Optimal Policy for Macro-Financial Stability*

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Abstract There is a new and now large literature analyzing government policies for financial stability based on models with endogenous borrowing constraints. These normative analyses build upon the concept of *constrained efficient allocation* where the social planner is constrained by the same borrowing limit that agents face. In this paper, we show that there exist at least one set of tools that implement the constrained efficient allocation that can also be used by a Ramsey planner to replicate an unconstrained allocation, achieving higher welfare. Constrained efficiency may lead to inaccurate characterizations of welfare maximizing policies relative to Ramsey optimal policy.

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1 Introduction

The recent global financial crisis has stimulated research on macroeconomic models with financial frictions. One of the most popular approaches builds upon the literature on sudden stops developed in the seminal work of Mendoza (2002, 2010), where financial frictions take the form of occasionally binding borrowing constraints. The virtue of this approach is its ability to distinguish between normal states of the economy (when the constraint is not binding) and crisis states (when the constraint binds). In particular, the amount of borrowing that agents undertake depends on the value of the collateral that is determined endogenously by a market price. For example, in Mendoza (2002), this market price is the real exchange rate while in Mendoza (2010) the is the price of physical capital. The presence of a market price in the borrowing constraint creates amplification mechanisms, known as debt-deflation spiral or asset fire sale, consistent with empirical features of sudden stop episodes. From a normative point of view, the presence of a market price in the borrowing constraint generates scope for policy intervention because agents do not take into account the effects of their choices on the market price when the collateral constraint binds. This distortion is referred to as pecuniary externality in the literature and several papers have studied the inefficiency associated with it and the corresponding scope for policy intervention in open and closed economy settings.[1]

This paper aims at strengthening the foundations of the normative analysis in this class of models. In many papers, the normative analysis builds upon the concept of *constrained efficient allocation*. The constrained efficient allocation is defined as a planning problem in which the planner faces the resource and technological constraints along with the borrowing limit. The analysis then proceeds by characterizing the constrained efficient allocation (henceforth SP) and by discussing how to implement it from a decentralized perspective. In this paper, we highlight an important element of fragility of this approach. In particular,

[1]For recent surveys of this literature see Erten, Korinek and Ocampo (2019) and Rebucci and Ma (2019).

we show that, in a typical specification of the model economy, there exists a set of tools that implements the SP allocation from a decentralized perspective that can also be used *optimally* by a Ramsey planner to replicate the allocation that would arise if the borrowing constraint were not present, i.e. the unconstrained allocation thereby achieving a much higher level of welfare.[2] This result shows that a standard Ramsey optimal policy approach is more robust than the SP approach typically used in the pecuniary externality literature because, whenever the policy tool can affect the price of the collateral, it attains all the welfare gains that are within reach of the policy instruments selected.[3] The result also highlights the importance to motivate the choice of the instrument assigned to the Ramsey planner or used to implement the SP allocation from the outset of the analysis.

In our analysis, we focus on a model economy used in Benigno et al. (2013, 2016), Bianchi (2011), Korinek (2018), Mendoza (2002), Schimitt-Grohe and Uribe (2016) and many other applications of this workhorse framework. This is a two-sector small open economy with traded and non-traded goods and in which agents have limited access to international capital market. Foreign borrowing is denominated in units of the tradable good, but it is leveraged on income generated in both sectors. Thus, the relative price of non-tradeable goods (which is typically interpreted as the real exchange rate in these models) affects the valuation of non-tradable income and hence the collateral value.[4]

We study both the production and endowment versions of this model economy. We first focus on the more general case of a production economy, and show that one tax scheme that can implement the SP allocation relies on the use of a tax on tradable consumption and a tax on firm non-tradable revenue along with lump-sum transfers to firms and households.

[2] As we will discuss later, our argument does not apply to all the possible combination of policy tools that implement the constrained efficient allocation. For the purpose of questioning the robustness of what we refer to as the standard approach in the literature it is sufficient to show that this is true at least for one case.

[3] Recent papers using this more robust approach, but computationally more demanding approach, Bianchi and Mendoza (2018), Devereux, Young and Yu (2018), Faia (2019). See Schmitt-Groh and Uribe (2017) for more details.

[4] This is a specification of the borrowing constraint that captures "balance sheet effects", a key feature of the capital structure of many emerging and advanced economies alike—e.g., Chang, Cespedes, and Velasco (2004) and Aghion, Bacchetta and Banerjee (2004).

The tax on tradable consumption mitigates the effect of the pecuniary externality on the borrowing decisions of the agents, while the tax on non-tradable profits reallocate resources between sectors as needed to restore efficiency, when the constraint binds.[5]

Next, we show that the *same* set of taxes can be used *optimally* by a Ramsey planner to achieve the unconstrained equilibrium, that is the allocation in which the borrowing constraint never binds. In this case, the tax on tradable consumption supports the relative price of non-tradable consumption in such a way that the borrowing constraint never binds in equilibrium, while the tax on firm profits ensures that, again, resource are allocated efficiently across sectors.

We then examine the special case of an endowment economy. In this context, our argument applies when the policy tool is given by the tax on tradeable consumption. While Benigno et al. (2016) show that a Ramsey planner with a tax on tradeable consumption can achieve the unconstrained equilibrium, the new result of this paper is to demonstrate that the same policy tool can implement the SP allocation, thus establishing another case in which the potential fragility of the approach emerges.[6]

While we derive our results in the context of a particular specification of the borrowing constraint and other features of the model economy, we emphasize its generality. The same considerations would arise in variants of our setting, including in alternative specifications in which the relative price of collateral is an asset price or in environments in which the borrowing constraint depends on the value of the collateral at the time of the repayment. Indeed the policy incentive from a Ramsey planner point of view, conditional on the set of policy tools assigned, whenever the policy tool can affect the price of the collateral, is to undo the effects of the borrowing constraint and to balance this incentive with the distortions created by the use of the policy tools.

[5]This tax scheme is neutral in the sense that is does not redistribute resources between households and firms.

[6]Indeed, Benigno et al. (2016) characterizes the constrained efficient allocation, but they do not discuss its implementation.

Related literature Our paper relates to several contributions in the literature. In the literature on pecuniary externalities in models with endogenous borrowing constraints, the most prominent examples of adoption of the SP approach are Lorenzoni (2008), Bianchi (2011), Benigno et al (2013), Davila (2014), Davila and Korinek (2018), and Jeanne and Korinek (2018). In the model of Lorenzoni (2008), agents are risk neutral and the asset price does not depend on the marginal utility of consumption so that it is not possible to influence it when the constraint is binding, thereby restricting the planner's ability to intervene only ex-ante.

Bianchi (2011), Benigno et al. (2016) and Korinek (2018), in particular, study the endowment version of the model economy analyzed here. Bianchi (2011) and Korinek (2018) implement the SP allocation with a tax on debt and lump-sum transfers, which is usually interpreted as capital control or macroprudential policy. Benigno et al. (2016) show that, in this economy, the state-contingent tax on *debt* that implements the SP allocation is the same as the one that a Ramsey planner would set if she/he is using the same instrument. This is because, in the case of an endowment economy, the tax on debt does not affect the collateral value so that, when the constraint binds, as long as there are lump-sum transfers, the solution of the SP planning problem is equivalent to that of the Ramsey optimal policy problem with the debt tax. In contrast, in this paper, we show that, also in the endowment case, if a tax on tradeable consumption can be used to implement the SP allocation, the same policy tool can achieve the unconstrained allocation when used optimally by a Ramsey planner.[7] Benigno et al. (2013) analyze the social planner allocation of the same economy studied in this paper, but does not discuss its implementation or the Ramsey planner problem.

Davila (2014) and Davila and Korinek (2017) acknowledge that the normative analysis of

[7]Benigno et al (2016) also show that a subsidy on non-traded consumption can achieve the unconstrained equilibrium. However, the *same* subsidy cannot implement the SP allocation. This is because, in the endowment economy, when the constraint binds, the SP allocation coincides with the competitive equilibrium one, and since the subsidy on non-traded consumption operates only when the constraint binds it cannot be used to replicate the SP allocation.

models with credit constraints that depend on endogenously determined asset prices hinges on the specific set up of the normative benchmark, but do not compare the constrained efficient allocation with Ramsey allocation and focus on the availability of ex-ante or ex-post transfers. In an environment with informational frictions, Di Tella (2018) considers a SP problem in which the planner is constrained by the same informational friction that private agents face. Kurlat (2018) also focuses on a SP problem in which the planner faces the same informational limitations as private agents. He incorporates such friction into a version of Lorenzoni's (2008) economy and, conducting a similar analysis, finds that the normative conclusions on fire sales are reversed. Neither of these studies, however, discuss the implementation of the SP allocation with specific tools.

Recent papers using the Ramsey approach include Bianchi and Mendoza (2018), Devereux, Young and Yu (2018), Faia (2019), and Jeanne and Korinek (2019). Bianchi and Mendoza (2018) and Devereux, Young and Yu (2018) compute the optimal markov policy in infinite horizon economies in which the borrowing constraint depends on an asset price. Faia (2019) considers a banking model where there is a possibility of bank runs and uses the constrained efficient approach to identify the externalites in the model, and the Ramsey approach to characterize the optimal policy under commitment, given the instrument assigned to the planner. Jeanne and Korinek (2019) solve analytically the optimal policy for the joint use of both ex ante and ex post interventions in a three-period model with an asset price constraint and compare commitment with discretion.

Other studies have focused on welfare-improving government interventions in the presence of multiple distortions in addition to pecuniary externalities. For instance, Brunnermeier and Sannikov (2015) show that restrictions to capital mobility can be welfare-improving in an economy with multiple goods, incomplete financial markets, and inefficient production, but they do not characterize the capital control policy that implements constrained efficiency. Cespedes, Chang and Velasco (2017) compare the transmission mechanism of alternative policy interventions in a model with an occasionally binding borrowing constraint, but don't discuss the implementation of the constrained efficient allocation or the computation of the optimal policy in their set up.

Our result on the ability of a given set of policy tools to undo the underlying distortions in the economy is reminiscent of the results in the paper by Correia, Nicolini and Teles (2008), in which the role of price stickiness for the design of monetary policy depends on the existence of alternative fiscal policy tools. Our paper is also naturally linked to the literature in which the *occasionally binding* constraint is the zero-lower-bound on interest rates. For example, Adams and Billi (2006 and 2007) study optimal monetary policy in a closed economy, New-Keynesian model in which there is zero-lower-bound on interest rates that binds only occasionally. The critical difference here, with respect to a sudden stop setting as in Mendoza (2002, 2010) is that the zero-bound constraint is fixed and does not evolve endogenously or depend on market prices.

Finally, other important related contributions that analyze financial frictions in infinite horizon macroeconomic models from a positive perspective as in the seminal contributions of Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999) are Iacoviello (2005), Curdia and Woodford (2010), Gertler and Karadi (2011), Gertler and Kiyotaki (2011). The key insight of our paper would apply also to the normative analysis these frameworks.

The rest of the paper is organized as follows. Section 2 describes the model and its competitive equilibrium. Section 3 sets up the SP problem and discusses its implementation with an unrestricted set of instruments. Section 4 analyzes the Ramsey planner problem. Section 5 discusses the endowment case. Section 6 concludes.

2 The Model and Its Competitive Equilibrium

In this section, we describe briefly our model set-up and discuss its key assumptions.

2.1 Households

There is a continuum of households $j \in [0,1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t^j - \frac{\left(H_t^j\right)^{\delta}}{\delta} \right)^{1-\rho} \right\}, \tag{1}$$

where C^j denotes the individual consumption basket and H^j the individual supply of labor for the tradable and non-tradable sectors ($H = H^T + H^N$). The elasticity of labor supply is δ, while ρ is the coefficient of relative risk aversion. For simplicity, in the remainder of this section, we omit the j subscript, but it is understood that all choices are made at the individual level. In what follows, we assume that $\beta < \frac{1}{1+i}$, where β is the discount factor and i is the real return on saving between period t and $t+1$.

The consumption basket, C_t, is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \tag{2}$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of tradable goods in the consumption basket. We normalize the price of tradable goods to 1. Denoting the relative price of nontradable goods by P^N, the aggregate price index is given by

$$P_t = \left[\omega + (1-\omega) \left(P_t^N\right)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}.$$

Households maximize utility subject to their budget constraint expressed in units of tradable consumption:

$$C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i)\, B_t, \tag{3}$$

where W_t is the wage in units of tradable goods, $B_{t+1} < 0$ denotes the debt position at the end of period t with gross real return $1+i$. Households receive profits, π_t, from firms' ownership, and their labor income is $W_t H_t$.

International financial markets are incomplete, and access to them is imperfect as in Mendoza (2002 and 2010). Specifically, the amount that each individual can borrow is limited by a fraction of his current *total* income:[8]

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\pi_t + W_t H_t\right]. \tag{4}$$

Unlike Mendoza (2010) or Bianchi and Mendoza (2018), we abstract from imposing a working capital requirement as our analysis is analytical rather than quantitative.

Households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first-order conditions of this problem are:

$$C_T : \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE} \tag{5}$$

$$C_N : \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE} P_t^N \tag{6}$$

$$B_{t+1} : \mu_t^{CE} = \lambda_t^{CE} + \beta\,(1+i)\, E_t\left[\mu_{t+1}^{CE}\right], \tag{7}$$

and

$$H_t : \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \left(H_t^{\delta-1}\right) = \mu_t^{CE} W_t + \frac{1-\phi}{\phi} W_t \lambda_t^{CE}, \tag{8}$$

where μ_t^{CE} is the multiplier on the period budget constraint, and λ_t^{CE} is the multiplier on

[8]For an discussion of the nature of the credit constraint we refer to Mendoza (2010), Bianchi (2011), Jeanne and Korinek (2018) and Bianchi and Mendoza (2018).

the international borrowing constraint. The presence of the borrowing constraint distorts directly two margins: the intertemporal margin, as the Euler equation (7) includes the term λ_t^{CE} which is positive when the constraint binds, and the labor supply choice (8). For future reference, note here that we can combine (5) and (6) to obtain the intratemporal allocation of consumption,

$$P_t^N = \frac{(1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}}}. \tag{9}$$

2.2 Firms

Firms produce tradable and non-tradable goods with a homogeneous labor input and the following decreasing return to scale technologies:

$$Y_t^N = A_t^N H_t^{1-\alpha^N}, \tag{10}$$

$$Y_t^T = A_t^T H_t^{1-\alpha^T}, \tag{11}$$

where A^N and A^T are the productivity levels, which are assumed to be random variables, in the non-tradable and tradable sector, respectively. The firm's problem is static, and current-period profits (π_t) are:

$$\pi_t = A_t^T \left(H_t^T\right)^{1-\alpha^T} + P_t^N A_t^N \left(H_t^N\right)^{1-\alpha^N} - W_t H_t. \tag{12}$$

The first-order conditions are:

$$W_t = \left(1-\alpha^N\right) P_t^N A_t^N \left(H_t^N\right)^{-\alpha^N}; \tag{13}$$

$$W_t = \left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}; \tag{14}$$

and taking the ratio of (13) over (14) we obtain:

$$P_t^N = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}}{\left(1-\alpha^N\right) A_t^N \left(H_t^N\right)^{-\alpha^N}}. \tag{15}$$

From this last expression we can see that the relative price of non-tradable goods determines the allocation of labor between the two sectors: for given productivity levels, a decrease (increase) in P_t^N drives down (up) the marginal product of non-tradables and induces a shift of labor toward (out of) the tradable sector.

2.3 Competitive Equilibrium

To determine the goods market equilibrium, combine the household budget constraint (3), the firm's profits (12), and the equilibrium condition in the nontradable sector to obtain the current account equation of our economy:

$$B_{t+1} = (1+i) B_t + A_t^T H_t^{1-\alpha^T} - C_t^T. \tag{16}$$

The nontradable equilibrium condition implies that

$$A_t^N \left(H_t^N\right)^{1-\alpha^N} = Y_t^N = C_t^N. \tag{17}$$

Finally, using (12) we can rewrite (4) as

$$B_{t+1} \geq -\frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N\right], \tag{18}$$

so that (16) and (18) determines the evolution of B_{t+1}.

Definition 1 (Competitive Equilibrium- CE) *The competitive equilibrium allocation of this economy is characterized by (16), (17) and (18) along with the first order conditions* *for the household (5), (7), (8) and (9), for the firms (15) and the following complementary slackness condition:*

$$\left(B_{t+1} + \frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N\right]\right)\lambda_t^{CE} = 0 \text{ with } \lambda_t^{CE} > 0.$$

2.4 Unconstrained Equilibrium

For later use, it is useful to define here the competitive equilibrium of the same economy without the borrowing constraint (18). We refer to this allocation as the "unconstrained equilibrium" (denoted with the UE superscript) and define it formally as follows.

Definition 2 (Unconstrained Equilibrium-UE) *The unconstrained allocation of our economy is defined as a decentralized equilibrium in which households maximize (1) subject to (2) and (3), and firms maximize profits (12) subject to (10).*

In this allocation financial markets are still incomplete in the sense that there are inefficient variations of consumption due to the lack of state contingent debt. To guarantee that the competitive equilibrium of the unconstrained economy has an ergodic distribution of debt with finite support under the assumption that $\beta(1+i) < 1$, in what follows, we assume the existence of a lower bound on debt which is strictly greater than the natural debt limit.

3 Constrained Efficiency

In this section we follow the social planner approach typically used in the literature. We first review the characterization of the constrained social planner allocation of our economy (SP from now on). We then show how this allocation can be implemented from a competitive equilibrium point of view, which is one of the contribution of this paper.

3.1 Definition and Analysis

The SP allocation is constrained in the sense that the planner faces the same borrowing limit that the private agents do, but from an aggregate country-wide perspective. It is also important to emphasize that this planner *does not* use any policy instrument, but simply allocates resources efficiently. The critical aspect of this planner problem is that the relative price that enters the borrowing constraint, by assumption, is determined by the same *pricing rule* (9) used in the competitive equilibrium allocation.

Definition 3 (Constrained Social Planner Problem–SP) *The planner chooses the optimal path of C_t^T, C_t^N, B_{t+1}, H_t^T, and H_t^N by maximizing (1) subject to the resource constraints (16) and (17), the aggregate borrowing constraint (18), and the pricing rule of the competitive equilibrium allocation (9).*

Importantly, the planner takes into account the effects of its decisions on P_t^N, and hence internalizes the pecuniary externality arising from the presence of this relative price in the borrowing constraint. To see this, we first rewrite (18) as

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[A_t^T\left(H_t^T\right)^{1-\alpha^T}+\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{1-\frac{1}{\kappa}}\right], \tag{19}$$

in which we substituted the production function and (9). The first-order conditions for the SP problem are:

$$C_T : \left(C_{j,t}-\frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}\left(\frac{\omega C}{C^T}\right)^{\frac{1}{\kappa}} = \mu_{1,t}^{SP}+ \tag{20}$$

$$-\frac{\lambda_t^{SP}}{\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega}\right)^{\frac{1-\kappa}{\kappa}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{\frac{\kappa-1}{\kappa}},$$

$$C_N : \left(C_{j,t}-\frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}C^{\frac{1}{\kappa}} = \mu_{2,t}^{SP}, \tag{21}$$

$$B_{t+1}: \mu_{1,t}^{SP} = \lambda_t^{SP} + \beta (1+i) E_t \left[\mu_{1,t+1}^{SP}\right], \tag{22}$$

$$H_t^T: \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \left(H_t^{\delta-1}\right) = \left(1-\alpha^T\right) \mu_{1,t}^{SP} A_t^T H_t^{-\alpha^T} + \frac{1-\phi}{\phi} \lambda_t^{SP} \left(1-\alpha^T\right) A_t^T H_t^{-\alpha^T}, \tag{23}$$

and

$$H_t^N: \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \left(H_t^{\delta-1}\right) = \left(1-\alpha^N\right) \mu_{2,t}^{SP} A_t \left(H_t^N\right)^{-\alpha^N} \tag{24}$$

$$+\frac{1-\phi}{\phi} \lambda_t^{SP} \frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}}} \frac{\kappa-1}{\kappa} \left(1-\alpha^N\right) \left(A_t^N\right)^{\frac{\kappa-1}{\kappa}} \left(H_t^N\right)^{\left(1-\alpha^N\right)\frac{\kappa-1}{\kappa}-1},$$

where $\mu_{1,t}^{SP}$ is the Lagrange multiplier on (16), $\mu_{2,t}^{SP}$ is the multiplier on (17) and λ_t^{SP} is the multiplier on (19). Benigno et al. (2013) provide an extensive discussion of these first order conditions relative to those in the competitive equilibrium. Here we note only that in equation (20), in choosing tradable consumption, the planner takes into account the effects that this choice has on the value of the collateral. This is the effect that is usually referred to in the literature as the "pecuniary externality."

3.2 Implementation with an Unrestricted Set of Instruments

We now discuss how to decentralize the SP allocation with an unrestricted set of policy instruments. In this economy, one can allow for the following set of instruments: a tax (or subsidy) on non-tradable and tradable consumption, τ_t^N and τ_t^T, respectively, which are usually interpreted in terms of exchange rate policy because they affect P_t^N directly; a tax (subsidy) on the amount that households borrow, τ_t^B, that is usually interpreted as capital controls;[9] a tax on wage income, denoted with τ^H, as well as lump-sum taxes (transfers) to the consumer, denoted T_t^C, which can be interpreted as traditional fiscal policy tools. On the production side of the economy, one can also allow for sector taxes on firms revenues rebated in lump-sum manner. We consider a distortionary tax (subsidy) on non-tradable

[9] As Bianchi (2011) showed, by introducing a domestic banking system that intermediates funds borrowed internationally to lend domestically, it is possible to use the same theoretical framework to analyze macro-financial regulation. In that context, τ_t^B is usually interpreted as a macro-prudential policy tool.

revenue, τ_t^D, rebated with a lump-sum transfer to the firm, T_t^D. A distortionary tax on tradable revenue could also be considered, but is equivalent to the distortionary tax on non-tradable revenue. Taxes on revenue can also be interpreted in term of fiscal policy.

Given this set of instruments, and assuming that the government balances the budget period by period we have:

$$\tau_t^B B_{t+1} + \tau^T C_t^T + \tau_t^N P_t^N C_t^N + \tau_t^D P_t^N Y_t^N + \tau^H W_t H_t = T_t^C + T_t^D. \tag{25}$$

The household budget constraint now is:

$$(1+\tau^T)C_t^T + (1+\tau^N)P_t^N C_t^N = \pi_t + (1-\tau^H)W_t H_t - (1-\tau^B)B_{t+1} + (1+i)\,B_t + T_t^C, \tag{26}$$

while the individual borrowing constraint becomes

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\pi_t + (1-\tau^H)W_t H_t\right]. \tag{27}$$

The firm's profit is now given by:

$$\pi_t = A_t^T \left(H_t^T\right)^{1-\alpha^T} + (1-\tau_t^D)P_t^N A_t^N \left(H_t^N\right)^{1-\alpha^N} - W_t H_t - T_t^D. \tag{28}$$

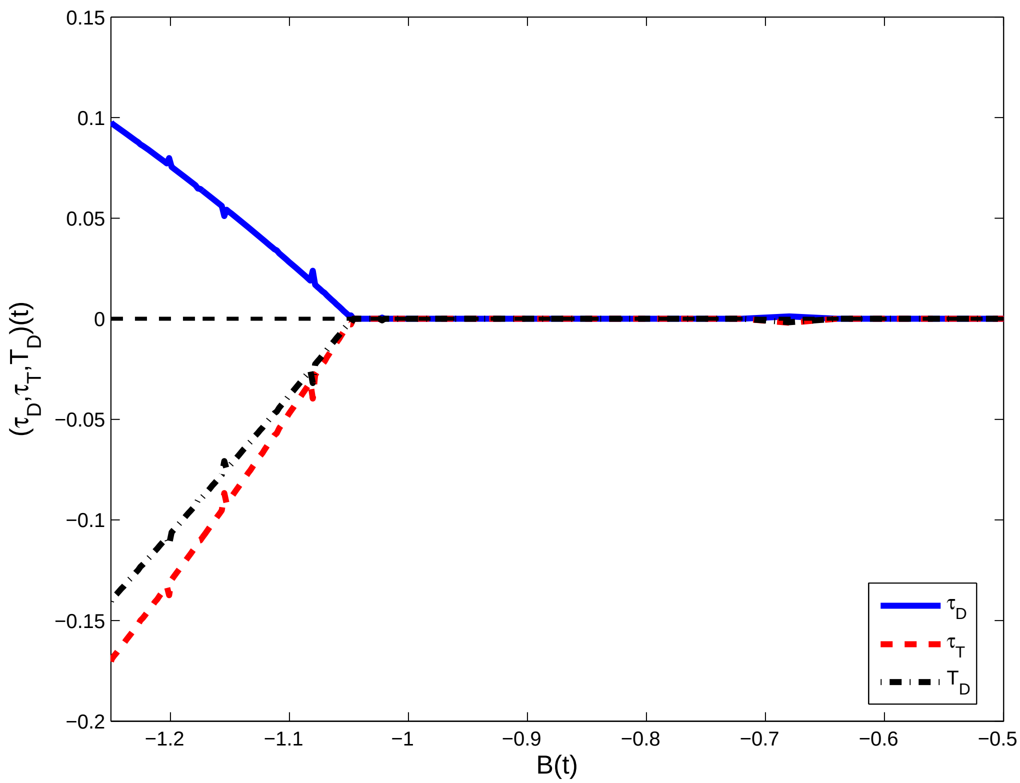
The following proposition characterizes how to decentralize the SP allocation with an unrestricted set of policy instruments.[10]

Proposition 1 (Implementation of the SP Allocation). *Given the following set of taxes (τ^B, τ^N, τ^T, τ^D, τ^H, T^D, T^C), there exists a combination of policy rules for the subset of taxes (τ^T, τ^D, T^D ,T^C) (for brevity, a tax scheme for this subset of instruments) that implements the constrained SP allocation in the competitive equilibrium of our economy. This tax scheme is time consistent.*

[10]Note that this result is neither in Benigno et al. (2013) nor in Benigno et al. (2016).

Proof. See Appendix.

Figure 1



Notes: The figure plots the policy rules for τ^T, τ^D, and T^D that implement the constrained efficient allocation. The plot assumes the same set of parameter values used by Benigno et a. (2013). Note that T^C is not plotted because it is determined from the government budget constraint. Borrowing decreases from left to right on the x-axis.

We note here that this proposition shows the existence of one possible tax scheme implementing the SP allocation, but this is not unique. As we show in appendix, it is possible to find other combinations of the available instruments that could replicate the same allocation. We focus on this particular tax scheme to highlight the potential fragility of the SP approach to the normative analysis of models with endogenous borrowing constraints that depend on market prices. Second, from the expressions of the policy rules for τ^T, τ^D, T^D, T^C plotted in Figure 1, we also can see that these taxes are used only when the constraint binds and should be interpreted as *ex-post* interventions in the sense of Benigno et al. (2016) and Jeanne and Korinek (2019).

As we can see, the tax scheme entails a subsidy on tradable goods and a tax on non-tradable revenue. The subsidy on tradable goods makes agents internalize the pecuniary externality. The tax on non-tradable revenue affects the intratemporal sector allocation of labor. Thus, by correcting the allocation of labor across sectors, the planner can relax the borrowing constraint when it binds by increasing the amount that it is produced in each sector.

4 Ramsey Optimal Policy

Thus far we saw that there exists a time-consistent tax scheme that can implement the constrained social planner allocation of our model economy. We will now show that the *same* subset of tools can be used optimally by a Ramsey planner to undo the constraint and thus achieve a higher level of welfare.

In a standard Ramsey problem, the planner maximizes the representative agent's utility given the resource constraint, the technological constraints, and the competitive allocation first order conditions for a given set of policy instruments. In order to compare the two approaches, we assign to the Ramsey planner the *same* subset of policy tools that implement the SP allocation, $\left(\tau^T, \tau^D, T^D, T^C\right)$.

Definition 4: Ramsey Planner Problem *For a given* $\{B_0\}$, *and assuming that* $\{A_t^T\}$ *and* $\{A_t^N\}$ *are Markov processes with finite strictly positive support, the Ramsey problem for* $\left(\tau^T, \tau^D, T^D, T^C\right)$ *is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t^j - \frac{\left(H_t^j\right)^{\delta}}{\delta} \right)^{1-\rho} \right\},$$

subject to (2), the agents resource constraints

$$(1+\tau^T)C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i)\, B_t + T_t^C, \tag{29}$$

the firms' definition of profits

$$\pi_t = A_t^T \left(H_t^T\right)^{1-\alpha^T} + (1-\tau_t^D) P_t^N A_t^N \left(H_t^N\right)^{1-\alpha^N} - W_t H_t - T_t^D \tag{30}$$

the government budget constraint

$$\tau_t^T C_t^T + \tau_t^D P_t^N Y_t^N = T_t^C + T_t^D,$$

the technological constraints (10) and (11), the non-tradeable goods market equilibrium condition (17), the borrowing constraint (18) the first order conditions of the household,

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE}(1+\tau_t^T) \tag{31}$$

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE} P_t^N \tag{32}$$

$$B_{t+1} : \frac{\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}}}{(1+\tau_t^T)} = \lambda_t^{CE} + \beta\,(1+i)\, E_t \left[\frac{\left(C_{t+1} - \frac{H_{t+1}^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_{t+1}^T\right)^{-\frac{1}{\kappa}} C_{t+1}^{\frac{1}{\kappa}}}{(1+\tau_{t+1}^T)} \right], \tag{33}$$

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \left(H_t^{\delta-1}\right) = \frac{\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}}}{(1+\tau_t^T)} W_t + \frac{1-\phi}{\phi} W_t \lambda_t^{CE}. \tag{34}$$

and the first order conditions of the firms,

$$W_t = (1-\tau_t^D)\left(1-\alpha^N\right) P_t^N A_t^N \left(H_t^N\right)^{-\alpha^N}, \tag{35}$$

$$W_t = \left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}, \tag{36}$$

Before proceeding, recall that, by taking the ratio of (32) to (31), we obtain

$$\frac{P_t^N}{(1+\tau_t^T)} = \frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}} \tag{37}$$

and by substituting (28), (11) and (17), the borrowing constraint becomes:

$$B_{t+1} + \frac{1-\phi}{\phi}\left[A_t^T (H_t^T)^{1-\alpha^T} + (1-\tau_t^D) P_t^N C_t^N - T_t^D\right] = 0.$$

The next proposition states the main result of the paper.

Proposition 2. *Given the set of taxes* $(\tau^T, \tau^D, T^D, T^C)$ *to the Ramsey planner as in definition (4) above, there exists an optimal time-consistent tax scheme that replicates the UE allocation.*

Proof. See the Appendix.

Importantly, note here that the quadruplet $(\tau_t^T, \tau_t^D, T_t^D, T_t^C)$ is the same set of taxes that decentralize the constrained social planner allocation. The critical difference is that when used optimally by the Ramsey planner, they can undo the constraint altogether, while the constrained social planner takes the borrowing constraint as given. Under the optimal Ramsey policy, τ_t^T removes the borrowing constraint altogether by affecting directly the market value of collateral entering it, P_t^N, while τ_t^D offsets the distortions created by τ_t^T. So if we allow a Ramsey planner to optimize over (τ_t^T, τ_t^D), given the behavior of the private sector, it is possible to replicate the unconstrained equilibrium in this economy. This result implies that, in the SP allocation, the tax scheme (τ_t^T, τ_t^D) is *suboptimal* in the sense that it does not achieves all the welfare gains that could be attained by using the same set of instruments.

Second, the policy rule for τ_T can be interpreted as a price support intervention, akin to an exchange rate intervention or an attempt to prop up the price of collateral. By taxing tradable goods, this policy increases the relative price of non-tradable goods. Critically, when the constraint binds, this supports the relative price of non-tradables, counteracting the debt-deflation spiral that would otherwise lead to a decline in tradable consumption and a fall in the relative price of nontradables.

Third, in equilibrium agents anticipate that policy will undo the constraint when this binds, and behave as if the constraint does not exist (i.e., like in the UE allocation). Eventually, in finite time, our economy will hit the borrowing constraint because agents are relatively impatient. When that happens, under the Ramsey optimal policy, $(\tau_t^T, \tau_t^D, T_t^D, T_t^C)$ will be set so that the multiplier on the borrowing constraint is zero (i.e., the constraint is just binding).

Finally, the result arises from the instrument's ability to affect the price of collateral on which the borrowing constraint is specified and therefore it has fairly general applicability. The substance of our main result would not change if we modify the borrowing constraint to include a working capital component, or if we consider a collateral constraint defined on an asset price, as long as the instrument assigned to the policy maker can affect the price of collateral when the borrowing constraint binds.

5 The Endowment Case

In this section we show that the same argument also applies to an endowment version of our model economy. In Benigno et al. (2016), we have shown how a tax on *tradeable* consumption can achieve the unconstrained allocation. Here we need to show that the same instrument can also implement the constrained efficient allocation.

We first characterize the social planner problem noting that the utility function no longer depends on the disutility of supplying labor. Specifically, the social planner maximizes

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t^j\right)^{1-\rho} \right\}, \tag{38}$$

subject to the same borrowing constraint that private agents face and the following market clearing conditions for tradable and nontradable goods:

$$B_{t+1} = Y_t^T + (1+r)\, B_t - C_t^T,$$

$$Y_t^N = C_t^N.$$

In specifying this problem, the equilibrium price of nontradables continues to be determined competitively according to the pricing rule (9) that serves also as a constraint on the planning problem. By substituting the relative price of nontradables, P_t^N, in the borrowing constraint with the competitive pricing rule (9) we can write the Lagrangian of the planning problem as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \frac{1}{1-\rho} (C_t)^{1-\rho} + \mu_{1,t}^{SP} \left(Y_t^T - B_{t+1} + (1+r)\, B_t - C_t^T \right) + \\ +\mu_{2,t}^{SP} \left(Y_t^N - C_t^N \right) + \lambda_t^{SP} \left(B_{t+1} + \frac{1-\phi}{\phi} \left[Y_t^T + \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega Y_t^N} \right)^{\frac{1}{\kappa}} Y^N \right] \right) \end{array} \right],$$

where $\mu_{1,t}^{SP}$, $\mu_{2,t}^{SP}$ and λ_t^{SP} denote the SP multipliers. The planner must choose the optimal path for C_t^T, C_t^N and B_{t+1}, and the first order conditions are:

$$C_T : u'(C_t^{SP}) C_{C^T}^{SP} + \lambda_t^{SP} \Sigma_t^{SP} = \mu_{1,t}^{SP}, \tag{39}$$

$$C_N : u'(C_t^{SP}) C_{C^N}^{SP} = \mu_{2,t}^{SP}, \tag{40}$$

$$B_{t+1} : \mu_{1,t}^{SP} = \lambda_t^{SP} + \beta\,(1+r)\, E_t \left[\mu_{1,t+1}^{SP} \right]. \tag{41}$$

$$\lambda_t^{SP} \left\{ B_{t+1}^{SP} + \frac{1-\phi}{\phi} \left[Y_t^T + P_t^N Y_t^N \right] \right\} = 0 \tag{42}$$

where $\Sigma_t^{SP} \equiv \frac{1-\phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left(\frac{(1-\omega)\left(C_t^T\right)}{\omega} \right)^{\frac{1}{\kappa}-1} \left(Y_t^N\right)^{\frac{\kappa-1}{\kappa}}$.

By combining the previous equations, we can characterize the social planner allocation with the following Euler equation,

$$u'(C_t^{SP})C_{C_t^T}^{SP} + \lambda_t^{SP}\Sigma_t^{SP} = \lambda_t^{SP} + \beta(1+r)E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP} + \lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]. \tag{43}$$

In order to implement the SP allocation with a tax con tradeable consumption we first note that the Euler equation in the decentralized equilibrium with a tax on traded consumption becomes

$$\frac{u'(C_t)C_{C_t^T}}{1+\tau_t^T} = \lambda_t + \frac{\beta\,(1+r)}{1+\tau_{t+1}^T}E_t\left[u'(C_{t+1})C_{C_{t+1}^T}\right]. \tag{44}$$

By combining (43) and (44) we find the tax schedule that equates the two margins is given by:

$$\frac{\tau_t^T - \tau_{t+1}^T}{1+\tau_{t+1}^T} = \frac{E_t[\lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]}{E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP}]} - \frac{\lambda_t^{SP}\Sigma_t^{SP}}{\beta(1+r)E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP}]}. \tag{45}$$

This expression says that the tax scheme that implement the SP is a *rule* that, at time t, for a given tax rate τ_t^T chosen at time $t-1$, selects a τ_{t+1}^T that has the same expression as the tax on borrowing in Bianchi (2011). Indeed in Bianchi (2011) we have that

$$\tau_t^B = \frac{E_t[\lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]}{E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP}]} - \frac{\lambda_t^{SP}\Sigma_t^{SP}}{\beta(1+r)E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP}]}$$

In Benigno et al. (2016) we have shown that a Ramsey planner that maximize agents' utility by choosing τ_t^T can achieve the unconstrained allocation. In contrast, as we discussed in the introduction, the tax on debt that implements the constrained efficient allocation cannot achieve the unconstrained equilibrium. The critical difference is that the tax on debt cannot influence the relative price when the constraint binds, while the tax on tradable consumption can.

6 Conclusions

In this paper we show that, in models with occasionally binding borrowing constraints in which the collateral value depends on market prices, there exists at least one relevant case in which the same combination of instruments that implements the constrained efficient allocation can also be used optimally by a Ramsey planner to achieve the unconstrained equilibrium where the constraint never binds in equilibrium. We established this in the context of a specific (albeit widely used) model economy, but the results have more general applicability. The result in fact applies whenever a policy instrument that is assigned to the planner can affect the market price determining the value of the collateral in the borrowing constraint. The result implies a potential lack of robustness of policy conclusions reached by adopting the constrained efficient allocation as a benchmark for the normative analysis in this class of models. This implies that a robust normative analysis in this class models requires explicit computations of the Ramsey optimal policy problems.

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A Appendix

A.1 Proof of Proposition 1

Proposition 1 (Implementation of the Constrained Efficient Allocation). *Given the following set of taxes (τ^B, τ^N, τ^T, τ^D, τ^H, T^D, T^C), there exists a combination of policy rules for the subset of taxes (τ^T, τ^D, T^D ,T^C) (for brevity, a tax scheme for this subset of instruments) that implements the constrained SP allocation in the competitive equilibrium of our economy. This tax scheme is time consistent.*

Proof. To prove the proposition above we seek a tax scheme that equates the first order conditions of the SP and CE allocations. When the constraint does not bind, it is easy to see by inspection that the first order conditions of the SP and the CE coincide. Therefore, there is no need to use any tax tools to equalize them. When the borrowing constraint binds, we can correct the distortion in the marginal utility of tradable consumption by using τ_t^T. In fact, by comparing (20) with (5), we have

$$\left(1+\tau_t^T\right)^{SP} = \left(1 - \frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega C^N}\right)^{\frac{1-\kappa}{\kappa}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{\frac{\kappa-1}{\kappa}}\right) < 1,$$

with the right-hand side evaluated at the SP allocation, which is a policy intervention that subsidizes consumption of tradable goods. Second, by setting $\tau^T = \left(\tau^T\right)^{SP}$ and $\tau^N = 0$ in (9), we can also implement the SP intratemporal allocation of consumption given by the ratio of (20) over (21). Third, note that once we set $\tau_t^B = 0$, the intertemporal allocation of consumption has the same expression for both the SP and the CE, in equations (22) and (7), respectively. Fourth, note that the intratemporal allocation of labor in the planner problem is given by:

$$\frac{\mu_{2,t}^{SP}}{\mu_{1,t}^{SP}} = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}}{\left(1-\alpha^N\right) A_t^N \left(H_t^N\right)^{-\alpha^N}} \frac{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right)}{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{2,t}^{SP}}\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\right)},$$

obtained from (23) and (24), which governs the sector allocation of labor. The corresponding condition in the competitive allocation is

$$P_t^N = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}}{\left(1-\tau^D\right)\left(1-\alpha^N\right) A_t^N \left(H_t^N\right)^{-\alpha^N}}.$$

It follows that, by setting τ_t^D such that

$$\frac{1}{\left(1-\tau^D\right)^{SP}} = \frac{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right)}{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{2,t}^{SP}}\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\right)}$$

where $\frac{1}{(1-\tau^D)^{SP}} < 1 (> 1)$ depending on the elasticity of intratemporal substitution $\kappa > 1 (< 1)$, we can also equalize this margin between the two allocations.

Note, finally, that when we use $\left(\tau^D\right)^{SP}$ and $\left(\tau^T\right)^{SP}$ as described above, when the constraint binds, we have that (18) can be written as

$$B_{t+1} = -\frac{1-\phi}{\phi}\left[Y_t^T + (1-\left(\tau_t^D\right)^{SP}) P_t^N Y_t^N - T_t^D\right],$$

with

$$\frac{P_t^N}{(1+\left(\tau^T\right)^{SP})} = \left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}}.$$

In general, therefore, when the constraint binds, the borrowing constraint in the SP will differ from the one in the CE because of $\left(\tau^D\right)^{SP}$ and $\left(\tau^T\right)^{SP}$. So we need to find a combination of taxes such that

$$B_{t+1} = -\frac{1-\phi}{\phi}\left[Y_t^T + \left(P_t^N\right)^{SP} Y_t^N\right].$$

To do so, denote with $\left(P_t^N\right)^{SP}$ the relative price of non-tradable in the social planner allocation:

$$\left(P_t^N\right)^{SP} = \left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}}.$$

so that

$$\left(P_t^N\right)^{SP}\left(1 + \left(\tau^T\right)^{SP}\right) = P_t^N$$

evaluating the price at the same (SP) allocation. This means that we can set T_t^D in the competitive equilibrium allocation such that

$$\left(T_t^D\right)^{SP} = -\left(\tau_t^D\right)^{SP}\left(1 + \left(\tau^T\right)^{SP}\right)\left(P_t^N\right)^{SP} Y_t^N.$$

Thus, the triplet $\left(\tau^D, \tau^T, T^D\right)^{SP}$, with T^C satisfying the government budget constraint, will be sufficient to replicate the SP allocation when the constraint binds.

The SP problem defined above is recursive. Therefore, the tax scheme $\{\tau^T, \tau^D, T^D, T^C\}^{SP}$ that decentralizes the SP allocation is time-consistent. **QED**.

A.2 Alternative Set of Taxes for SP Implementation

Another way to decentralize the SP allocation is to use the following set of distortionary taxes: a tax on tradable consumption, a tax on nontradable consumption, a tax on new debt, a tax on labor income, and a tax on tradable output; the government budget constraint is assumed to be satisfied via a lump-sum tax/transfer. In this world we have the following conditions for a competitive equilibrium:

$$u_{1,t} - \left(1+\tau_t^T\right)\mu_t = 0$$

$$u_{3,t} + \left(1-\tau_t^H + \frac{\max\{\lambda_t, 0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\left(1-\tau_t^D\right)\mu_t\left(1-\alpha_T\right)A_t^T\left(H_t^T\right)^{-\alpha_T} = 0$$

$$u_{3,t} + \left(1-\tau_t^H + \frac{\max\{\lambda_t, 0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\frac{u_{2,t}}{1+\tau_t^N}\left(1-\alpha_N\right)A^N\left(H_t^N\right)^{-\alpha_N} = 0$$

$$\max\{-\lambda_t, 0\}^2 - \left(B_{t+1} + \frac{1-\phi}{\phi}\left(\left(1-\tau_t^D\right)A_t^T\left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t}}{\mu_t\left(1+\tau_{Nt}\right)}A^N\left(H_t^N\right)^{1-\alpha_N}\right)\right) = 0$$

$$\beta\left(1+r\right)E_t\left[\mu_{t+1}\right] - \left(1-\tau_t^B\right)\mu_t + \max\{\lambda_t, 0\}^2 = 0.$$

The first equation determines τ_t^T

$$\tau_t^T = \frac{\mu_{1t} - \mu_t - \max\{\lambda_t, 0\}^2\frac{1-\phi}{\phi}\frac{1}{\kappa}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1-\kappa}{\kappa}}}{\mu_t}$$

the last determines τ_t^B

$$\tau_t^B = \frac{\beta\left(1+r\right)E_t\left[\mu_{1,t+1} - \mu_{t+1}\right] - \left(\mu_{1,t} - \mu_t\right)}{\mu_t},$$

and the middle three jointly determine $\left(\tau_t^H, \tau_t^N, \tau_t^D\right)$:

$$\left(1 + \frac{\max\{\lambda_t, 0\}^2}{\mu_{1t}}\frac{1-\phi}{\phi}\right)\mu_{1t} = \left(1-\tau_t^H + \frac{\max\{\lambda_t, 0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\left(1-\tau_t^D\right)\mu_t \tag{46}$$

$$1 + \frac{\max\{\lambda_t, 0\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{47}$$

$$= \left(\frac{1-\tau_t^H}{1+\tau_t^N} + \frac{\max\{\lambda_t, 0\}^2}{\left(1+\tau_t^N\right)\mu_t}\frac{1-\phi}{\phi}\right)$$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{48}$$

$$= -\tau_t^D A_t^T\left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t}A^N\left(H_t^N\right)^{1-\alpha_N}}{\mu_t\left(1+\tau_t^N\right)}.$$

Since $\mu_{1,t} = \mu_t$ (the marginal value of wealth is equalized) the expressions simplify:

$$\tau_t^T = \frac{-\max\left\{\lambda_t, 0\right\}^2}{\mu_{1,t}} \frac{1-\phi}{\phi} \frac{1}{\kappa} \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}} \left(C_t^T\right)^{\frac{1-\kappa}{\kappa}} \tag{49}$$

$$\tau_t^B = 0 \tag{50}$$

$$\tau_t^D = \frac{-\tau_t^H}{1-\tau_t^H + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}} \tag{51}$$

$$1 + \frac{\max\left\{\lambda_t, 0\right\}^2}{u_{2,t}} \frac{1-\phi}{\phi} \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}} \tag{52}$$

$$= \left(\frac{1-\tau_t^H}{1+\tau_t^N} + \frac{\max\left\{\lambda_t, 0\right\}^2}{\left(1+\tau_t^N\right)\mu_t} \frac{1-\phi}{\phi}\right)$$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}} \tag{53}$$

$$= -\tau_t^D A_t^T \left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t} A^N \left(H_t^N\right)^{1-\alpha_N}}{\mu_t \left(1+\tau_t^N\right)}.$$

A lump-sum tax/transfer can then be used to balance the government budget constraint. Note that the third and fourth equations can be used to define $\left(\tau_t^D, \tau_t^N\right)$ entirely in terms of τ_t^H, so that solving the system of equations can be reduced to solving one nonlinear equation in τ_t^H:

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}} = \frac{\tau_t^H A_t^T \left(H_t^T\right)^{1-\alpha_T}}{1-\tau_t^H + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}} +$$
$$\frac{u_{2,t} A^N \left(H_t^N\right)^{1-\alpha_N} \left(1-\tau_t^H + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)}{\mu_t \left(1 + \frac{\max\{\lambda_t,0\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}}\right)}.$$

The LHS of this equation is constant in terms of τ_t^H and the first term on the RHS is strictly increasing in τ_t^H, but the second term is strictly decreasing so no guarantee of uniqueness can be obtained. However, at least one $\tau_t^H < 1$ exists that solves this equation, as the LHS is strictly positive and the RHS can be made both negative (setting $\tau_t^H < 0$) and arbitrarily positive (setting τ_t^H close to 1).

A.3 Proof of Proposition 2

Proposition 2. *Given the set of taxes* $(\tau^T, \tau^D, T^D, T^C)$ *to the Ramsey planner as in definition (4) above, there exists an optimal time-consistent tax scheme that replicates the UE allocation.*

Proof. To see this, we will first find the tax scheme for the assigned set of tools that implements the UE allocation. We then prove this is Ramsey optimal and time-consistent. Focus first on the tradable good tax, τ_t^T, as a policy tool that can undo the borrowing constraint. Let τ_t^T be such that $P_t^{N,CE} = (1+\tau_t^T)P_t^{N,UE}$, with $T_t^D = \tau_t^D P_t^{N,CE} C_t^N$, so that the borrowing constraint is not binding

$$B_{t+1}^{UE} + \frac{1-\phi}{\phi}\left[A_t^T (H_t^{T,UE})^{1-\alpha^T} + (1+\tau_t^T)P_t^{N,UE}C_t^{N,UE}\right] > 0.$$

However, since τ_t^T affects also the intertemporal allocation of resources (33), we need to find a constant τ_t^T such that the intertemporal margin is not distorted.

To do so, we first note that, by setting $\lambda_t^{CE} \equiv 0$ and τ_t^T so that

$$\frac{1}{1+\tau_t^T} = \frac{\beta(1+r)E_t\left[\frac{u'(C_{t+1}^{UN})C_{C_{t+1}^T}^{UN}}{1+\tau_{t+1}^T}\right]}{E_t[u'(C_{t+1}^{UN})C_{C_{t+1}^T}^{UN}]}, \tag{54}$$

the Euler equations of the Ramsey problem and the unconstrained equilibrium coincide. It follows that the tax rate τ_t^T that satisfies (33) must be constant, otherwise the intertemporal margin would be distorted. Note now that, by inspection of the UE allocation, the non-tradable price has a strictly positive lower limit. Therefore, there exists a lower bound of τ^T, $\underline{\tau}^T$, compatible with the strictly positive lower limit on the relative price of non-tradables, such that the borrowing constraint (18) is always satisfied for any $\tau^T \geqslant \underline{\tau}^T$. Thus, any constant tax policy of the form $\tau_t^T \equiv \tau^T \geqslant \underline{\tau}^T$ can be part of the tax schedule replicating the UE allocation.

Now, if the borrowing constraint is not binding, $\lambda_t^{CE} = 0$, so that all the other equilibrium conditions will be identical to those in the UE allocation, except for the one determining the labor demand in the non-tradable sector, which is affected by P_t^N. Indeed we have that:

$$\begin{aligned} W_t^{CE} &= (1-\tau_t^D)\left(1-\alpha^N\right) P_t^{N,CE} A_t^N \left(H_t^{N,CE}\right)^{-\alpha^N} \\ W_t^{UE} &= (1-\alpha^N) P_t^{N,UE} A_t^N (H_t^{N,UE})^{-\alpha^N}. \end{aligned}$$

Since $P_t^{N,CE} = (1+\tau_t^T)P_t^{N,UE}$, if we set τ_t^D such that

$$(1-\tau_t^D) = \frac{1}{(1+\tau_t^T)},$$

we will have $W_t^{CE} = W_t^{UE}$ because, when evaluated at the unconstrained equilibrium, the two taxes cancel each other. Given that $T_t^D = \tau_t^D P_t^{N,UE} C_t^N$, the government budget constraint can always clear by using T_t^C so that

$$T_t^C = \tau_t^T C_t^{T,UE} + \tau_t^D P_t^{N,UE} Y_t^{N,UE} - T_t^D.$$

Since the labor demand in the non-tradeable sector is the only condition indirectly distorted by τ_t^T, the tax scheme above for $\tau_t^T, \tau_t^D, T_t^D, T_t^C$ achieves the UE allocation.

Note now that, given the assigned policy tools, the Ramsey planner has no incentive to deviate from the tax scheme above at *any* point in time. In fact, all welfare gains that can be reached have been achieved because there are no additional instruments that can be used to complete the markets: τ_t^T is used to undo the constraint, while all other available tools are used to undo the distortions created by τ_t^T. Therefore, the tax scheme above for $\tau_t^T, \tau_t^D, T_t^D, T_t^C$ is Ramsey optimal and also time consistent. **QED**