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Optimal Policy for Macro-Financial Stability

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Abstract In this paper we study whether policy makers should wait to intervene until a financial crisis strikes or rather act in a preemptive manner. We study this question in a relatively simple dynamic stochastic general equilibrium model in which crises are endogenous events induced by the presence of an occasionally binding borrowing constraint as in Mendoza (2010). First, we show that the *same* set of taxes that replicates the constrained social planner allocation could be used optimally by a Ramsey planner to achieve the first best unconstrained equilibrium: in both cases without any precautionary intervention. Second, we show that the extent to which policymakers should intervene in a preemptive manner depends critically on the set of policy tools available and what these instruments can achieve when a crisis strikes. For example, in the context of our model, we find that, if the policy tools is constrained so that the first best cannot be achieved and the policy maker has access to only one tax instrument, it is always desirable to intervene before the crisis regardless of the instrument used. If however the policy maker has access to two instruments, it is optimal to act only during crisis times. Third and finally, we propose a computational algorithm to solve Markov-Perfect optimal policy for problems in which the policy function is not differentiable.

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1 Introduction

The global financial crisis and ensuing great recession of 2007-2009 have started a debate on the role of policy for the stability of the financial system and hence the economy as a whole (i.e., macro-financial stability). This debate in advanced economies revolves around the role of macroeconomic and regulatory policies in causing the global crisis and how the conduct of such policies should be designed in the future to prevent a recurrence of such events. In emerging economies, the resurgence of strong capital inflows from advanced economies has also focused the discussion on the role of such policies in ensuring the financial stability of these countries in the face of highly volatile capital flows.

A key question in this debate is whether policy makers should wait until a crisis strikes to intervene or rather act during normal times in a precautionary or preemptive manner. While a seemingly overwhelming consensus has emerged that preemptive intervention would have been desirable (e.g., Bianchi, 2011; Bianchi and Mendoza, 2010; Jeanne and Korinek, 2011), investigation of this important policy question has thus far been hampered by the absence of computational tools to address it. Indeed, optimal policy in models *a la* Mendoza (2010) has not been studied yet because the decision rules in that framework are not differentiable, and available computational methods do not apply. The normative analysis this class of models therefore is usually conducted by comparing the competitive equilibrium with the constrained social planner allocation, with the planner facing the same credit friction as private agents do. Policy is then designed by seeking a set of policy rules that decentralizes the planner allocation.

In this paper we study the question above in a relatively simple dynamic stochastic general equilibrium model with credit frictions *a la* Mendoza (2010) in which normal times alternate with crisis times endogenously. More specifically, there are three main contributions of our work.

First, we show that the *same* set of taxes that replicates the constrained social planner allocation could be used by a Ramsey planner to achieve the first best unconstrained equilibrium. In both cases there is no need to rely on preempitve intervention. This suggests that when the policy maker has the right set of tools to deal with the financial crisis, there is no need of any precautionary intervention. This also shows that, a Ramsey optimal policy approach is more reliable than the social planner approach currently used in the literature because it attains all the welfare gains that are within reach of the policy instruments selected.

Second, we show that the extent to which policymakers should intervene in a preemptive manner depends critically on the set of policy tools available. Specifically, in light of the results above, we then study numerically more general policy problems in which the government has a restricted set of tools and cannot achieve the first best and chooses taxes optimally in a time-consistent (i.e., Markov-Perfect) equilibrium. For example, in the numerical analysis of our model, we find that, if the policy maker has access to only one tax instrument, it is always desirable to intervene before the crisis regardless of the instrument used. If however the policy maker has access to two instruments, it is optimal to act only during crisis times. In the first case the policymaker has an ineffective set of tools to manage the crisis so that it is optimal to try and prevent the crisis from occurring by acting in a prudential manner. In the second case, the policymaker can use one instrument to address the crisis when one occurs to its full potential and the second instrument to contain the side effects of such interventions. The key insight of this second contribution of our work is that the optimal design of the policy intervention before a financial crisis strikes depends on the effectiveness of the policy tools during the financial crisis. In other words, the interaction between ex-post (crisis times) and ex-ante (normal times) policy behavior is a crucial aspect of the policy design problem: the ex-ante optimal policy intervention depends on the policy design during crises times.

Third and finally, since in this environment available solution methods do not apply, we contribute to the literature from a methodological perspective, by developing a computational algorithms that solves for the global optimal Markov-Perfect equilibrium of the economy. To our knowledge this is the first paper that computes optimal policy in the context of dynamic general equilibrium model with occasionally binding credit constraint.[1]

The model we use to address this issue is now fairly standard for the study of macro-financial stability. This is an economy in which there are both crisis and non-crisis states, in which a crisis event is an endogenous outcome (e.g., Mendoza, 2010). In this environment, financial crises are events in which a financial friction (e.g., an international borrowing constraint) becomes binding endogenously, depending on agents' choices, the state of the economy, and policy decisions as well. When the constraint does not bind the model economy exhibits normal business cycle fluctuations. Specifically, our endogenous borrowing credit constraint is embedded in a standard two-sector (tradable and non-tradable goods) small open economy in which foreign borrowing is denominated in units of the tradable good, but it is leveraged on income generated in both sectors.[2] Thus, the relative price of non-tradeable good affects the value of non tradable income. This is a specification of the borrowing constraint that captures "liability dollarization" a key feature of the capital structure of emerging markets (e.g., Krugman (1999), Aghion, Bacchetta and Banerjee (2004)).[3]

The competitive equilibrium of this economy is not efficient because of the presence of the relative price of non-tradable goods in the borrowing constraint. This inefficiency is usually referred to as a pecuniary externality (or as price or credit externality) in the literature: individual agents take prices as given and do not internalize the effect of their individual decisions on the market price that enters the specification of the financial friction.[4]

As we noted above, in the existing literature, scope for policy intervention is identified by comparing the competitive equilibrium with the social planner allocation in which the planner is subject to the same collateral constraint but takes into account the effects of its actions on equilibrium prices. The policy analysis is then conducted by choosing the policy instruments that implement the efficient allocation (e.g., see for instance Bianchi

[1] On optimal policy in models in which the economy is in a financial crisis see Braggion, Christiano, Roldos (2009), Caballero and Panageas (2007), Cúrdia (2007), Caballero and Krishnamurthy (2005), Hevia (2008), among others.

[2] See for example Mendoza (2002) and Bianchi (2011).

[3] The latest wave of crises in Europe is striking evidence of the importance of such feature.

[4] For details see Bianchi (2011), Bianchi and Mendoza (2010), Caballero and Lorenzoni (2008), Chang, Cespedes, and Velasco (2012), Jeanne and Korinek (2010), and Lorenzoni (2008)) who analyze the same kind of externality.

(2011) on debt taxes or capital controls). The social planner approach provides a useful normative benchmark, but it does not allow for interaction between the policy maker and the private sector, thus omitting a fundamental aspect of typical macroeconomic stabilization problems—e.g., Kydland and Prescott (1977). In broad terms, the key contribution of this paper is to allow for this interaction in the analysis of one of the most important questions in the ongoing policy debate.

In this paper, in contrast, we determine an optimal policy equilibrium for an economy that is subject to an occasionally binding collateral constraint. We do that in the context of a Markov-Perfect (or "time-consistent") equilibrium since the presence of the borrowing constraint creates an incentive for the policy maker to deviate from past promises when the constraint is binding. As Chari and Kehoe (2010) noted, this is a more realistic assumption than "commitment". Therefore our approach is more closely related to the problem faced by actual governments that are unable to commit in advance to a given set of bail out policies.

To solve for optimal policy in this model we develop a global solution method. That is, we solve for a policy rule across both states of the world, when the constraint binds and when it does not. Such an approach enforces that optimal policy rule away from the crisis periods is designed with full knowledge of what the rule will be when the economy enters a crisis state. This is true for both the policy maker and the agents in the economy. This solution method, while computationally costly, is critical for understanding the interaction between precautionary behavior on the part of the private sector with precautionary behavior on the part of the policy maker. The technical challenge in solving such a model is that the constraint binds only occasionally, and the policy functions are not differentiable. Existing methods (e.g., Klein, Krusell, and Rios-Rull, 2009) do not apply under this assumption.

We consider several tax instruments in our analysis. In the numerical analysis we focus on a distortionary tax on non-traded consumption that can be interpreted in terms of exchange rate policy (or more generally as a price support policy) and a tax on debt that can be interpreted as a control on capital flows. We also consider alternative tax tools such as a tax on tradable consumption, a tax on labor income, a tax on firm profits, as well as alternative combinations of these instruments to illustrate the robustness of the results.

We first derive the constrained-efficient allocation that is usually used in the current literature as a normative benchmark. Then we show how a tax on tradable consumption and a tax on non-tradable firms profits used only when the constraint binds can be used to decentralize it (i.e. we show that there is no need for precautionary policy to replicate the constrained efficient allocation). In fact when the constraint binds, private agents cannot borrow the desired amount, and since movements in the relative price of non-tradable are inefficient, resources are not allocated efficiently between the two sectors. The tax on tradable consumption mitigates the effect of the pecuniary externality on borrowing decisions and the tax on non-tradable profits reallocate resources properly.

The problem in following the constrained social planner approach to policy design is that the use of the distortionary tax tool is not optimized. Indeed, we show that the same set of policy tools could be used optimally by a Ramsey planner to achieve the first best unconstrained equilibrium. In this case, the tax on tradable consumption supports the relative price of non tradable in such a way that the borrowing constraint never binds in equilibrium, while the tax on non-firm profits ensures that resource allocation is not distorted. In the unconstrained equilibrium the commitment to intervene removes the constraints and the optimal tax scheme is never used in equilibrium. Moreover, as these promised interventions are time-consistent, the Ramsey planner's commitment is fully credible. In contrast, the implementation of the social planner allocation with the same set of taxes requires that they would be used when the constraint binds, which may raise feasibility issues. Based on these results, and in light of the fact that Ramsey optimal policy is generally not time-consistent, we advocate the use of a Markov-Perfect optimal policy approach to the normative analysis of this class of models. In practice, given the set of policy instrument selected, it is useful to follow an approach to policy design that can attains all the welfare gains that are within their reach.

We then compute numerically the optimal time-consistent policy for given sets of tax instruments. When we allow for two policy tools (a tax on non-tradable consumption and the tax on new debt) there is no need to intervene in normal times as the combined used of the two taxes allows to mitigate the costs of using distortionary taxation in crises times. In contrast, when we allow for only one policy tool, (a tax on non-tradable consumption or a tax on debt) the policy maker cannot limit the costs of using distortionary policy tools in crises times and as such intervenes in normal times to limit their occurrence. We find that similar results hold for the tax on tradable consumption and the tax on wages.

This paper is related to the recent strand of literature that has built on the seminal work by Mendoza (2010) and has studied the normative properties in model economies subject to sudden stops. Bianchi (2011) uses an endowment version of our economy and finds that without production the competitive equilibrium always entails more borrowing relative to the social planner allocation, and that a prudential capital control (i.e., a prudential tax on borrowing) can replicate the social planner allocation. Benigno et al (2012) show that in a production economy the extent to which private agents borrow too little or too much depend on parameter values, but they don't discuss issues related to the implementation of the constrained efficient allocation. Benigno et al (2012b) compare alternative tax instruments in the same economy analyzed by Bianchi (2011) and find that taxes on consumption (i.e., real exchange rate interventions) dominate capital controls as a policy tool because they can achieve the first best while capital controls can achieve only the second best. Cespedes, Chang and Velasco (2012) compare the transmission mechanism of alternative policy interventions in a model with an occasionally binding financial friction, but don't discuss the implementation of the constrained efficient allocation or the computation of optimal policy in tier set up. Jeanne and Korinek (2011) and Bianchi and Mendoza (2010) analyze models in which the price externality arises because agents fail to internalize the effect of their decisions on an asset price rather than the relative price of non-tradable goods like in our model and compare competitive equilibrium with the constrained social planner allocation. Their policy implications are similar to those of Bianchi (2011). While computing optimal policy in models with asset-price entering the borrowing constraint presents additional computational challenges, the general lessons for policy design of our analysis apply also to those set ups.

To our knowledge, there are no contributions in the literature on the analysis of optimal policy in an environment in which a borrowing constraint both binds occasionally and is endogenous to the decisions in the model. Adams and Billi (2006a and 2006b) study optimal monetary policy in a closed economy, new Keynesian model in which there is zero lower bound on interest rates that binds only occasionally. Their zero-bound constraint is fixed and does not evolve endogenously. In terms of solution technique for the optimal policy problem we set up, our method is related to Klein, Krusell, and Rios-Rull (2009): the main difference is that our algorithm does not require that the policy functions are differentiable (which in general would not hold in our environment due to the occasionally-binding constraint) but only that they are continuous.

The rest of the paper is organized as follows. Section 2 describes the model we use and its competitive equilibrium. Section 3 derives a set of normative results analytically. In particular, it derives the social planner allocation and the first best unconstrained equilibrium of the model and discusses how to decentralize them via tax policy interventions. Section 4 define the equilibria that we compute numerically and describes the solution methods we propose, and discusses some properties of the tax instruments we consider that are useful to interpret the numerical results. Section 5 discusses the parametrization of the model. Section 6 studies optimal policy numerically in our model environment. Section 7 concludes.

2 The Model and the Competitive Equilibrium

In this section, we describe our model set-up. The model that we use is a relatively simple, two-sector (tradable and non-tradable) production small open economy, in which financial markets are not only incomplete but also imperfect as in Mendoza (2010) and Benigno et al (2012). In what follows we describe the competitive equilibrium allocation with the set of taxes that we are going to use in our normative analysis.

2.1 Households

There is a continuum of households $j \in [0, 1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{1-\rho} \right\}, \tag{1}$$

with C_j denoting the individual consumption basket and H_j the individual supply of labor for the tradable and non-tradable sectors ($H_j = H_j^T + H_j^N$). The assumption of perfect substitutability between labor services in the two sectors ensures that there is a unique labor market. For simplicity we omit the j subscript for the remainder of this section, but it is understood that all choices are made at the individual level. The elasticity of labor supply is δ, while ρ is the coefficient of relative risk aversion. In (1), the preference specification follows from Greenwood, Hercowitz and Huffman (GHH, 1988).[5]

[5] In the context of a one-good economy this specification eliminates the wealth effect from the labor supply choice. Here we emphasize that in a multi-good economy, the sectoral allocation of consumption will affect the labor supply decision through relative prices (See Benigno et al., 2012 for more details).

The consumption basket, C_t, is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{\frac{\kappa-1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}}. \tag{2}$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of tradable goods in the consumption basket. We normalize the price of tradable goods to 1. The relative price of the nontradable goods is denoted by P^N. The aggregate price index is then given by

$$P_t = \left[\omega + (1-\omega)\left(P_t^N\right)^{1-\kappa}\right]^{\frac{1}{1-\kappa}},$$

where we note that there is a one-to-one link between the aggregate price index P and the relative price P^N.

Households maximize utility subject to their budget constraint, which is expressed in units of tradable consumption. The constraint each household faces is:

$$(1+\tau_t^T)C_t^T + (1+\tau_t^N)P_t^N C_t^N = \pi_t + W_t H_t - T_t^C - (1-\tau_t^B)B_{t+1} + (1+i)\,B_t, \tag{3}$$

where W_t is the wage in units of tradable goods, $B_{t+1} < 0$ denotes the debt position at the end of period t with gross real return $1+i$. Households receive profits, π_t, from owning the representative firm. Their labor income is given by $W_t H_t$. We denote with $\tau_t^B > 0 (< 0)$ a tax (subsidy) on the amount that households borrows, with $\tau_t^N, \tau_t^T > 0(< 0)$ a tax (subsidy) on non-tradeable and tradeable consumption respectively, and with $T_t^C > 0(< 0)$ lump-sum taxes (transfers) to the consumer.[6]

International financial markets are incomplete, and access to them is imperfect. The asset menu includes only a one-period bond denominated in units of tradable consumption. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current *total* income:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[\pi_t + W_t H_t\right]. \tag{4}$$

Note first that the value of the collateral is endogenous in this model and it depends on the current realization of profit and wage income. And a necessary condition for defining a crisis event in the context of this class of models is to have the constraint binding. Second note that this constraint captures a balance sheet effect (e.g., Krugman, 1999, and Aghion, Bacchetta and Banerjee, 2004) since foreign borrowing is denominated in units of tradables while the income that can be pledged as collateral is generated also in the non-tradable sector.

Following the related literature on pecuniary externalities and prudential policies, we do not explicitly derive the credit constraint as the outcome of an optimal contract between lenders and borrowers. However, we can interpret the constraint above as the outcome of an interaction between lenders and borrowers in which lenders are not willing to permit

[6] Although at this stage we do not explicitly set up taxes on wage income, in the numerical analysis we will consider also this possibility.

borrowing beyond a certain limit. This limit depends on the parameter ϕ, which measures the tightness of the financial friction, and on current income that could be used as a proxy of future income.[7] At the empirical level, a specification in terms of current income is consistent with evidence on the determinants of access to credit markets (e.g., Jappelli 1990) and lending criteria and guidelines used in mortgage and consumer financing.[8]

Households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T, B_{t+1}, and H_t. The first-order conditions of this problem are the following:

$$C_T : \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t(1+\tau_t^T) \tag{5}$$

$$C_N : \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t P_t^N(1+\tau_t^N) \tag{6}$$

$$B_{t+1} : (1-\tau_t^B)\mu_t^{CE} = \lambda_t^{CE} + \beta\left(1+i\right) E_t\left[\mu_{t+1}^{CE}\right], \tag{7}$$

and

$$H_t : \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho} \left(H_{j,t}^{\delta-1}\right) = \mu_t^{CE} W_t + \frac{1-\phi}{\phi} W_t \lambda_t^{CE}. \tag{8}$$

where μ_t is the multiplier on the period budget constraint, and λ_t^{CE} is the multiplier on the international borrowing constraint. The presence of the borrowing constraint distorts directly two margins: the intertemporal margin, as the Euler equation (7) includes a term ($\lambda_t^{CE} > 0$) when the constraint binds, and the labor supply choice (8) since when the constraint binds agents are willing to supply additional units of labor.

We can combine (5) and (6) to obtain the intratemporal allocation of consumption, and (5) with (8) to obtain the labor supply schedule, and summarize the first order conditions of the household as:

$$\frac{(1+\tau_t^N)P_t^N}{(1+\tau_t^T)} = \frac{(1-\omega)^{\frac{1}{\kappa}} \left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}}} \tag{9}$$

$$\left(H_{j,t}^{\delta-1}\right) = \left(\frac{\omega C}{C^T}\right)^{\frac{1}{\kappa}} W_t \left(1 + \frac{1-\phi}{\phi}\frac{\lambda_t^{CE}}{\mu_t^{CE}}\right), \tag{10}$$

It is important to note here that τ_t^N and τ_t^T affect the determination of the relative price directly. For example, for given consumption of tradeable and non-tradeable, a decrease of in τ_t^N implies an increase in the relative price of non-tradeable goods and hence a higher value of the collateral in units of tradable consumption. On the other hand, the channel through

[7] A constraint expressed in terms of future income, which could be the outcome of the interaction between lenders and borrowers in a limited commitment environment, would introduce further computational difficulties that we need to avoid for tractability since future consumption choices would affect current borrowing decisions.

[8] The substance of our normative results will not change if we modify the borrowing constraint to include a working capital component that will have more realistic implications in terms of business cycle moments and financial crisis dynamics. To keep the normative analysis as simple as possible we abstract from this component here.

which τ^B works, depends on the constraint being binding or not. When the constraint is not binding, τ^B reduces the amount that agents borrow. When the constraint binds, since the amount of borrowing is determined by the endogenous limit, τ^B affects the value of the Lagrange multiplier associated with the constraint, λ_t^{CE}. This can be seen by re-arranging the first order condition for debt as:

$$\lambda_t^{CE} = (1-\tau_t^B)\mu_t^{CE} - \beta\,(1+i)\,E_t\left[\mu_{t+1}^{CE}\right] > 0.$$

For example, for given future marginal utility of tradeable consumption $\left(E_t\left[\mu_{t+1}^{CE}\right]\right)$, an increase in τ_t^B will tend to decrease the value of the multiplier, λ_t^{CE} and the amount of labor that is supplied in crisis times for given real wages (10).

The tax on tradable consumption, τ_t^T, works similarly to the tax on nontradable consumption; the main difference is that, in general, changes in the tax rate between two periods affect the intertemporal path of consumption of tradables. As we shall see, however, in the Markov-Perfect equilibrium that we compute numerically these two taxes (the one on tradable consumption and the one on nontradable consumption) have equivalent implications.

2.2 Firms

Firms produce tradable and non-tradable goods with a variable labor input and the following decreasing return to scale technologies:

$$\begin{aligned} Y_t^N &= A_t^N H_t^{1-\alpha^N}, \\ Y_t^T &= A_t^T H_t^{1-\alpha^T}, \end{aligned}$$

where A^N and A^T are the productivity levels, which are assumed to be random variables, in the non-tradable and tradable sector, respectively. The firm's problem is static and current-period profits (π_t) are:

$$\pi_t = A_t^T\left(H_t^T\right)^{1-\alpha^T} + (1-\tau_t^D)P_t^N A_t^N\left(H_t^N\right)^{1-\alpha^N} - W_t H_t - T_t^D.$$

In some cases, we shall allow for distortionary taxation (subsidy) on non-tradeable firm's revenue, $\tau_t^D > 0$ (< 0) and we allow for lump-sum taxes or transfer to the firm T_t^D, so that the first-order conditions for labor demand in the two sectors are given by:

$$W_t = (1-\tau_t^D)\left(1-\alpha^N\right)P_t^N A_t^N\left(H_t^N\right)^{-\alpha^N}, \tag{11}$$

$$W_t = \left(1-\alpha^T\right)A_t^T\left(H_t^T\right)^{-\alpha^T}, \tag{12}$$

and the value of the marginal product of labor equals the wage in units of tradable goods (W_t). By taking the ratio of (11) over (12) we obtain:

$$P_t^N = \frac{\left(1-\alpha^T\right)A_t^T\left(H_t^T\right)^{-\alpha^T}}{(1-\tau_t^D)\left(1-\alpha^N\right)A_t^N\left(H_t^N\right)^{-\alpha^N}}. \tag{13}$$

From this last expression we can see that the relative price of non-tradable goods determines the allocation of labor between the two sectors: for given productivity levels, a decrease (increase) in P_t^N drives down (up) the marginal product of non-tradables and induces a shift of labor toward (out of) the tradable sector. Similarly an increase in τ_t^D tends to shift labor out of the tradable sector towards the non-tradable one.

2.3 Government Budget Constraint

The government follows a balanced budget rule and has access to lump-sum transfer or taxes:

$$\tau_t^B B_{t+1} + \tau_t^N P_t^N C_t^N + \tau_t^D P_t^N Y_t^N = T_t^C + T_t^D.$$

Note that in this class of models, in which the pecuniary externality make the competitive equilibrium inefficient, the presence of lump-sum taxes or transfer simplifies the policy problem without making it trivial: in particular, as we will see later, access to lump-sum taxes does not guarantee that the equilibrium will be time-consistent.

2.4 Competitive Equilibrium

To determine the goods market equilibrium, combine the household budget constraint, the government budget constraint and the firm's profits with the equilibrium condition in the nontradable good market to obtain the current account equation of our small open economy:

$$C_t^T = A_t^T H_t^{1-\alpha^T} - B_{t+1} + (1+i)\, B_t. \tag{14}$$

The nontradable goods market equilibrium condition implies that

$$C_t^N = Y_t^N = A_t^N \left(H_t^N\right)^{1-\alpha^N}. \tag{15}$$

Finally, using the definitions of firms' profits and wages, the credit constraint implies that the amount that the country, as a whole, can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left[Y_t^T + (1-\tau_t^D) P_t^N Y_t^N - T_t^D\right], \tag{16}$$

so that (14) and (16) determines the evolution of the foreign borrowing.

The competitive equilibrium allocation is then characterized by (14), (16), (15) along with the first order conditions for the household (5), (7), (9) and (10), and firms (13) and the complementary slackness condition:[9]

$$\left(B_{t+1} + \frac{1-\phi}{\phi}\left[Y_t^T + (1-\tau_t^D) P_t^N Y_t^N - T_t^D\right]\right)\lambda_t^{CE} = 0 \text{ with } \lambda_t^{CE} > 0.$$

[9] The properties of the competitive equilibrium of this economy are well known (See for instance Mendoza, 2002 and Benigno et al. 2012).

3 Normative Analysis: Constrained Social Planner and First Best

In this section we derive a few normative results analytically. These results provide useful benchmarks for the numerical optimal policy analysis we report below and are important in their own right. In particular, we first define the social planner (SP) equilibrium and discuss its implementation with a given set of instruments. We then show that the same set of instruments that implement the SP can achieve the first best when used optimally by a Ramsey planner.

3.1 The Social Planner Problem and Its Implementation

We first turn to the characterization of the social planner allocation and its implementation. This allocation is typically used as a benchmark for policy design in this class of models. Our planner problem is constrained in the sense that the she/he faces the same borrowing constraint that the private agents do, but from an aggregate country-wide perspective.

We shall also emphasize here, for clarity, that the social planner *does not* set any tax instrument, but simply allocates resources directly. In fact, the planner chooses the optimal path of C_t^T, C_t^N, B_{t+1}, H_t^T and H_t^N by maximizing (1) subject to the resource constraints (14) and (15), the international borrowing constraint from an aggregate perspective ((16) with taxes equal to zero), and the pricing *rule* of the competitive equilibrium allocation (9) in which $\tau^N = \tau^T = 0$. Therefore, the key difference between the two allocations (competitive and social planner ones) is that the planner takes into account the effects of its decision of prices and hence internalizes the pecuniary externality that is present in the model.

How we determine the relative price that enters the financial friction (in our case P_t^N) in the social planner allocation can affect the policy analysis significantly. By constraining the social planner problem with the pricing rule of the competitive equilibrium we follow the "constrained efficiency" definition of Kehoe and Levine (1993). An alternative way that has been used in the related literature (Bianchi (2011), Bianchi and Mendoza (2011)) is to adopt the concept of "conditional efficiency" from Kehoe and Levine (1993). With conditional efficiency the planner problem is constrained by the competitive equilibrium pricing function (i.e., $P_t^N = f^{CE}(B_t, A_t^N, A_t^T)$), in which f^{CE} is the policy function for P_t^N obtained from solving the competitive equilibrium allocation.

This second alternative has been adopted in the literature mainly for computational reasons in models in which the key market price is a forward-looking variable like an asset price (see Benigno et al 2012b for more details). It is evident that with the second alternative, by construction, equilibrium prices in the competitive and social planner allocation are identical for a given set of exogenous and endogenous states. As a result, the scope for divergence between the two allocations is limited by construction, and the states of the world in which the constraint binds are almost "efficient" by definition: i.e., financial crises become efficient outcomes. To avoid this crucial pitfall, we use constrained rather than conditional efficiency. Constrained efficiency is also consistent with a standard Ramsey approach to optimal taxation.[10]

Since in the social planner problem there are no taxes or subsidies, we rewrite (16) as

$$B_{t+1} \geqslant -\frac{1-\phi}{\phi}\left[A_t^T\left(H_t^T\right)^{1-\alpha^T}+\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{1-\frac{1}{\kappa}}\right]. \tag{17}$$

We can then obtain the first-order conditions for the planner's problem as

$$C_T:\left(C_{j,t}-\frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}\left(\frac{\omega C}{C^T}\right)^{\frac{1}{\kappa}}=\mu_{1,t}^{SP}+ \tag{18}$$

$$-\frac{\lambda_t^{SP}}{\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega}\right)^{\frac{1-\kappa}{\kappa}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{\frac{\kappa-1}{\kappa}},$$

$$C_N:\left(C_{j,t}-\frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}C^{\frac{1}{\kappa}}=\mu_{2,t}^{SP}, \tag{19}$$

$$B_{t+1}:\mu_{1,t}^{SP}=\lambda_t^{SP}+\beta(1+i)E_t\left[\mu_{1,t+1}^{SP}\right], \tag{20}$$

$$H_t^T:\left(C_t-\frac{H_t^{\delta}}{\delta}\right)^{-\rho}\left(H_t^{\delta-1}\right)=\left(1-\alpha^T\right)\mu_{1,t}^{SP}A_t^TH_t^{-\alpha^T}+\frac{1-\phi}{\phi}\lambda_t^{SP}\left(1-\alpha^T\right)A_t^TH_t^{-\alpha^T}, \tag{21}$$

and

$$H_t^N:\left(C_t-\frac{H_t^{\delta}}{\delta}\right)^{-\rho}\left(H_t^{\delta-1}\right)=\left(1-\alpha^N\right)\mu_{2,t}^{SP}A_t\left(H_t^N\right)^{-\alpha^N} \tag{22}$$

$$+\frac{1-\phi}{\phi}\lambda_t^{SP}\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\left(1-\alpha^N\right)\left(A_t^N\right)^{\frac{\kappa-1}{\kappa}}\left(H_t^N\right)^{\left(1-\alpha^N\right)\frac{\kappa-1}{\kappa}-1},$$

where $\mu_{1,t}^{SP}$ is the Lagrange multiplier on (14), $\mu_{2,t}^{SP}$ is the Lagrange multiplier on (15) and λ_t^{SP} is the multiplier on (17).

An extensive discussion on the comparison between the FOCs in the CE and the SP of this model can be found in Benigno et al. (2012). Here we note only that in equation (18), in choosing tradable consumption, the planner takes into account the effects that this choice has on the value of the collateral. This effect is usually referred to as "pecuniary externality".

We now discuss how to implement the social planner allocation in a decentralized equilibrium. The following proposition holds.

Proposition 1. *Given the set of available tax policy tools ($\tau^B, \tau^N, \tau^T, \tau^D, T^D, T^C$), there exists a combination of policy rules for (τ^T, τ^D, T^D, T^C) (a tax scheme for brevity) that can implement the social planner equilibrium of our production economy. These policy rules are time consistent.*

[10] Adopting one criterion versus the other also affects the normative analysis in terms of implementation of the social planner problem, as the combination of taxes and transfers that implements the social planner allocation differ depending on the definition of efficiency adopted.

In order to implement the social planner equilibrium we compare the first order conditions of the two allocations and seek policy rules for our tools that equalize them. Note first that we can correct the distortion in the marginal utility of tradable consumption by using the tax on tradable goods: in fact by comparing (18) with (5) we have that

$$\left(1+\tau_t^T\right)^{SP} = \left(1 - \frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega C^N}\right)^{\frac{1-\kappa}{\kappa}}\left(A_t^N\left(H_t^N\right)^{1-\alpha^N}\right)^{\frac{\kappa-1}{\kappa}}\right) < 1,$$

where the right hand side is evaluated at the social planner allocation, and the policy intervention subsidizes the consumption of tradable goods. Second, by setting $\tau^T = \left(\tau^T\right)^{SP}$ and $\tau^N = 0$ into (9) we obtain that the intratemporal allocation of consumption in the social planner allocation (the ratio of (18) over (19)) is also implemented. Third, note that the intertemporal allocation of consumption has the same expression for both the planner (equation (20)) and the competitive equilibrium (equation (7)) once we set $\tau_t^B = 0$. So there is no need to set a tax on international borrowing to implementation the social planner equilibrium in our economy. Fourth, note that the intratemporal allocation of labor is modified in the planner problem. In fact from (21) and (22), we have that:

$$\frac{\mu_{2,t}^{SP}}{\mu_{1,t}^{SP}} = \frac{\left(1-\alpha^T\right)A_t^T\left(H_t^T\right)^{-\alpha^T}}{\left(1-\alpha^N\right)A_t^N\left(H_t^N\right)^{-\alpha^N}}\frac{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right)}{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{2,t}^{SP}}\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\right)},$$

which governs how labor is allocated between the tradables and nontradables sectors. The corresponding condition in the competitive allocation is

$$P_t^N = \frac{\left(1-\alpha^T\right)A_t^T\left(H_t^T\right)^{-\alpha^T}}{\left(1-\tau^D\right)\left(1-\alpha^N\right)A_t^N\left(H_t^N\right)^{-\alpha^N}}.$$

It follows that, τ_t^D can be used to equalize this margin between the two allocations by setting τ^D such that

$$\frac{1}{(1-\tau^D)^{SP}} = \frac{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right)}{\left(1+\frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{2,t}^{SP}}\frac{(1-\omega)^{\frac{1}{\kappa}}\left(C_t^N\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}\left(C_t^T\right)^{-\frac{1}{\kappa}}}\frac{\kappa-1}{\kappa}\right)}$$

where $\frac{1}{(1-\tau^D)^{SP}} < 1 (> 1)$ depending on the elasticity of intratemporal substitution $\kappa > 1 (< 1)$.

Note finally that when we use $\left(\tau^D\right)^{SP}$ and $\left(\tau^T\right)^{SP}$ as described above, we will have that, when the constraint binds, it will be affected by the tax scheme in the decentralized equilibrium. In fact we can rewrite the borrowing constraint in the decentralized equilibrium as

$$B_{t+1} = -\frac{1-\phi}{\phi}\left[Y_t^T + (1-\left(\tau_t^D\right)^{SP})P_t^N Y_t^N - T_t^D\right]$$

with

$$\frac{P_t^N}{\left(1+\left(\tau^T\right)^{SP}\right)} = \left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}}.$$

In general, the expression for the borrowing constraint will differ between the competitive and the social planner allocation because of the presence of taxes in the competitive equilibrium one. So we need to find the a combination of taxes such that

$$B_{t+1} = -\frac{1-\phi}{\phi}\left[Y_t^T + \left(P_t^N\right)^{SP} Y_t^N\right].$$

To do so, denote with $\left(P_t^N\right)^{SP}$ the relative price of non-tradable in the social planner allocation:

$$\left(P_t^N\right)^{SP} = \left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}}.$$

so that

$$\left(P_t^N\right)^{SP}\left(1+\left(\tau^T\right)^{SP}\right) = P_t^N$$

evaluating the price at the same (SP) allocation. This means that we can set T_t^D in the competitive equilibrium allocation such that

$$\left(T_t^D\right)^{SP} = -\left(\tau_t^D\right)^{SP}\left(1+\left(\tau^T\right)^{SP}\right)\left(P_t^N\right)^{SP} Y_t^N.$$

So the triplet $\left(\tau^N, \tau^T, T^D\right)^{SP}$ with T^C satisfying the government budget constraint will be sufficient to replicate the SP allocation when the constraint binds.

When the constraint does not bind, it is easy to see by inspection that the first order conditions of the social planner and the competitive equilibrium are the same so that there is no need to use any tax tools.

The social planner problem above is recursive. Therefore the tax scheme that implements it in a decentralized equilibrium is time-consistent. QED.

To understand the intuition behind the implementation of the social planner equilibrium, it is important to note that this planner is constrained by the international borrowing constraint—i.e., he takes the constraint as given. Thus, the planner makes consumption and labor decisions with the aim of limiting the consequences of the constraint. Essentially, the social planner takes into account the effect of the pecuniary externality on the marginal utility of tradable consumption and on the reallocation of labor across sectors. When the constraint binds, private agents consume less tradable consumption than the social planner so that in order to restore efficiency the planner subsidizes tradable consumption. When the constraint binds, the planner also allocates sectoral labor depending on the intratemporal elasticity of substitution κ. If tradables and nontradables consumption are complements ($\kappa < 1$) then the implementation of the social planner problem requires to set $\tau^D > 0$, i.e. to tax the revenues from non-tradable production to shift production towards tradable goods relative to the competitive equilibrium. By doing this, the planner limits the consequences of the binding constraint by increasing the relative price of non-tradables. In contrast, in the competitive equilibrium, private agents will just increase total labor supply without taking into account the effect of the pecuniary externality on the allocation of resources within sectors.

Three remarks are in order here. First, as Figure A1 in the appendix shows, the set of policy rules that implement the social planner allocation of our economy is a set of *ex post* interventions that do not include any tax on borrowing or any *ex ante* intervention. Despite the fact that the set of policy rules that implement the social planner allocation entails interventions only in crisis times when the borrowing constraint binds, the SP allocation and the CE allocation differs also in normal times: this difference arises because *ex ante* behavior takes into account *ex post* intervention; an intuition that, as we shall see, carry over to our numerical optimal policy results below.

Second, this tax scheme is not unique. In fact it is possible to design an alternative tax policy scheme with no lump-sum transfers or lump-sum taxes that also implements the social planner allocation (shown in the appendix). In both cases though we note that there is no need to adopt a policy of capital controls ($\tau^B = 0$) or ex ante policy interventions.

Third, as we noted, the social planner problem is recursive and the triplet $\left(\tau^N, \tau^T, T^D\right)^{SP}$ that decentralizes the allocation is time-consistent. Therefore, the tax scheme that implements the social planner allocation therefore is a set of time consistent intervention in crises times (i.e. when the borrowing constraint binds).

3.2 The Unconstrained First Best Equilibrium

In the previous section we showed that there exist a tax scheme that can implement the social planner allocation. Here we first characterize the unconstrained equilibrium (i.e. the first best) and then we show how the same set of taxes that implement the social planner allocation, can also replicate the unconstrained first best when set optimally by a Ramsey planner.

We define the first best for our small open, economy as the unconstrained allocation in which households and firms maximize utility and profits subject to budget and technological constraints.[11] Let us use the superscript "FB" to denote the first-best unconstrained optimal allocation. Then we can rewrite the first order conditions of the household (7), (10), (9) as

$$\left(C_t^{FB} - \frac{\left(H_t^{FB}\right)^{\delta}}{\delta}\right)^{-\rho} \left(\frac{\omega C_t^{FB}}{C_t^{T,FB}}\right)^{\frac{1}{\kappa}} = \mu_t^{FB} \tag{23}$$

$$B_{t+1} : \mu_t^{FB} = \beta \left(1+i\right) E_t \left[\mu_{t+1}^{FB}\right], \tag{24}$$

$$P_t^{N,FB} = \left(\frac{1-\omega}{\omega} \frac{C_t^{T,FB}}{C_t^{N,FB}}\right)^{\frac{1}{\kappa}}, \tag{25}$$

$$(H_t^{FB})^{\delta-1} = \left(\frac{\omega C_t^{FB}}{C_t^{T,FB}}\right)^{\frac{1}{\kappa}} W_t^{FB}. \tag{26}$$

[11] We note here that there are still inefficient variations in consumption due to lack of state contingent debt.

From the firms' maximization problem we obtain

$$P_t^{N,FB} = \frac{\left(1-\alpha^T\right) A_t^T \left(H_t^{T,FB}\right)^{-\alpha^T}}{\left(1-\alpha^N\right) A_t^N \left(H_t^{N,FB}\right)^{-\alpha^N}}, \tag{27}$$

$$W_t^{FB} = (1-\alpha^N) P_t^{N,FB} A_t^N (H_t^{N,FB})^{-\alpha^N}. \tag{28}$$

The following proposition shows that there exists a different, optimal schedule for the same set of taxes that can replicate the first best.

Proposition 2. *Let* $\tau_t^N \leq \bar{\tau}_t^N$ *where* $\bar{\tau}_t^N > -1$ *such as* $P_t^{N,CE} = \frac{1}{1+\bar{\tau}_t^N} P_t^{N,FB}$ *solves the following borrowing constraint equation (if there is no solution then let* $\bar{\tau}_t^N = \infty$*)*

$$B_{t+1}^{FB} + \frac{1-\phi}{\phi}\left[A_t^T (H_t^{T,FB})^{1-\alpha^T} + \frac{1}{1+\bar{\tau}_t^N} P_t^{N,FB} C_t^{N,FB}\right] = 0,$$

and let $\tau_t^D = \tau_t^N$*. Then this pair of tax policy rules replicates the first best unconstrained optimal in the competitive equilibrium solution. These tax rules are also time-consistent optimal policy rules.*

Proof. We only need to check that the first best solution satisfies all the equilibrium conditions. From the latter specification along with the relative price condition (9) and the wage equation (11), the competitive equilibrium solution satisfies

$$\begin{aligned} W_t &= (1-\tau_t^D)\left(1-\alpha^N\right) P_t^N A_t^N \left(H_t^N\right)^{-\alpha^N} \\ &= (1-\alpha^N)\left(\frac{1-\omega}{\omega}\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\kappa}} A_t^N (H_t^N)^{-\alpha^N} \end{aligned}$$

since the 2 taxes cancel out each other. Note this wage equation recovers the wage in the first best unconstrained solution, W_t^{FB}, if we use $C_t^{T,FB}$, $C_t^{N,FB}$, and $H_t^{N,FB}$ in places of C_t^T, C_t^N, and H_t^N. In addition, since the first best solution does not make the borrowing constraint binding under the current setting of τ_t^N, the Lagrange multiplier $\lambda_t \equiv 0$ under the first best solution. So the (10) and (7) hold under the first best unconstrained solution. Since these are the only conditions distorted by the taxes and the borrowing constraint, we conclude that we achieve the first best unconstrained solution via the tax schedule on τ_t^N and τ_t^D.

Since the tax schedule achieves the first best unconstrained optimal, the government has no incentive to deviate from the schedule at any point of time. Therefore such tax schedule is also a time consistent optimal policy. *Q.E.D.*

Corollary: *There exists a combination of* (τ_t^D, τ_t^T) *that replicates the first best unconstrained allocation and it is time-consistent.*

Let the optimal non-tradable consumption tax be τ_t^N. It is easy to see that if we set $\frac{1}{1+\tau_t^T} = 1+\tau_t^N$, we achieve the first best unconstrained allocation and $\lambda_t \equiv 0$ for all t. Now, since the tax on tradable consumption affects also the intertemporal allocation of resources, we need to show that the tax policy that replicates the unconstrained first best equilibrium is constant so that the intertemporal margin is not affected. By comparing Euler equations in both the unconstrained and the constrained competitive equilibrium, and using $\lambda_t \equiv 0$, it is sufficient to find τ_t^T so that

$$\frac{1}{1+\tau_t^T} = \frac{E_t\left[\frac{u'(C_{t+1}^{FB})C_{C_{t+1}^T}^{FB}}{1+\tau_{t+1}^T}\right]}{E_t[u'(C_{t+1}^{FB})C_{C_{t+1}^T}^{FB}]} \tag{29}$$

and the international borrowing constraint is satisfied, in order for the competitive equilibrium to achieve the unconstrained first-best allocation.

First note that a constant tax policy will satisfy (29). Secondly, by inspection of the first-best unconstrained allocation, we can see that the non-tradable price has a strictly positive lower limit. Therefore there exists $\underline{\tau}^T$ such that the borrowing constraint is always satisfied for any $\tau^T \geqslant \underline{\tau}^T$. Thus, any constant tax policy of the form $\tau_t^T \equiv \tau^T \geqslant \underline{\tau}^T$ is an optimal policy such that the competitive equilibrium replicates the first best unconstrained allocation.

As in the previous proposition, such policy is time-consistent as there are no incentives to deviate from it. QED.

From this analysis it follows that if we allow a Ramsey planner to choose the optimal combination of taxes on tradable consumption and on non-tradable profits, it is possible to replicate the first best unconstrained equilibrium. In fact the Ramsey planner can undo the borrowing constraint altogether by affecting the key market price that enters it with the consumption tax and use the other available policy tools to undo the distortions created by the consumption taxes.

Three remarks are in order here. First, we note that (τ_t^D, τ_t^T) is the same set of policy tools that we used to decentralize the social planner allocation. The key difference is that when used optimally, they undo the constraint altogether, while the social planner was taking the borrowing constraint as given. Therefore financial crises never occurs in this equilibrium. This result implies that in the social planner allocation the combination of taxes (τ_t^D, τ_t^T) is suboptimal. Based on these results, and in light of the fact that Ramsey optimal policy is generally not time-consistent, below we shall advocate the use of a Markov-Perfect optimal policy approach to the normative analysis of this class of models. In practice, given the set of policy instrument selected, it is useful to follow an approach to policy design that can attains all the welfare gains that are within reach of the policy instruments selected.

Second, our combination of policy tools requires a commitment to a certain tax schedules when the economy enters a crisis state. However, since in the first best unconstrained equilibrium crises cease to occur, this is just a promise to intervene in the case of a financial crisis, rather than an actual intervention like in the case of the social planner allocation in which crisis continue to occur. Moreover, since the promised interventions are time-consistent, such a commitment is fully credible. In contrast, the implementation of the social planner allocation with the same taxes requires that they are actually used when the constraint binds. And this may rise traditional feasibility issues.

Third and finally, like in the case of the implementation of the social planner allocation, the tax schedules that replicate the first best do not rely a the tax on debt, τ^B, or any other actual ex ante policy intervention. As we shall see below, however, once we limit the set of available tools in such a way that the first best cannot be reached, optimal policy will contain precautionary components every time that the instruments assigned to the policy maker cannot address the crisis effectively.

4 Solution Methods

In this section we describe the global solution methods that we use to compute the equilibria that we consider. We will present results for three different equilibria in section 4. The first is the competitive equilibrium of the model where there is no intervention on the part of the government. The second is the social planner equilibrium that is the benchmark used in the related literature. The third is a Markov-Perfect optimal policy equilibrium that is time-consistent.[12] The competitive and social planner solution algorithms are those used by Benigno at al. 2012) so here we simply summarize them. The algorithm for the solution of the Markov-Perfect optimal policy problem is a novel contribution of this paper, and we provide more details.

We focus on Markov-Perfect optimal policy, which is time-consistent, because it is more relevant for policymakers lacking credible commitment devices as the massive policy interventions during the global financial crisis have shown.[13] In fact, the tax schedules that we discussed in the previous section to implement the social planner allocation or the first best allocation are time-consistent. But as we shall see below, in general, Ramsey optimal policy in this class of models is time-inconsistent.

4.1 Competitive Equilibrium and Social Planner Solutions

As in Benigno et al. (2012) the algorithm for the solution of the competitive equilibrium is derived from Baxter (1990) and Coleman (1991), and involves iterating on the functional equations that characterize a recursive competitive equilibrium in the states $\left(B_t, A_t^T\right)$. The key step is to transform the complementary slackness conditions on the borrowing constraint into a set of nonlinear equations that can be solved using standard methods (in particular, a modified Powell's method). In particular, the expression $\max\left\{\lambda_t^*, 0\right\}^2$ replaces the Lagrange multiplier λ_t (so that that $\max\left\{-\lambda_t^*, 0\right\}^2 = 0$ if the constraint binds and $\lambda_t^* > 0$) while the single nonlinear equation

$$\max\left\{-\lambda_t^*, 0\right\}^2 = B_{t+1} + \frac{1-\varphi}{\varphi}\left(A_t^T\left(H_t^T\right)^{1-\alpha_T} + P_t^N A^N\left(H_t^N\right)^{1-\alpha_N}\right).$$

[12]While the tax schedules that we discussed in the previous section, that implement the social planner allocation or the first best allocation, are time-consistent in general optimal policy in this class of models is time-incosistent. Here we shall focus on Markov-Perfect optimal policy which is time-consistent.

[13]As Chari and Kehoe (2009) noted, "discretion" is a more realistic description than "commitment" of the incentives faced by actual policy makers in this model environment.

replaces the set of complementary slackness conditions:

$$\lambda_t \geq 0$$

$$B_{t+1} + \frac{1-\varphi}{\varphi}\left(A_t^T \left(H_t^T\right)^{1-\alpha_T} + P_t^N A \left(H_t^N\right)^{1-\alpha_N}\right) \geq 0$$

$$\lambda_t \left(B_{t+1} + \frac{1-\varphi}{\varphi}\left(A_t^T \left(H_t^T\right)^{1-\alpha_T} + P_t^N A \left(H_t^N\right)^{1-\alpha_N}\right)\right) = 0.$$

Now, define

$$\eta_{t+1} = E_t\left[\mu_{t+1}\right]$$

the expected value of the marginal utility of tradable consumption; then fix a grid for B, guess a function $\eta_{t+1} = G_\eta\left(B_{t+1}, A_t^T\right)$, and solve for $\left\{\lambda_t^*, \mu_t, B_{t+1}, C_t^T, C_t^N, H_t^T, H_t^N, P_t^N\right\}$ at each value of $\left(B_t, A_t^T\right)$. To evaluate G_η at values of B_{t+1} that are not on the grid for B we parametrize G_η using a cubic spline and solve the resulting system of nonlinear equations with a standard solver. The solution for μ_t is then used to update the G_η function to convergence.[14] Given the solution for the equilibrium decision rules, we can compute the equilibrium value of the lifetime utility by solving the functional equation

$$V\left(B_t, A_t^T\right) = \frac{1}{1-\rho}\left(C_t - \frac{1}{\delta}\left(H_t\right)^\delta\right)^{1-\rho} + \beta E\left[V\left(B_{t+1}, A_{t+1}^T\right)|A_t^T\right],$$

which defines a contraction mapping and thus has a unique solution.[15]

For the social planner equilibrium, following Benigno et al. (2012), we solve a standard dynamic programming problem with Bellman equation

$$V^{SP}\left(B_t, A_t^T\right) = \max_{C_t^T, C_t^N, H_t^T, H_t^N, B_{t+1}, P_t^N}\left\{\frac{1}{1-\rho}\left(C_t - \frac{1}{\delta}\left(H_t\right)^\delta\right)^{1-\rho} + \beta E\left[V^{SP}\left(B_{t+1}, A_{t+1}^T\right)|A_t^T\right]\right\}$$

subject to the resource constraints, the borrowing constraint, the definition of C_t, and the equation that determines P^N:

$$C_t^T = (1+r)\,B_t + A_t^T\left(H_t^T\right)^{1-\alpha_T} - B_{t+1} \text{ and } C_t^N = A^N\left(H_t^N\right)^{1-\alpha_N}$$

$$B_{t+1} \geq -\frac{1-\varphi}{\varphi}\left(A_t^T\left(H_t^T\right)^{1-\alpha_T} + P_t^N A^N\left(H_t^N\right)^{1-\alpha_N}\right)$$

$$P_t^N = \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(\frac{C_t^N}{C_t^T}\right)^{-\frac{1}{\kappa}}.$$

Again we again parametrize the function V^{SP} using a cubic spline to evaluate it at values of B_{t+1} that are not on the grid for B, and solve the maximization problem using feasible sequential quadratic programming.

[14] Note also that $\lambda_t = \max\left\{\lambda_t^*, 0\right\}^2 \geq 0$, $\max\left\{-\lambda_t^*, 0\right\}^2 \geq 0$, and $\max\left\{\lambda_t^*, 0\right\}^2 \max\left\{-\lambda_t^*, 0\right\}^2 = 0$ so the complementary slackness conditions are satisfied.

[15] This functional equation gives us lifetime utility only in equilibrium. To obtain lifetime utility outside equilibrium, we would need to solve the household problem separating individual debt b from aggregate debt B; a calculation that is straightforward to do.

4.2 Markov-Perfect Optimal Policy Solution

In section 5 below, we report Markov-Perfect optimal policy (or optimal policy for brevity) for several alternative sets of taxes. In general, we shall focus on optimal policy for cases in which the set of policy tools is constrained in an arbitrary way so that it is not possible to replicate the first best unconstrained equilibrium. We note here that Ramsey optimal policy in this class of models can be time-inconsistent, even though the tax schedules that we discussed in the previous section, which can implement the social planner allocation or the first best allocation, are time-consistent. For this reason, in the numerical optimal policy analysis we shall focus on Markov-Perfect equilibria which are time-consistent.[16]

The optimal policy solution algorithm is always the same. So here we explain only the solution for the case in which the Ramsey planner has two specific instruments as an example, a tax on debt and a tax on non-tradable consumption. Optimal policy with one of these two instruments simply sets to zero the other instrument in all equations below. Optimal policy with other sets of instruments simply uses different model equations correspondingly modified.

While the presentation in this section is model-specific, the optimal policy solution algorithm can be applied in a wider set of cases for models in which the credit constraint is occasionally binding.

Broadly speaking, an optimal policy is a state contingent tax plan that maximizes the lifetime utility of the representative agent. Thus, the optimization problem of the current government is given by

$$V^{OP}\left(B_t, A_t^T\right) = \max_{x_t} \left\{ \frac{1}{1-\rho} \left(C_t - \delta^{-1} H_t^{\delta}\right)^{1-\rho} + \beta E\left[V^{OP}\left(B_{t+1}, A_t^T\right) | A_t^T\right] \right\} \tag{30}$$

subject to

$$C_t^T = (1+R) B_t + A_t^T \left(H_t^T\right)^{1-\alpha_T} - B_{t+1} \tag{31}$$

$$C_t^N = A^N \left(H_t^N\right)^{1-\alpha_N} \tag{32}$$

$$\left(1-\tau_t^B\right)\mu_t = \beta(1+R) G_\eta\left(B_{t+1}, A_{t+1}^T\right) + \max\left\{\lambda_t^*, 0\right\}^2 \tag{33}$$

$$\mu_t = \left(C_t - \delta^{-1}\left(H_t\right)^{\delta}\right)^{-\rho} C_t^{-1} \omega^{\frac{1}{\kappa}} \left(C_t^T\right)^{-\frac{1}{\kappa}} \tag{34}$$

$$P_t^N = \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{C_t^N}{C_t^T}\right)^{-\frac{1}{\kappa}} \frac{1}{1+\tau_t^N} \tag{35}$$

$$\left(C_t - \delta^{-1}\left(H_t\right)^{\delta}\right)^{-\rho} \left(H_t\right)^{\delta-1} = P_t^N \left(1-\alpha_N\right) A^N \left(H_t^N\right)^{-\alpha_N} \left(\mu_t + \frac{1-\varphi}{\varphi} \max\left\{\lambda_t^*, 0\right\}^2\right) \tag{36}$$

$$P_t^N \left(1-\alpha_N\right) A^N \left(H_t^N\right)^{-\alpha_N} = A_t^T \left(1-\alpha_T\right) \left(H_t^T\right)^{-\alpha_T} \tag{37}$$

$$\max\left\{-\lambda_t^*, 0\right\}^2 = B_{t+1} + \frac{1-\varphi}{\varphi} \left(A_t^T \left(H_t^T\right)^{1-\alpha_T} + P_t^N A^N \left(H_t^N\right)^{1-\alpha_N}\right) \tag{38}$$

$$-\tau_t^B B_{t+1} + \tau_t^N P_t^N C_t^N + T_t = 0, \tag{39}$$

[16] As Chari and Kehoe (2009) noted, "discretion" is a more realistic description than "commitment" of the incentives faced by actual policy makers in this model environment.

where $x_t = \left\{\tau_t^B, \tau_t^N, C_t^T, C_t^N, H_t^T, H_t^N, P_t^N, \lambda_t^*, \mu_t\right\}\left(B_t, A_t^T\right)$. These constraints are the complete set of conditions describing the competitive equilibrium of the model.

A formal definition of a Markov-Perfect Ramsey optimal policy equilibrium can now be stated. To simplify such definition, let's first redefine the competitive equilibrium.

Definition 1. *A recursive competitive equilibrium, given the government policy functions* $\left(\tau^N, \tau^B, T\right)\left(B, A^T\right)$*, is an equilibrium value function* $V^{OP}\left(B, A^T\right)$ *and a set of equilibrium functions* $\left(C^T, C^N, H^T, H^N, B', P^N, \lambda^*, \mu, T\right)\left(B, A^T\right)$ *such that*

1. $\left(C^T, C^N, H^T, H^N, B', P^N, \lambda^*, \mu, T\right)\left(B, A^T\right)$ *solve equations (31)-(39) and*
2. $V^{OP}\left(B, A^T\right)$ *solves*

$$V^{OP}\left(B, A^T\right) = \frac{1}{1-\rho}\left(C\left(B, A^T\right) - \delta^{-1}\left(H^T\left(B, A^T\right) + H^N\left(B, A^T\right)\right)^{\delta}\right)^{1-\rho} + \tag{40}$$

$$\beta E\left[V^{OP}\left(B'\left(B, A^T\right), A^{T\prime}\right)|A^T\right] \tag{41}$$

given $\left(C^T, C^N, H^T, H^N, B', P^N, \lambda^*, \mu\right)\left(B, A^T\right)$.

Definition 2. ***A Markov-Perfect optimal policy equilibrium*** *is a recursive competitive equilibrium as defined above in which* $\left(\tau^N, \tau^B, T\right)\left(B, A^T\right)$ *satisfies*

$$\left(\tau^N, \tau^B\right)\left(B, A^T\right) \in \underset{\tau^N, \tau^B, T}{\operatorname{argmax}}\left\{\begin{array}{c} \frac{1}{1-\rho}\left(C\left(B, A^T\right) - \delta^{-1}\left(H^T\left(B, A^T\right) + H^N\left(B, A^T\right)\right)^{\delta}\right)^{1-\rho} + \\ \beta E\left[V^{OP}\left(B'\left(B, A^T\right), A^{T\prime}\right)|A^T\right] \end{array}\right\} \tag{42}$$

given $\left(C^T, C^N, H^T, H^N, B', P^N, \lambda^*, \mu, V^{OP}\right)\left(B, A^T\right)$.

Two remarks are in order here. First, this definition corresponds to the "compact" equilibrium definition of Krusell (2002) and Klein, Krusell, and Ríos-Rull (2009)—that is, it does not explicitly write the competitive equilibrium prices and quantities as a function of the government policy tools. As a result, the objective function depends on the policy instruments only implicitly through the set of constraints on the problem. In a more explicit definition of the equilibrium the tax instruments would be arguments of the endogenous variables (such as for instance $C^T\left(B, A^T, \tau^N, \tau^B, T\right)$). Then one would specify that the taxes are chosen so as to maximize household utility, so that for example $\tau^i = \psi^i\left(B, A^T\right)$; substitution then yields $C^T\left(B, A^T, \psi^N\left(B, A^T\right), \psi^B\left(B, A^T\right), T\left(B, A^T\right)\right)$. Here we adopted a compact equilibrium definition for ease of notation without any loss of generality and without imposing any particular restriction on our problem.

Second, there is an important issue of commitment (time inconsistency) versus discretion. Time inconsistency arises whenever future governments may want to choose different optimal policies than the ones chosen today (or in the past). In our set up, the incentive to deviate tomorrow from the optimal policy chosen today arises because the constraints faced by tomorrow's government may change if the borrowing limits binds. A way to see how a time inconsistency problem arises in our set up is to look at the government decision problem at time $t-1$, when the Euler equation is one of the constraint on our optimal policy problem and it involves a decision variable at time t, namely μ_t:

$$\mu_{t-1}(1-\tau^B) = \beta\,(1+R)\,E_{t-1}\,[\mu_t] + \lambda_{t-1}.$$

Under commitment, this constraint represents a promise regarding the value of the marginal utility of tradable consumption at time t, μ_t. Under discretion, the government chooses μ_t at time t without taking such promise into account; that is, the implicit promise made at time $t-1$ regarding marginal utility at time t might not be respected. This means that government decisions at time t need not to internalize how they affect μ_{t-1}. As a result, the optimal plans under commitment will generally differ from those under discretion in our model set.[17]

The computational algorithm we set up is based on backward iteration and in practice consists of three steps. First, we guess the value function $V^{OP,0}\left(B, A^T\right)$ and the marginal utility function $\mu^0\left(B, A^T\right)$. Second, we solve an optimization step to find government taxes and the corresponding resource allocations by the agent. Third, we update the value function given the results of the optimization step. The last two steps are iterated till convergence. Thus:

1. Guess $V^{OP,0}\left(B, A^T\right)$ and $\mu^0\left(B, A^T\right)$, and compute

$$\begin{aligned} G_V\left(B'.A^T\right) &= E\left[V^{OP,0}\left(B', A^{T\prime}\right)|A^T\right] \\ G_\eta\left(B', A^T\right) &= E\left[\mu^0\left(B', A^{T\prime}\right)|A^T\right]. \end{aligned}$$

2. Solve the constrained maximization problem for $\left(\tau^N, \tau^B, C^T, C^N, B', \lambda^*, \mu, P^N, H^T, H^N, T\right)$ as functions of $\left(B, A^T\right)$. In this step we impose nonnegativity constraints on those variables that require them (consumption, labor supply, and prices). The constraint that the argument of the utility function is positive is imposed as well:

$$\left(\omega^{\frac{1}{\kappa}}\left(C^T\right)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}}\left(C^N\right)^{\frac{\kappa-1}{\kappa}}\right)^{\frac{\kappa}{\kappa-1}} - \delta^{-1}\left(H^T + H^N\right)^{\delta} > 0.$$

3. Finally, update the value function using

$$\begin{aligned} V^{OP,1}\left(B, A^T\right) &= \frac{1}{1-\rho}\left(C\left(B, A^T\right) - \delta^{-1}\left(H^T\left(B, A^T\right) + H^N\left(B, A^T\right)\right)^{\delta}\right)^{1-\rho} + \\ &\quad \beta G_V\left(B'\left(B, A^T\right).A^T\right) \end{aligned}$$

   and $\mu^1\left(B, A^T\right)$, and repeat to convergence.

[17] Another way to see the same issue is the following. Marcet and Marimon (2009) show how to construct a state space in which the commitment problem remains recursive by adding past multipliers as state variables. Our assumption here is that the state space $\left(B_t, A_t^T\right)$ limits the set of variables that the current government assumes the future government will take into account in its decisions. That is, we rule out trigger-strategy equilibria (see Bernheim, Ray, and Yeltekin 1999).

Our algorithm locates an infinite-horizon equilibrium (if it exists) that is the limit of a sequence of finite-horizon equilibria, but nothing guarantees that it is unique. Since the operator defined above is not a contraction mapping, nor guaranteed to be monotone, there are no known conditions under which it converges; indeed, it does not converge for some parameter values and from some initial conditions (in general we use the value function for the competitive equilibrium as an initial condition, which technically means we are assuming that the government "at the end of time" will impose no taxes). However, in every case in which it converged for a given set of parameter values, it always converged to the same equilibrium. In addition, we found no evidence of multiple solutions to the optimization problem above. As an aside, smooth equilibria of the sort considered by Klein, Krusell, and Ríos-Rull (2009) do not exist in this class of models, because the policy functions are not differentiable at the point where the constraint binds exactly (that is, where $\lambda^* \left(B, A^T\right) = 0$ in our model).

4.3 Welfare Measure

In section 5 we will rank equilibria under alternative optimal policies by calculating the welfare gain relative to the competitive equilibrium associated with that policy. As usual, welfare is quantified as the percent of consumption that the representative agent is willing to pay at every date and state to move from one equilibrium to the other. In our environment, this quantity is constructed by calculating the welfare gain at each state. The state specific gains are then aggregated using the unconditional probability of being in each state. This measure takes into account the effects of any transitional dynamics associated with the fact that the ergodic distributions for the competitive equilibrium and the different policy equilibria may not have the same support.[18]

Let $V^{SP}\left(B_t, A_t^T\right)$ denote lifetime utility in the social planning or an optimal policy allocation. We first solve the dynamic functional equation

$$v\left(B_t, A_t^T; \chi\right) = \frac{1}{1-\rho}\left(C_t - \delta^{-1}\left(H_t\right)^{\delta}\right)^{1-\rho} + \beta E\left[v\left(B_{t+1}, A_{t+1}^T; \chi\right) | A_t^T\right]$$

where $v\left(B_t, A_t^T; \chi\right)$ is the lifetime utility obtained from evaluating the competitive equilibrium decision rules with an extra χ percent of tradable consumption given freely to the representative household.[19] This functional equation defines a contraction mapping, so it has a unique solution. From the solution of this problem, we can compute the solution to the nonlinear equation

$$V^{SP}\left(B_t, A_t^T\right) = v\left(B_t, A_t^T; \chi\left(B_t, A_t^T\right)\right),$$

which yields the percent increase in tradable consumption that, state-by-state, renders the representative agent indifferent between the competitive equilibrium and the social planner, or any particular optimal policy allocation computed.

[18] Welfare gain and losses are computed as a percentage of tradable consumption. The ranking of the allocations would not change if we were to express welfare gains and losses as a percentage of overall consumption.

[19] Note that the household is not maximizing utility using these decision rules once the extra χ percent of tradable consumption is endowed.

4.4 Equivalence of Alternative Taxes in a Markov Perfect Equilibrium

We now discuss some properties of alternative taxes that are useful in order interpret our numerical results for the case in which the set of policy tools is constrained. It is important to remember here that we are focusing on time-consistent equilibria in which decisions at time t are taken as given from the perspective of time $t-1$. So the results below about the equivalence of alternative tax instruments hold in the Markov-Perfect Equilibria that we compute, but might not hold in a commitment equilibrium. To do so, we first analyze how the presence of the borrowing constraint distorts the different margins of the competitive equilibrium allocation. Once the margins are identified, we will examine how alternative taxes can be used to improve upon the competitive equilibrium allocation.

If we combine (18) and (19) from the social planner problem, we obtain:

$$\frac{\mu_{2,t}^{SP}}{\mu_{1,t}^{SP}} = \left(\frac{(1-\omega)C^T}{\omega C^N}\right)^{\frac{1}{\kappa}} \left(1 - \frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega C^N}\right)^{\frac{1-\kappa}{\kappa}}\right),$$

which is the condition that governs the intratemporal allocation of consumption between tradables and nontradables in the social planner equilibrium. Note here that this condition is distorted when the constraint binds and implies a lower marginal utility of nontradables consumption compared to the equivalent condition that would arise in the competitive equilibrium with taxes:

$$P_t^N = \left(\frac{(1-\omega)C^T}{\omega C^N}\right)^{\frac{1}{\kappa}} \frac{1}{(1+\tau_t^N)}.$$

From this comparison it follows that we can offset the distortion caused by the borrowing constraint on the intratemporal consumption allocation by using τ_t^N to equalize the two margins. To do so we can set the tax as follows:

$$\frac{1}{(1+\tau_t^N)} = \left(1 - \frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}\kappa}\frac{1-\phi}{\phi}\frac{(1-\omega)}{\omega}\left(\frac{(1-\omega)\left(C_t^T\right)}{\omega C^N}\right)^{\frac{1-\kappa}{\kappa}}\right) < 1,$$

which implies $\tau_t^N > 0$. From this expression, it is immediate to see that a tax on tradables consumption would operate similarly to a nontradable consumption tax. This implies that in the Markov-Perfect Ramsey optimal policy equilibrium that we will compute below, a tax on tradable consumption (τ^T), a tax on nontradable consumption (τ^N) or a combination of the two (τ^T, τ^N) generates the exactly same allocation.

We now examine how a binding borrowing limit distorts the labor supply decision. From (21) and (22) we obtain

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho}\left(H_t^{\delta-1}\right) = \left(1-\alpha^T\right)\mu_{1,t}^{SP} A_t^T H_t^{-\alpha^T}\left(1 + \frac{1-\phi}{\phi}\frac{\lambda_t^{SP}}{\mu_{1,t}^{SP}}\right),$$

where

$$\lambda_t^{SP} = \mu_t^{SP} - \beta\left(1+i\right)E_t\left[\mu_{t+1}^{SP}\right]$$

While in the competitive equilibrium allocation we have

$$\left(C_{j,t}-\frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}\left(H_{j,t}^{\delta-1}\right)=\left(1-\alpha^{T}\right)A_{t}^{T}\left(H_{t}^{T}\right)^{-\alpha^{T}}\mu_{t}^{CE}\left(1+\frac{1-\phi}{\phi}\frac{\lambda_{t}^{CE}}{\mu_{t}^{CE}}\right).$$

where

$$\lambda_{t}^{CE}=(1-\tau_{t}^{B})\mu_{t}^{CE}-\beta\left(1+i\right)E_{t}\left[\mu_{t+1}^{CE}\right].$$

First note here that when the constraint binds ($\lambda_t^{CE}>0$), τ_t^B influences the labor supply by altering the value of the multiplier λ_t^{CE}. In fact, for given exogenous states, when the constraint binds, the amount that can be borrowed is not affected by τ^B. In this sense, when the constraint binds, τ_t^B operates like a tax on wages (i.e., τ^L) since both instruments affect the same margin, i.e. the household labor supply decision.[20] A similar argument holds for the case in which the constraint does not bind since in a time-consistent equilibrium the Ramsey planner takes $E_t\left[\mu_{t+1}\right]$ as given. This implies that in our Markov-perfect optimal policy equilibrium τ^B, τ^L or a combination of the two (τ^B,τ^L) would also generate the same allocation.

5 Parameter Values

The model is calibrated at quarterly frequency on Mexican data. There are several reasons to focus on Mexico. First Mexico is an advanced emerging market economy whose experience is particularly relevant for the main issue addressed in the paper. Mexico experienced three major financial crises since 1980: the 1982 debt crisis; the well known "Tequila crisis" in 1994-1995; and the third one in 2008-09, during the global financial crisis, perhaps less severe than the previous two but nonetheless leading Mexico to seek (or accept) IMF financial assistance. Second, Mexico is a well functioning, relatively large, market-based economy in which production in both the tradable and non-tradable sectors of the economy goes well beyond the extraction of natural resources such as oil or other commodities. Indeed, Mexico is an OECD economy whose experience is relevant also for the advanced economies struggling with financial crises like those in the euro zone. Third and finally, there is a substantial body of previous quantitative work on Mexico, starting from Mendoza (1991), which greatly facilitates the choice of the parameter values of the model. In particular, we choose model parameters following the work of Mendoza (2002, 2010) and Kehoe and Ruhl (2008) to the extent possible, and use available data where necessary to complement or update their choices.

The specific set of parameter values that we use are reported in Table 1. We note here that in our calibration of the competitive allocation we set all the distorting policy tools to zero. The elasticity of intertemporal substitution is set to standard value of $\rho=2$, like

[20] A tax on wages would alter only the labor supply margin in the competitive equilibrium as

$$\left(C_{j,t}-\frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho}\left(H_{j,t}^{\delta-1}\right)=\left(1-\tau^{L}\right)\left(1-\alpha^{T}\right)A_{t}^{T}\left(H_{t}^{T}\right)^{-\alpha^{T}}\mu_{t}^{CE}\left(1+\frac{1-\phi}{\phi}\frac{\lambda_{t}^{CE}}{\mu_{t}^{CE}}\right).$$

in Mendoza (2002, 2010). We set then the world interest rate to $i = 0.01587$, which yields an annual real rate of interest of about 6.5 percent like in Mendoza (2002): a value that is between the 5 percent of Kehoe and Ruhl (2008) and the 8.6 percent of Mendoza (2010).

The elasticity of intratemporal substitution in consumption between tradables and non-tradables is an important parameter in the analysis. But there is a good degree of consensus in the literature on its value. We follow Ostry and Reinhart (1992), who estimates a value of $\kappa = 0.760$ for developing countries. This is a conservative assumption compared to the value of 0.5 used by Kehoe and Ruhl (2008) that is closer to the one assumed for an advanced, more closed economy like the United States.

Estimates of the wage elasticity of labor supply in Mexico are uncertain at best (Mendoza, 2002 and 2010). We set the value of $\delta = 1.75$, close to the value of 1.84 adopted by Mendoza (2010). The labor share of income, $(1 - \alpha^T)$ and $(1 - \alpha^N)$ is set to 0.66 in both tradable and non tradable sectors: a standard value, close to that used by Mendoza (2002), and consistent with empirical evidence on the aggregate share of labor income in GDP in household survey of Garcia-Verdu (2005).

The shock to tradable total factor productivity specified as

$$\log\left(A_t^T\right) = \rho_A \log\left(A_{t-1}^T\right) + \varepsilon_t,$$

where ε_t is an iid $\mathrm{N}(0, \sigma_A^2)$ innovation. The parameters of this process are set to $\rho_A = 0.537$ and $\sigma_A = 0.0134$, which are the first autocorrelation and the standard deviation of aggregate total factor productivity reported by Mendoza (2010). Both the average value of A^T and the constant A^N are set to one.

The remaining three model parameters—the share of tradable consumption in the consumption basket (ω), the credit constraint parameter (ϕ), and the discount factor (β)—are set by iterating on a routine that minimizes the sum of squared differences between the moments in the ergodic distribution of the competitive equilibrium of the model and three data targets. The data targets are a C^N/C^T ratio of 1.643, a 35 percent debt-to-GDP ratio, and an unconditional probability of capital flow reversal of 2 percent per quarter. The targeted C^N/C^T ratio is the value implied by the following ratios estimated by Mendoza (2002): Y^T/Y^N =0.648, C^T/Y^T =0.665, and C^N/Y^N =0.708.[21] The debt-to-GDP target is Mexico's average net foreign asset to annual GDP ratio, from 1970 to 2008, in an updated version of the Lane and Milesi-Ferretti (2006) data set.

The target for the unconditional probability of capital flow reversal is more difficult to pin down. Despite a significant body of empirical work, there is no consensus in the literature on how to define financial crises empirically, and hence no accepted measure of their unconditional probability. By focusing on Mexico, we can pin down this target simply and unambiguously, measuring it as the relative frequency, on a quarterly basis, of Mexico's crisis years over the period 1975-2010. This assumes that, as generally accepted, 1982, 1995, and 2009 were crisis years for Mexico. The resulting 2 percent is very close to the 1.9 percent implied by the empirical analysis of Jeanne and Ranciere (2010) over the period 1975-2003, who use an "absolute" definition of crisis as a current account reversals larger than 5 percent of GDP. Our choice is also similar to the 2.2 percent value implied by Calvo, Izquierdo, and Mejía (2008) for the period 1990-2004, based on a "relative" definition

[21] Ratios computed with updated data are essentially the same.

of financial crisis as current account reversals larger than two standard deviations. The two percent value, however, is at the low-end of the range of values estimated in these studies by pooling data for the whole sample of emerging markets considered.

In order to compute the frequency of financial crisis in the ergodic distribution of the model consistent with the empirical literature above we define it as an event in which : (a) $\lambda_t > 0$ (i.e. the international borrowing constraint is binding) and (b) $(B_{t+1} - B_t) > 2\sigma(B_{t+1} - B_t)$ (i.e. the current account or changes in the net foreign asset position in a given period exceed two times its standard deviation). The first criterion is a purely model based definition of crisis. The second criterion allows us to consider only model events in which there are large current account reversals, in line with the aforementioned empirical literature.[22] [23]

With the targets above we obtain $\omega = 0.3526$, $\beta = 0.9717$, and $\phi = 0.415$. The implied value of ω is slightly higher than in Mendoza (2002) and slightly lower than assumed by Kehoe and Ruhl (2008). The implied annual value of β yields an annual discount factor of 0.8915, only slightly lower than in Kehoe and Ruhl (2008).[24] The implied value of ϕ is lower than in Mendoza (2002), who however calibrates it to the deterministic steady state of the model, and there are no standard benchmarks for this model parameter in the literature.

The quantitative performance of the model from a positive perspective is discussed at length in Benigno et al. (2012), where we emphasize its virtues and limits. In particular one of the main drawback of our model is that it implies a counterfactual increase of labor when the constraint binds. As we noted earlier, however, introducing a working capital constraint would help in this direction but would not alter the normative analysis which is the focus of this paper.

6 Optimal Policy

In this section we compute optimal policy for our economy when the set of instruments available limits the possibility of replicating the unconstrained equilibrium. As we outlined in the previous section we compute Markov-Perfect optimal policy for alternative policy tools. We first examine a two instrument case and then consider a set of optimal policy problems in which the policy maker has access to one instrument only. The two instruments that we consider in the baseline case are a tax on non-tradable consumption, τ_t^N (i.e., exchange rate policy), and a tax on debt, τ_t^B (i.e., a direct control on capital flows). We then also report results for a tax on wage income τ^L and a tax on tradable consumption

[22] The definition of financial crisis typically used in the empirical literature focuses on large capital flows reversals because some smaller ones may be due to terms of trade changes or other factors Jeanne and Rancière (2010), for instance, excludes commodity importers and oil producers, while Calvo Izquierdo, and Mejía (2008) add other criteria to the second definition we use above.

[23] Note that balance of payment data may have trends, and hence the empirical literature focuses on changes in the current account, or the first difference of the capital flows. As our model has no trend growth, we focus on the current account rather than its change. We obtain similar results when we define the sudden stop with respect to changes in the current account.

[24] This value is not comparable to the one assumed by Mendoza (2002) as he uses an elastic discount factor specification. In our model, the presence of the borrowing constraint removes the necessity to introduce any device to induce a stationary ergodic distribution of foreign borrowing.

τ^T.

6.1 Two Instruments

Figure 1 reports the decision rules for τ_t^N and τ_t^B (where a positive value is a tax and a negative value is a subsidy), as well as the decision rule for the lump sum transfer, T_t, when the policy maker can use the two tax policy instruments simultaneously. These functions are plotted for the average value of the tradable productivity shock A^T, but qualitatively they do not change in response to changes in A^T (instead they simply shift up and down). Figure 1 also reports the decision rules for borrowing (B_{t+1}), the marginal utility of tradable consumption (μ_t), and the Lagrange multiplier on the collateral constraint (λ_t). Figure 2 reports the decision rules for the relative price of non tradable (P_t^N), total labor supply (H_t), and sector labor allocation (H_t^T and H_t^N, respectively). Figure 3 reports the decision rules for the real wage (W_t), total consumption (C_t), and tradable and non tradable consumption respectively (C_t^N, C_t^T). In addition to the decision rules for the optimal policy allocation, denoted OP(τ_t^N,τ_t^B), we also report the rules for the social planner allocation (SP) and the competitive equilibrium allocation (CE) in which all taxes are set equal to zero.

The optimal policy rules for τ_t^N and τ_t^B are highly non-linear. If the policy maker has access to both a consumption tax and a tax on debt as instruments, the optimal intervention is no intervention before the constraint binds and a tax on debt combined with a subsidy on non-tradable consumption when the constraint binds (Figure 1). Thus, in states of the world in which the constraint is not binding the optimal policy is "passive", i.e., $\tau_t^N = \tau_t^B = 0$. In states of the world in which the constraint is binding, the optimal policy is "active" with a subsidy to non-tradable consumption and a tax on borrowing. Interestingly, the lump sum transfer is always zero.

When the constraint binds, as we saw in Sections 2 and 3, subsidizing non-tradable consumption increases the demand of non-tradable goods (even more than in the CE) and reduces it in the tradable sector (even more than in the SP) (Figure 2). The relative price of nontradable increases (but not as much as in the SP) increasing the value of the collateral and hence borrowing (Figure 2 and 1). When the constraint binds, since the amount of borrowing is determined by the endogenous limit, taxing debt affects the labor supply margin and tends to decrease the amount of labor that is supplied in crisis times for given real wages (10).

The resource allocation under optimal policy is not only quantitatively but also qualitatively different than the social planner one. Labor in the OP(τ_t^N,τ_t^B) allocation is higher than in the CE allocation in the non-tradable sector and lower than in the CE in the tradable sector. In contrast, in the SP allocation, in which total labor is lower than in the OP(τ_t^N,τ_t^B) allocation, labor is lower than in the CE both in the tradable and the non-tradable sector. As a result, the relative price of non-tradable, the real wage, total consumption, and tradable consumption are higher in the SP than both the CE and the OP(τ_t^N,τ_t^B) allocation. Non tradable consumption, instead, is lower in the SP allocation than both in the CE and the OP(τ_t^N,τ_t^B) allocation.

When the constraint does not bind the optimal policy is "no policy action". To understand this result, it is helpful to preview the results for the one instrument case below.

In that situation, the government acts before the constraint binds (i.e., in states in which the constraint does not bind today but may bind with positive probability tomorrow). In that case, the government uses the only tool it has available to reduce the crisis frequency because it cannot effectively mitigate the costs associated with the use of distortionary policy tools during the crisis if one does occur.

In contrast, with two instruments the government could limit these costs. Specifically, by subsidizing nontradable consumption with τ_t^N the government affects the marginal product of labor in the non-tradable sector (see (11)) and decreases the demand for non-tradable labor for any given wage. The government then mitigates this effect by decreasing the total supply of labor through an increase in τ^B. Given the set of tools available, therefore, there is no need to distort the allocation today to ameliorate the allocation tomorrow even when there is a positive probability that the constraint binds tomorrow; in effect, the government today understands that the government tomorrow will act in the event of a crisis and therefore does not need to distort the economy today (when the constraint is not binding).[25]

As in the SP allocation, the OP(τ_t^N,τ_t^B) allocation displays more borrowing and a lower probability of a crisis than the CE allocation. Table 2 reports the ergodic mean of debt in units of tradable consumption. It shows that average borrowing under OP(τ_t^N,τ_t^B) is more than in the CE allocation but less than in the SP allocation. This is consistent with the plot of the decision rules in Figure 1. Table 3 reports the unconditional probability of a financial crisis. Under OP(τ_t^N,τ_t^B), the probability of crisis is lower than in the SP allocation and is close to zero. This suggests that what matters is not necessarily limiting the probability of a crisis but allocating resources more efficiently as in the SP allocation.

Welfare is higher under OP(τ_t^N,τ_t^B) than in the CE. Nonetheless, OP(τ_t^N,τ_t^B) achieves only a fraction of the welfare gains in the SP. Table 4, which reports the welfare gain for all the alternative policy regimes considered compared to the CE equilibrium, shows this. Indeed, Table 4 shows that the welfare gains from OP(τ_t^N,τ_t^B) are one order of magnitude smaller than those in the SP. Note however here that even the welfare gains under SP are very small, consistent with the finding of the rest of the literature.

Consider now optimal policy under alternative pairs of policy instruments. We allow for a tax on wage income, τ^L, as well as a tax on tradable consumption, τ^T, and we consider the following possible combination of taxes: OP(τ^T, τ^B), OP(τ^N, τ^L), OP(τ^T, τ^L), and OP(τ^L, τ^B). Figure 4 reports the decision rules for alternative pairs of policy instruments and the corresponding lump sun transfers, while Figure 5 reports key variables of the corresponding allocations.

When we use alternative pairs of two policy instruments we find that, consistently with the results in Section 4.4, the following allocations coincide exactly, except for the behavior of the lump sum transfer: OP(τ^N, τ^B)=OP(τ^T, τ^B)=OP(τ^N, τ^L)=OP(τ^T, τ^L). These equivalences hold because, as shown in section 4.4, the tax on tradable and non-tradable consumption operates on the same margin, and the tax on debt and the wage tax operate on the labor margin in the same way when the constraint binds.

[25] Note here that the OP(τ_t^N,τ_t^B) allocation remains distant from the SP one. Indeed, as we showed in section 3, it is not possible to achieve the SP with these two specific instruments. OP(τ_t^N,τ_t^B) however satisfies (42).

Finally note that, consistent with the results in section 3, none of these combinations of optimal policies replicates the social planner.

6.2 One Instrument

Consider now a set of optimal policy problems when the policy maker has access to one instrument only. We consider all instruments discussed above, used one at a time: τ_t^N or τ_t^B, τ_t^T or τ_t^L. Figure 6 reports the decision rules for τ_t^N or τ_t^B as well as the for the corresponding lump sum transfer. The benchmark here is the optimal policy problem with two instruments that we discussed above, OP(τ^N, τ^B).

There is a key difference relative to OP(τ^N, τ^B). If the policy maker has only one instrument, the optimal policy is always a tax before the constraint binds and a subsidy while the constraint binds, regardless of the instrument used. Thus, the optimal Ramsey policy continues to be non linear, but now has a precautionary component regardless of the instrument used. These interventions can be interpreted as "leaning against the wind" policies in normal times, either against the real exchange rate or debt, and "bail outs" in crisis times, either in the domestic good market or the international capital market.

The reason for this qualitative difference relative to optimal policy with two instruments is the following. In our production economy, the inefficiency caused by the pecuniary externality distorts different private agents' decisions. As we explained above, when the constraint binds, if the policy maker has two instruments, it is optimal to tax debt and subsidize non-tradable consumption. With one instrument only, when the constraint binds, the policy maker faces bigger costs in the use of τ^N and τ^B respectively since it cannot undo the costs associated with the use of one distortionary tool at a time As a consequence, anticipating this trade off in crisis states, she/he acts before the crisis in a way that lowers the unconditional probability of a crisis happening in the first place. With two instruments, instead, the government can correct the allocation enough to remove the incentive to intervene in a precautionary manner. That is, with two instruments, the welfare cost of intervening *ex ante* is larger than the expected value of the gain stemming from entering a crisis period with the allocation induced by such *ex ante* intervention.

Notwithstanding these similarities, there is an important difference in the effects and the working of optimal policy with alternative instruments. Welfare gains with τ_t^N or τ_t^T are one order of magnitude higher than with τ_t^B or τ_t^L even though the associated probability of crisis is lower with τ_t^B or τ_t^L (Tables 3 and 4). Related to this, we can see that borrowing (and hence tradable consumption that is not reported) is lower under OP(τ_t^B) than OP(τ_t^N) (Table 3).

Finally, by comparing Figure 6 with Figure 1, we can see that the size of the tax interventions, for both instruments, is much smaller with one instrument than in the case in which there are two instruments. The reason is again is associated with the costs of using one instrument at a time without the possibility of mitigating its consequences on agents' private decisions.

7 Conclusions

In this paper we study optimal tax policy in a production economy in which a borrowing constraint binds only occasionally and the financial crisis is an endogenous event. We first show analytically that when the set of policy tools is unconstrained, the same combination of policy instruments that can achieve the constrained efficient allocation could be used optimally by a Ramsey planner to replicate the unconstrained first best equilibrium. We then consider the more realistic case in which the set of policy tools is constrained, so that the first best cannot be achieved. To study this case we propose a numerical solution algorithm that computes optimal Markov-Perfect policy in this environment where the policy function is not differentiable and available methods do not apply. In the numerical analysis of our model, we find that, if the government has two instruments (i.e., a consumption tax and a tax on debt), it is optimal to intervene only during financial crises and not to intervene in normal times. In contrast, when the policy maker has only one policy tool, we find that the optimal policy has a precautionary component regardless of the specific instruments used to intervene.

These results provides a novel rationale for macro-prudential policies. In fact, in broad terms, our analysis suggests that the scope for preemptive policies depends on the effectiveness of crisis resolution policies and their interaction is crucial for the policy design problem. The more effective (ineffective) are ex post policies, there less significant (important) is the scope for ex ante policies. While most of the related literature has focused on the quantity of debt as an indicator for policy intervention (by referring to credit booms or overborrowing) , our analysis suggests that the allocation of resources might be more relevant than the overall quantity of debt. As a consequence policy tools that help allocate resources more efficiently are more effective than the ones aimed at limiting the total amount of available borrowed resources.

One important questions raised by our analysis is whether optimal policy under commitment may differ significantly than under discretion, including in its prudential component. We regard this as a fruitful area of future research.

References

[1] **Adam, Klaus, and Roberto M. Billi.** 2006. "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates." *Journal of Money, Credit and Banking* 38(7): 1877-1905.

[2] **Adam, Klaus, and Roberto M. Billi. 2007.** "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates." *Journal of Monetary Economics* 54(3): 728-52.

[3] **Aghion, Philippe, Philippe Bacchetta, and Abhijit Banerjee.** 2004. "Financial Development and the Instability of Open Economies." *Journal of Monetary Economics* 51(6): 1077-1106.

[4] **Baxter, Marianne.** 1990. "Approximating Suboptimal Dynamic Equilibria: An Euler Equation Approach." *Journal of Monetary Economics* 28(2): 173-200.

[5] **Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young.** 2012a. "Financial Crises and Macro-Prudential Policies." *Journal of International Economics*, forthcoming.

[6] **Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci, and Eric R. Young.** 2012b. "Capital Controls or Exchange Rate Policy? A Pecuniary Externality Perspective." Federal Reserve Bank of St. Louis Working Paper 2012-025A.

[7] **Bernheim, B. Douglas, Debraj Ray, and Şevin Yeltekin. 1999.** "Self-Control, Savings, and the Low Asset Trap." Unpublished.

[8] **Bianchi, Javier. 2011.** "Overborrowing and Systemic Externalities in the Business Cycle." *American Economic Review* 101(7): 3400-26.

[9] **Bianchi, Javier, and Enrique G. Mendoza.** 2010. "Overborrowing, Financial Crises and 'Macro-Prudential' Taxes," National Bureau of Economic Research Working Paper 16091.

[10] **Braggion, Fabio, Lawrence J. Christiano, and Jorge Roldos.** 2009. "Optimal Monetary Policy in a 'Sudden Stop''*Journal of Monetary Economics,* 56, pp. 582-595. Journal of Monetary Economics 56, pp. 582-595." National Bureau of Economic Research Working Paper 13254.

[11] **Caballero, Ricardo J., and Arvind Krishnamurthy.** 2005. "Inflation Targeting and Sudden Stops." In *The Inflation-Targeting Debate*, edited by Ben S. Bernanke and Michael Woodford, 423-46. University of Chicago Press.

[12] **Caballero, Ricardo J., and Stavros Panageas.** 2007. "A Global Equilibrium Model of Sudden Stops and External Liquidity Management." Unpublished.

[13] **Calvo, Guillermo A., Alejandro Izquierdo, and Luis-Fernando Mejía.** 2008. "Systemic Sudden Stops: The Relevance Of Balance-Sheet Effects And Financial Integration." National Bureau of Economic Research Working Papers 14026.

[14] **Chang, Roberto, Luis Cespedes and Andres Velasco** 2012 "Financial Intermediation, Exchange Rates, and Unconventional Policy in an Open Economy", unpublished manuscript.

[15] **Chari, V.V., and Patrick J. Kehoe.** 2009. "Bailouts, Time Inconsistency and Optimal Regulation." Unpublished.

[16] **Coleman, Wilbur John, II.** 1991. "Equilibrium in a Production Economy with an Income Tax." *Econometrica* 59(4): 1091-1104.

[17] **Cúrdia, Vasco.** 2007. "Optimal Monetary Policy under Sudden Stops." Federal Reserve Bank of New York Staff Report No. 278.

[18] **Eaton, Jonathan, and Mark Gersovitz.** 1981. "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies* 48(2): 289-309.

[19] **García-Verdú, Rodrigo.** 2005. "Factor Shares from Household Survey Data." Banco de México Working Paper 2005-05.

[20] **Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman.** 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review* 78(3): 402-17.

[21] **Hevia, Constantino.** 2008. "Optimal Policy with Sudden Stops." Unpublished.

[22] **Jeanne, Olivier, and Anton Korinek.** 2011. "Managing Credit Booms and Busts: A Pigouvian Taxation Approach." Unpublished.

[23] **Jeanne, Olivier, and Romain Rancière.** 2011. "The Optimal Level of International Reserves for Emerging Market Countries: A New Formula and Some Applications." *Economic Journal* 121(555): 905-30.

[24] **Kehoe, Timothy J., and David K. Levine.** 1993. "Debt-Constrained Asset Markets." *Review of Economic Studies* 60(4): 865-88.

[25] **Kehoe, Timothy J., and Kim J. Ruhl.** 2008. "Are Shocks to the Terms of Trade Shocks to Productivity?" *Review of Economic Dynamics* 11(4): 804-19.

[26] **Kim, Jinill, Sunghyun Kim and Robert Kollmann,** 2008, "Solving the Incomplete Markets Model with Aggregate Uncertainty Using a Perturbation Method, "Journal of Economic Dynamics and Control, forthcoming.

[27] **Klein, Paul, Per Krusell, and Jose-Victor Rios-Rull** 2008. "Time-consistent public policy." Review of Economic Studies 75(3), 789-808.

[28] **Krugman, Paul.** 1999. "Balance Sheets, the Transfer Problem, and Financial Crises." *International Tax and Public Finance* 6(4): 459-72.

[29] **Krusell, Per** 2002. "Time-consistent redistribution." European Economic Review 46(4-5), 755-769.

[30] **Lane, Phillip R., and Gian Maria Milesi-Ferretti.** 2007. "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004." *Journal of International Economics* 73(2): 223-50.

[31] **Lorenzoni, Guido.** 2008. "Inefficient Credit Booms." *Review of Economic Studies* 75(3): 809-833.

[32] **Marcet, Albert, and Ramon Marimon.** 2009. "Recursive Contracts." Unpublished.

[33] **Mendoza, Enrique G.** 1991. "Real Business Cycles in a Small Open Economy." *American Economic Review* 81(4): 797-818.

[34] **Mendoza, Enrique G.** 2002. "Credit, Prices, and Crashes: Business Cycles with a Sudden Stop." In *Preventing Currency Crises in Emerging Markets*, edited by Sebastian Edwards and Jeffrey A. Frankel, 335-92. National Bureau of Economic Research.

[35] **Mendoza, Enrique G.** 2010. "Sudden Stops, Financial Crises and Leverage: A Fisherian Deflation of Tobin's q." *American Economic Review* 100(5): 1941-66.

[36] **Ostry, Jonathan D., and Carmen Reinhart.** 1992. "Private Saving and Terms of Trade Shocks: Evidence from Developing Countries." *IMF Staff Papers* 39(3): 495-517.

8 Appendix – Alternative Set of Taxes to implement Social Planner

Another way to decentralize the SP allocation is to use the following set of distortionary taxes: a tax on tradable consumption, a tax on nontradable consumption, a tax on new debt, a tax on labor income, and a tax on tradable output; the government budget constraint is assumed to be satisfied via a lump-sum tax/transfer. In this world we have the following conditions for a competitive equilibrium:

$$u_{1,t} - \left(1+\tau_t^T\right)\mu_t = 0$$

$$u_{3,t} + \left(1 - \tau_t^L + \frac{\max\left\{\lambda_t, 0\right\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\left(1-\tau_t^D\right)\mu_t\left(1-\alpha_T\right)A_t^T\left(H_t^T\right)^{-\alpha_T} = 0$$

$$u_{3,t} + \left(1 - \tau_t^L + \frac{\max\left\{\lambda_t, 0\right\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\frac{u_{2,t}}{1+\tau_t^N}\left(1-\alpha_N\right)A^N\left(H_t^N\right)^{-\alpha_N} = 0$$

$$\max\left\{-\lambda_t, 0\right\}^2 - \left(B_{t+1} + \frac{1-\phi}{\phi}\left(\left(1-\tau_t^D\right)A_t^T\left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t}}{\mu_t\left(1+\tau_{Nt}\right)}A^N\left(H_t^N\right)^{1-\alpha_N}\right)\right) = 0$$

$$\beta\left(1+r\right)E_t\left[\mu_{t+1}\right] - \left(1-\tau_t^B\right)\mu_t + \max\left\{\lambda_t, 0\right\}^2 = 0.$$

The first equation determines τ_t^T

$$\tau_t^T = \frac{\mu_{1t} - \mu_t - \max\left\{\lambda_t, 0\right\}^2\frac{1-\phi}{\phi}\frac{1}{\kappa}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1-\kappa}{\kappa}}}{\mu_t}$$

the last determines τ_t^B

$$\tau_t^B = \frac{\beta\left(1+r\right)E_t\left[\mu_{1,t+1} - \mu_{t+1}\right] - \left(\mu_{1,t} - \mu_t\right)}{\mu_t},$$

and the middle three jointly determine $\left(\tau_t^L, \tau_t^N, \tau_t^D\right)$:

$$\left(1 + \frac{\max\left\{\lambda_t, 0\right\}^2}{\mu_{1t}}\frac{1-\phi}{\phi}\right)\mu_{1t} = \left(1 - \tau_t^L + \frac{\max\left\{\lambda_t, 0\right\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)\left(1-\tau_t^D\right)\mu_t \tag{43}$$

$$1 + \frac{\max\left\{\lambda_t, 0\right\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{44}$$

$$= \left(\frac{1-\tau_t^L}{1+\tau_t^N} + \frac{\max\left\{\lambda_t, 0\right\}^2}{\left(1+\tau_t^N\right)\mu_t}\frac{1-\phi}{\phi}\right)$$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\left(A^N\left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}} \tag{45}$$

$$= -\tau_t^D A_t^T\left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t}A^N\left(H_t^N\right)^{1-\alpha_N}}{\mu_t\left(1+\tau_t^N\right)}.$$

Since $\mu_{1,t} = \mu_t$ (the marginal value of wealth is equalized) the expressions simplify:

$$\tau_t^T = \frac{-\max\{\lambda_t, 0\}^2}{\mu_{1,t}} \frac{1-\phi}{\phi} \frac{1}{\kappa} \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}} \left(C_t^T\right)^{\frac{1-\kappa}{\kappa}} \tag{46}$$

$$\tau_t^B = 0 \tag{47}$$

$$\tau_t^D = \frac{-\tau_t^L}{1-\tau_t^L + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}} \tag{48}$$

$$1 + \frac{\max\{\lambda_t, 0\}^2}{u_{2,t}} \frac{1-\phi}{\phi} \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}} \tag{49}$$

$$= \left(\frac{1-\tau_t^L}{1+\tau_t^N} + \frac{\max\{\lambda_t, 0\}^2}{\left(1+\tau_t^N\right)\mu_t} \frac{1-\phi}{\phi}\right)$$

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}} \tag{50}$$

$$= -\tau_t^D A_t^T \left(H_t^T\right)^{1-\alpha_T} + \frac{u_{2,t} A^N \left(H_t^N\right)^{1-\alpha_N}}{\mu_t \left(1+\tau_t^N\right)}.$$

A lump-sum tax/transfer can then be used to balance the government budget constraint. Note that the third and fourth equations can be used to define $\left(\tau_t^D, \tau_t^N\right)$ entirely in terms of τ_t^L, so that solving the system of equations can be reduced to solving one nonlinear equation in τ_t^L:

$$\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{\kappa-1}{\kappa}} \left(C_t^T\right)^{\frac{1}{\kappa}} = \frac{\tau_t^L A_t^T \left(H_t^T\right)^{1-\alpha_T}}{1-\tau_t^L + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}} +$$

$$\frac{u_{2,t} A^N \left(H_t^N\right)^{1-\alpha_N} \left(1-\tau_t^L + \frac{\max\{\lambda_t,0\}^2}{\mu_t}\frac{1-\phi}{\phi}\right)}{\mu_t \left(1 + \frac{\max\{\lambda_t,0\}^2}{u_{2,t}}\frac{1-\phi}{\phi}\left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}\left(A^N \left(H_t^N\right)^{1-\alpha_N}\right)^{\frac{-1}{\kappa}}\left(C_t^T\right)^{\frac{1}{\kappa}}\right)}.$$

The LHS of this equation is constant in terms of τ^L and the first term on the RHS is strictly increasing in τ^L, but the second term is strictly decreasing so no guarantee of uniqueness can be obtained. However, at least one $\tau^L < 1$ exists that solves this equation, as the LHS is strictly positive and the RHS can be made both negative (setting $\tau^L < 0$) and arbitrarily positive (setting τ^L close to 1).

Table 1. Model Parameters

Structural parameters	**Values**
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.760$
Intertemporal substitution and risk aversion	$\rho = 2$
Labor supply elasticity	$\delta = 1.75$
Credit constraint parameter	$\phi = 0.415$
Labor share in production	$1 - \alpha^T = 1 - \alpha^N = 0.66$
Relative weight of tradable and non-tradable goods	$\omega = 0.3526$
Discount factor	$\beta = 0.9717$
Exogenous variables	**Values**
World real interest rate	$i = 0.01587$
Steady state productivity level	$A^N = A^T = 1$
Productivity process	
Persistence	$\rho_{\varepsilon^T} = 0.5370$
Volatility	$\sigma_{\varepsilon^T} = 0.0134$
Average values in the ergodic distribution	**Values**
Net foreign assets (or minus foreign borrowing)	$B = -0.914$
Qaurterly GDP	$Y = 0.6486$
Qaurterly Tradable GDP	$Y^T = 0.2544$
Qaurterly Non-Tradable GDP	$Y^N = 0.3942$

Table 2. Ergodic Mean of Debt
(In units of tradable consumption unless noted)

	CE	**SP**	**OP**(τ_N, τ_B) 1/	**OP**(τ_N) 2/	**OP**(τ_B) 3/
B_{t+1}	-1.17	-1.22	-1.20	-1.19	-1.17

Table 3. Quarterly crisis probabilities
(In percent, unconditional)

CE	**SP**	**OP**(τ_N, τ_B) 1/	**OP**(τ_N) 2/	**OP**(τ_B) 3/
1.96	1.63	0.09	0.60	0.00

Table 4. Welfare gain of moving from the CE
(In percent of permanent consumption)

	Overall	**In crisis states**
CE	*na*	*na*
SP	0.18%	0.22%
OP($\tau_{N,B}$) 1/	0.04%	0.05%
OP(τ_N) 2/	0.02%	0.03%
OP(τ_B) 3/	0.003%	0.005%

1/ Allocation equivalent to OP(τ_T, τ_B), OP(τ_N, τ_H), and OP(τ_T, τ_H)

2/ Allocation equivalent to OP(τ_T)

3/ Allocation equivalent to OP(τ_H, τ_B), and OP(τ_H)

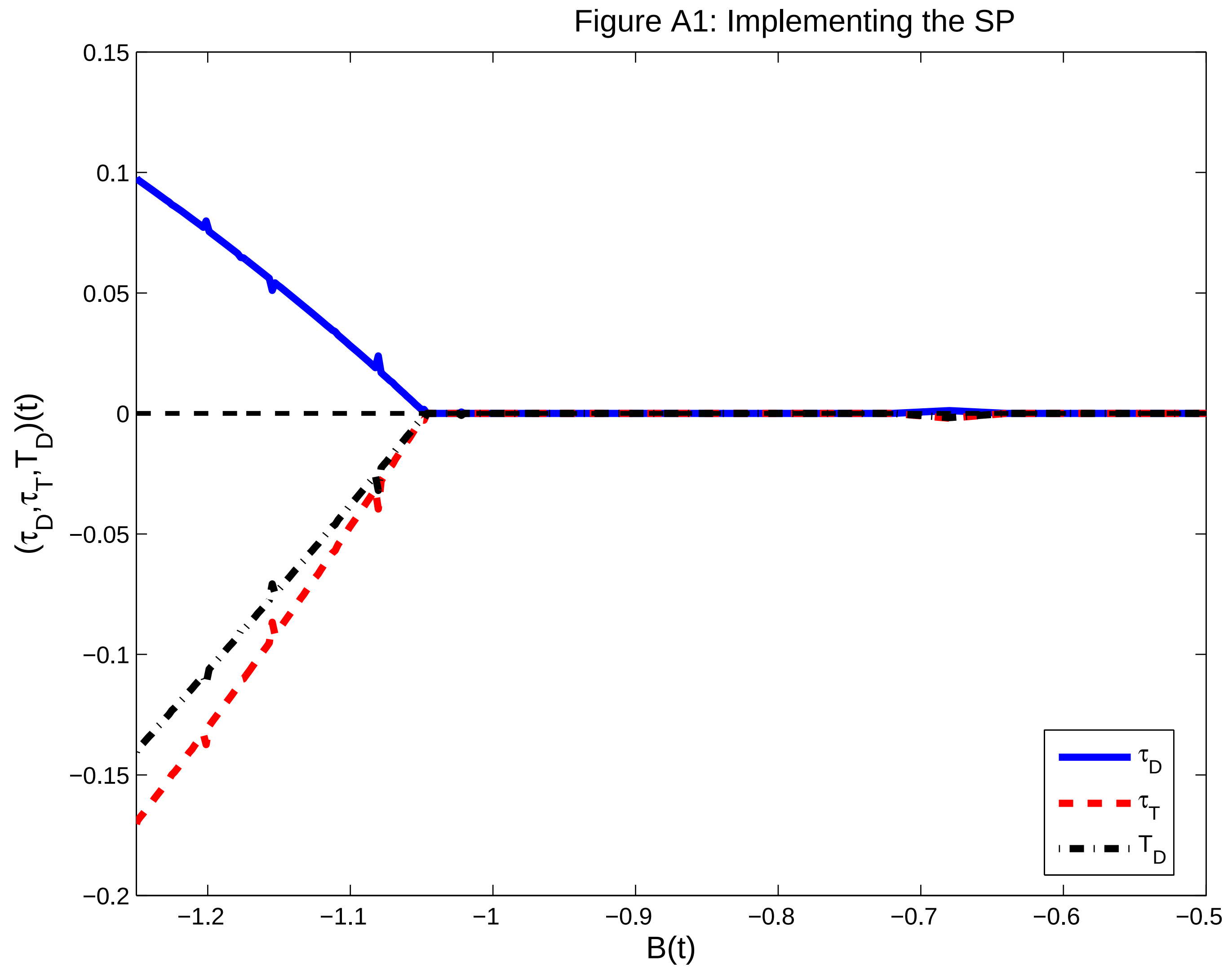

Figure A1: Implementing the SP
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−1.2
−1.1
−1
−0.9
−0.8
−0.7
−0.6
−0.5
B(t)
$(\tau_D, \tau_T, T_D)(t)$
τ_D
τ_T
T_D

Figure 1: Optimal Policy With Two Instruments

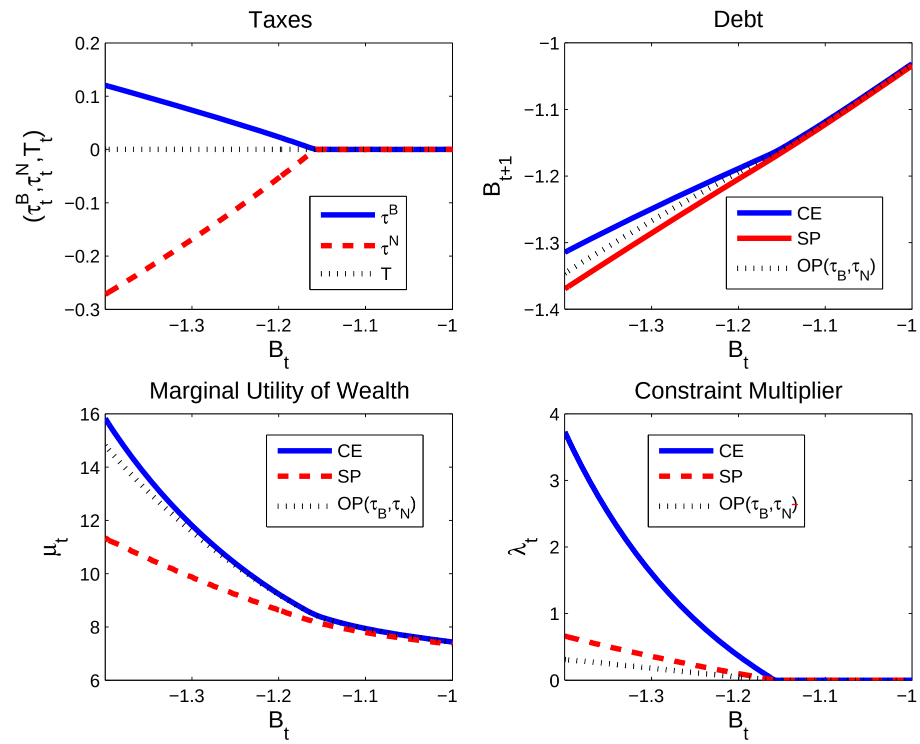

Figure 2: Optimal Policy With Two Instruments

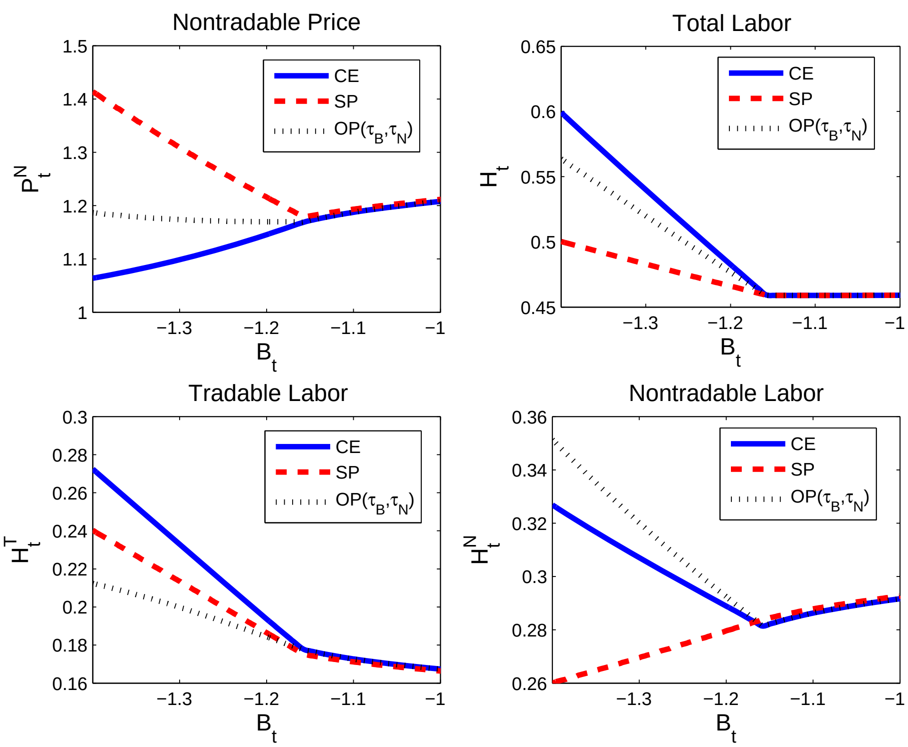

Figure 3: Optimal Policy With Two Instruments

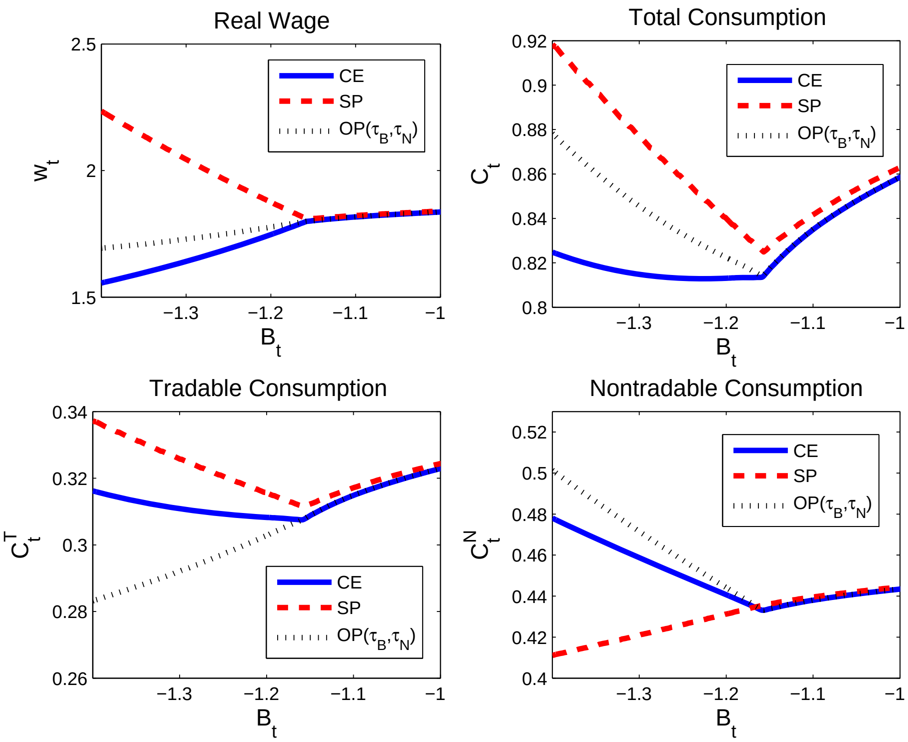

Figure 4: Optimal Policy With Two Instruments

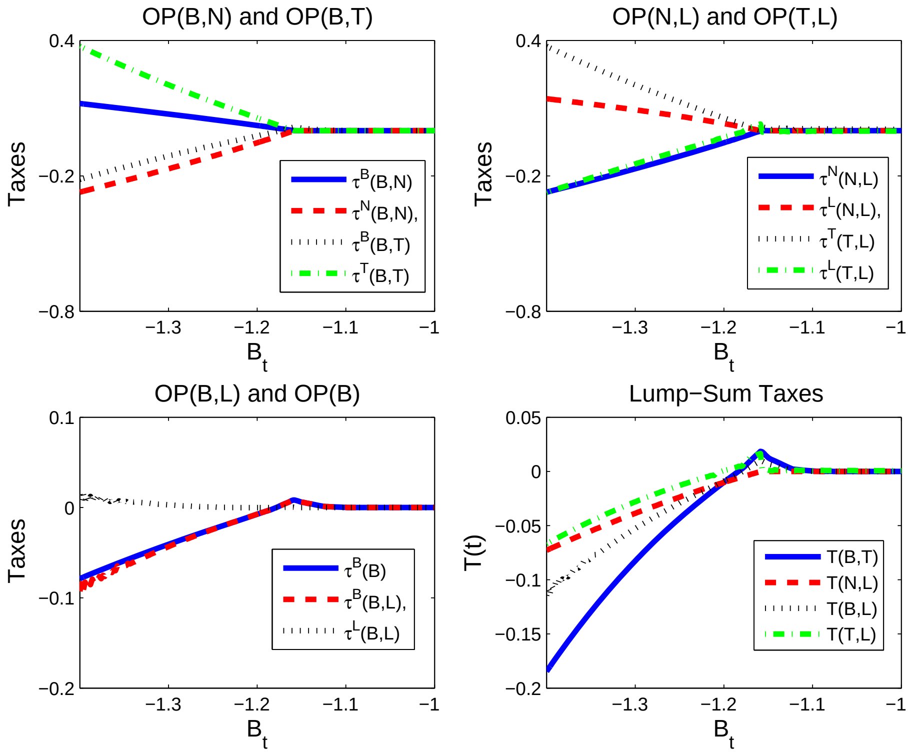

Figure 5: Optimal Policy With Two Instruments

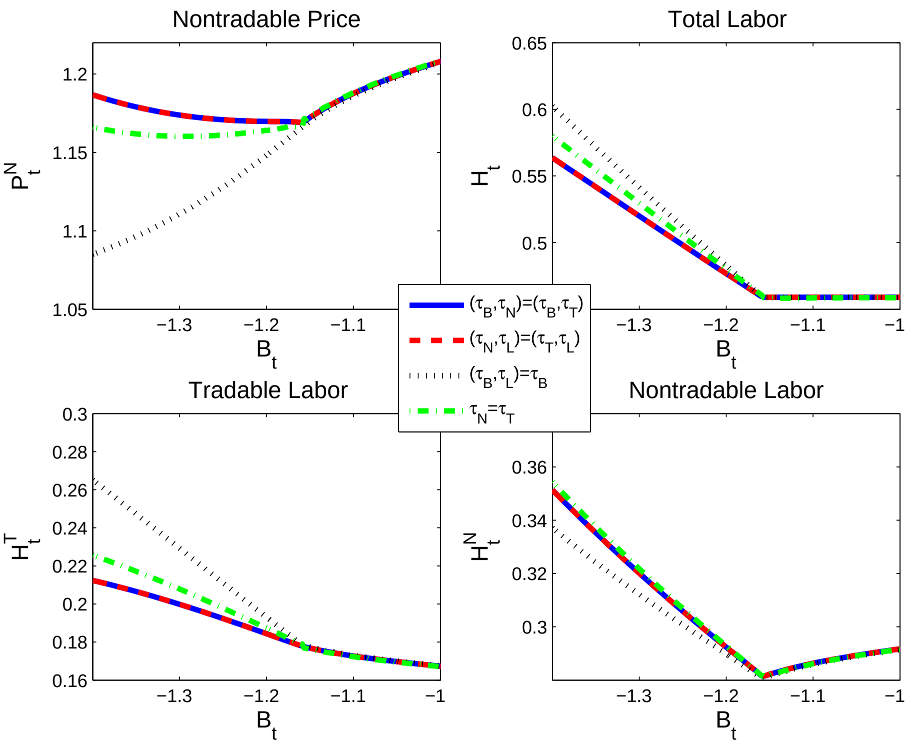

Figure 6: Optimal Policy With One Instrument

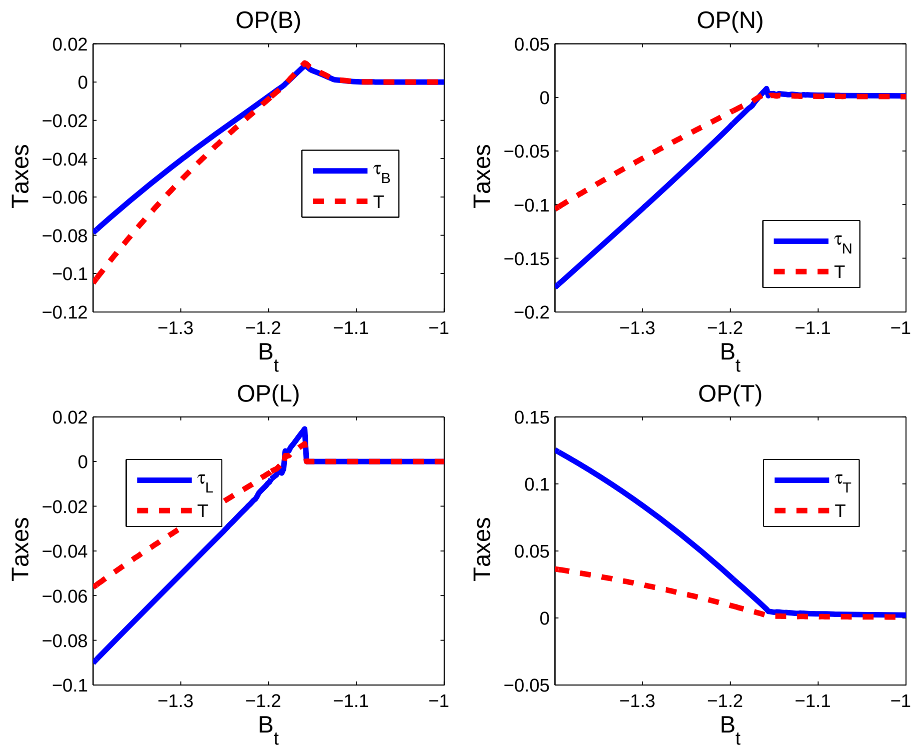

Figure 7: Optimal Policy With One Instrument

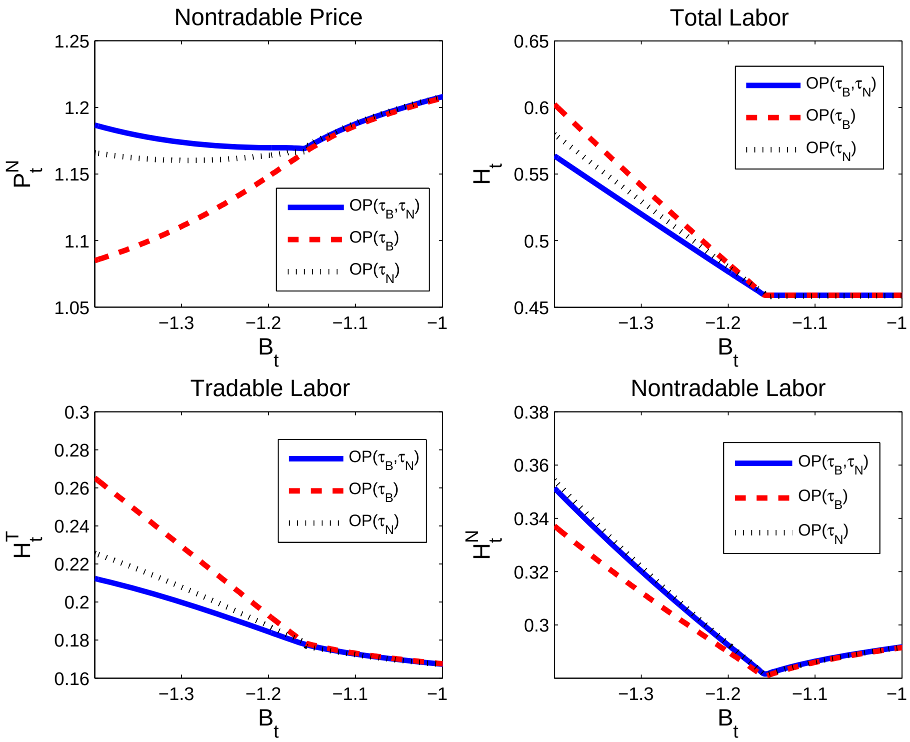

Figure 8: Optimal Policy With One Instrument