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The stationary wealth distribution under progressive taxation ☆

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A B S T R A C T

This paper considers the long-run distribution of capital holdings in a model with complete asset markets and progressive taxation. Households are assumed to be heterogeneous in their labor market productivity. We show that this model is capable of producing a nondegenerate determinate wealth distribution. However, it also predicts that capital and labor income will be negatively correlated. These results are robust to the introduction of elastic labor supply and borrowing constraints.

1. Introduction

This paper studies the long-run distribution of wealth in a model with three features: heterogeneous consumers, progressive income taxation, and complete asset markets. There have been several papers that derive the properties of long-run wealth distributions in models with complete markets. One prominent example is Becker (1980), which shows that an economy with heterogeneous discount factors leads to a collapse of the wealth distribution—only the most patient household has wealth above the minimum in the long run. Wang (1996) illustrates how an economy with heterogeneous risk aversion coefficients also has a degenerate long-run distribution of wealth, while Coen-Pirani (2004) shows that the assumption of expected utility can be critical for degeneracy.[1] Finally, Sarte (1997) shows that heterogeneity of discount factors may not lead to a degenerate wealth distribution if taxation is progressive.

Continuing this line of research, we examine the joint distribution of income and wealth under the following assumptions: homogeneous preferences, permanent exogenous wage differences, progressive taxation, and complete markets. Permanent differences in wages across individuals is supported by a large econometric literature; recent macroeconomic contributions are Flodén and Lindé (2001) and Heathcote et al. (2004), but there are many others. Furthermore, a wide range of models that study inequality in complete markets—such as Caselli and Ventura (2000), Krusell and Ríos-Rull (1999),

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[1] Two other notable papers, Chatterjee (1994) and Krusell and Ríos-Rull (1999), prove that the economy features no wealth mobility along the transition to a steady state.

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and Flodén (in press)—generate heterogeneity through permanent wage differences. We ask what predictions these models make when confronted with progressive income taxation, a feature common to all developed economies.[2]

We find that the stationary wealth distribution can be determinate and have wealth held by more than one type of agent; all agents, however, must face the same marginal tax rate. Under the additional assumption that the tax rate function is *marginal-rate progressive* (the marginal tax rate is monotone increasing in income), the stationary distribution of *income* is degenerate: all agents earn the same level of income in the stationary state. Moreover, wealth declines with productivity, implying that the covariance between wealth and income is 0 and that labor income and capital income are perfectly negatively correlated. These findings are quite robust, as permitting elastic labor supply or imposing an exogenous borrowing constraint does not reverse the sign of the correlation. In fact, including an exogenous borrowing limit leads to a nondegenerate long-run distribution of income in which high income agents are at the borrowing limit; that is, there appears a negative correlation between income and wealth as well. Our results stand in stark contrast to the data: both the correlation of labor income and capital income and the correlation of income and wealth are positive in the Survey of Consumer Finances. The existing study that links progressive taxation to inequality under complete markets—Sarte (1997)—does not make a distinction between sources of income and therefore cannot detect the anomalous correlations that we uncover.[3]

Our findings suggest that results from complete markets models with heterogeneity in wages and proportional income taxation are likely not robust to deviations toward nonlinear tax schedules; if the flat tax economy (which has an indeterminate wealth distribution) is calibrated to match the data, the stationary distribution of wealth for a model with a nearby progressive tax system will be strikingly different. This non-robustness would be of particular concern for complete market political economy models of redistribution, such as Krusell and Ríos-Rull (1999) and Azzimonti et al. (2006), where agents vote over flat tax systems.[4] In the data, high wage individuals tend to be wealthy as well, and they earn a higher percentage of their income from assets than individuals with lower wages do.[5] This distribution suggests that high wage individuals should prefer labor income taxes to capital income taxes; however, in our model high wage individuals would prefer capital taxes because they have very low wealth. As shown in Domeij and Heathcote (2004), in an incomplete market economy the lower wage households prefer capital taxes; we expect they do in the data as well, because low wage households do not hold many assets.

We do not view this paper as establishing a puzzle regarding either the correlation of income and wealth or that between labor income and capital income. Within a model environment based on Aiyagari (1994), the correlation between wealth and income is typically positive.[6] Rather, we see our results as uncovering a disturbing lack of robustness in complete market models with homogeneous preferences. To get the anomalous results we find here all we need is labor income to be distinct from capital income and wages to be heterogeneous. Whenever these conditions are satisfied, households respond by trading off one type of income for the other in order to minimize their marginal tax rate. Given the empirical evidence, we find neither requirement unreasonable.

We conclude the main part of the paper with a discussion on how to reconcile the model with the cross-sectional data. As suggested by our comment above, this reconciliation requires discount factor heterogeneity. The data imposes structure on the joint distribution of discount factors, wages, and marginal income tax rates. Specifically, the data can be reconciled with the model under two circumstances: either wages and discount factors are negatively correlated or wages and discount factors are positively correlated and the marginal tax rate function is sufficiently flat near the steady state level of income for a given individual. We find evidence in the literature that the first condition is not satisfied; generally, discount factors and wages are positively correlated. Estimates of US tax functions imply the second condition may be satisfied—Gouveia and Strauss (1994, 1999) find that US marginal tax rates are nearly the same for all levels of income once all exemptions and credits are taken into account. Thus, the model with heterogeneous discount factors may indeed be suitable for policy experiments, provided that risk sharing is not of primary importance.[7]

Of course abandoning the assumption of complete markets is also a potential resolution—in models with incomplete markets, Krusell and Smith (1998) and Hendricks (2007) show that preference heterogeneity can go a long way towards accounting for the concentration of wealth in the US; a different approach is taken in Castañeda et al. (2003). Since complete market economies are much easier to study analytically than incomplete market ones, it seems important to determine

[2] These papers establish the presence of permanent differences across individuals in an OLG setting. In a dynastic setting, mean reversion across generations implies that quantitatively the importance of permanent wage differences may be small. For our question we do not need these permanent differences to be quantitatively large, only that they exist.

[3] Sarte (1997) combines human and physical capital into one aggregate, yielding an *Ak*-style model where capital and labor income cannot be distinguished.

[4] Our results could also impact the results of Ramsey optimal taxation in the presence of heterogeneous households, such as Flodén (in press). Optimal policy problems where progressive taxation is inherently valued are a new development—one early contribution is Golosov et al. (2006) contains a summary of the recent contributions.

[5] Budría Rodríguez et al. (2002) note that the fraction of income derived from labor and transfers varies over the wealth distribution from 98.2 percent in the lowest quintile to 60.0 in the highest and from 98.9 in the lowest 1 percent to 34.5 in the highest.

[6] The correlation between capital and labor income may still be a problem, however. It is also unknown whether our results would hold when asset markets are complete but enforcement frictions prevent agents from completely smoothing their marginal utility. We think they will, however, because it is known that enforcement friction models with production typically accumulate enough capital that the frictions effectively disappear.

[7] One experiment that we think would fall into this class is the effect of progressive income taxation on growth—see Li and Sarte (2003).

when we should and should not use them. Obviously, such models are of no use for studying risk sharing given the wide array of papers that find evidence of incomplete insurance in the data. We show here that the complete market model without preference heterogeneity may not be useful even for questions where risk sharing is a secondary concern.

2. The model

The model economy is populated by three groups of agents: households, firms, and the government.

2.1. Households

There is a unit continuum of households. Each household is endowed with 1 unit of time which it devotes entirely to labor activities. Households are heterogeneous with respect to their labor productivity ε. Let all households with the same level of labor productivity constitute a type and assume that there are a finite number of types indexed by $i \in I$. A type i can be summarized by two values: the level of labor productivity ε_i and the corresponding measure $\psi_i \in (0,1)$. Preferences for a type-i household give rise to a time-separable lifetime utility function

$$U_i = \sum_{t=0}^{\infty} \beta^t u(c_{it}), \tag{2.1}$$

where $\beta \in (0,1)$. We assume that $u(c)$ is strictly increasing in c, strictly concave, and continuously-differentiable as many times as needed.

The household maximizes this function while facing the period-t budget constraint

$$c_{it} + k_{i,t+1} \leqslant y_{it} + k_{it} - \tau(y_{it}) + TR_t, \tag{2.2}$$

where

$$y_{it} = w_t \varepsilon_i + r_t k_{it} \tag{2.3}$$

is household i's period-t income, and w_t and r_t are the wage rate and rental rate, respectively. $\tau(y): \mathcal{R}_+ \to \mathcal{R}_+$ is the total income tax paid by a household with income y. We restrict this function to satisfy $\tau(y) \leqslant y$ for all y with equality only if $y = 0$, so that the tax burden is never too large for consumption to be positive. k_{it} is the stock of physical capital owned by the household at the beginning of period t and TR_t is a lump-sum transfer from the government. In our model, dynasties cannot purchase insurance against their permanent productivity difference. Since the presence or absence of these markets does not affect the steady state outcomes, we ignore them. Our assumption of complete markets then amounts to the ability to insure any subsequent variations in ε. Finally, we impose a non-binding lower limit on assets:

$$k_{i,t+1} \geqslant k^b$$

for some large negative k^b (such as one that equals or exceeds the natural debt limit). In a later section we will consider cases where this constraint may be binding.

2.2. Production and government

A stand-in firm rents labor and capital from the households and produces output according to a constant-returns-to-scale production function $F(K_t, N_t)$. We assume F is continuously differentiable, strictly increasing, strictly concave, obeys the Inada conditions, and satisfies $F(K,0) = F(0,N) = 0$. Factor prices are determined within a competitive market, so each factor is paid its marginal product.[8] The pricing conditions are thus

$$w_t = F_N(K_t, N_t), \tag{2.4}$$

$$r_t = F_K(K_t, N_t) - \delta, \tag{2.5}$$

where $\delta \in [0,1]$ is a depreciation rate on capital. The government's only activity is to tax income and redistribute the tax revenues to the households as a lump-sum transfer each period. The period-t government budget constraint is therefore

$$TR_t = \sum_{i \in I} \tau(y_{it}) \psi_i.$$

Since the presence of government consumption is irrelevant for our results, we omit it.

[8] The assumption of competitive markets is not critical to our results so long as households do not have market power. For example, monopolistically-competitive firms that charge constant markups on their differentiated goods can be accommodated without altering the results.

2.3. Equilibrium conditions

Differentiating the lifetime utility function of each type with respect to $k_{i,t+1}$ yields the following optimality conditions for all i, t:

$$-u_c(c_{it}) + \beta u_c(c_{i,t+1})\big[\big(1 - \tau_y(y_{i,t+1})\big)r_{t+1} + 1\big] = 0, \tag{2.6}$$

$$r_t k_{it} + w_t \varepsilon_i + k_{it} - \tau(y_{it}) - k_{i,t+1} - c_{it} + TR_t = 0, \tag{2.7}$$

where $\tau_y(y)$ is the marginal tax rate function. Given an initial wealth distribution $\{\underline{k}_{i,t}\}_{i\in I}$ along with a transversality condition on $k_{i,t}$, these equations determine the period-t equilibrium levels of consumption, c_{it}, and savings, $k_{i,t+1}$, for each household of type i. For (2.6) to satisfy the second-order condition for a maximum, we require $\tau_{yy}(y) \geqslant 0$; we maintain this assumption in all subsequent sections.

In equilibrium the labor, asset, and goods markets must clear:

$$K_t = \sum_{i\in I} k_{it}\psi_i, \tag{2.8}$$

$$N_t = \sum_{i\in I} \varepsilon_i \psi_i, \tag{2.9}$$

$$\sum_{i\in I} c_{it}\psi_i + K_{t+1} = F(K_t, N_t) + (1-\delta)K_t. \tag{2.10}$$

We define a competitive equilibrium in the usual way: households maximize, firms maximize, the government budget constraint is satisfied, and markets clear.

3. Steady state

We proceed now by analyzing the steady-state distribution produced by the economy. Formally, we have the following definition.

Definition 1. A steady state for this economy is a list $\{c_i, k_i, y_i\}$ such that

(i) $k_{i,t+1} = k_{it} = k_i$ and $c_{i,t} = c_i$ satisfy (2.6) and (2.7),
(ii) the firm maximizes by choosing $K = \sum_{i\in I}\psi_i k_i$ and $N = \sum_{i\in I}\psi_i\varepsilon_i$,
(iii) the government budget constraint is satisfied, and
(iv) the goods market clearing condition $\sum_{i\in I}\psi_i c_i + \delta K = F(K,N)$ is satisfied.

A steady state is therefore a distribution over asset holdings such that households remain at those levels. This definition differs from the one used in incomplete market models (such as Aiyagari, 1994) in that we require individual as well as aggregates to remain constant over time. We do not provide any proof that the steady state exists or is unique, although we have not encountered any examples where these properties did not hold. We now provide definitions for two terms that we will employ in this section: the notions of degenerate and indeterminate steady state distributions.

Definition 2. The steady state distribution of wealth $\{k_i\}$ is *degenerate* (in the limit) if $k_i \to k^b$ for all but one i. The steady state distribution of income $\{y_i\}$ is *degenerate* if $y_i = y$ $\forall i$.

Definition 3. A steady state distribution $\{x_i\}$ is *indeterminate* if there exist i and j such that the model only pins down the sum $x_i + x_j$.

That is, degenerate wealth distributions involve the concentration of wealth in the hands of only one agent, with all other agents converging to their debt limit, while degenerate income distributions have mass at a single point. Since K must be positive and the natural debt limit is nonpositive, a degenerate wealth distribution involves $k_i > 0$ for only one i. Indeterminate wealth distributions arise when the model does not have enough independent equations to identify every agent's asset holdings (see Krusell and Ríos-Rull, 1999, for a demonstration that the model with constant marginal tax rates is indeterminate).

3.1. The long-run income distribution

In a steady state, Eqs. (2.6)–(2.7) can be simplified to

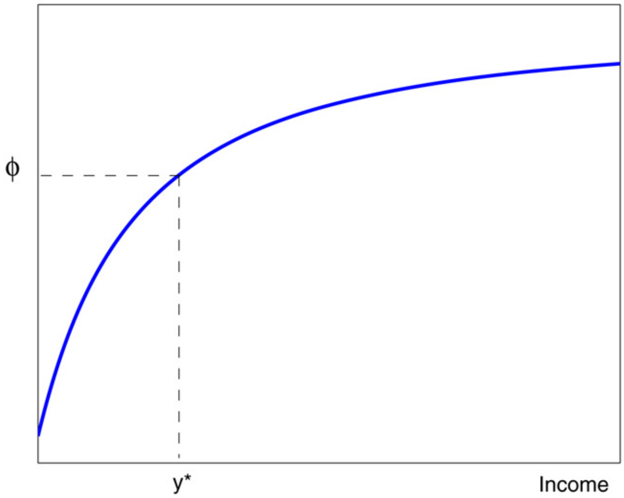

Fig. 1. Strictly increasing marginal tax function.

$$\beta\big[\big(1-\tau_y(y_i)\big)r+1\big]=1, \tag{3.1}$$

$$y_i-\tau(y_i)-c_i+TR=0. \tag{3.2}$$

By rewriting (3.1) as

$$\tau_y(y_i)=\frac{r-\beta^{-1}+1}{r}=\phi, \tag{3.3}$$

it is evident that there can be only one marginal tax rate, ϕ, which all households face in the long run. This equilibrium condition relates the steady state distribution of income directly to the shape of the marginal tax function. Depending upon the shape of $\tau_y(y)$, (3.3) could be satisfied for more than one value of y. Define

$$\Upsilon=\big\{y:\ \tau_y(y)=\phi\big\}.$$

In order for a household to satisfy its optimality condition for saving, its steady state income must be a member of Υ; Υ is therefore the support of the steady state income distribution. Notice that if $\tau(y)$ is proportional to income, then $\tau_y(y)$ is the same for all income values and so (3.3) is satisfied for arbitrary income distributions (taxes are flat and therefore both the income and wealth distributions are indeterminate). Our first proposition analyzes the case where $\tau_y(y)$ is strictly increasing (that is, $\tau(y)$ is a *marginal-rate progressive* tax function).

Proposition 1. *If $\tau_y(y)$ is strictly increasing the long-run income distribution is degenerate.*

Proof. For all $y\in\Upsilon$,

$$y=\tau_y^{-1}(\phi).$$

When $\tau_y(y)$ is strictly increasing, its inverse maps each marginal tax rate in the range of $\tau_y(y)$ to a unique income level. Therefore ϕ can only be associated with one y. $\square$

On the other hand, if the function is not strictly increasing then $\tau_y(y)=\phi$ may hold for more than one y. To illustrate, Fig. 1 shows a long-run equilibrium under a continuous, strictly increasing marginal tax rate function. At ϕ, Υ is a singleton, so all households have y^* units of income. In contrast, Fig. 2 shows an equilibrium where the marginal tax function has steps: at ϕ, the interval $[y_{\text{low}}, y_{\text{high}}]$ comprises Υ and the income distribution is indeterminate within it.

3.2. The long-run wealth distribution

When the income distribution is degenerate the wealth distribution is determinate—associated with each individual i is a unique k_i. We establish here some properties of the wealth distribution.

Proposition 2. *If $\tau_y(y)$ is strictly increasing k and ε are negatively correlated.*

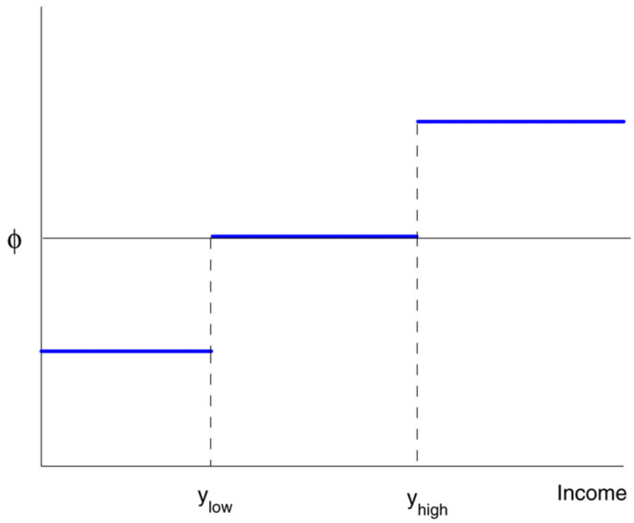

Fig. 2. Marginal tax function with steps.

Proof. Consider two households i and j, and without loss of generality let $\varepsilon_i > \varepsilon_j$. By Proposition 1, the income distribution is degenerate. Therefore i and j must have the same level of income in the long-run:

$$y_i = \tau_y^{-1}(\phi) = y_j.$$

By the definition of income, $w\varepsilon_i + rk_i = w\varepsilon_j + rk_j$. Thus,

$$\varepsilon_i - \varepsilon_j = -\frac{r}{w}(k_i - k_j).$$

Since $\varepsilon_i > \varepsilon_j$, $k_i < k_j$. □

In the steady state, wealth is held by low wage households. The following corollary establishes a key implication of this result: labor and asset income must be perfectly negatively correlated. The intuition is straightforward—since in the long run all households have the same income, households who earn high labor income have low capital income.

Corollary 1. *Under the assumptions above, labor income and asset income are perfectly negatively correlated.*

Proof. From above, if $\varepsilon_i > \varepsilon_j$ then $w\varepsilon_i > w\varepsilon_j$. Since $y_i = y_j$, $rk_i < rk_j$. Since income is equated across i, $w\varepsilon$ and rk have a correlation of -1. □

If the tax function is only weakly increasing, indeterminacy can obtain. For example, if the tax function consists of a finite number of steps, the income distribution is indeterminate over the bounded interval corresponding to marginal tax rate ϕ. If indeterminacy does obtain, our results do not apply since the correlation between income and wealth can be arbitrarily chosen.

4. Robustness

Our results on the long run distribution of wealth are quite robust. In particular, they are robust to the introduction of elastic labor supply and exogenous limits on asset holdings. They also obtain in a wide class of models that feature preference heterogeneity, including (but not limited to) models with habit formation.

4.1. Elastic labor supply

The conclusion that capital income and labor income are negatively correlated when the marginal tax rate is strictly increasing arises because all households must have the same long-run income. Augmenting the model to allow households to choose the number of hours will not change this outcome. In the long-run the hours decision can only affect the composition of income, as total income is still pinned down by (2.6). As long as total income is equalized, households with high (low) labor income must have low (high) capital income, leaving the correlation negative. Unlike in the basic model, however, wealth could be positively correlated with wages, although that result requires leisure to be a Giffen good.[9] This result does not require households to have the same preferences for leisure or the utility function to be separable in consumption and leisure.

It is important to note that elastic labor supply does alter Proposition 2 slightly if negative labor supply is not permitted. Assuming that the utility function does not possess a very large income effect relative to the substitution effect so that hours are a non-decreasing function of ε, low ε types will work less and hold greater wealth in the steady state than high ε types. If hours for a type with productivity ε^* reach the lower bound of 0, then all types with lower labor productivity will have the same wealth and therefore income as the ε^* type.

4.2. Exogenous borrowing limit

An exogenous borrowing constraint relaxes the strict equality of the optimality condition, so that possibly not all households have the same long-run income level. We now assume that households face a lower bound for assets k_b which is a function of labor productivity. Let $k_b(\varepsilon) = b\varepsilon$ with $b \leqslant 0$ and domain $(0, \infty)$. The optimality condition for savings in the steady state is

$$\beta\big[\big(1 - \tau_y(y_i)\big)r + 1\big] \leqslant 1, \tag{4.1}$$

with equality if $k_i > k_b(\varepsilon_i)$. Our proposition handles both the inelastic and elastic labor supply cases.

Proposition 3. *Let $\bar{k}(\varepsilon)$, be the long run asset holdings of a household with productivity ε in the absence of an exogenous borrowing limit. If $b > -\frac{w}{r}$, there exists ε^* such that $\bar{k}(\varepsilon) > b\varepsilon$ for all $\varepsilon < \varepsilon^*$ and $\bar{k}(\varepsilon) = b\varepsilon$ for all $\varepsilon \geqslant \varepsilon^*$. If $b \leqslant -\frac{w}{r}$, then $\bar{k}(\varepsilon) > b\varepsilon$.*

Proof. If some households do not supply positive hours, then those types must have positive wealth. Therefore if $\bar{k}(\varepsilon)$ and $k_b(\varepsilon)$ intersect, this intersection occurs at an ε type that supplies positive hours. From Proposition 2, $\bar{k}(\varepsilon)$ is linear with slope $-\frac{w}{r}$ for households that supply positive hours. Since $k_b(\varepsilon)$ is also linear, $\bar{k}(\varepsilon)$ and $k_b(\varepsilon)$ intersect no more than once. Because $\bar{k}(\varepsilon)$ is decreasing in ε, the lowest ε type must hold a positive asset position and therefore $\bar{k}(\varepsilon) > k_b(\varepsilon)$ for some ε. When $b > -\frac{w}{r}$, the two lines intersect. When $b \leqslant -\frac{w}{r}$, they do not intersect. $\square$

Proposition 4. *Constrained households have high income.*

Proof. Since the marginal tax function is strictly monotone increasing, by Proposition 1 all households with $k_i > k_b$ have the same level of income y^*. Now consider a household i that is constrained. With the possible exception of a set of measure zero, household i's optimality condition for savings holds with strict inequality, so that

$$\beta\big[\big(1 - \tau_y(y_i)\big)r + 1\big] < \beta\big[\big(1 - \tau_y(y^*)\big)r + 1\big].$$

Therefore

$$\tau_y(y^*) < \tau_y(y_{it})$$

and

$$y^* < y_i. \qquad \square$$

If an exogenous borrowing constraint is imposed, it can only bind for high income households. Because capital income for these households must be less than the capital income of all other households, they must have greater labor income and therefore higher effective wages. The existence of borrowing-constrained households does increase the correlation between capital income and labor income, because households at the borrowing limit all have the same amount of capital income but those with greater ε have greater labor income and thus greater total income. Quantitatively, for a wide range of experiments we found the increase very small and the correlation never became positive. An immediate corollary of the above result is that income and wealth are negatively correlated, since constrained households have low wealth and high income.

Corollary 2. *The correlation between k and y is negative if some households are constrained.*

[9] If k is increasing in $w\varepsilon$, then h must be decreasing because total income is constant. Thus, an increase in the price of leisure must generate a higher demand for leisure, or leisure is Giffen.

Table 1
SCF statistics.

Correlation (S.E. $\times 10^3$)	1992	1995	1998	2001	2004
$\rho(y, k)$	0.33 (0.091)	0.46 (0.079)	0.61 (0.062)	0.53 (0.070)	0.59 (0.062)
$\rho(w\varepsilon h, rk)$	0.24 (0.096)	0.43 (0.082)	0.14 (0.097)	0.20 (0.093)	0.15 (0.092)
$\rho(y, w\varepsilon)$	0.08 (0.102)	0.53 (0.072)	0.06 (0.098)	0.18 (0.094)	0.17 (0.092)

5. Empirical facts

The robust predictions of the model—the zero covariance between income and wealth and between capital income and labor income—are grossly inconsistent with the data. Using the 1992, 1995, 1998, 2001, and 2004 waves of the Survey of Consumer Finances, we measure the correlation for income and wealth and the correlation between labor and capital income across the population. Our measures of income, wealth, labor income, and capital income are those used by Budría Rodríguez et al. (2002) and are deflated using the CPI; Table 1 displays the key correlations.[10] In all years, the correlation between income and wealth is positive, and for the last three waves it has been very close to 0.60. Labor and capital income are also positively correlated, though the correlation is weaker.[11] Finally, wages and income are positively correlated in each of the five waves.

From this table we clearly see that the model is inconsistent with the data along all three dimensions; given the robustness of this result, we assert that the model is fundamentally incapable of replicating the joint distribution of income, wealth, capital income, and labor income. To make this point as clearly as possible, we repeat ourselves—in the presence of progressive taxation and permanent differences in productivity across households, the growth model with homogeneous preferences cannot be made consistent with this data unless it predicts that the wealth distribution is indeterminate. In the next subsection we discuss relaxing the homogeneous preference assumption to accommodate heterogeneous discount factors; we then show that this approach may be a promising direction to proceed.

6. Preference heterogeneity

Reconciling the model with the data requires some additional heterogeneity in the steady state Euler equations. In the steady state many forms of preference heterogeneity vanish, leaving them irrelevant for the question at hand.[12] In order for either the correlation between income and wealth or the correlation between capital income and labor income to be positive in the steady state, (4.1) must hold with equality for some y_i greater than the minimum value in the population. Heterogeneity in the discount factor is a possible avenue to reconcile model and data, since β does not disappear from the steady state Euler equation. The idea of combining heterogeneity in discount factors with progressive taxation is not new to this paper; as noted in the Introduction, Sarte (1997) uses heterogeneous discount factors to get a nondegenerate distribution of wealth. But Sarte (1997) does not distinguish sources of income, so his paper does not address whether the distribution of capital and labor income is consistent with the data.[13]

Consider two households with discount factors $\beta_j < \beta_i$. The model predicts that $y_j < y_i$ (more patient households have higher income). Thus, it is possible that this household could have both high labor income and high asset income. A positive correlation between β and ε is not sufficient to get capital income and labor income to have a positive correlation. The gap between the labor incomes $w(\varepsilon_i - \varepsilon_j) > 0$ exceeds the gap in capital income $r(k_i - k_j)$.[14] Unfortunately, this condition can be satisfied for either $k_i - k_j < 0$ or $k_i - k_j > 0$, so that the correlation between income and wealth can be of any sign; which result obtains in the model depends on the shape of the inverse of the marginal tax function. If the marginal tax function is flat enough in the region of interest (so that the inverse marginal tax function is steep), then incomes will be further apart (for a given difference in β). Since the gap in labor incomes is fixed, a wide enough gap in income requires a positive gap between capital holdings.

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[10] Specifically, income includes all pre-tax income sources and all transfers. Wealth is defined as all financial and non-financial assets less all debt. Labor income is total wages and salaries. Capital income is total income less labor income, business and farm income, and transfers. The numbers in parentheses are the standard errors, which we compute using the population weights.

[11] This correlation would rise if we exclude retirees, since their labor income is typically very low.

[12] As noted above, habit formation preferences would fall into the class of preferences that do not matter here, even if the habit parameter is heterogeneous across households. Heterogeneous preferences for leisure or elasticities of labor supply would also not alter the results (in the case where labor supply is elastic).

[13] The argument for the two features are "opposite" in the two papers as well. Sarte (1997) uses progressive taxation to rescue the model from the degenerate wealth distribution predicted by heterogeneous discount factors, while we use heterogeneous discount factors to rescue the model from anomalous income and (nondegenerate) wealth distributions.

[14] In the steady state,

$$y_i - y_j = w(\varepsilon_i - \varepsilon_j) + r(k_i - k_j) > 0.$$

Therefore,

$$w(\varepsilon_i - \varepsilon_j) > r(k_i - k_j).$$

We now formalize the notion of "flat enough" marginal tax functions. For a given rental rate

$$y = f(\beta) \equiv \tau_y^{-1}\left(1 - \frac{\beta^{-1} - 1}{r}\right)$$

is the long-run income of a household with discount factor β. If the marginal tax rate function $\tau_y(y)$ is strictly monotone-increasing, long-run income is increasing with β. Consider two households i, j with $\beta_i > \beta_j$, and let $\Delta x = x_i - x_j$ denote the difference between household i variables and household j variables. Simple algebra implies

$$\Delta k = \frac{1}{r}\big(\Delta f(\beta) - w\Delta\varepsilon\big). \tag{6.1}$$

Therefore wealth will rise with income if and only if $\Delta f(\beta) > w\Delta\varepsilon$; that is, if the change in total income produced in the steady state by higher β is greater than the change in labor income. This condition is certainly satisfied if ε is not positively correlated with β. If ε is positively correlated with β, then wealth could still rise with income if the slope of f is sufficiently steep. Under such a case, small changes in β would correspond to large increases in long-run total income.[15] We summarize the discussion with a formal proposition.

Proposition 5. *Let $\Delta f(\beta) > w\Delta\varepsilon$. Then wealth and income are positively correlated.*

We find evidence in the literature for a positive correlation between discount factors and wages. Becker and Mulligan (1997) use a life-cycle model with uncertainty to infer discount factors from household wealth data in the 1992 wave of the SCF and document a positive relationship between discount factors and income. Similarly, Lawrance (1991) estimates discount factors using the Euler equation and finds a statistically-significant positive relationship between the discount rate and education as well as between the discount rate and average labor income, controlling for education.[16] With respect to the marginal tax function, Gouveia and Strauss (1994, 1999) find that the US marginal effective income tax rate function is close to flat for a large fraction of observed US incomes, once the myriad of exemptions and credits are taken into account. The combination of this positive relationship and the nearly-flat tax function for the US suggests that discount factor heterogeneity may be successful at matching the joint distribution of capital income, labor income, and wealth observed in the data, although this statement is clearly quantitative and model-dependent.

7. Conclusion

This paper studies the long-run distributions of income and wealth under three features: progressive income taxation, heterogeneous permanent labor productivity, and complete asset markets. When discount factors are equal across households, all households must face the same marginal tax rate in the long-run. When each income level is associated with a unique marginal tax rate, this condition implies that all households have the same level of long-run income. Furthermore, the wealth distribution is determinate and wealth decreases with labor income. Therefore, the correlation between labor income and capital income in the model is negative, a prediction which is strongly at odds with the data. The result is very robust, as it survives elastic labor supply and binding borrowing constraints. Furthermore, introducing a lower limit on asset holdings higher than the natural debt limit yields a second empirically-unreasonable prediction, namely that income and wealth are negatively correlated.

While it has been well-established that data on consumption rejects complete markets as a model environment for studying risk sharing, complete markets models with heterogeneous agents are still widely used to study other questions related to inequality (we noted some papers in the Introduction). The complete markets assumption often has the advantage of providing analytic tractability, and to the degree to which the model's predictions come close to the data it can serve to provide valuable intuition. The findings of this paper, however, suggest that for questions related to income and wealth inequality, predictions based upon the complete markets assumption are unlikely to correspond well to the data. Including heterogeneity in the rate of time preference may possibly reconcile the complete markets assumption with the observed income and wealth distributions, but this result requires a relatively-flat tax function. Thus, any such model may not be reliable for policy experiments that change the progressivity of the tax function.

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[15] For intuition, notice that as $f'(\bullet)$ increases, the corresponding tax function approaches a flat tax. In that case, very small increases in β imply very large increases in wealth, as in Becker (1980).

[16] Samwick (1998) finds a negative relationship between discount factors and income, but attributes this finding to model misspecification based on Becker and Mulligan (1997).

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