Optimal Policy for Macro-Financial Stability*

Gianluca Benigno
Federal Reserve Bank of New York
LSE and CEPR

Huigang Chen
Uber Technologies Inc.

Christopher Otrok
University of Missouri
Federal Reserve Bank of St. Louis

Alessandro Rebucci
Johns Hopkins University
CEPR and NBER

Eric R. Young
University of Virginia

This Draft: August 6, 2019

Abstract There is a recent and now large literature analyzing government policies for financial stability based on models with endogenous borrowing constraints. The normative analysis of these models builds upon the concept of constrained efficient allocation in which the social planner faces the same borrowing constraint as individual agents. In this paper, we show that the set of policy tools that implement the constrained efficient allocation can be used by a Ramsey planner to achieve higher welfare and replicate the unconstrained allocation. Thus the constrained social planner approach may lead to an inaccurate characterization of welfare-maximizing policies relative to the Ramsey approach for the same policy tool. We illustrate this point in a well-known model environment and show that the drawback of the social planner approach arises because of the policy instrument's ability to affect the collateral value in the borrowing constraint.

JEL Classification: E61, F38, F44, H23

Keywords: Constrained efficiency, Financial crises, Macroprudential policies and capital controls, Pecuniary externalities, Ramsey optimal policy, Social planner.

*This paper is a substantially revised version of Benigno, Chen, Otrok, Rebucci and Young (2012). Earlier versions of this paper also circulated under the title "Optimal Policy with Occasionally Binding Credit Constraints" and "Optimal Stabilization Policy in a Model with Endogenous Sudden Stops". We thank numerous seminar and workshop participants and discussants, as well as Javier Bianchi, Anton Korinek, Enrique Mendoza, and Olivier Jeanne for comments and numerous discussions. The views expressed in this paper are exclusively those of the authors and not those of the institutions with which they are affiliated.

1 Introduction

The recent global financial crisis has stimulated research on macroeconomic models with financial frictions. One of the most popular approaches builds upon the literature on sudden stops initially developed in the seminal work of Mendoza (2002, 2010), where financial frictions take the form of occasionally binding borrowing constraints. The virtue of this approach is its ability to distinguish between normal states of the economy (when the constraint is not binding) and crisis states (when the constraint binds). In particular, the amount of borrowing that agents are allowed to undertake depends on the value of the collateral that is determined endogenously by a key market price that enters into the borrowing constraint. For example, in Mendoza (2002), this key market price is the real exchange rate while in Mendoza (2010) the key market price is the price of physical capital. The presence of a market price in the collateral constraint creates a mechanism of financial amplification known as debt-deflation spiral or fire asset sale which is consistent with empirical features of sudden stop episodes. From a normative point of view, the presence of a relative market price in the borrowing constraint generates scope for policy intervention because agents do not take into account the effects that their choices have on the market price when the collateral constraint binds. This distortion is usually referred to as pecuniary externality and several papers have studied the inefficiency associated with it and the corresponding scope for policy intervention.[1]

The paper aims at strengthening the foundations of the normative analysis of this class of models. In many papers, the normative analysis builds upon the concept of *constrained efficient allocation*. The constrained efficient allocation is defined as a social planner problem in which the planner is constrained by the resource and technological constraints along with the borrowing constraint. The analysis then proceeds by characterizing the constrained

[1]See for instance, among others, Lorenzoni (2008), Bianchi (2011), Benigno, Chen, Otrok, Rebucci and Young (2011, 2013 and 2016), Bianchi and Mendoza (2018), Korinek (2018), and Jeanne and Korinek (2018).

efficient allocation and by discussing how to implement it from a decentralized perspective. In this paper, we highlight a drawback of this approach. Indeed, we show that, in our model economy, the set of tools that implements the constrained efficient allocation from a decentralized perspective can also be used *optimally* by a Ramsey planner to replicate the allocation that would arise if the borrowing constraint were not present, i.e. the unconstrained allocation. Our analysis relies on a standard Ramsey planner approach. In particular, we show that, when we allow the Ramsey planner to maximize welfare with a set of instruments that implement the constrained efficient allocation, she/he can attain the unconstrained allocation, thereby reaching a much higher level of welfare than in the constrained efficient allocation. This result shows that a standard Ramsey optimal policy approach is more robust than the social planner approach typically used in the pecuniary externality literature because it attains all the welfare gains that are within reach of the policy instruments selected.

In our analysis, we focus on a production version of the endowment economy described in Mendoza (2002), Korinek (2018), Bianchi (2011), and Benigno et al. (2016), as in Benigno et al. (2013). This is a a two-sector small open economy that produces traded and non-traded goods and in which agents have limited access to international capital market. From the perspective of the small open economy, foreign borrowing is denominated in units of the tradable good, but it is leveraged on income generated in both sectors. Thus, the relative price of non-tradeable good (which is typically interpreted as the real exchange rate in these models) affects the value of non-tradable income and hence the collateral value.[2]

We first show that, in the model economy, one particular tax scheme that implements the constrained efficient allocation relies on the use of a tax on tradable consumption and a tax on firm non-tradable revenues along with lump-sum transfers to firms and the household.[3] Moreover, we find that this combination of taxes is used only when the constraint

[2]This is a specification of the borrowing constraint that captures "balance sheet effects", a key feature of the capital structure of many emerging and advanced economies alike—e.g., Chang, Cespedes, and Velasco (2004) and Aghion, Bacchetta and Banerjee (2004).

[3]This tax scheme is neutral in the sense that is does not redistribute resources between households and

binds so that there is no need for precautionary policy to replicate the constrained efficient allocation.[4] When the constraint binds, private agents cannot borrow the desired amount, movements in the relative price of non-tradable are inefficient, and resources are not allocated efficiently between the two sectors of the economy. The tax on tradable consumption mitigates the effect of the pecuniary externality on the borrowing decisions of the agents, while the tax on non-tradable profits reallocate resources between sectors as needed to restore efficiency.

Next, we show that the same set of taxes can be used *optimally* by a Ramsey planner to achieve the unconstrained equilibrium, that is the allocation in which the borrowing constraint never binds. In this case, the tax on tradable consumption supports the relative price of non-tradable consumption in such a way that the borrowing constraint never binds in equilibrium, while the tax on firm profits ensures that, again, resource are allocated efficiently across sectors. This result means that a Ramsey optimal policy approach is more robust than the social planner approach often used in the literature because it attains all the welfare gains that are within reach of the policy instruments selected.

While we derive our results in the context of a particular model economy, we emphasize its generality. The same considerations arise in variations of our setting, including in alternative frameworks in which the relative price of collateral is an asset price or in environments in which the borrowing constraint depends on the value of the collateral at the time of the repayment.[5] Indeed the incentive from a Ramsey planner point of view, conditional on the set of policy tools assigned, is to undo the effects of the borrowing constraint and to balance this incentive with the distortions created by the use of the policy tools.

Our paper is closely related to several strands of the literature. In the literature on

firms.

[4]We label as precautionary a policy that would be put in place in states of the world in which the constraint is not binding.

[5]Specifying the borrowing constraint so that the amount that agents can borrow depends on the value of the collateral at the time of repayment complicates substantially the analysis since time-consistency considerations would arise. We abstract from these issues to present our results in the simplest possible environment.

pecuniary externality in models with endogenous borrowing constraint, the most prominent examples of contributions that adopt the normative criterion of constrained efficiency are Lorenzoni (2008), Bianchi (2011), Benigno et al (2013), Bianchi and Mendoza (2018), Davila (2014), Davila and Korinek (2018), Di Tella (2018), and Jeanne and Korinek (2018). In the model of Lorenzoni (2008), agents are risk neutral and the asset price does not depend on the marginal utility of consumption so that it is not possible to influence the asset price when the constraint is binding, thereby restricting the palnner's ability to intervene only ex-ante.

In Benigno et al. (2013) analyze the social planner problem of the same production economy studied in this paper, but do not discuss the implementation of the allocation in a decentralized equilibrium using government taxes and subsidies. In Benigno et al. (2016), we show how, in the endowment version of the model economy analyzed in this paper, the social planner problem is equivalent to a Ramsey policy problem with a tax on debt and lump-sum transfers as in Bianchi (2011) and Korinek (2018). In the case of an endowment economy analyzed in these latter papers, the tax on debt does not affect the relative price that enters the collateral so that, when the constraint binds, as long as there are lump-sum transfers, the solution of the constrained efficient planning problem is equivalent to Ramsey policy problem with a tax on debt as instrument. Similarly, in Bianchi and Mendoza (2018), in an economy in which the collateral constraint is expressed in terms of the value of an asset which is in fixed supply and under lump-sum transfers and a tax on debt, the Ramsey planner problem is equivalent to the constrained social planner allocation. [6]

Davila (2014) and Davila and Korinek (2017) acknowledge that the normative analysis of models with credit constraints that depend on endogenously determined asset prices hinges on the specific set up of the normative benchmark, but do not compare the constrained effi-

[6]This is because the tax on debt enters only in the Euler equation which is not imposed as a constraint in the social planner problem. Indeed, in that set up, once the optimal allocation is characterized, it is possible to use the Euler equation to retrieve the state-contingent tax on debt that implements the constrained social planner allocation and, at the same time, maximize the utility of the Ramsey planner that has the same instruments—See Bianchi and Mendoza (2018), Proposition 1.

cient allocations with Ramsey allocations and focus on the availability of ex-ante or ex-post transfers. [7] Di Tella (2018), in an environment with informational frictions, considers a constrained social planner problem where the planner is constrained by the same informational friction that private agents face. In his case, though, as the nature of the inefficiency differs from the aforementioned papers, the scope of the social planner is to allocate risk efficiently among consumers and intermediaries.

Kurlat (2018) also focuses on a constrained social planner problem in which the planner faces the same informational limitations as private agents. He incorporates his model of fire sales into a simplified version of Lorenzoni's (2008) economy and conducts a similar normative analysis finding that, in an environment with informational frictions, the normative conclusions on fire sales are reversed.

In a related strand of the literature Cespedes, Chang and Velasco (2017) compare the transmission mechanism of alternative policy interventions in a model with an occasionally binding borrowing constraint, but don't discuss the implementation of the constrained efficient allocation or the computation of the optimal policy in their set up. Other important connected contributions that analyze financial frictions in infinite horizon macroeconomic models from a positive perspective as in the seminal contributions of Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999) are Iacoviello (2005), Curdia and Woodford (2010), Gertler and Karadi (2011), Gertler and Kiyotaki (2011) and Brunnermeier and Sannikov (2014, 2015).

Our paper is naturally linked to the literature in which the occasionally binding constraint is the zero-lower-bound on interest rates. For example, Adams and Billi (2006 and 2007) study optimal monetary policy in a closed economy, New-Keynesian model in which there is zero-lower-bound on interest rates that binds only occasionally. The key difference here, with respect to a sudden stop setting as in Mendoza (2002, 2010) is that

[7] Jeanne and Korinek (2008) on the other hand focus on the optimal policy of ex-ante and ex-post policy tools in a model similar to Lorenzoni (2008) by solving a Ramsey problem both ex-ante and ex-post and allowing for different policy tools depending on the state (ex-ante or ex-post).

the zero-bound constraint is fixed and does not evolve endogenously or depend on market prices.

Finally, our result on the ability of a given set of policy tools to undo the underlying distortions in the economy is reminiscent of the results in the paper by Correia, Nicolini and Teles (2008), in which the role of price stickiness for the design of monetary policy depends on the existence of alternative fiscal policy tools.

The rest of the paper is organized as follows. Section 2 describes the model we use and its competitive equilibrium. Section 3 sets up the social planner problem and discusses its implementation with an unrestricted set of instruments. Section 4 analyzes the Ramsey planner problem. Section 5 concludes.

2 The Model and Its Competitive Equilibrium

In this section, we describe our model set-up and discuss its key assumptions. The model is a two-sector (tradable and non-tradable) production small open economy, in which financial markets are not only incomplete but also imperfect as in Mendoza (2002, 2010) and Benigno et al (2013).

2.1 Households

There is a continuum of households $j \in [0,1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t^j - \frac{\left(H_t^j\right)^{\delta}}{\delta} \right)^{1-\rho} \right\}, \tag{1}$$

with C^j denoting the individual consumption basket and H^j the individual supply of labor for the tradable and non-tradable sectors ($H = H^T + H^N$). The assumption of perfect substitutability between labor services in the two sectors ensures that there is a unique labor market. For simplicity we omit the j subscript for the remainder of this section, but it is understood that all choices are made at the individual level. The elasticity of labor supply is δ, while ρ is the coefficient of relative risk aversion. In (1), the preference specification follows from Greenwood, Hercowitz and Huffman (GHH, 1988).[8] In what follows, we will assume that β, the subjective discount factor is such that $\beta < \frac{1}{1+i}$, where i is real return on saving between period t and $t+1$, so that agents in this economy are relatively impatient.

The consumption basket, C_t, is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}. \tag{2}$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of tradable goods in the consumption basket. We normalize the price of tradable goods to 1. The relative price of the nontradable goods is denoted by P^N. The aggregate price index is then given by

$$P_t = \left[\omega + (1-\omega)\left(P_t^N\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}},$$

where we note that there is a one-to-one link between the aggregate price index P and the relative price P^N.

Households maximize utility subject to their budget constraint, which is expressed in units of tradable consumption. The constraint each household faces is:

$$C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i) B_t, \tag{3}$$

where W_t is the wage in units of tradable goods, $B_{t+1} < 0$ denotes the debt position at the end of period t with gross real return $1+i$. Households receive profits, π_t, from firms'

[8] In a one-good economy this specification eliminates the wealth effect from the labor supply choice. In a multi-goods economy, however, the sectoral allocation of consumption will affect the labor supply decision through relative prices (See Benigno et al., 2013 for more details).

ownership. Their labor income is given by $W_t H_t$.

International financial markets are incomplete, and access to them is imperfect. The asset menu includes only a one-period bond denominated in units of tradable consumption. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current *total* income:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\pi_t + W_t H_t\right]. \tag{4}$$

A few remarks are in order here on the working of this constraint.[9] First, note that the value of the collateral is endogenous and depends on the current realization of profit and wage income. As in the literature on sudden stop (e.g., Mendoza, 2010), a crisis event is identified with the state in which the constraint binds. Second, note that this constraint captures a balance sheet effect–e.g., Chang, Cespedes, and Velasco (2004) and Aghion, Bacchetta and Banerjee (2004) since foreign borrowing is denominated in units of tradables while the income that can be pledged as collateral is generated also in the non-tradable sector. Third, unlike Mendoza (2010), we abstract from imposing a working capital requirement for simplicity as our analysis is analytical rather than quantitative.[10]

Households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first-order conditions of this problem are the following:

$$C_T : \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE} \tag{5}$$

$$C_N : \left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE} P_t^N \tag{6}$$

[9] For a discussion of the nature of the credit constraint we refer to Mendoza (2010), Bianchi (2011), Benigno et al. (2013) and Jeanne and Korinek (2011).

[10] Similarly, a constraint expressed in terms of future income, which could be the outcome of the interaction between lenders and borrowers in a limited commitment environment, would introduce further computational difficulties that we need to avoid for tractability, since future consumption choices would affect current borrowing decisions.

$$B_{t+1} : \mu_t^{CE} = \lambda_t^{CE} + \beta\left(1+i\right) E_t\left[\mu_{t+1}^{CE}\right], \tag{7}$$

and

$$H_t : \left(C_t - \frac{H_t^{\delta}}{\delta}\right)^{-\rho} \left(H_t^{\delta-1}\right) = \mu_t^{CE} W_t + \frac{1-\phi}{\phi} W_t \lambda_t^{CE}. \tag{8}$$

where μ_t^{CE} is the multiplier on the period budget constraint, and λ_t^{CE} is the multiplier on the international borrowing constraint. The presence of the borrowing constraint distorts directly two margins: the intertemporal margin, as the Euler equation (7) includes a term ($\lambda_t^{CE} > 0$) when the constraint binds, and the labor supply choice (8) since when the constraint binds agents are willing to supply additional units of labor. Note here for future reference that we can combine (5) and (6) to obtain the intratemporal allocation of consumption, and (5) with (8) to obtain the labor supply schedule, and summarize the first order conditions of the household as:

$$P_t^N = \frac{\left(1-\omega\right)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}}} \tag{9}$$

$$\left(H_{j,t}^{\delta-1}\right) = \left(\frac{\omega C}{C^T}\right)^{\frac{1}{\kappa}} W_t \left(1 + \frac{1-\phi}{\phi}\frac{\lambda_t^{CE}}{\mu_t^{CE}}\right). \tag{10}$$

2.2 Firms

We now turn to the production side of our economy. Firms produce tradable and non-tradable goods with a variable labor input and the following decreasing return to scale technologies:

$$\begin{aligned} Y_t^N &= A_t^N H_t^{1-\alpha^N}, \\ Y_t^T &= A_t^T H_t^{1-\alpha^T}, \end{aligned} \tag{11}$$

where A^N and A^T are the productivity levels, which are assumed to be random variables, in the non-tradable and tradable sector, respectively. The firm's problem is static and current-period profits (π_t) are:

$$\pi_t = A_t^T \left(H_t^T\right)^{1-\alpha^T} + P_t^N A_t^N \left(H_t^N\right)^{1-\alpha^N} - W_t H_t. \tag{12}$$

The first-order conditions for labor demand in the two sectors are given by:

$$W_t = \left(1-\alpha^N\right) P_t^N A_t^N \left(H_t^N\right)^{-\alpha^N}, \tag{13}$$

$$W_t = \left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}, \tag{14}$$

where the value of the sector, marginal product of labor equals the wage in units of tradable goods (W_t). By taking the ratio of (13) over (14) we obtain:

$$P_t^N = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}}{\left(1-\alpha^N\right) A_t^N \left(H_t^N\right)^{-\alpha^N}}. \tag{15}$$

From this last expression we can see that the relative price of non-tradable goods determines the allocation of labor between the two sectors: for given productivity levels, a decrease (increase) in P_t^N drives down (up) the marginal product of non-tradables and induces a shift of labor toward (out of) the tradable sector.

2.3 Competitive Equilibrium

To determine the goods market equilibrium, combine the household budget constraint, the firm's profits, the equilibrium condition in the nontradable good market to obtain the current account equation of our economy:

$$B_{t+1} = (1+i) B_t + A_t^T H_t^{1-\alpha^T} - C_t^T. \tag{16}$$

The nontradable goods market equilibrium condition implies that

$$A_t^N \left(H_t^N\right)^{1-\alpha^N} = Y_t^N = C_t^N. \tag{17}$$

Finally, using the definitions of firms' profits and wages, the credit constraint implies that the amount that the country, as a whole, can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N\right], \tag{18}$$

so that (16) and (18) determines the evolution of the foreign borrowing.

The competitive equilibrium allocation is then characterized by (16), (17) and (18) along with the first order conditions for the household (5), (7), (9) and (10), the firms (15), and the following complementary slackness condition:

$$\left(B_{t+1} + \frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N\right]\right)\lambda_t^{CE} = 0 \text{ with } \lambda_t^{CE} > 0.$$

2.4 Unconstrained Equilibrium

We will see below that, for a Ramsey planner endowed with a certain combination of policy instruments, it is possible to achieve an allocation that is identical to the competitive equilibrium of the economy without the borrowing constraint (18). We will refer to this allocation as the "unconstrained equilibrium" (denoted with UE superscript) and define it formally as follows.

Definition 1 *The unconstrained allocation of our economy is defined as a decentralized equilibrium in which households maximize (1) subject to (2) and (3), and firms maximize profits (12) subject to (11).*

This equilibrium characterizes an allocation in which financial markets are only incomplete, so that there are inefficient variations in consumption due to the lack of state contingent debt. We also point out that, since agents are assumed to be impatient, in the deterministic steady state of the UE equilibrium, the allocation will converge to the natural debt limit.[11] In our stochastic economy without borrowing constraint, however, agents engage in precautionary saving so that the probability of hitting the natural debt limit is zero. In what follows, we assume the existence of a lower bound on debt which is strictly greater than the natural debt limit to guarantee that the competitive equilibrium allocation without credit constraint has an ergodic distribution of debt with finite support under the assumption that $\beta(1+i) < 1$.

For future reference, finally, note that we can rewrite the UE first order conditions of the household (7), (10), (9) as

$$\left(C_t^{UE} - \frac{\left(H_t^{UE}\right)^{\delta}}{\delta}\right)^{-\rho} \left(\frac{\omega C_t^{UE}}{C_t^{T,UE}}\right)^{\frac{1}{\kappa}} = \mu_t^{UE} \tag{19}$$

$$B_{t+1} : \mu_t^{UE} = \beta\left(1+i\right) E_t\left[\mu_{t+1}^{UE}\right], \tag{20}$$

$$P_t^{N,UE} = \left(\frac{1-\omega}{\omega}\frac{C_t^{T,UE}}{C_t^{N,UE}}\right)^{\frac{1}{\kappa}}, \tag{21}$$

$$(H_t^{UE})^{\delta-1} = \left(\frac{\omega C_t^{UE}}{C_t^{T,UE}}\right)^{\frac{1}{\kappa}} W_t^{UE}; \tag{22}$$

while the firms' maximization problem is

$$P_t^{N,UE} = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^{T,UE}\right)^{-\alpha^T}}{\left(1-\alpha^N\right) A_t^N \left(H_t^{N,UE}\right)^{-\alpha^N}}, \tag{23}$$

$$W_t^{UE} = (1-\alpha^N) P_t^{N,UE} A_t^N (H_t^{N,UE})^{-\alpha^N}. \tag{24}$$

[11] In our model, this level equals (minus) the annuity value of the lowest tradable endowment value.

3 Constrained Efficiency

The constrained efficient allocation (also called social planner allocation) is typically used in the literature as a benchmark to conduct the normative analysis in this class of models. In this section we follow this conventional approach (e.g., Lorenzoni, 2008). We first review the characterization of the constrained social planner allocation of this production economy (e.g., Benigno et al. 2013), then show how this allocation can be implemented from a competitive equilibrium point of view, which is one of the contribution of this paper.

3.1 Definition and Analysis

The allocation is constrained in the sense that the planner faces the same borrowing constraint that the private agents do, but from an aggregate country-wide perspective. It is also important to emphasize here, for clarity, that this planner *does not* use any policy instrument, but simply allocates resources efficiently. The key aspect of the constrained social planner problem is that the price that enters the borrowing constraint is determined by the *pricing rule* as in the competitive equilibrium allocation (see (9)).

Definition 2 (Social Planner Problem) *The planner chooses the optimal path of C_t^T, C_t^N, B_{t+1}, H_t^T, and H_t^N by maximizing (1) subject to the resource constraints (16) and (17), the international borrowing constraint from an aggregate perspective (18), and the pricing rule of the competitive equilibrium allocation (9).*

The key difference relative to the competitive equilibrium is that the planner takes into account the effects of its decisions on market prices, and hence internalizes the pecuniary externality arising from the presence of the relative price of non-traded goods in the borrowing constraint. To see this, we first rewrite (18) as

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[A_t^T\left(H_t^T\right)^{1-\alpha^T}+\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{1-\frac{1}{\kappa}}\right], \tag{25}$$

in which we substituted the production function and (9). The first-order conditions for the constrained social planner's problem (SP) are given by

$$C_T : \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}\left(\frac{\omega C}{C^T}\right)^{\frac{1}{\kappa}} = \mu_{1,t}^{SP} + \tag{26}$$

$$-\frac{\lambda_t^{SP}}{\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega}\right)^{\frac{1-\kappa}{\kappa}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{\frac{\kappa-1}{\kappa}},$$

$$C_N : \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}C^{\frac{1}{\kappa}} = \mu_{2,t}^{SP}, \tag{27}$$

$$B_{t+1} : \mu_{1,t}^{SP} = \lambda_t^{SP} + \beta(1+i)E_t\left[\mu_{1,t+1}^{SP}\right], \tag{28}$$

$$H_t^T : \left(C_t - \frac{H_t^{\delta}}{\delta}\right)^{-\rho}\left(H_t^{\delta-1}\right) = \left(1-\alpha^T\right)\mu_{1,t}^{SP}A_t^T H_t^{-\alpha^T} + \frac{1-\phi}{\phi}\lambda_t^{SP}\left(1-\alpha^T\right)A_t^T H_t^{-\alpha^T}, \tag{29}$$

and

$$H_t^N : \left(C_t - \frac{H_t^{\delta}}{\delta}\right)^{-\rho}\left(H_t^{\delta-1}\right) = \left(1-\alpha^N\right)\mu_{2,t}^{SP}A_t\left(H_t^N\right)^{-\alpha^N} \tag{30}$$

$$+\frac{1-\phi}{\phi}\lambda_t^{SP}\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\left(1-\alpha^N\right)\left(A_t^N\right)^{\frac{\kappa-1}{\kappa}}\left(H_t^N\right)^{\left(1-\alpha^N\right)\frac{\kappa-1}{\kappa}-1},$$

where $\mu_{1,t}^{SP}$ is the Lagrange multiplier on (16), $\mu_{2,t}^{SP}$ is the Lagrange multiplier on (17) and λ_t^{SP} is the multiplier on (25).

Benigno et al. (2013) provide an extensive discussion on the comparison between these first order conditions and those in the competitive equilibrium. Here we note only that in equation (26), in choosing tradable consumption, the planner takes into account the effects that this choice has on the value of the collateral. This is the effect that is usually referred to in the literature as the "pecuniary externality".

3.2 Implementation with an Unrestricted Set of Instruments

We now discuss how to decentralize the social planner allocation with an unrestricted set of policy instruments. By this we mean that the planner can choose freely from the menu of all policy tools available in this economy. Thus, before proceeding, we need to discuss the menu of available taxes that can be imposed in this economy, the government budget, and how these taxes modify the individual budget and the borrowing constraint, as well as the firm's profit.

In this economy we can consider the following set of taxes: a tax (or subsidy) on non-tradable and tradable consumption, which we denote with τ_t^N and $\tau_t^T > 0(< 0)$, respectively; a tax (subsidy) on the amount that households borrow, denoted $\tau_t^B > 0(< 0)$; a tax on wage income, denoted with $\tau^H > 0$; as well as lump-sum taxes (transfers) to the consumer, denoted $T_t^C > 0(< 0)$. On the production side of the economy, one can allow for sector taxes on firms revenues rebated in lump-sum manner. For instance, we will consider a distortionary tax (subsidy) on non-tradable revenue, $\tau_t^D > 0(< 0)$ rebated with a lump-sum transfer to the firm, T_t^D.[12]

This is an exhaustive list of taxes that can be considered in our model economy. Assuming that the government balances the budget period by period, its constraint is given by:

$$\tau_t^B B_{t+1} + \tau^T C_t^T + \tau_t^N P_t^N C_t^N + \tau_t^D P_t^N Y_t^N + \tau^H W_t H_t = T_t^C + T_t^D. \quad (31)$$

The household budget constraint, becomes:

$$(1+\tau^T)C_t^T + (1+\tau^N)P_t^N C_t^N = \pi_t + (1-\tau^H)W_t H_t - (1-\tau^B)B_{t+1} + (1+i)\, B_t + T_t^C, \quad (32)$$

[12] A distortionary tax on tradable revenue could also be considered, but is equivalent to the distortionary tax on non-tradeable revenue.

while the individual borrowing constraint becomes

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\pi_t + (1-\tau^H)W_t H_t\right]. \tag{33}$$

And the firm's profit is:

$$\pi_t = A_t^T \left(H_t^T\right)^{1-\alpha^T} + (1-\tau_t^D)P_t^N A_t^N \left(H_t^N\right)^{1-\alpha^N} - W_t H_t - T_t^D. \tag{34}$$

It is important to note here that τ_t^N and τ_t^T affect the determination of the relative price directly, regardless of whether the constraint binds or not. For example, for given consumption of tradables and non-tradables, a decrease of τ_t^N implies an increase in the relative price of non-tradable goods, and hence a higher value of the collateral in units of tradable consumption. Here, note also that the tax on tradable consumption, τ_t^T, works similarly to the tax on nontradable consumption, τ_t^N in terms of its effects on the relative price of non-tradeable goods; the main difference is that τ_t^T also influences the intertemporal path of tradeable consumption.

The channel through which τ_t^B works depends on the constraint being binding or not. When the constraint is not binding, τ^B reduces the amount that agents borrow. When the constraint binds, since the amount of borrowing is determined by the endogenous limit, τ^B affects the value of the Lagrange multiplier associated with the constraint, λ_t^{CE}. This can be seen by re-arranging the first order condition for debt as:

$$\lambda_t^{CE} = (1-\tau_t^B)\mu_t^{CE} - \beta(1+i)E_t\left[\mu_{t+1}^{CE}\right] > 0.$$

For example, for given future marginal utility of tradeable consumption ($E_t\left[\mu_{t+1}^{CE}\right]$), an increase in τ_t^B will tend to decrease the value of the multiplier, λ_t^{CE}, and the amount of labor that is supplied in crisis times for given real wages (10).

The following proposition characterizes how to decentralize the social planner allocation with an unrestricted set of policy instruments.

Proposition 1 (Implementation of the Constrained Efficient Allocation). *Given the following set of available taxes (τ^B, τ^N, τ^T, τ^D, τ^H, T^D, T^C), there exists a combination of policy rules for the subset of instruments (τ^T, τ^D, T^D ,T^C) (for brevity, a tax scheme for this subset of instruments) that implements the constrained social planner allocation in the competitive equilibrium of our economy. This tax scheme is time consistent.*

Proof. See appendix.

Several remarks are in order. First this proposition shows the existence of one possible set of instruments that implements the constrained social planner allocation, but this tax scheme is not unique. As we show in appendix, is possible to find other combinations of the available instruments that could replicate the same allocation. As we will discuss below, we focus on this particular tax scheme to highlight the problems that can arise in using this efficiency benchmark for the normative analysis of pecuniary externalities in models with borrowing constraints that depend on market prices.

Second, note that the tax on borrowing, τ^B, is not needed to implement the constrained social planner allocation. This highlights the importance of motivating the choice of the instruments given to the planner as nothing in the economy justifies preferring to use one or another instrument. This is a choice that cannot be justified within the context of the theoretical framework analyzed, but rather needs to be motivated with the normative question formulated and its practical relevance for the question at hand. Our analysis highlights the importance to consider this choice in the context of all the instruments that can be used in a given model.

Third, from the expressions of the policy rules for τ^T, τ^D, T^D, T^C reported in appendix, we can see that these taxes are used only when the constraint binds. Using the terminology adopted in the literature, (e.g, Benigno et al (2016) and Jeanne and Korinek (2017)) they are *ex post* interventions. In Figure 1, we plot the policy rules for $\{\tau^T, \tau^D, T^D\}$, assuming the same set of plausible parameter values used by Benigno et a. (2013).[13] As we can see, this tax scheme requires a subsidy on tradable goods and a tax on tax on non-tradable revenue. The subsidy on tradable goods makes agents internalize the pecuniary externality. The tax on non-tradable revenue affects the intratemporal sector allocation of labor. By correcting the allocation of labor between the two sectors, the planner relaxes the borrowing constraint when it binds by increasing the amount that it is produced in each sector.

Fourth, as we noted in the proposition's proof, this planner problem is recursive, and thus the quadruplet $\{\tau^T, \tau^D, T^D, T^C\}^{SP}$ that decentralizes the allocation is time-consistent.

Fifth and finally, in our production economy, all subsidies and taxes can be used both ex-ante and ex-post, but here, under the tax scheme that we are considering, they are used only ex-post. Of course, the implementation of this ex-post tax scheme might be constrained by practical considerations, even if it is time-consistent. For example, Benigno et al (2016) in the endowment version of this economy show that, when lump-sum taxation is not available to the planner, the optimal policy changes and entails not only ex-post actions but also ex-ante, prudential interventions. The importance of ex-post policy tools stems from their role in affecting the key price that enters the borrowing constraint.[14]

4 Ramsey Optimal Policy

Thus far we saw that there exists a time-consistent tax scheme that can implement the constrained social planner allocation of our model economy. We will now show that the *same* subset of tools can be used optimally to remove the constraint altogether, and hence achieve the unconstrained allocation, by designing a standard Ramsey problem.

[13]Note that T^C is not plotted because it is determined from the government budget constraint.

[14]One way to limit the extent to which policy tools could be used to affect the key market price in the borrowing constraint is to adopt the definition of conditional efficiency in the design of the social planner problem. Under a conditional efficient social planner problem, the determination of the price that enters the borrowing constraint occurs through the pricing function as in the competitive equilibrium allocation (as opposed to the pricing rule as in the constrained efficient allocation). This implies that the price in the competitive equilibrium allocation and the price in the conditionally efficient social planner problem coincide limiting the scope for policy intervention when the constraint binds.

In the standard Ramsey problem, the planner maximizes the representative agent's utility given the resource constraint, the technological constraints and the first order conditions that characterize the competitive equilibrium allocation for a given set of policy instruments. In our normative analysis, the Ramsey planner will use $\left(\tau^T, \tau^D, T^D, T^C\right)$, the same subset of policy tools that can implement the constrained social planner allocation.[15]

Definition 3: Ramsey planner *For a given* $\{B_0\}$ *and assuming that* $\{A_t^T\}$ *and* $\{A_t^N\}$ *are Markov processes with finite strictly positive support, the Ramsey problem for* $\left(\tau^T, \tau^D, T^D, T^C\right)$ *is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_t^j - \frac{\left(H_t^j\right)^\delta}{\delta} \right)^{1-\rho} \right\},$$

subject to (2), the agents resource constraints

$$(1+\tau^T)C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i)\, B_t + T_t^C, \tag{35}$$

the firms' definition of profits

$$\pi_t = A_t^T \left(H_t^T\right)^{1-\alpha^T} + (1-\tau_t^D) P_t^N A_t^N \left(H_t^N\right)^{1-\alpha^N} - W_t H_t - T_t^D \tag{36}$$

the government budget constraint

$$\tau_t^T C_t^T + \tau_t^D P_t^N Y_t^N = T_t^C + T_t^D,$$

the technological constraints

$$Y_t^T = A_t^T \left(H_t^T\right)^{1-\alpha^T}, Y_t^N = A_t^N \left(H_t^N\right)^{1-\alpha^N} \tag{37}$$

[15]See Benigno et al. (2012) for the numerical solution of this problem and a quantitative characterization of some of the policy tools discussed here.

the non-tradeable goods market equilibrium condition

$$C_t^N = Y_t^N, \tag{38}$$

the borrowing constraint

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[Y_t^T + P_t^N Y_t^N\right], \tag{39}$$

the first order conditions of the household,

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE}(1+\tau_t^T) \tag{40}$$

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t^{CE} P_t^N \tag{41}$$

$$B_{t+1} : \frac{\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}}}{(1+\tau_t^T)} = \lambda_t^{CE} + \beta\left(1+i\right) E_t\left[\frac{\left(C_{t+1} - \frac{H_{t+1}^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_{t+1}^T\right)^{-\frac{1}{\kappa}} C_{t+1}^{\frac{1}{\kappa}}}{(1+\tau_{t+1}^T)}\right], \tag{42}$$

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho}\left(H_t^{\delta-1}\right) = \frac{\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}}}{(1+\tau_t^T)} W_t + \frac{1-\phi}{\phi} W_t \lambda_t^{CE}. \tag{43}$$

and the first order conditions of the firms:

$$W_t = (1-\tau_t^D)\left(1-\alpha^N\right) P_t^N A_t^N \left(H_t^N\right)^{-\alpha^N}, \tag{44}$$

$$W_t = \left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}, \tag{45}$$

Before proceeding, recall that, by taking the ratio of (41) to (40), we obtain

$$\frac{P_t^N}{(1+\tau_t^T)} = \frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}} \tag{46}$$

and by substituting (34), (37) and (38), the borrowing constraint becomes:

$$B_{t+1} + \frac{1-\phi}{\phi}\left[A_t^T (H_t^T)^{1-\alpha^T} + (1-\tau_t^D) P_t^N C_t^N - T_t^D\right] = 0.$$

The next proposition states the main point of the paper.

Proposition 2. *Given the set of taxes $(\tau^T, \tau^D, T^D, T^C)$ to the Ramsey planner as in definition (3), there exists a time-consistent tax scheme that replicates the unconstrained allocation.* **Proof.** See appendix.

Several remarks are in order here. First, note that the quadruplet $(\tau_t^T, \tau_t^D, T_t^D, T_t^C)$ is the same set of taxes that decentralize the constrained social planner allocation. The key difference is that when used optimally, they can undo the constraint altogether, while the constrained social planner takes the borrowing constraint as given. Under the optimal Ramsey policy, τ_t^T removes the borrowing constraint altogether by affecting directly the market price that enters in it, P_t^N, while τ_t^D offsets the distortions created by τ_t^T. So if we allow a Ramsey planner to optimize over $(\tau_t^D, \tau_t^T,)$ given the behavior of the private sector, it is possible to replicate the unconstrained equilibrium. This result implies that in the constrained social planner allocation the tax scheme (τ_t^D, τ_t^T) is suboptimal in the sense that it does not achieves all the welfare gains that could be attained by using the same instrument.

Second, the policy can be interpreted as a price support intervention, akin to an exchange rate intervention or an attempt to prop up the price of collateral. By taxing traded goods, this policy increases the relative price of non-traded goods. Crucially, when the constraint binds, this supports the relative price of non-tradables, counteracting the debt-deflation spiral that would otherwise lead to a decline in tradable consumption and a fall in the relative price of nontradables.

Third, in equilibrium agents anticipate that policy will undo the constraint when this binds and will behave as if the constraint does not exist (i.e. like in the unconstrained allocation). Eventually (i.e. in finite time) our economy will hit the borrowing constraint because agents are relatively impatient. When that happens, under the optimal policy, $(\tau_t^T, \tau_t^D, T_t^D, T_t^C)$ will be set so that the multiplier on the constraint is zero (i.e. the constraint is just binding).

Fourth, like in the case of the implementation of the social planner allocation, the tax schedules that replicate the unconstrained allocation does not rely a the tax on debt, τ^B, and just rely on policy intervention only when the economy hits the borrowing limit. More generally though, once we limit the set of available tools in such a way that the unconstrained allocation cannot be reached, optimal policy will contain a precautionary components every time that the instruments assigned to the policy maker cannot eliminate the consequences of the constraint being binding.

Fifth and finally, the result arises from the instrument's ability to affect the price of collateral on which the borrowing constraint is specified and therefore has fairly general applicability. The substance of our normative results, in fact, would not change if we modify the borrowing constraint to include a working capital component that would have more realistic implications in terms of business cycle moments and financial crisis dynamics, or if we consider a collateral constraint defined on an asset price, as long as the instrument assigned to the policy maker can affect the price of collateral. Here, we abstracted from these considerations, to keep the analysis as transparent and simple as possible.

5 Conclusions

In this paper we show that, in models with occasionally binding borrowing constraints in which the collateral value depends on market prices, the same combination of instruments that implements the constrained efficient allocation can also be used optimally by a Ramsey planner to achieve the unconstrained equilibrium–i.e., an allocation in which the constraint never binds in equilibrium. We established this in the context of a specific, widely used, model economy, but the results have more general applicability. The result in fact applies whenever a policy instrument that is assigned to the planner can affect the market price determining the value of the collateral in the borrowing constraint.

The result implies possible lack of robustness of any policy conclusions reached by adopting a constrained efficient allocation as a benchmark for the normative analysis in this class of models. A robust normative analysis in this class models requires explicit computations of Ramsey optimal policy problems, despite the significantly more challenging computational difficulties. In other words, in this class of models, it is useful to follow an approach to policy design that can attains all the welfare gains that are within reach of the policy instruments selected. The paper also highlights the importance of discussing and motivating carefully the choice of the instruments assigned to the Ramsey planner from the outset of the analysis, with particular attention to their implications for the price of collateral.

References

[1] Adam K. and R. M. Billi. (2006), "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates," *Journal of Money, Credit and Banking* 38(7): pp. 1877-1905.

[2] Adam K. and R. M. Billi (2007), "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates," "Journal of Monetary Economics", 54(3): pp. 728-52.

[3] Aghion, P. , P. Bacchetta, and A. Banerjee (2004). "Financial Development and the Instability of Open Economies." *Journal of Monetary Economics* 51(6): 1077-1106.

[4] Bianchi, J. (2011), "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review* 101(7), pp. 3400-3426.

[5] Bianchi, J. and E.G. Mendoza (2018), ""Optimal Time-Consistent Macroprudential Policy,"," *Journal of Political Economy* 126, no. 2 (April 2018): 588-634.

[6] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E.R. Young (2011), "Revisiting Overborrowing and Its Policy Implications," *Analysis and Economic Policies Series*, Vol. 14, Central Bank of Chile.

[7] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E.R. Young (2012), "Optimal Policy for Macro-financial Stability, " CEPR DP No. 9223 and IDB WP No. 361.

[8] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E.R. Young (2013), "Financial Crises and Macro-Prudential Policies," *Journal of International Economics*, Vol. 89(2), pp. 453-470.

[9] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E. R. Young (2016). "Capital Controls or Exchange Rate Policy? A Pecuniary Externality Perspective." *Journal of Monetary Economics*, Vol. 84, pp. 147-165.

[10] Bernanke, B., M. Gertler, and S. Gilchrist (1999). "The financial accelerator in a quantitative business cycle framework," Chapt. 21, p. 1341-1393 in Taylor, J. B. and M. Woodford (eds.) *Handbook of Macroeconomics*, vol. 1, Part C, Elsevier.

[11] Brunnermeier, M. and Sannikov, Y. (2014) "A Macroeconomic Model with a Financial Sector", *American Economic Review*, Vol. 104, No. 2 pp. 379-42.

[12] Brunnermeier, M. and Sannikov, Y. (2015) "International Credit Flows and Pecuniary Externality", American Economic Journal: Macroeconomics 7.1 (2015): 297-338.

[13] Carlstrom C. and T. Fuerst (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, vol. 87(5), pp. 893-910.

[14] Céspedes L. F., R. Chang, and A. Velasco (2004), "Balance Sheets and Exchange Rate Policy," *American Economic Review*, Vol. 94, No. 4, pp. 1183-1193.

[15] Cespedes F. L., Chang, R. Chang, and A. Velasco (2017) "Financial Intermediation, Exchange Rates, and Unconventional Policy in an Open Economy, *Journal of International Economics*, 108(S1), pp. 76-86.

[16] Correia, I., J. P. Nicolini, P. Teles (2008) "Optimal Fiscal and Monetary Policy: Equivalence Results", *Journal of Political Economy* 168, 1, pp.141-170.

[17] Curdia V. and M. Woodford, "Credit Spreads and Monetary Policy", *Journal of Money Credit, and Banking*, Volume42, Issues1, September 2010, pp. 3-35.

[18] Davila E. (2014), "Dissecting Fire Sales Externalities", Manuscript, New York University and Yale University.

[19] Davila E. and A. Korinek (2018), "Pecuniary Externalities in Economies with Financial Frictions", *The Review of Economic Studies*, Volume 85, Issue 1, pp. 352-395.

[20] Di Tella S. (2019), "Optimal Regulation of Financial Intermediaries" *American Economic Review*, 109(1): pp. 271-313.

[21] Gertler M and N. Kiyotaki (2015), "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy," *American Economic Review*, VOL. 105, NO. 7, pp. 2011-43.

[22] Iacoviello M., (2005) "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle", *American Economic Review*, VOL. 95, NO. 3, pp. 739-764.

[23] Kiyotaki N. and J. Moore (1997), "Credit Cycles", *Journal of Political Economy*, Vol. 105, No. 2 (April 1997), pp. 211-248.

[24] Jeanne, O. and A. Korinek (2012), "Managing Credit Booms and Busts: A Pigouvian Taxation Approach,", *Journal of Monetary Economics*, forthcoming.

[25] Jeanne, O. and A. Korinek (2018), "Macroprudential Regulation versus Mopping Up after the Crash", mimeo.

[26] Korinek, A. (2018), "Capital Flows to Emerging Markets: An Externality View ", *Journal of International Economics* 111, pp. 61-80.

[27] Kurlat P. (2018), "How I Learned to Stop Worrying and Love Fire Sales", *Manuscript*, Stanford University.

[28] Lorenzoni, G. (2008), "Inefficient Credit Booms," *Review of Economic Studies* 75(3), pp. 809-833.

[29] Mendoza, E. G. (2002). "Credit, Prices, and Crashes: Business Cycles with a Sudden Stop." In *Preventing Currency Crises in Emerging Markets*, edited by Sebastian Edwards and Jeffrey A. Frankel, 335-392. National Bureau of Economic Research.

[30] Mendoza, E.G. (2010), "Sudden Stops, Financial Crises and Leverage: A Fisherian Deflation of Tobin's Q,"*American Economic Review* 100(5), pp. 1941-1966.

A Appendix

A.1 Proof of Proposition 1

Proposition 1 (Implementation of the Constrained Efficient Allocation). *Given the following set of available taxes (τ^B, τ^N, τ^T, τ^D, τ^H, T^D, T^C), there exists a combination of policy rules for the subset of instruments (τ^T, τ^D, T^D ,T^C) (for brevity, a tax scheme for this subset of instruments) that implements the constrained social planner equilibrium of our economy. This tax scheme is time consistent.*

Proof. In order to implement the social planner equilibrium above we compare the first order conditions of the two allocations and seek policy rules for our taxes that equalize them. Note first that we can correct the distortion in the marginal utility of tradable consumption by using the tax on tradable goods: in fact by comparing (26) with (5) we have that

$$\left(1+\tau_t^T\right)^{SP} = \left(1 - \frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega C^N}\right)^{\frac{1-\kappa}{\kappa}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{\frac{\kappa-1}{\kappa}}\right) < 1,$$

where the right hand side is evaluated at the social planner allocation, and the policy intervention subsidizes the consumption of tradable goods.

Second, by setting $\tau^T = \left(\tau^T\right)^{SP}$ and $\tau^N = 0$ into (9) we obtain that the intratemporal allocation of consumption in the social planner allocation—the ratio of (26) over (27)—is also implemented.

Third, note that the intertemporal allocation of consumption has the same expression for both the planner—equation (28)—and the competitive equilibrium—equation (7)—once we set $\tau_t^B = 0$.[16] Fourth, note that the intratemporal allocation of labor is modified in the

[16] So there is no need to set a tax on international borrowing to implementation the social planner equilibrium in our economy.

planner problem. In fact from (29) and (30), we have that:

$$\frac{\mu_{2,t}^{SP}}{\mu_{1,t}^{SP}} = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}}{\left(1-\alpha^N\right) A_t^N \left(H_t^N\right)^{-\alpha^N}} \frac{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right)}{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{2,t}^{SP}}\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\right)},$$

which governs how labor is allocated between the tradable and nontradable sectors. The corresponding condition in the competitive allocation is

$$P_t^N = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^T\right)^{-\alpha^T}}{\left(1-\tau^D\right)\left(1-\alpha^N\right) A_t^N \left(H_t^N\right)^{-\alpha^N}}.$$

It follows that, τ_t^D can be used to equalize this margin between the two allocations by setting τ^D such that

$$\frac{1}{(1-\tau^D)^{SP}} = \frac{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right)}{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{2,t}^{SP}}\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\right)}$$

where $\frac{1}{(1-\tau^D)^{SP}} < 1 (> 1)$ depending on the elasticity of intratemporal substitution $\kappa > 1 (< 1)$.

Finally, note that when we use $\left(\tau^D\right)^{SP}$ and $\left(\tau^T\right)^{SP}$ as described above, we have that, when the constraint binds, it is affected by the tax scheme in the decentralized equilibrium. In fact we can rewrite the borrowing constraint in the decentralized equilibrium as

$$B_{t+1} = -\frac{1-\phi}{\phi}\left[Y_t^T + (1-\left(\tau_t^D\right)^{SP})P_t^N Y_t^N - T_t^D\right]$$

with

$$\frac{P_t^N}{(1+(\tau^T)^{SP})} = \left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}}.$$

In general, the expression for the borrowing constraint will differ between the competitive and the constrained social planner allocation because of the presence of taxes in the competitive equilibrium one. So we now need to find the a combination of taxes such that

$$B_{t+1} = -\frac{1-\phi}{\phi}\left[Y_t^T + \left(P_t^N\right)^{SP} Y_t^N\right].$$

To do so, denote with $\left(P_t^N\right)^{SP}$ the relative price of non-tradable in the social planner allocation:

$$\left(P_t^N\right)^{SP} = \left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}}.$$

so that

$$\left(P_t^N\right)^{SP}\left(1+\left(\tau^T\right)^{SP}\right) = P_t^N$$

evaluating the price at the same (SP) allocation. This means that we can set T_t^D in the competitive equilibrium allocation such that

$$\left(T_t^D\right)^{SP} = -\left(\tau_t^D\right)^{SP}\left(1+\left(\tau^T\right)^{SP}\right)\left(P_t^N\right)^{SP} Y_t^N.$$

So the triplet $\left(\tau^D, \tau^T, T^D\right)^{SP}$ with T^C satisfying the government budget constraint will be sufficient to replicate the SP allocation when the constraint binds.

When the constraint does not bind, it is easy to see by inspection that the first order conditions of the social planner and the competitive equilibrium are the same and there is no need to use any tax tools to equalize them.

The social planner problem defined above is recursive. Therefore the tax scheme that implements it in a decentralized equilibrium is time-consistent.

QED.

A.2 Alternative Set of Taxes for SP Implementation

Another way to decentralize the SP allocation is to use the following set of distortionary taxes: a tax on tradable consumption, a tax on nontradable consumption, a tax on new debt, a tax on labor income, and a tax on tradable output; the government budget constraint is assumed to be satisfied via a lump-sum tax/transfer. In this world we have the following conditions for a competitive equilibrium:

$$u_{1,t} - \left(1+\tau_t^T\right)\mu_t = 0$$
$$u_{3,t} + \left(1-\tau_t^H + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\left(1-\tau_t^D\right)\mu_t\left(1-\alpha_T\right)A_t^T\left(H_t^T\right)^{-\alpha_T} = 0$$
$$u_{3,t} + \left(1-\tau_t^H + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\frac{u_{2,t}}{1+\tau_t^N}\left(1-\alpha_N\right)A^N\left(H_t^N\right)^{-\alpha_N} = 0$$
$$\max\{-\lambda_t,0\}^2 - \left(B_{t+1} + \frac{1-\phi}{\phi}\left(\left(1-\tau_t^D\right)A_t^T\left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t}}{\mu_t\left(1+\tau_{Nt}\right)}A^N\left(H_t^N\right)^{1-\alpha_N}\right)\right) = 0$$
$$\beta\left(1+r\right)E_t\left[\mu_{t+1}\right] - \left(1-\tau_t^B\right)\mu_t + \max\{\lambda_t,0\}^2 = 0.$$

The first equation determines τ_t^T

$$\tau_t^T = \frac{\mu_{1t} - \mu_t - \max\{\lambda_t,0\}^2\frac{1-\phi}{\phi}\frac{1}{\kappa}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1-\kappa}{\kappa}}}{\mu_t}$$

the last determines τ_t^B

$$\tau_t^B = \frac{\beta\left(1+r\right)E_t\left[\mu_{1,t+1} - \mu_{t+1}\right] - \left(\mu_{1,t} - \mu_t\right)}{\mu_t},$$

and the middle three jointly determine $\left(\tau_t^H, \tau_t^N, \tau_t^D\right)$:

$$\left(1+\frac{\max\left\{\lambda_t,0\right\}^2}{\mu_{1t}}\frac{1-\phi}{\phi}\right)\mu_{1t}=\left(1-\tau_t^H+\frac{\max\left\{\lambda_t,0\right\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\left(1-\tau_t^D\right)\mu_t \tag{47}$$

$$1+\frac{\max\left\{\lambda_t,0\right\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{48}$$
$$=\left(\frac{1-\tau_t^H}{1+\tau_t^N}+\frac{\max\left\{\lambda_t,0\right\}^2}{\left(1+\tau_t^N\right)\mu_t}\frac{1-\phi}{\phi}\right)$$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{49}$$
$$=-\tau_t^D A_t^T\left(H_t^T\right)^{1-\alpha_T}+\frac{u_{2,t}A^N\left(H_t^N\right)^{1-\alpha_N}}{\mu_t\left(1+\tau_t^N\right)}.$$

Since $\mu_{1,t}=\mu_t$ (the marginal value of wealth is equalized) the expressions simplify:

$$\tau_t^T=\frac{-\max\left\{\lambda_t,0\right\}^2}{\mu_{1,t}}\frac{1-\phi}{\phi}\frac{1}{\kappa}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1-\kappa}{\kappa}} \tag{50}$$

$$\tau_t^B=0 \tag{51}$$

$$\tau_t^D=\frac{-\tau_t^H}{1-\tau_t^H+\frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}} \tag{52}$$

$$1+\frac{\max\left\{\lambda_t,0\right\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{53}$$
$$=\left(\frac{1-\tau_t^H}{1+\tau_t^N}+\frac{\max\left\{\lambda_t,0\right\}^2}{\left(1+\tau_t^N\right)\mu_t}\frac{1-\phi}{\phi}\right)$$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{54}$$
$$=-\tau_t^D A_t^T\left(H_t^T\right)^{1-\alpha_T}+\frac{u_{2,t}A^N\left(H_t^N\right)^{1-\alpha_N}}{\mu_t\left(1+\tau_t^N\right)}.$$

A lump-sum tax/transfer can then be used to balance the government budget constraint. Note that the third and fourth equations can be used to define $\left(\tau_t^D, \tau_t^N\right)$ entirely in terms of τ_t^H, so that solving the system of equations can be reduced to solving one nonlinear equation in τ_t^H:

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}}=\frac{\tau_t^H A_t^T\left(H_t^T\right)^{1-\alpha_T}}{1-\tau_t^H+\frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}}+$$
$$\frac{u_{2,t}A^N\left(H_t^N\right)^{1-\alpha_N}\left(1-\tau_t^H+\frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)}{\mu_t\left(1+\frac{\max\{\lambda_t,0\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}}\right)}.$$

The LHS of this equation is constant in terms of τ_t^H and the first term on the RHS is strictly increasing in τ_t^H, but the second term is strictly decreasing so no guarantee of uniqueness can be obtained. However, at least one $\tau_t^H < 1$ exists that solves this equation, as the LHS is strictly positive and the RHS can be made both negative (setting $\tau_t^H < 0$) and arbitrarily positive (setting τ_t^H close to 1).

A.3 Proof of Proposition 2

Proposition 2. *Given the set of taxes $(\tau^T, \tau^D, T^D, T^C)$ to the Ramsey planner as in definition (3), there exists a time-consistent tax scheme that replicates the unconstrained allocation.*

Proof. To see this, focus first on the tradable good tax, τ_t^T, as a policy tool to undo the borrowing constraint. Let τ_t^T be such that $P_t^{N,CE} = (1+\tau_t^T)P_t^{N,UE}$ with T_t^D such that $\tau_t^D P_t^{N,CE} C_t^N = T_t^D$ so that the borrowing constraint is not binding

$$B_{t+1}^{UE} + \frac{1-\phi}{\phi}\left[A_t^T (H_t^{T,UE})^{1-\alpha^T} + (1+\tau_t^T)P_t^{N,UE}C_t^{N,UE}\right] > 0.$$

However, since τ_t^T affects also the intertemporal allocation of resources (42) we need to show that there is a constant τ_t^T such that the intertemporal margin is not distorted by the tax.

To do so, we first note that, by setting $\lambda_t^{CE} \equiv 0$ and τ_t^T so that

$$\frac{1}{1+\tau_t^T} = \frac{\beta(1+r)E_t\left[\frac{u'(C_{t+1}^{UN})C_{C_{t+1}^T}^{UN}}{1+\tau_{t+1}^T}\right]}{E_t[u'(C_{t+1}^{UN})C_{C_{t+1}^T}^{UN}]}, \tag{55}$$

the Euler equations of the Ramsey problem and the unconstrained equilibrium coincide. It follows that the tax rate τ_t^T that satisfies (42) must be constant (otherwise the intertemporal margin would be distorted). By inspection of the unconstrained allocation, the non-tradable price has a strictly positive lower limit. Therefore there exists A $\underline{\tau}^T$, this is the lower level of the tax on tradables compatible with the strictly positive lower limit on the relative price of non-tradables), such that the borrowing constraint (39) is always satisfied for any $\tau^T \geqslant \underline{\tau}^T$. Thus, any constant tax policy of the form $\tau_t^T \equiv \tau^T \geqslant \underline{\tau}^T$ can be part of the optimal policy plan replicating the unconstrained allocation.

If the borrowing constraint is not binding, $\lambda_t^{CE} = 0$, so that all the other equilibrium conditions will be identical to those in the unconstrained allocation, except for the one that determines the labor demand in the non-tradable sector, as it is affected by the relative price of non-tradable goods. Indeed we have that:

$$\begin{aligned} W_t^{CE} &= (1-\tau_t^D)\left(1-\alpha^N\right) P_t^{N,CE} A_t^N \left(H_t^{N,CE}\right)^{-\alpha^N} \\ W_t^{UE} &= (1-\alpha^N) P_t^{N,UE} A_t^N (H_t^{N,UE})^{-\alpha^N}. \end{aligned}$$

Since $P_t^{N,CE} = (1+\tau_t^T)P_t^{N,UE}$, $W_t^{CE} = W_t^{UE}$, we need to set the tax revenue, τ_t^D, as

$$(1-\tau_t^D) = \frac{1}{(1+\tau_t^T)},$$

so that, when evaluated at the unconstrained equilibrium, the two taxes cancel each other.

The government budget constraint can clear by using T_t^C so that

$$T_t^C = \tau_t^T C_t^{T,UE} + \tau_t^D P_t^{N,UE} Y_t^{N,UE} - T_t^D,$$

where $\tau_t^D P_t^{N,UE} C_t^N = T_t^D$. Since these are the only conditions distorted by the taxes and the borrowing constraint, we conclude that we achieve the unconstrained solution via the tax schedule on τ_t^T,τ_t^D, T_t^D, T_t^C.

This tax scheme achieves the unconstrained allocation and, given the available policy tools, the Ramsey planner has no incentive to deviate from the schedule at any point in time.[17] Therefore such tax schedule is also a time consistent optimal policy. Indeed, the tax on tradeable goods is used to undo the constraint while all the other available policy tools are used to undo the distortion created by the tax on traded goods. QED

[17]Indeed, there are no additional instruments that can be used to address the incomplete market inefficiency.