include Ventura (1999), Castañeda et al. (1998), Díaz-Giménez and Pijoan-Mas (2006), Conesa and Krueger (2006), and Conesa et al. (2009). All of these papers begin with the presumption that insurance markets are absent (as in Aiyagari, 1994); progressive taxation therefore has beneficial insurance properties, as it reduces the variance of labor income. It turns out that a robust prediction of these models is that aggregate activity and welfare respond positively to “flattening” the tax code. Table 1 illustrates how assumptions about the nature of income and wealth inequality lead to different predictions about the effects of flattening the tax code. In contrast, we approach the problem from the other end of the spectrum — we ask how progressive tax reform would effect the economy in a model without any uncertainty where inequality is entirely due to immutable heterogeneity in preferences and endowments.

Constructing a model that matches the US distributions of income and wealth not based on idiosyncratic risk is difficult given the results we found in Carroll and Young (2009). In the absence of discount factor heterogeneity, deterministic models with progressive taxation predict that the stationary distribution will have a negative correlation between income and wealth and between capital and labor income, both of which are inconsistent with US data. Elastic labor supply and/or borrowing constraints do not affect those results.

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1 Meh (2005) considers how progressive taxation affects the economy in a world with risky saving via entrepreneurial activity.

2 Elastic labor supply and/or borrowing constraints do not affect those results.

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doi:10.1016/j.jedc.2011.06.004
match the distributions of assets, income, and labor hours in the model to those in the data. We use this model to investigate the response of the economy to changes in the progressivity of the income tax code.\footnote{Saez (2002) uses a similar model in which the only heterogeneity is in initial wealth.}

The qualitative and quantitative implications of tax reforms in our model are quite different than those in the incomplete market literature. We conduct three revenue-neutral tax reform experiments and find that flattening the tax code – reducing the progressivity of the income tax – tends to reduce aggregate capital and labor input, rather than increase it; the decrease is also quantitatively large. More progressive marginal tax schedules can lead to steady states with as much as 47% more aggregate capital and 40% more labor input. The results of our other experiments, though less pronounced, consistently find that tax reforms with more progressive schedules increase aggregate capital and labor input. We also find that more progressivity generally decreases income inequality but increases wealth inequality. These changes occur without any change in the average tax rate in the economy, since we impose revenue neutrality on our experiments. With respect to labor input, progressivity increases labor input because it reallocates labor from less productive to more-productive agents, generating output gains even though labor supply – measured by raw hours – actually declines. The driving factor behind both of these findings comes from the heterogeneous response of households to changes in marginal income tax rates. Households with identical discount factors have the same level of long-run income, and thus face the same marginal income tax rate; however, these same households may differ with respect to their labor productivity and disutility of labor. When this occurs, these households will face different marginal utility costs from adjusting saving and labor. As a result, they will not choose the same composition of income between labor income and capital income.

Our results have two implications. First, endogenizing the extent to which the private sector can provide insurance – as in Krueger and Perri (2011) or Ábrahám and Carceles-Poveda (2009) – may be critical for understanding whether progressive taxation increases or decreases aggregate activity. In our model, the government does not crowd out insurance markets because they are already complete, while in the incomplete market literature the government also does not crowd out insurance because there are no such markets. If the government cannot provide insurance without crowding out existing sources, then the welfare gains from adjusting the tax code may be significantly smaller than those found in the literature to date.

Second, the extent to which inequality is driven by preferences vs. endowments also will play a role in determining the effects of progressive taxation, as they do in determining the effects of eliminating the business cycle (see Krusell et al., 2009).\footnote{Davila et al. (2005) and Athreya et al. (2009) show that optimal taxation in a model of uninsurable idiosyncratic risk has a progressive aspect to it. The extent of this progressivity seems likely to be related to the preferences vs. endowments issue as well.} Castañeda et al. (2003) show that a model with only labor efficiency shocks (and stochastic mortality) can match the main facts about US wealth, earnings, and consumption inequality, without any preference heterogeneity. To make the model fit the data, they argue that direct estimates of the wage process are unreliable, since they do not include the wealthiest households (who also contribute disproportionately to earnings). They do not attempt to match the income process for those individuals who can be observed, however, and also do not include any preference heterogeneity. It is an open question what part preference heterogeneity plays in US inequality, and our results suggest that it is potentially important for policy questions.\footnote{Badel and Huggett (2010) show that preference shocks in an OLG setting can replicate some key facts of inequality in the US.} It is also an open question how to calibrate a model in a way that separately identifies preference heterogeneity and shocks, but again based on our results it appears that such a model may be needed to measure the effects of progressivity.

2. Model

The model economy is composed of three sectors: a stand-in firm, a government, and a unit measure of heterogeneous households.

2.1. Households

The economy is populated by a finite set $I$ of households of unit measure. These households differ \textit{ex ante} along three dimensions: their discount factor $\beta$, their permanent labor productivity $\epsilon$, and their disutility from labor $B$. Every

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household is endowed with 1 unit of discretionary time which it may allocate to leisure, \( l \), or labor, \( h \). Any household \( i \in I \) has preferences over consumption, \( c \), and leisure which are described by the following lifetime utility function:

\[
U_i = \sum_{t=0}^{\infty} \beta^t \left[ \log(c_{it}) + B_i \frac{g_{it}}{1-\sigma} \right], \quad B_i > 0.
\]

(2.1)

\( \sigma \geq 0 \) is the inverse of the Frisch elasticity of leisure and is assumed uniform across households.\(^6\) While we do not focus on sustained growth, the utility function is consistent with a balanced-growth path along which leisure and labor supply are constant.

2.2. Firm

Each period, a stand-in firm uses capital and labor input to produce output according to a production technology \( F(K,N) \).

Let \( F(K,N) \) be strictly concave and increasing in \( K \) and \( N \) and \( F(0,N) = F(K,0) = 0 \). Output may be consumed or invested toward future capital. The firm rents inputs from the households through perfectly competitive markets. Letting the production technology be Cobb–Douglas with capital share parameter \( z \in [0,1] \), profit-maximization implies that each input is paid its marginal product:

\[
\begin{align*}
\ell_t &= F_1(K,N) = zK^{z-1}N^{1-z}, \\
\ell_t &= F_2(K,N) = (1-z)K^zN^{-z}.
\end{align*}
\]

We assume that each period the stock of capital depreciates by a factor \( \delta \in [0,1] \).

2.3. Government

Each period, the government collects revenue from a tax on income, \( \tau(y) \) and purchases \( G_t \) goods which do not enter the households’ utility functions. Let \( \tau(y) : \mathbb{R}_+ \rightarrow (0,1) \) be continuous and monotone increasing. Let \( I(i) \) be the measure of agents of type \( i \). Any surplus revenue is rebated back to the households via a lump-sum transfer, \( T_t \), so that the government’s budget constraint,

\[
T_t = \sum_{i \in I} \tau(y_{it})I(i) - G_t
\]

(2.2)

is satisfied each period. We abstract from the presence of government debt.

2.4. Equilibrium

Each household \( i \) maximizes (2.1) by choice of consumption, leisure, and savings \( k_{it+1} \), while respecting its budget and time constraints

\[
\begin{align*}
c_{it} + k_{it+1} &= y_{it} - \tau(y_{it}) + T_t + k_{it}, \\
\ell_{it} + h_{it} &\leq 1, \\
c_{it} &\geq 0, \\
\ell_{it} &\geq 0, \\
h_{it} &\geq 0,
\end{align*}
\]

(2.3)\( \text{to} \quad (2.4)\)\( \text{to} \quad (2.5)\)\( \text{to} \quad (2.6)\)\( \text{to} \quad (2.7)

where

\[
y_{it} = w_{it}\ell_{it} + (r_t - \delta)k_{it}
\]

(2.8)

is household income. Given the behavior of the firm and the government and a population density \( I(i) \), an equilibrium can be defined as a set of household decisions \( \{(c_{it},\ell_{it},k_{it+1})_{t=0}^{\infty}\}_{i \in I} \), market prices \( \{w_t,r_t\}_{t=0}^{\infty} \), and government policies \( \{G_t,T_t\}_{t=0}^{\infty} \) such that for any \( t \in \{0,1,\ldots\} \):

1. For every \( i \in I \), \( (c_{it},\ell_{it},k_t) \) maximizes (2.1).
2. \( \{w_t,r_t\} \) clear the labor and capital markets:

\[
K_t = \sum_{i \in I} k_{it} I(i),
\]

\(^6\) We can match the same distributions if we assume heterogeneity in \( \sigma \) and homogeneity in \( B \).
\[ N_i = \sum_{i \in I} h_i c_i J(i). \]

3. The goods market clears
\[ \left[ \sum_{i \in I} c_i J(i) \right] + \left[ \sum_{i \in I} k_{i(t+1)} J(i) \right] + G_i = F(K_i, N_i) + (1 - \delta)K_i. \]

4. At \( \{G_i, T_i\} \) the government’s budget is balanced.

The household’s optimization problem has a continuous, concave objective function and a compact and convex constraint set (assuming a lower bound on assets that can be made nonbinding). Along with (2.3) and a transversality condition for each \( i \in I \), the following system of equations describes an equilibrium:
\[
\begin{align*}
\frac{c_{i(t+1)}}{c_i} &= \beta_i [1 + (1 - \tau'(y_i))(r_i - \delta)], \\
0 &\geq \frac{1}{c_i}(1 - \tau'(y_i))we_i - B_i(1 - h_a)^{-\sigma}.
\end{align*}
\tag{2.9}
\]
if \( h_a > 0 \) then the second condition is an equality.

2.5. Steady state

Suppose \( \tau' \) strictly positive. Eqs. (2.9), (2.10), (2.3), and (2.8) simplify in the steady state to
\[
1 = \beta_i [1 + (1 - \tau'(y_i))(r - \delta)],
\tag{2.11}
\]
\[
0 \geq \frac{1}{c_i}(1 - \tau'(y_i))we_i - B_i(1 - h_i)^{-\sigma} \text{ with eq. if } h_i > 0,
\tag{2.12}
\]
\[
c_i = y_i - \tau(y_i) + T,
\tag{2.13}
\]
\[
y_i = we_i h_i + (r - \delta)k_i.
\tag{2.14}
\]
For a given rental rate, (2.11) pins down the long-run marginal tax rate for each household. Since \( \tau'(y) \) is strictly increasing, each marginal tax rate is associated with a unique level of income, so (2.11) identifies the long-run distribution of income. Given \( \tau(y) \), household \( i \)'s long run income is a function only of \( \beta_i \) and \( r \):
\[
y_i(\beta_i, r; \tau') = [\tau']^{-1} \left( 1 - \frac{\beta_i^{-1} - 1}{r - \delta} \right) \equiv \theta(\beta_i, r; \tau').
\tag{2.15}
\]
The following properties of \( \theta \) are immediate: \( \partial \theta / \partial \beta > 0 \), \( \partial \theta / \partial r > 0 \), and \( \partial \theta / \partial \tau' < 0 \).

Because for a given tax function and transfer, a household’s steady-state consumption depends only upon its income, the hours supplied by a household can be expressed as a function of its preferences and market prices:
\[
h_i \left( \beta_i, \frac{B_i}{c_i}, r; \tau' \right) = \begin{cases} 0 & \text{if } A_i > \frac{c_i}{B_i} \\ 1 - \left( \frac{B_i}{c_i} \right)^{1/\sigma} & \text{otherwise,} \end{cases}
\]
where
\[
A_i = \frac{c \theta(\beta_i, r; \tau')}{[1 - \tau'(\theta(\beta_i, r; \tau'))]w} = \frac{\theta(\beta_i, r; \tau') - \tau(\theta(\beta_i, r; \tau')) + T}{\frac{\beta_i^{-1} - 1}{r - \delta}} w.
\]

Note that what matters for hours is not productivity per se, but rather productivity relative to the disutility parameter.\(^7\)

Finally, the long-run wealth of each household is also determined by preferences and prices through (2.14), so
\[
k_i \left( \beta_i, \frac{B_i}{c_i}, e_i, r; \tau' \right) = \frac{\theta(\beta_i, r; \tau') - we_i h_i \left( \frac{B_i}{c_i}, r; \tau' \right)}{r - \delta}.
\]
The level of productivity plays a direct role in determining the asset holdings of each household.

\(^7\) Hours have not been expressed as a function of the wage because equilibrium \( w \) can be written as a function of \( r \) and \( z \) under the assumptions about \( F \).
2.5.1. **The role of household characteristics for steady-state income, wealth, and hours**

Holding market prices and government policy fixed, we now examine how long-run income, wealth and hours are affected by β_i, ε_i, and B. That is, these derivatives are interpersonal comparisons between agents with differing characteristics.

- **The discount factor:** Given market prices, β_i identifies long-run income y_i. Larger β implies larger y. Turning to hours and wealth,

\[
\frac{\partial h}{\partial β} = -\frac{1}{σ} \left( A_i B_i \right)^{(1/σ)-1} \frac{B_i}{ε_i} \left( 1 - \frac{ε_t}{ε} \right) \frac{β_i^{-1} - 1}{β_i} \frac{w}{w + (θ - τ(θ) + T) \frac{w}{β_i (r - δ)}} < 0
\]

and

\[
\frac{\partial k}{\partial β} = \frac{1}{r - δ} \left[ \frac{ε_t w h}{\frac{\partial h}{\partial β}} - \frac{w e}{\frac{\partial h}{\partial β}} \right] > 0.
\]

More patient households will save more and work less, all other things equal.

- **Labor productivity:** In the steady state, income is independent of ε, and therefore so is consumption. For hours and wealth,

\[
\frac{\partial h}{\partial ε} = \frac{1}{σ} (A_i B_i)^{1/σ} e^{-(1/σ)-1} > 0
\]

and

\[
\frac{\partial k}{\partial ε} = - \frac{w e}{r - δ} \frac{wh}{ε} < 0,
\]

so hours rise with ε and wealth declines. Since high ε implies that leisure is expensive, agents choose to finance their income more heavily from labor than assets.

- **Disutility of labor:** As one would expect, increases in B decrease steady-state hours and increase steady-state wealth. Since with high B leisure is cheap, agents choose to finance their income more heavily from assets than labor.

- **Frisch elasticity:** The parameter σ (the reciprocal of the Frisch elasticity of leisure) only affects the sensitivity of the responses of hours and wealth, not the direction. As σ → ∞, h^σ → 1, so that hours are unresponsive to changes in either parameters or prices. All changes in income in this limiting case would come from changes in capital income.

2.5.2. **The response of steady-state income, wealth, and hours to changes in prices and fiscal policy**

While a nice feature of our model is its ability to well-approximate the joint distribution of income, wealth, and hours in the US, as is common with models of its kind, we cannot obtain unambiguous results in general equilibrium analytically; the main obstacle is that different types adjust their capital stocks in different directions in response to changes in taxes. Consequently, our results for the general equilibrium effects are obtained using numerical methods. Nevertheless, it is helpful to do some partial equilibrium comparative statics on the steady state to aid the reader’s intuition.

- **Increase in τ(y):** (or a decrease in T): Holding prices fixed, an increase in τ(y) (or a decline in T) does not affect long-run income; A_i falls. Hours weakly rise for each household.\(^8\) Because income is fixed, wealth moves in the opposite direction as hours so it weakly falls.

- **Increase in τ(θ):** If the marginal tax rate rises, then according to (2.11) y_i falls. For households with discount factors not less than 1/(1 + r - δ), hours weakly rise in response to an increase in the marginal tax rate.

\[
\frac{\partial h}{\partial τ} = -\frac{1}{σ} \left( A_i B_i \right)^{(1/σ)-1} \frac{B_i}{ε_i} \left( 1 - \frac{ε_t}{ε} \right) \frac{\partial (τ(θ))}{\partial τ} \left( \frac{β_i^{-1} - 1}{β_i} \right) \frac{w}{w + (θ - τ(θ) + T) \frac{w}{β_i (r - δ)}} < 0
\]

where the second equality is true because

\[
\frac{\partial (τ(θ))}{\partial τ} = 1 - \frac{β_i^{-1} - 1}{r - δ}.
\]

by (2.15). Households that have discount factors less than 1/(1 + r - δ) will be converging to the natural borrowing limit where hours approach 1. Decreasing the return to savings by increasing the marginal tax rate will not induce these households to reverse their behavior. Therefore as long as β_i is sufficiently large, steady-state wealth decreases with τ.\(^9\)

---

\(^8\) For households at the lower bound on hours after the A_i decrease, the multiplier on the hours constraint is smaller after the tax change.

\(^9\) In our experiments no household has a β less than 1/(1 + r - δ).
• Increase in r: An increase in the steady-state rental rate leads to a rise in income for every household. The size of this increase for any given household will depend upon what part of the marginal tax function the household faces. In order for (2.11) to be satisfied for any particular household i, the after-tax return on savings in the new steady state must be equal to what it was in the initial steady state. Therefore, an r increase must be offset by an increase in τ(yi) (i.e., income must rise). If a particular household is in a region of the marginal tax function where its derivative is near zero (near the upper bound), then a large increase in yi will be necessary to satisfy (2.11). On the other hand, if the derivative is large, then only a small increase in yi will restore equality in (2.11). Therefore, an increase in r will tend to increase income inequality.

In addition, the long run response of hours is weakly negative and that of long-run wealth is strictly positive. Putting aside the trivial case where hours are 0 before and after the tax change

\[
\frac{\partial h}{\partial r} = \left(1 - \left(\frac{A_i B_i}{e_i}\right)^{1/\sigma}\right) = \frac{1}{\sigma} \left(\frac{A_i B_i}{e_i}\right)^{(1/\sigma) - 1} \left(\frac{\partial A_i B_i}{\partial r e_i}\right) < 0.
\]

Including households at the lower bound on hours, \(\partial h/\partial r \leq 0\). Since long-run income is unambiguously greater and labor income is weakly decreasing, long-run wealth is a strictly increasing function of r. This result ensures that a steady state exists.

3. Numerical experiments

In order to find quantitative results for the model, we conduct a series of revenue-neutral tax experiments. We select the following functional form for each household’s tax bill:

\[
τ(y) = v_0 (y − (y^{−τ_1} + v_2)^{−(1/τ_1)}) + v_3 y.
\]

The first term is the functional form Gouveia and Strauss (1994) assumed to estimate the effective personal income tax function using 1989 US tax return data.\(^\text{10}\) The second term, \(v_3 y\), captures other tax revenues that are not modeled but are paid by households in the data (e.g., excise taxes, estate taxes, property taxes). In the interest of focusing on the progressivity of the personal income tax, the combined burden of these other taxes is assumed to be a linear function of income.

\(v_0\) sets the upper bound on the marginal personal income tax rate, with the highest possible value of \(τ(y)\) being \(v_0 + v_3\). \(v_1\) changes the curvature of the function; the exact way in which it does so will be clear after the first experiment. Finally, \(v_2\) adjusts \(τ(y)\) for the unit of measure of income. Throughout all experiments its value remains fixed. It is important to remember that because of revenue neutrality, the average tax rate is unchanged across experiments. The wide range of steady-state distributions and of their corresponding moments strongly suggests that ignoring the distributional effects of tax changes may not be innocuous.

We choose not to model labor income and capital income as separate tax bases for two reasons. First, this paper makes comparisons to other papers on tax reform which do not distinguish between these two bases. Second, because there is no risk in the model, the appropriate tax rates are those placed on income generated from (nearly) risk-free assets like interest on bank savings accounts, certificates of deposit, and government bonds. The US tax code, the code to which the model is calibrated, does not subject these sources to capital gains taxation but rather to general income tax. We also do not attempt to decompose our results into the effects caused by altering the progressivity of capital income or labor income separately. Our tax function is not additive, so that it is not possible to use separate tax functions for different sources of income but keep the marginal tax schedule on total income the same as in the benchmark economy.

We do not compute transitional dynamics in our model. As the reader will see below, changes in progressivity have quantitatively large effects in our model, particularly for high-β types. These types also converge very slowly to their new steady state, due to the relative flatness of the tax function they face. The implication is that transitions would be extremely lengthy and, given the number of different types we need to capture the joint distribution of hours, income, and wealth, would require far too much memory for even 64-bit addressing to handle. The length of the transition also implies that the reader should interpret our large effects as likely to arise in the very distant future, so that perhaps they do not seem unreasonable.

3.1. Calibration

To initialize the model, we calibrate to income, wealth, hours, and analysis weight data from the Survey of Consumer Finances 1992, 1995, 1998, 2001, and 2004. The total number of households used for the experiment is 15,437. After deflating the data by the 1992 GDP deflator, we normalize aggregate income to 1, wealth to 3, and hours to 0.33. We set

\(^{10}\) This functional form has been used in a number of quantitative studies of progressive taxation, including Castañeda et al. (1998), Conesa and Krueger (2006), and Conesa et al. (2009). An alternative smooth specification is used in Sarte (1997), Li and Sarte (2003), and Carroll (2011), while a more detailed nonsmooth function is used in Ventura (1999).
$\alpha = 0.36$ and $\sigma = 2$. Government spending is 20% of aggregate income and transfers are 10%. We also fix $v_0$ and $v_1$ to 0.258 and 0.768 as estimated by Gouveia and Strauss (1994). We set the remaining parameters, so that the steady state of our model matches specific aggregate statistics at the annual frequency. $v_3 = 0.0855$ which implies that 71.5% of tax revenue is raised through the progressive personal income tax.\footnote{This is the average fraction of tax revenue from personal income taxes for the years 1992–2004. Our measure is taken from the Office of Management and Budget Historical Table 2.2. We do not include Social Insurance and Retirement Receipts in our calculation, so we exclude FICA taxes.} Depreciation is set to $\delta = 0.05$, so that investment is 15% of income. $\psi_i$ for each household is set to the household’s population weight in the SCF. We then normalize these weights, so that $\sum_i \psi_i = 1$.

$(\beta, \varepsilon, B_i)$, $r$, and $v_2$ are solved for jointly. The preference parameters are backed out from the first-order conditions and definition of income for each household. A difficulty with this method arises when a survey household works zero hours. Because the intratemporal condition is not binding there are infinitely many possible solutions to the system. Specifically, it is impossible to back out $\varepsilon$ and $B_i$ directly. To address this problem, we first solve for the household characteristics of working households and regress in logs $\varepsilon_i$ on age, education, and race (reported in the survey). In addition, we construct the cdf of $B_i$. Then if we encounter a non-working household, we use its reported age, education, and race along with the regression equation to get a predicted $\varepsilon_i$. Given $\varepsilon_i$, we find the value $B_{i,\text{min}}$ for which the nonnegativity constraint on hours is just binding (the constraint binds and the multiplier is zero). To select a $B_i$, we draw from the cdf of $B_i$ truncated below at $B_{i,\text{min}}$. Finally, $r$ and $v_2$ are adjusted to clear the market for capital goods and the government budget constraint. We ignore solutions on the downward sloping side of the Laffer curve.

The scatter plots in Figs. 1–3 display the relationship between the calibrated values for preferences and labor productivities. Looking first across discount factors, the model is backing out from the data a nonlinear relationship between $\beta$ and the other characteristics. Although $\varepsilon$ tends to rise with $\beta$ at low $\beta$-values, there is a very sharp increase in the observed productivities among the most patient households. This is due to the shape of the marginal tax function. Because it becomes relatively flat at high levels of income, large variation in income at the upper end will map into nearly the same marginal tax rates. Since the marginal tax rates identify $\beta$ through the Euler equation, these households will have discount factors that are almost, though not exactly, equal. Even though these households do not vary much in terms of discount factors, they do vary greatly in their labor productivity because of large differences in individual wealth and hours worked. These differences show up in $\varepsilon$ and $B$.

It is important to note that the scatter plots in Figs. 1–3 do not take into account the population weights of the households. In Table 2, we report the mean values $\beta$, $\varepsilon$, and $B$ and, because of the clear nonlinear relationship between some of these parameters, the cross-correlations of their logs weighted by population. The numbers confirm the relationships that we would expect. Labor productivity is positively correlated with discount factors which can be directly interpreted as high-income households also tend to have high salaries. Labor disutility is negatively correlated with $\beta$, implying that high-income households...
Finally, labor productivity and labor disutility are modestly positively correlated, because there is a positive correlation in the data between income and wealth but a negative correlation between hours and wealth. Households with high income tend to have high \( \varepsilon \) values, but do not work that much more on average than low-income households, which the model matches by assigning to these households a relatively high value for \( B \).

3.1.1. A note on the measurement of tax progressivity

We now discuss the measurement of progressivity. It is not clear exactly how to characterize a tax change as "progressive" or "regressive" when the underlying distribution of income changes. As a result, it is not straightforward to compare the progressiveness of two tax functions and so there are several methods used in the literature, and these
measures may not agree on the ranking of tax schedules. We choose to report the Kakwani (1977) index which is the tax Gini coefficient minus the income Gini coefficient. A higher value of the index corresponds to greater progressivity. This index is particularly well-suited to our problem because the income distribution is not fixed in our experiments. To see this, suppose two tax functions, \( t_A \) and \( t_B \), are associated with two different steady-state income distributions \( G_A \) and \( G_B \), and without loss of generality assume that \( G_{t_A} > G_{t_B} \), where \( G_t \) is the Gini coefficient of the tax burden in the steady state resulting from \( t \). In words, the tax burden in steady state \( A \) is more unequal than it is in \( B \). Now there are two possible reasons why the tax burden may be more unequal. First, \( t_A \) may place higher average tax rates on high income households than \( t_B \). Alternatively, \( G_A \) may have a greater fraction of high income households than \( G_B \), so that most of the tax burden rests with these high income households (even if their average rate is lower than under \( t_B \)). The Kakwani index corrects for these differences in income inequality.

In Tables 4, 7, and 9, we report the Gini coefficients of income and of wealth as well as two Kakwani index calculations. Because it is unclear whether a tax change which seems more progressive initially will end up being more progressive in the resulting steady state, we provide two Kakwani index measures. \( \text{kakpre} \) measures the Kakwani index of the tax change using the initial income distribution, while \( \text{kakpost} \) measures the index given the long-run distribution of income. In our experiments, the direction of progressivity never reverses (i.e., the ordering of tax reforms according to progressivity resulting from \( \text{kakpre} \) is preserved under \( \text{kakpost} \)). Nevertheless, in most cases the degree of progressivity is significantly diminished in the long run as households respond to the new tax code. \( \text{kakpost} > \text{kakpre} \) in only the final experiment, and this ordering occurs because the Gini coefficient of the tax burden hits its upper bound of 1 while income inequality increases in the new steady state.

### 3.1.2. Experiment 1: increase in the curvature of the personal income tax

In this experiment we increase the value of \( v_1 \) and adjust \( v_3 \) to balance the government budget constraint. \( v_1 \) alters the degree of progressivity of the tax function: when \( v_1 = 0 \) the personal income tax is flat, but when \( v_1 > 0 \), average tax rates and marginal tax rates rise with income. \( \text{Fig. 4} \) displays the marginal personal income tax function for several \( v_1 \) values in the range explored in the experiment. A higher \( v_1 \) reduces the marginal tax rate on low incomes and induces a more rapid rise to the highest rate, \( v_0 \).\(^{12}\) This figure, however, does not account for revenue neutrality’s effect on the total tax bill. \( \text{Fig. 5} \)

\(^{12}\) In the limit as \( v_1 \rightarrow \infty \), the marginal tax function approaches a flat tax with an exemption at low-income levels. See Conesa and Krueger (2006).
shows the marginal tax bill function which reflects the changes both in $v_1$ and $v_3$. When revenue neutrality is imposed it is not immediately clear which tax is more progressive. A $v_1$ of 0.768 places a higher total marginal tax rate on low income than greater $v_1$ values do. It should be stressed that due to general equilibrium effects on tax revenue, the steady-state marginal tax function after the policy change may imply lower marginal tax rates on all households, due to the adjustment of $v_3$ required to clear the government budget constraint. Since this reduction occurs over a (potentially) long transition, some households do face increased marginal tax rates initially as shown in Fig. 4.13

Turning to the effects of tax policy changes on the long-run levels of aggregate variables. We report the percentage changes in Table 3. $v_1$ has a hump-shaped relationship with income, capital, labor input, and wages. Fig. 6 plots the steady-state levels of aggregate income, capital and consumption over $v_1$. All three exhibit dramatic responses to changes in $v_1$. For example, increasing $v_1$ from 0.768 to 2.0 causes aggregate wealth to increase by 41.1%. Even at the highest value of $v_1$, capital is still 43% above the baseline level. Responses in the labor market are plotted in Fig. 7, average hours decline by as much as 29.4% meaning that higher progressivity in the personal income tax causes a substitution in production from less-productive to more-productive households. Interestingly, this pattern persists at high values of $v_1$ even though the average wage falls sharply in this region. The large increase in labor input comes from the upper 4% of productive households. Figs. 8 and 9 plot the average hours within each percentile of the $\omega$-distribution for the baseline and $v_1 = 2.0$ cases. Notice that hours fall for nearly the entire economy, rising somewhat only at the upper end. Clearly

---

**Table 3**

Percentage change in aggregate values.

<table>
<thead>
<tr>
<th>Experiment 1: % change</th>
<th>Y</th>
<th>K</th>
<th>C</th>
<th>H</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1 = 0.768$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$v_1 = 1.0$</td>
<td>17.4</td>
<td>21.1</td>
<td>21.7</td>
<td>–11.4</td>
<td>16.1</td>
</tr>
<tr>
<td>$v_1 = 2.0$</td>
<td>37.6</td>
<td>41.1</td>
<td>47.0</td>
<td>–27.6</td>
<td>36.4</td>
</tr>
<tr>
<td>$v_1 = 3.0$</td>
<td>41.7</td>
<td>43.0</td>
<td>52.2</td>
<td>–29.4</td>
<td>41.3</td>
</tr>
</tbody>
</table>

---

13 We investigate a log-linear approximation to the resulting steady-state outcomes of our policy experiments and in each case find local saddle stability in the face of very highly persistent small shocks to $v_0$ and $v_1$. We cannot prove or demonstrate numerically that our model converges to a new steady state for truly permanent large changes in the tax parameters, but based on related results in Carroll (2011) for a model with inelastic labor supply we conjecture that it will converge monotonically, but slowly.
average hours fall, but the impact on labor input of the upper 4% is much more significant. To make this point more clearly, we plot the average labor input within each percentile of \( \varepsilon \) in Figs. 10 and 11. Now the difference at the upper tail is starkly apparent, especially in the top 1%. Even within the top 1%, positive association between hours and labor efficiency is evident. Notice that labor input for this group increases by an even larger proportion than its hours. The more-productive households in this group increase their hours more relative to the less productive.

One advantage of the modeling technique used here is the ability to characterize behavior at the extremes of the income and wealth distributions. Figs. 12 and 13 show the breakdown in income and wealth across the steady-state distribution for some values of \( n_1 \). There are several conclusions that can be made about \( n_1 \) increases. First, increasing \( n_1 \) reduces income inequality. The Gini coefficient of income is cut by more than 50% when \( n_1 \) increases from its baseline to 3.0. In general this decline is caused because low and middle-income households' marginal total tax rates are reduced the most. However, not all high income households reduce their income. Finally, the interest rate is lower in the \( n_1 = 3.0 \) steady state. As discussed previously, this reduction leads to lower income inequality. Wealth inequality, on the other hand, increases. Plots of the income, wealth, and tax burden Gini coefficients across steady states are shown in Fig. 14, and summary statistics for this case are presented in Table 4.

For comparison purposes, we also compute this experiment using \( T \) to clear the government budget constraint instead of \( n_3 \); this experiment disentangles the pure progressivity effect from the required change in the flat tax rate. Table 5 contrasts the results from the two experiments a specific value of \( n_1 \) (it turns out that computing the new steady state is significantly more difficult in the case, for reasons that are unclear to us, so we concentrate on the case with the largest \( n_1 \) that we were able to solve). It is clear that this change has little impact on our results; the only difference is a slightly larger decline in income inequality and a corresponding slightly larger increase in wealth inequality. Tax revenue is increased by roughly 15% (about 4% of initial GDP), which generates the increase in \( T \), and the aggregate quantities of income and wealth are not substantially different.
3.1.3. Experiment 2: shift between personal income taxation and other tax sources

This experiment increases the fraction of tax revenue raised with the progressive income tax relative to the flat tax by increasing $v_0$ and reducing $v_3$. Figs. 15 and 16 compare the baseline marginal personal income tax functions and marginal tax bill functions for several values of $v_0$ in the experiment. As $v_0$ increases the marginal personal tax function rotates upward, however, the marginal total tax function rotates downward which is consistent with our finding from the previous experiment that higher progressivity is associated with more aggregate activity. In fact, in the steady state where $v_3 = 0$, aggregate income is 11.6% larger and the capital stock is 14.5% larger than in the baseline case. Percent changes for all aggregates are displayed in Table 6. In Figs. 17 and 18, the steady-state values of the economy’s aggregates are plotted for the whole range of $v_0$ in the experiment. There is basically a linear relationship between $v_0$ and the aggregate variables. As in Experiment 1 (though to a lesser degree), average hours falls and total labor input rises so once again more progressivity is effecting a substitution from less productive workers to more-productive workers.

The qualitative results for inequality are also similar to those from Experiment 1. Fig. 19 plots the steady-state Gini coefficients of income, wealth, and taxes. The income Gini falls slightly as the total tax schedule tilts towards the progressive income tax schedule. In contrast, wealth inequality rises significantly (roughly 16%). In this case, the large increase in wealth inequality comes primarily from large negative asset positions taken by moderately patient households with very high labor productivity. Summary statistics on inequality are presented in Table 7.

As above, we compare the results to a case where the transfer adjusts to clear the government budget constraint. Table 5 shows that the same results obtain here as in Experiment 1: income inequality falls a little more and wealth inequality rises a little more, with little difference in aggregates from the case where $v_3$ adjusts.

\[14\] At $v_0 = 0.32, v_3 = 0.$
Fig. 8.

Fig. 9.

Fig. 10.
3.1.4. Experiment 3: flat tax with an exemption

In our final experiment, we consider a tax function like that from Conesa and Krueger (2006) who find that the optimal progressive income tax combines a flat tax with an exemption level for income (we make no claims here about the optimality of this change). In our case, this function is approximated by letting $n = 1.15$ In keeping with that paper we eliminate the linear schedule (i.e., $n_3 = 0$) and adjust $n_0$ to balance the government’s budget. There are two marginal tax rates under this system — one is zero and the other is $t = n_0$, with a level of income $y = 1$ that determines the switch. Incomes above $y$ pay a tax bill $t \cdot y$, while incomes below $y$ pay nothing.

Fig. 11.

Technical, the limiting marginal tax function maps $y = 1$ to $t(1) = y_0 y_2 / (1 + y_2)$. With the exception of this single point, the limiting marginal tax function and the two-step function used in this experiment are identical.
Wealth Distribution by Percentile

Fig. 13.

Gini Coefficient

Fig. 14. The response of inequality to progressivity.

Table 4

<table>
<thead>
<tr>
<th>$\nu_1$</th>
<th>$G_y$</th>
<th>$G_k$</th>
<th>$Kak_{pre}$</th>
<th>$Kak_{post}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.768</td>
<td>49.4</td>
<td>74.5</td>
<td>0.045</td>
<td>0.045</td>
</tr>
<tr>
<td>1.0</td>
<td>41.6</td>
<td>85.2</td>
<td>0.075</td>
<td>0.064</td>
</tr>
<tr>
<td>2.0</td>
<td>26.3</td>
<td>88.9</td>
<td>0.144</td>
<td>0.095</td>
</tr>
<tr>
<td>3.0</td>
<td>23.8</td>
<td>93.4</td>
<td>0.165</td>
<td>0.118</td>
</tr>
</tbody>
</table>
Proposition 3.1. Given the tax function described above and a set of types each with a discount factor from \( \beta_1, \beta_2, \ldots, \beta_N \), where \( 1 > \beta_1 > \beta_2 > \cdots > \beta_N > 0 \), in any steady state with positive government expenditures

\[
\beta_2 \leq \frac{1 + (1 - \tau)(r - \delta)}{1 + r - \delta} \beta_1.
\]

Further, any type with discount factor \( \beta_1 \) has income \( y_1 > 1 \). Types with discount factors less than \( \beta_2 \) have income equal to \( -T \). Any type with discount factor \( \beta_2 \) will have \( y_2 \leq 1 \).

Proof. A necessary condition for a steady state is that

\[
1 \geq \beta_n [1 + (1 - \tau(y_n))(r - \delta)] \quad \forall n,
\]

where

\[
\tau(y) = \begin{cases} 
0 & \text{if } y \leq 1, \\
\tau > 0 & \text{if } y = 1.
\end{cases}
\]

First, from the household’s budget constraint any type \( n \) with assets approaching the borrowing limit must have steady-state income approaching \( -T \). Thus if \( y_n > -T \)

\[
1 = \beta_n [1 + (1 - \tau(y))(r - \delta)].
\]
Table 6
Percentage change in aggregates.

<table>
<thead>
<tr>
<th>$v_0$</th>
<th>Experiment 2: % change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y$</td>
</tr>
<tr>
<td>0.258</td>
<td>–</td>
</tr>
<tr>
<td>0.27</td>
<td>1.3</td>
</tr>
<tr>
<td>0.29</td>
<td>5.1</td>
</tr>
<tr>
<td>0.32</td>
<td>11.6</td>
</tr>
</tbody>
</table>

Fig. 16. The response of aggregate variables to changes in progressivity.
Fig. 18. The response of aggregate variables to changes in progressivity.

Fig. 19. The response of inequality to progressivity.

Table 7

<table>
<thead>
<tr>
<th></th>
<th>$G_w$</th>
<th>$G_k$</th>
<th>$K_{a_{pre}}$</th>
<th>$K_{a_{post}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0 = 0.258$</td>
<td>49.4</td>
<td>74.5</td>
<td>0.045</td>
<td>0.045</td>
</tr>
<tr>
<td>$v_0 = 0.27$</td>
<td>48.6</td>
<td>78.6</td>
<td>0.049</td>
<td>0.048</td>
</tr>
<tr>
<td>$v_0 = 0.29$</td>
<td>47.8</td>
<td>82.8</td>
<td>0.054</td>
<td>0.052</td>
</tr>
<tr>
<td>$v_0 = 0.32$</td>
<td>46.7</td>
<td>86.5</td>
<td>0.064</td>
<td>0.058</td>
</tr>
</tbody>
</table>
Second, since government expenditures are positive by budget balance \( \tau(y) = \tau \) for at least one type, implying that there are households with income greater than 1 in the steady state. The steady-state Euler equation for any such household is

\[
1 = \beta_n[1 + (1 - \tau)(r - \delta)].
\]

Given \( r \) and \( \tau \), this condition can only be satisfied for one \( \beta_n \). It is easy to see that this \( \beta_n = \beta_1 \). If it were satisfied for some other \( \beta_n \), then

\[
1 < \beta_n[1 + (1 - \tau(y))(r - \delta)],
\]

which violates (3.1). Therefore

\[
1 = \beta_n[1 + (1 - \tau)(r - \delta)]
\]

and

\[
1 > \beta_n[1 + (1 - \tau)(r - \delta)], \quad n > 1.
\]

Note that (3.1) is satisfied for \( n > 1 \) when

\[
\beta_n \leq \frac{1 + (1 - \tau)(r - \delta)}{1 + r - \delta} \beta_1,
\]

implying

\[
\beta_n \leq \frac{1}{1 + r - \delta}.
\]

To complete the proof, we will now show that a steady state cannot exist for \( \beta_n \geq (1 + (1 - \tau)(r - \delta))\beta_1/(1 + r - \delta) \). Assume not. Then there must exist a \( \beta_i \) such that

\[
\beta_i = \frac{1}{1 + r - \delta} \left( \frac{1}{1 + r - \delta} \right) [1 + (1 - \tau)(r - \delta)]
\]

\[
0 < \eta < 1.
\]

Either \( y_s \leq 1 \) or \( y_s > 1 \). If \( y_s \leq 1 \), then

\[
1 \geq \beta_n[1 + r - \delta] \geq \frac{\eta(1 + (1 - \tau)(r - \delta))}{1 + r - \delta} \frac{1}{1 + (1 - \tau)(r - \delta)} + (1 - \eta) \frac{1}{1 + (1 - \tau)(r - \delta)}
\]

which is a contradiction because \( \tau > 0 \). Therefore \( y_s > 1 \), and

\[
1 = \beta_i[1 + (1 - \tau)(r - \delta)] = \frac{\eta(1 + (1 - \tau)(r - \delta))}{1 + r - \delta} + (1 - \eta) \frac{1}{1 + (1 - \tau)(r - \delta)}
\]

which is also a contradiction. Therefore \( y_s \) does not exist implying that there can be no \( \beta_i \) in a steady state.

Define an intermediate \( \beta \) type as any type \( n \) for which

\[
\beta_i[1 + (1 - \tau)(r - \delta)] < \beta_n < \beta_1.
\]

As shown above, the existence of an intermediate \( \beta \) type rules out a steady state. To see this, note that \( y = 1 \) is the only potential steady-state income value for any intermediate \( \beta \) type. At \( y = 1 \), however, consumption growth must be positive because this type discounts the future less than the market pays for deferred consumption (\( \beta_n > 1/(1 + r - \delta) \)) so income rises. Any increase in income discontinuously increases the marginal tax rate, so that the market no longer sufficiently rewards this type for postponing consumption. Consumption growth will be less than one so income will fall. If the number of intermediate \( \beta \) types is greater than 1, then the most patient of them will have the largest consumption growth in the initial period and will converge the slowest back toward an income of 1.

The wealth distribution in this case would have the most patient type holding the largest share of wealth (possibly an extremely large share). Intermediate \( \beta \) types could have positive or negative wealth depending upon their labor income. All other types have assets approaching the natural borrowing limit (meaning their consumption and leisure both approach zero). We find that intermediate types exist in our calibrated economy, thus the reported findings for this experiment are not from a steady state. They should be interpreted instead as reporting features of an economy with a joint income and wealth distribution that is (not quite) the limiting distribution from the sequence of steady states associated with a sequence of \( v_1 \) as \( v_1 \) approaches infinity.

In the (limiting) steady state of our numerical experiment, \( \tau = 14.5\% \). Table 8 reports the percentage changes in the steady state aggregates. Aggregate income rises by 203% while the capital stock increases by 266%. The extreme rise in these values is caused almost entirely by the behavior of the most patient household. This household has income equal to 40,672 times the
average and wealth equal to \(166,512\) times the average. Hours increase only \(11\%\), but total labor input surges by \(185\%\). The big increase in labor input is not caused by the most patient household but rather by the highly productive among the other households. Hours for these households rise in response to a zero tax rate, leading to large increases in labor income. To maintain an income level below \(y\), this additional labor income is balanced by very large negative asset positions. With households in this economy taking such extreme positions, it is not surprising that inequality increases significantly. The Gini coefficient of income rises by \(41.4\%\) to \(0.7\), and the Gini coefficient of wealth rises from \(0.745\) to nearly \(1\). The extreme inequality that results from this experiment is consistent with known results regarding models with heterogeneous discount factors and no taxation, such as Becker (1980) and Smith (2009): every type except the most patient converges to the lower bound on consumption. Table 9 presents the summary statistics on inequality.

### 3.2. Explanation of results

In all three cases, increasing the progressivity of the tax schedule leads to increases in aggregate capital, aggregate labor input, and, as a consequence, aggregate income. At first, this result seems surprising given that a more progressive tax places a relatively heavier tax burden upon the wealthy. The key is that since the income tax falls on both means of income (capital and labor) households cannot avoid taxes by changing the source of their income. For a given tax policy, each household's long run total income is set by its discount factor and the rate of return on savings. The only thing to be resolved is the composition of long run income between labor income and capital income.\(^{17}\)

#### 3.2.1. Change in hours

As shown above, long-run hours for a household \(i\) are determined by the following equation:

\[
    h_i = \begin{cases} 
        0 & \text{if } A_i > \frac{e_i}{B_i} \\
        1 - \left(\frac{A_i}{B_i} \frac{e_i}{\tilde{e}_i} \right)^{1/\sigma} & \text{otherwise}, 
    \end{cases}
\]

where

\[
    A_i = \frac{c_i (1-\tau_i(y_i)) w}{\left(\frac{\beta_i^{-1}}{1-r_0}\right)^{\frac{\tau_i(y_i)}{1-\delta}} - \omega_i}.
\]

Since \(B_i/e_i\) is fixed for the household, the direction of its labor response to tax reform depends only on \(A_i\). If \(A_i\) increases (decreases), then hours fall (rise). In words, \(A_i\) is the ratio of the long-run disposable income of household \(i\) to the long-run factor price ratio scaled by the household’s rate of time preference. The numerator captures the wealth effect (if disposable income increases, then all else equal hours decline) while the denominator is the substitution effect (all else equal a rise in

\(^{16}\) To give some perspective to this number, the average wealth in the US is around \(180,000\) (inclusive of illiquid retirement portfolios and housing). Our wealthiest household therefore has a wealth of nearly \(30\) billion. Currently there are only three individuals on the Forbes' billionaires list with total wealth greater than \(30\) billion.

\(^{17}\) The explanations in the following subsections are specific to the numerical experiments above. Assumptions regarding the directions of changes in aggregate variables are consistent with their changes in the experiments. The following explanations should not be considered a rigorous analytical study of general equilibrium in the model. As discussed above, such a general equilibrium study would not deliver unambiguous analytical results. For instance, because household types respond to tax changes in opposite ways, any results conclusions about the impact on aggregate variables would depend critically upon the relative measure of those types as well as the strength of their responses.
the wage relative to the return on capital causes hours to rise). The rate of time preference $\beta^{-1} - 1$ makes the wealth effect relatively more important for highly patient households. The magnitude by which hours move depends both upon the size of the change in $A_i$ and the ratio of labor disutility to labor effectiveness. A household that is highly productive relative to its disutility for labor will change hours less. Clearly, if this ratio is very small then the household will not work at all.

The fact that highly patient (or equivalently, high income) households place more importance on the wealth effect, along with strong tendency for the very highly productive to have high income, explains why aggregate effective labor input rises even though average hours falls. Making the tax code more progressive causes the income distribution to shrink inward. Income rises for the poor and middle-class causing them to work less. At the same time income for the rich falls. Because the wealth effect is particularly strong for them, their hours increase. Although, on average the rich tend to have lower $B_i/e_i$, the dampening effect this heterogeneity produces only partially mitigates their rise in hours. Moreover, again because they tend to be highly productive workers, the positive labor response induces a large upward movement in effective labor input.\footnote{In experiment 1, for example, the top 5% most productive households contribute a very small positive amount to total hours worked, but they add significantly to effective labor. These households increase the economy’s total hours by less than 1% of its initial steady-state value. Those extra hours, however, have a very large impact on effective labor, adding 52% of its initial steady-state value.}

\subsection*{3.2.2. Change in wealth}

Whether long-run wealth rises or falls for household $i$ depends upon the change in total income relative to labor income and the change in the interest rate. To see this, consider the definition of income in a steady state

$$y_i = w_i h_i + (r - \delta)k_i.$$ 

Since steady-state income can be expressed as $\theta(i, r) \equiv \theta_i$, $k_i$ can be related back to fundamentals:

$$k_i = \frac{1}{r - \delta} \left\{ \theta_i - w_i \mathbb{1}_{[A_i < k_i]} \left[ 1 - \left( \frac{A_i B_i}{e_i} \right)^{1/\sigma} \right] \right\},$$

(3.2)\hspace{1cm}

$$k_i = \frac{1}{r - \delta} \left\{ \theta_i - w_i \mathbb{1}_{[A_i < k_i]} \left[ 1 - \left( \frac{\theta_i - \gamma(\theta_i) + T B_i}{(\beta_i^{-1} - 1) w_i/e_i} \right)^{1/\sigma} \right] \right\},$$

(3.3)

where a value of 1 for the indicator function corresponds to a positive supply of hours.

For any household, compare the long-run wealth holdings in the pre-tax reform and post-tax reform steady states and label these $k_1$ and $k_2$, respectively. Using similar notation to indicate the steady-state values of other variables the absolute change in wealth is

$$k_2 - k_1 = \frac{\theta_2 - w_2 h_2 e}{r_2} - \frac{\theta_1 - w_1 h_1 e}{r_1}.$$ 

(3.4)

Consider first the case where hours in the pre-reform steady state are positive (i.e., $h_1 > 0$). In this case we can express all of the post-reform steady-state variables as multiples of their pre-reform values:

$$\theta_2 = (1 + \lambda_o)\theta_1,$$

(3.5)

$$w_2 = (1 + \lambda_w)w_1,$$

(3.6)

$$r_2 = (1 + \lambda_r)r_1,$$

(3.7)

$$h_2 = (1 + \lambda_h)h_1.$$ 

(3.8)

Substituting (3.5)–(3.8) into (3.4) produces the following condition for the change in steady-state wealth:

$$k_2 - k_1 = \frac{(\lambda_o - \lambda_r)\theta_1 + [\lambda_o - \lambda_w - \lambda_h - \lambda_r]w_1 h_1 e}{(1 + \lambda_r)r_1}.$$ 

For households with positive hours in the initial steady state, post-reform wealth will be greater if and only if

$$(\lambda_o - \lambda_r) > (\lambda_w + \lambda_h + \lambda_r - \lambda_o) w_1 h_1 e,$$

(3.9)

and smaller if and only if

$$(\lambda_o - \lambda_r) < (\lambda_w + \lambda_h + \lambda_r - \lambda_o) w_1 h_1 e.$$ 

(3.10)

In each experiment $\lambda_r < 0$, $\lambda_w > 0$, $|\lambda_r| > |\lambda_w|$, and, for high $\beta$ types, $|\lambda_r|$ and $|\lambda_w|$ are very small relative to $|\lambda_o|$. Furthermore, note that $w_1 h_1 e/\theta_1 > 0$ by definition. Consider households with relatively high discount factors. Because steady-state income is ordered by $\beta$, these households must also have high income. A progressive tax reform will lead to a lower long-
run income for this group (i.e., $\lambda_d < 0$) and as stated above the decline is large in magnitude relative to the decline in $r$. From the analysis above regarding hours, it is clear that hours will rise so $\lambda_h > 0$. Taken together, the signs of the $\lambda$’s imply that (3.10) holds and so steady-state wealth falls within this group. By similar reasoning, nearly all low- and middle-income households increase their wealth because $\lambda_d > 0$, $\lambda_h < 0$, and $\lambda_r$ are relatively small in magnitude compared to $\lambda_d$ and $\lambda_h$. There is a very small measure of households in the upper-middle income region for which the $\lambda_h \approx 0$ and $\lambda_d \approx 0$. Among these households wealth may rise or fall slightly depending upon the proportion of pre-reform labor income to pre-reform total income.

Finally, turning to households which worked zero hours pre-reform

$$k_2 - k_1 = \frac{(\lambda_d - \lambda_r) \theta_1 - (1 + \lambda_w) w_1 h_2 e}{(1 + \lambda_d) r_1}.$$ 

Again, from examining the hours supply decision, households for which $h_2 > 0$ must experience sufficiently large movements in disposable income. In our experiments this large change never occurs, so that hours before and after the reform remain zero. This result should not be surprising given that few high-income households supply zero hours in the pre-reform steady state, and our method for choosing $B$ is unlikely to produce households that are marginal with respect to their hours choice. Since $h_2 = 0$ is the relevant case, whether wealth rises or falls after reform for any household depends upon the sign of $\lambda_d$ (because $\lambda_r < 0$). Only a very small measure of high-income households with no labor income decrease wealth. All other zero hours households increase their wealth. In general, because the marginal tax function is much flatter at high levels of income, the change in $\theta_1$ will be larger in magnitude for the rich and thus so will the change in wealth. That said, because the income distribution is skewed, the rich make up a small fraction of the total economy. On net, in all of our experiments the dissaving of the rich is more than offset by the saving of other households.

4. Conclusion

In this paper we have investigated the consequences of altering the progressivity of the tax code in a model with heterogeneous household but no idiosyncratic risk. As we noted in the Introduction, our results are qualitatively different from those found in models with incomplete asset markets, as well as quantitatively. Thus, we argue that more attention must be paid to derive the insurance opportunities available to households, either in terms of borrowing limits or missing insurance markets. Some papers take steps in this direction, such as Krueger and Perri (2011) or Ábrahám and Carceles-Poveda (2009), but more work is needed.

We want to point out "progressive" tax reforms – that is, changes in the tax function that induce more progressivity relative to the estimated U.S. tax function – would enjoy strong political support. Carroll (2011) investigates the source for this support; it would be of considerable interest to investigate this issue in the models that endogenize risk sharing. It would also be of interest to extend the results from Davila et al. (2005) and Athreya et al. (2009) regarding the constrained efficient allocations to the model considered here.

Acknowledgments

We would like to thank the participants of the Conference in Honor of Steve Turnovksy in Vienna for comments, particularly our discussant Timo Trimborn and the editor Ken Kletzer. Young thanks the Bankard Fund for Political Economy at the University of Virginia for financial support. All errors are our responsibility. The opinions expressed here are those of the authors only and do not reflect the positions of the Federal Reserve Bank of Cleveland or the Federal Reserve System.

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