# Infrequent Random Portfolio Decisions in an Open Economy Model[1]


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**Abstract**

Motivated by evidence of portfolio frictions at the level of both households and global mutual funds, we analyze a two-country DSGE model of the global equity market where investors have a constant probability of making a new portfolio decision. There are both dividend and financial shocks, which lead to exogenous portfolio shifts. The model is solved with a global solution method that combines a Taylor projection method with the modified Shepard's inverse-weighting interpolation. Intuition is developed by deriving an approximated portfolio expression. A numerical illustration shows that the portfolio friction significantly affects the behavior of asset prices, expected excess returns and portfolios in response to financial shocks. For the same size financial shock, the impact is much larger with the friction. The model with the friction can account better for the observed excess return predictability and other moments involving excess returns and portfolio shares.

JEL classification: F30, F41, G11, G12

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# 1 Introduction

The response of portfolio allocation to changes in expected returns is a critical element of open economy macro models. The extent of the portfolio response affects asset prices as well as capital flows, which in turn affects business cycles. Most models of international capital flows driven by portfolio choice are frictionless models where all investors reallocate their portfolios each period based on changes in expected returns and risk.[1] Recently Bacchetta, Tièche and van Wincoop (2020), from here on BTW, provide evidence on the importance of portfolio frictions for international portfolio choice. Similar to Gârleanu and Pederson (2013), they extend the standard Markowitz frictionless mean-variance portfolio to allow for portfolio frictions in the form of costly deviations from benchmark portfolios (lagged portfolio share and buy-and-hold portfolio share). They apply the resulting portfolio expression to international equity portfolios of U.S. mutual funds and find strong evidence of portfolio frictions. These frictions lead to a more gradual portfolio response, and weaker initial response, to changes in expected returns. Giglio et al. (2019) provide related evidence, although not for international data. Using a survey of U.S. based Vanguard investors, they document a response of equity portfolio shares to expected returns that is too weak to make sense in the context of frictionless models. They further provide evidence that changes in expected returns have limited explanatory power for when investors trade, but help predict the direction and the magnitude of trading conditional on its occurrence. They suggest that this can be captured by introducing infrequent random trading, à la Calvo.[2]

An important contribution of the paper is to develop and solve a model with infrequent random portfolio decision making, which has not been done before. We introduce the Calvo type portfolio friction in a two-country DSGE model for the global equity market. Infinitely lived investors can hold equity of both countries and a risk-free bond. Each period with probability $p$ they choose a new portfo-

[1]Examples of recent DSGE models of capital flows based on portfolio choice include Benhima and Cordonier (2020), Davis and van Wincoop (2018), Devereux and Sutherland (2007, 2010), Didier and Lowenkron (2012), Evans and Hnatkovska (2012, 2014), Gabaix and Maggiori (2015), Hnatkovska (2010) and Tille and van Wincoop (2010a,b, 2014).

[2]Other papers that have documented portfolio inertia by households include Ameriks and Zeldes (2004), Bilias et al. (2010), Brunnermeier and Nagel (2008), Mitchell et al. (2006). However, these papers do not consider the portfolio response to changes in expected returns.

lio and otherwise hold their portfolio allocation constant. The Taylor projection method developed in Levintal (2018) is used to obtain a global solution. An additional contribution is to develop intuition about optimal portfolio decisions in this framework and the effect of the portfolio friction on asset prices, excess returns and portfolios. Finally, in a numerical illustration we show that the model is more consistent with data on excess returns and portfolio shares than the frictionless model.

To develop intuition, we derive an approximate expression of the equity portfolio share in the Home country that uses techniques related to Campbell and Viceira (1999). This approximation of the optimal portfolio is equal to sum of three terms: a term proportional to the expected present discounted value of all future excess returns, a term that depends on exogenous portfolio shocks, and a hedge term that depends on the present discounted value of risk associated with future asset returns. We show numerically that the first two terms account for almost all of the fluctuations in portfolios by agents making new portfolio decisions. Making infrequent portfolio decisions implies that investors have longer effective horizons when they do make a portfolio decision. Their portfolio choice therefore depends on expectations of expected excess returns further into the future.

A key implication of the portfolio friction is that it leads to a weaker, and more gradual, portfolio response to changes in expected excess returns. This is the case both because a limited fraction of investors make new portfolio decisions each period and because those that do make a new portfolio decision have longer effective horizons and are therefore less responsive to expected excess returns in the near future. The weaker portfolio response implies that financial shocks, associated with exogenous portfolio shifts, have a much bigger impact on asset prices, expected excess returns and portfolio shares. Examples of financial shocks are portfolio shifts due to changes in risk-aversion or the risk-bearing capacity of financial institutions, liquidity trade, noise trade or FX intervention. Since portfolios are less sensitive to expected excess returns under the portfolio friction, a larger change in equilibrium expected excess returns is needed to clear the market. This implies larger changes in asset prices. Equilibrium portfolio shares will also change more.[3]

[3]In frictionless models, a portfolio shift from the US to the rest of the world causes a very small (third-order) drop in the relative US asset price, leading to an equally small increase in its expected excess return that reverses the flows and generates equilibrium. Tille and van Wincoop (2014) show that first-order changes in portfolio shares are associated with third-order changes

Gabaix and Maggiori (2015) and Itskhoki and Muhkin (2019) have recently emphasized the importance of financial shocks for global capital markets.[4] Itskhoki and Muhkin (2019) show that they account for the disconnect between exchange rates and observed macro fundamentals. Gabaix and Maggiori (2015) show that financial shocks can have large effects on exchange rates and capital flows in a model where global financiers who intermediate financial flows have limited risk-bearing capacity. They cite a variety of evidence consistent with the view that financial shocks are an important driver of exchange rates and capital flows.[5]

The larger equilibrium expected excess returns as a result of the portfolio friction are consistent with excess return predictability based on on international equity return differentials. In Table 1 we report results from a panel regression of monthly US minus foreign stock returns on the differential in the log earning-price ratio. The panel includes 73 foreign countries and is based on a half century of data, from January 1970 to February 2019. There is strong evidence of predictability of international return differentials at horizons of 1, 3, 12 and 48 months. Analogous that what is typically found for excess returns of stocks over bonds, the predictability coefficient and $R^2$ increase with the horizon.[6] Such predictability of international equity return differentials is analogous to the well-known predictability of international short-term bond return differentials by the interest differential, also known as the forward discount puzzle or Fama puzzle.

Assuming a constant probability $p$ of making a new portfolio decision has similarities to Calvo price setting, but is more complex. The similarity is most apparent in the approximated portfolio expression that we derive. The portfolio depends

in expected returns because expected excess returns are divided by second order moments (e.g. the variance of the excess return) in optimal portfolios.

[4]In a similar spirit, the literature on limited arbitrage has documented various cases of demand shocks, e.g., see Gromb and Vayanos (2010).

[5]Two of the papers they cite are Blanchard et al. (2015) and Hau, Massa, and Peress (2010). Blanchard et al. (2015) provide evidence that foreign exchange intervention, an example of a financial shock, has a significant effect on exchange rates. Hau, Massa, and Peress (2010) show that after a change in the weights of the MSCI World Equity Index, countries that see their weight increase experience capital inflows and a currency appreciation. The portfolio response to the re-indexing is itself evidence of portfolio frictions as it suggests that it is costly to deviate from the benchmark. Also relevant is Rey (2013), who finds that changes in the VIX (a measure of stock price risk or risk-aversion) affects asset prices and cross-border financial flows.

[6]See Campbell, Lo and MacKinlay (1997) for a textbook discussion and Cochrane (2007) for further evidence.

on the present discounted value of expected future excess returns and risk. Under Calvo price setting the optimal price depends on the present discounted value of expected future marginal costs. In both cases the discount factor is the same, equal to $1-p$ times the time discount rate.

But adding the friction to portfolio choice significantly complicates the solution. It raises the number of state variables from 5 to 15 and control variables from 9 to 15. Seven of the additional state variables are associated with lagged portfolio decisions, one is lagged relative wealth, and the other two are lagged financial shocks; none of these terms matter if portfolios adjust immediately. The additional control variables include variables that summarize the expected products of future excess returns and stochastic discount factors (beyond the next period) that enter the portfolio Euler equations. The Taylor projection method used to solve the model involves local linear solutions at many nodes of the ergodic state space, which are then combined into a global solution through modified Shepard's inverse-weighting interpolation.

Our approach differs from the related literature on infrequent portfolio adjustment. Most of this literature assumes that agents adjust their portfolios in a staggered way every $T$ periods.[7] In empirical applications this has the drawback that it generates a significant discontinuity in the impulse response to shocks that happens $T$ periods after the shock. This occurs because the initial group of infrequent traders who change their portfolio at the time of the shock will change their portfolio again $T$ periods later, with predictable certainty. The anticipation of this adjustment by other traders significantly affects the equilibrium. The constant probability setup that we adopt here implies more smoothness as the agents who change their portfolio at the time of a shock will change their portfolio again at varying dates in the future. It also has the advantage of a cleaner aggregation. We do not need to keep track of all generations of agents by the time of their last portfolio decision. Only portfolios from one period ago enter in the state space.[8]

[7]For recent contributions, see Abel et. al (2007), Bacchetta and van Wincoop (2010), Bogousslavsky (2016), Chien et al. (2012), Duffie (2010), Greenwood et al. (2018) and Hendershott et al. (2013). Earlier papers examine the impact of infrequent portfolio adjustments taking the process of asset returns as exogenous, e.g. see Lynch (1996) or Gabaix and Laibson (2002).

[8]An alternative approach to model gradual portfolio adjustment is to assume a cost of adjusting portfolios, as in Vayanos and Woolley (2012), Gârleanu and Pedersen (2013), Bacchetta and van Wincoop (2019) and Bacchetta, Tièche and van Wincoop (2020). This significantly simplifies the portfolio problem, but is of course more ad hoc.

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 discusses the global solution method. Section 4 discusses an approximation of the optimal portfolio in order to develop intuition. Section 5 contains a numerical illustration. Section 6 concludes.

# 2 Model

There are two countries, Home ($H$) and Foreign ($F$). There is a single good. In both countries there is a continuum of agents on the interval $[0, 1]$ who have infinite lives and make decisions about consumption and portfolio allocation. Agents of both countries can hold three assets: Home and Foreign equity and a risk-free bond.

## 2.1 Infrequent Decision Making

The key aspect of the model is infrequent decision making about consumption and portfolios. Analogous to Calvo price setting, we assume that agents make new decisions with a probability $p$. However, infrequent decision making only affects portfolio choice: we assume an intertemporal elasticity of substitution of 1, which implies that optimal consumption is a constant fraction of wealth. Agents therefore do not need to rethink their consumption choice. For portfolio choice we assume that the fraction $1 - p$ of agents that does not make new portfolio decisions will hold their portfolio shares constant until the time comes that they make a new portfolio decision.[9]

## 2.2 Assets

Agents can invest in Home equity, Foreign equity and a one-period risk-free bond. The number of equity shares is normalized to 1 in both countries, while bonds are

[9] An alternative, not explored here, is that agents hold the quantity of asset holdings constant. This is analogous to a buy-and-hold portfolio, in which case there is no rebalancing. In our specification, even the agents that do not make new portfolio decisions still trade to rebalance their portfolio. This can for example be achieved by investing in a mutual fund. While in reality a combination of both is realistic, this would significantly complicate our analysis. The findings by Bacchetta, Tièche and van Wincoop (2020) for foreign investment by US mutual funds imply more sluggishness in deviating from past portfolio shares than from a buy-and-hold portfolio.

in zero net supply. The gross interest rate on the bond is denoted $R_t$. The returns on Home and Foreign equity from $t$ to $t+1$ are

$$R_{H,t+1} = \frac{Q_{H,t+1} + D_{H,t+1}}{Q_{H,t}} \tag{1}$$

$$R_{F,t+1} = \frac{Q_{F,t+1} + D_{F,t+1}}{Q_{F,t}}, \tag{2}$$

where $Q_{H,t}$ and $Q_{F,t}$ are the prices of Home and Foreign equity shares and $D_{H,t}$, $D_{F,t}$ are dividends. Dividends follow an AR process in logs:

$$d_{H,t+1} = (1-\rho_d)d + \rho_d d_{H,t} + \epsilon^d_{H,t} \tag{3}$$

$$d_{F,t+1} = (1-\rho_d)d + \rho_d d_{F,t} + \epsilon^d_{F,t}. \tag{4}$$

The vector $(\epsilon^d_{H,t}, \epsilon^d_{F,t})'$ of dividend shocks has a $N(0,\Omega^d)$ distribution.

## 2.3 Budget Constraints

We focus mostly on describing Home agents. For Foreign agents we simply need to replace the $H$ with an $F$. Consider agent $i$ in the Home country who makes a new portfolio decision at time $t$. First some notation is in order. Let $W^i_{H,t}$ be the wealth of the agent at the start of period $t$ and $cw^i_{H,t}$ the fraction of wealth that is consumed. The remainder is then invested in the three assets. A fraction $z^i_{HH,t}$ is invested in Home equity and $z^i_{HF,t}$ in Foreign equity. The remainder is invested in the bond. We also denote $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$ as the average portfolio shares of all Home agents that make new portfolio decisions at time $t$. In equilibrium $z^i_{HH,t} = \tilde{z}_{HH,t}$ and $z^i_{HF,t} = \tilde{z}_{HF,t}$ for investors making new portfolio decisions at time $t$. But we will make this substitution only after deriving the first-order conditions for agent $i$. For Foreign agents we denote the fractions allocated to the Home and Foreign equity as $z^i_{FH,t}$ and $z^i_{FF,t}$.

Wealth of agent $i$ making a new consumption and portfolio decision at time $t$ then evolves according to

$$W^i_{H,t+1} = (1 - cw^i_{H,t})W^i_{H,t}R^{p,H,i}_{t+1} \tag{5}$$

where the portfolio return is

$$\begin{aligned} R^{p,H,i}_{t+1} \quad = \quad & R_t + z^i_{HH,t}(R_{H,t+1} - R_t) + z^i_{HF,t}(e^{-\tau_{H,t}}R_{F,t+1} - R_t) + \\ & + \tilde{z}_{HF,t}(1 - e^{-\tau_{H,t}})R_{F,t+1}. \end{aligned} \tag{6}$$

Here $\tau_{H,t}$ is a tax on Foreign investment, which will be discussed below. The aggregate of this tax across all Home agents making a portfolio decision at time $t$ is reimbursed through the last term of (6). This assures that it will only affect portfolio allocation, not overall wealth accumulation. For Foreign agents the tax applies to the Home asset.

Wealth of Home agent $i$ who does not make new consumption/portfolio decisions at time $t$, and last made new decisions at $t-j$, evolves according to

$$W^i_{H,t+1} = (1 - cw^i_{H,t-j})W^i_{H,t}R^{p,H,i,t-j}_{t+1} \tag{7}$$

where the portfolio return is

$$\begin{aligned} R^{p,H,i,t-j}_{t+1} &= R_t + z^i_{HH,t-j}(R_{H,t+1} - R_t) + z^i_{HF,t-j}(e^{-\tau_{H,t-j}}R_{F,t+1} - R_t) + \\ &\quad + \tilde{z}_{HF,t-j}(1 - e^{-\tau_{H,t-j}})R_{F,t+1}. \end{aligned} \tag{8}$$

The portfolio return has an extra $t-j$ superscript to denote when consumption/portfolio decisions were last made. The consumption-wealth ratio and portfolio shares are those chosen at $t-j$. The tax on Foreign investment is also at $t-j$ as it is held constant until a new portfolio decision is made.

After deriving the portfolio Euler equations, we will substitute $z^i_{HH,t} = \tilde{z}_{HH,t}$ and $z^i_{HF,t} = \tilde{z}_{HF,t}$. The same is done for portfolio shares prior to time $t$. The portfolio return of agents who last made a portfolio decision at time $t-j$ is then

$$R^{p,H,t-j}_{t+1} = R_t + \tilde{z}_{HH,t-j}(R_{H,t+1} - R_t) + \tilde{z}_{HF,t-j}(R_{F,t+1} - R_t) \tag{9}$$

The tax $\tau_{H,t-j}$ no longer enters.

## 2.4 Financial Shocks

Equation (6) introduces a cost $\tau_{H,t}$ of investing in Foreign equity that reduces the return that Home agents earn on Foreign equity. There is an analogous cost $\tau_{F,t}$ of investing in Home equity by Foreign agents. These costs play two roles. First, their mean level can be set to generate realistic average portfolio home bias. Second, changes in these costs generate exogenous portfolio shifts, which we will refer to as financial shocks. We assume that they follow AR processes:

$$\tau_{H,t} = \tau + \rho_\tau(\tau_{H,t-1} - \tau) + \epsilon^\tau_{H,t} \tag{10}$$

$$\tau_{F,t} = \tau + \rho_\tau(\tau_{F,t-1} - \tau) + \epsilon^\tau_{F,t} \tag{11}$$

where the vector $(\epsilon^{\tau}_{H,t}, \epsilon^{\tau}_{F,t})'$ of financial shocks has a $N(0, \Omega^{\tau})$ distribution. Financial shocks, defined as exogenous portfolio shifts unrelated to endogenous changes in expected returns and risk, can be introduced in many other ways. In the literature they sometimes are modeled in the form of noise trade, liquidity trade, hedge trade, time-varying risk-bearing capacity or time-varying investment opportunities.[10] We do not wish to take a strong stand on what the exact origin of these portfolio shocks is.

## 2.5 Bellman Equations

Agents are assumed to have Rince preferences, which for any agent (from Home or Foreign) we can write as

$$\ln(V_t) = \max_{c_t, \mathbf{z}_t} \left\{ (1-\beta)\ln(c_t) + \beta \ln\left( \left[ E_t V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right) \right\}, \tag{12}$$

where $c_t$ is consumption and $\mathbf{z}_t$ the vector of portfolio shares. This implies an intertemporal elasticity of substitution (IES) of 1 and a rate of risk aversion of $\gamma$.

Let $V_t^{n,i}$ be the value function of Home agent $i$ who makes new consumption/portfolio decisions at time $t$. Similarly, $V_t^{o,i,t-j}$ is the value function of Home agent $i$ who does not make new decisions at time $t$ and who last made a consumption/portfolio decisions at $t-j$. Here $o$ stands for "old". For either of these agents, there is a probability $p$ that they make a new portfolio decision at $t+1$ and a probability $1-p$ that they do not. We can then write the Bellman equations for these respective agents as

$$\begin{aligned} \ln(V_t^{n,i}) &= \max_{cw^i_{H,t}, z^i_{HH,t}, z^i_{HF,t}} \Bigg\{ (1-\beta)\ln(cw^i_{H,t} W^i_{H,t}) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left( pE_t \left(V_{t+1}^{n,i}\right)^{1-\gamma} + (1-p)E_t\left(V_{t+1}^{o,i,t}\right)^{1-\gamma} \right) \Bigg\} \quad (13) \\ ln(V_t^{o,i,t-j}) &= (1-\beta)\ln(cw^i_{H,t-j} W^i_{H,t}) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left( pE_t \left(V_{t+1}^{n,i}\right)^{1-\gamma} + (1-p)E_t\left(V_{t+1}^{o,i,t-j}\right)^{1-\gamma} \right). \quad (14) \end{aligned}$$

The value functions will be proportional to the wealth of the agent. We will

[10]For different ways of modeling the portfolio shocks, see Bacchetta and van Wincoop (2006), Dow and Gorton (1995), Gabaix and Maggiori (2015), He and Wang (1995), Spiegel and Subrahmanyam (1992) and Wang (1994).

therefore write

$$V_t^{n,i} = W_{H,t}^i e^{f^n(S_t)} \tag{15}$$
$$V_t^{o,i,t-j} = W_{H,t}^i e^{f^o(S_t, z_{HH,t-j}^i, z_{HF,t-j}^i, \tilde{z}_{HF,t-j}, \tau_{H,t-j})}. \tag{16}$$

Here $S_t$ is a vector of aggregate state variables, which will be defined below. Apart from the wealth of the agent, the value function of an agent making new consumption/portfolio decisions at time $t$ only depends on the aggregate state $S_t$ through the function $f^n$. The value function of an agent who last made a portfolio decision at $t-j$ also depends, through the function $f^o$, on the portfolio shares at time $t-j$, $z_{HH,t-j}^i$ and $z_{HF,t-j}^i$.[11] It also depends on $\tilde{z}_{HF,t-j}$ and $\tau_{H,t-j}$, as they affect portfolio returns $R_{t+s}^{p,i,H,t-j}$ until new portfolio decisions are made. The functions $f^n$ and $f^o$ evaluated at their respective state variables at time $t$ are also denoted $f_{H,t}^n$ and $f_{H,t}^{o,i,t-j}$ for Home agents.

Substituting (15) and (16) into (13) and (14), and using the wealth accumulation equations, we can write the Bellman equations as

$$\begin{aligned} f_{H,t}^n &= \max_{cw_{H,t}^i, z_{HH,t}^i, z_{HF,t}^i} \Bigg\{ (1-\beta) ln(cw_{H,t}^i) + \beta \ln(1 - cw_{H,t}^i) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left( E_t \left( pe^{(1-\gamma)f_{H,t+1}^n} + (1-p)e^{(1-\gamma)f_{H,t+1}^{o,i,t}} \right) \left( R_{t+1}^{p,H,i} \right)^{1-\gamma} \right) \Bigg\} \end{aligned} \tag{17}$$

$$\begin{aligned} f_{H,t}^{o,i,t-j} &= (1-\beta) ln(cw_{H,t-j}^i) + \beta \ln(1 - cw_{H,t-j}^i) + \\ &\quad \frac{\beta}{1-\gamma} \ln\left( E_t \left( pe^{(1-\gamma)f_{H,t+1}^n} + (1-p)e^{(1-\gamma)f_{H,t+1}^{o,i,t-j}} \right) \left( R_{t+1}^{p,H,i,t-j} \right)^{1-\gamma} \right). \end{aligned} \tag{18}$$

When the individual-specific portfolio shares $z_{HH,t-j}^i$ and $z_{HF,t-j}^i$ are evaluated at the equilibrium portfolio shares $\tilde{z}_{HH,t-j}$ and $\tilde{z}_{HF,t-j}$ for agents last making portfolio decisions at $t-j$, we omit the $i$ supercript and write

$$f_{H,t+1}^{o,t-j} = f^o(S_{t+1}, \tilde{z}_{HH,t-j}, \tilde{z}_{HF,t-1}, \tilde{z}_{HF,t-j}, \tau_{H,t-j})$$

It is also useful to define

$$\lambda_{HH,t+1}^t = \frac{\partial f_{H,t+1}^{o,i,t}}{\partial z_{HH,t}^i} \tag{19}$$

$$\lambda_{HF,t+1}^t = \frac{\partial f_{H,t+1}^{o,i,t}}{\partial z_{HF,t}^i}. \tag{20}$$

[11]In principle the lagged consumption wealth decision $cw_{H,t-j}^i$ should enter as well, but we will see that this remains constant over time.

These derivatives are again evaluated by setting the agent $i$-specific portfolio shares equal to $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$.

## 2.6 Portfolio Euler Equations

Maximizing the right hand side of (17) with respect to $cw^i_{H,t}$, $z^i_{HH,t}$ and $z^i_{HF,t}$, using the portfolio return (6), gives three first-order conditions. The first-order condition with respect to the consumption-wealth ratio simply gives $cw^i_{H,t} = 1-\beta$. Agents therefore always consume a fraction $1-\beta$ of their wealth, so that the infrequent decision making only matters for portfolio choice. Home agent $i$ then invest $\beta W^i_{H,t}$ in the three assets.

For the portfolio Euler equations it is useful to define scaled stochastic discount factors:

$$m^{n,t-j}_{H,t+1} = \left[R^{p,H,t-j}_{t+1}\right]^{-\gamma} e^{(1-\gamma)f^n_{H,t+1}}$$
$$m^{o,t-j}_{H,t+1} = \left[R^{p,H,t-j}_{t+1}\right]^{-\gamma} e^{(1-\gamma)f^{o,t-j}_{H,t+1}}.$$

These are scaled stochastic discount factors for an agent who last made portfolio decisions at $t-j$, conditional on the agent respectively making a new portfolio decision at $t+1$ and not making a new portfolio decision at $t+1$. We also define an unconditional stochastic discount factor as $m^{t-j}_{H,t+1} = pm^{n,t-j}_{H,t+1} + (1-p)m^{o,t-j}_{H,t+1}$.[12]

After taking the derivatives of (17) with respect to $z^i_{HH,t}$ and $z^i_{HF,t}$, and then setting these portfolio shares equal to $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$, we obtain the following portfolio Euler equations

$$E_t m^t_{H,t+1}(R_{H,t+1} - R_t) + (1-p)E_t m^{o,t}_{H,t+1} R^{p,H,t}_{t+1} \lambda^t_{HH,t+1} = 0 \qquad (21)$$
$$E_t m^t_{H,t+1}(e^{-\tau_{H,t}} R_{F,t+1} - R_t) + (1-p)E_t m^{o,t}_{H,t+1} R^{p,H,t}_{t+1} \lambda^t_{HF,t+1} = 0. \qquad (22)$$

The first terms in (21)-(22) are the expected excess returns discounted with the pricing kernel. When agents make new portfolio decisions each period ($p=1$), equating these first terms to zero gives the portfolio Euler equations. The second term applies when $p<1$, so it specifically relates to infrequent portfolio decisions. It captures the impact of future expected returns and risk beyond period $t+1$,

[12]The SDF for Rince preferences is $[c_t/c_{t+1}][V^{1-\gamma}_{t+1}/E_t V^{1-\gamma}_{t+1}]$. After substituting the solution for consumption, wealth accumulation, (15)-(16), and multiplying by $\beta E_t R^{p,H,t-j}_{t+1} m^{t-j}_{H,t+1}$, the scaled discount factors are obtained.

which affect $\lambda^t_{HH,t+1}$ and $\lambda^t_{HF,t+1}$. Knowing that they may not get an opportunity to change their portfolio allocation again for some time, agents that make portfolio decisions at time $t$ need to incorporate beliefs about expected returns and risk beyond time $t+1$.

We can then also write the Bellman equations as

$$e^{(1-\gamma)f^n_{H,t}/\beta} = \alpha E_t m^t_{H,t+1} R^{p,H,t}_{t+1} \tag{23}$$
$$e^{(1-\gamma)f^{o,t-1}_{H,t}/\beta} = \alpha E_t m^{t-1}_{H,t+1} R^{p,H,t-1}_{t+1} \tag{24}$$

where $\alpha = (1-\beta)^{(1-\gamma)(1-\beta)/\beta}\beta^{1-\gamma}$. These are for an agent who last made a portfolio decision at time $t$ and $t-1$.

In the first-order conditions $\lambda^t_{HH,t+1}$ and $\lambda^t_{HF,t+1}$ play an important role. Their values one period earlier, so $\lambda^{t-1}_{HH,t}$ and $\lambda^{t-1}_{HF,t}$, will be control variables to be solved as a function of the state at time $t$. Expressions for them can be obtained by considering an agent who last made a portfolio decision at time $t-1$, but does not make a new portfolio decision at time $t$. Taking derivatives of (18) for $j=1$ with respect to $z^i_{HH,t-1}$ and $z^i_{HF,t-1}$, and then setting the agent $i$ portfolio shares equal to $\tilde{z}_{HH,t-1}$ and $\tilde{z}_{HF,t-1}$, we have

$$E_t R^{p,H,t-1}_{t+1}\left(m^{t-1}_{H,t+1}\lambda^{t-1}_{HH,t} - \theta m^{o,t-1}_{H,t+1}\lambda^{t-1}_{HH,t+1}\right) = \beta E_t m^{t-1}_{H,t+1}(R_{H,t+1} - R_t) \tag{25}$$
$$E_t R^{p,H,t-1}_{t+1}\left(m^{t-1}_{H,t+1}\lambda^{t-1}_{HF,t} - \theta m^{o,t-1}_{H,t+1}\lambda^{t-1}_{HF,t+1}\right) = \beta E_t m^{t-1}_{H,t+1}(e^{-\tau_{H,t-1}}R_{F,t+1} - R_t) \tag{26}$$

where $\theta = \beta(1-p)$. While we will not do so, one can use these to write the portfolio Euler equations (21)-(22) as equating an expected present discounted value of all future excess returns, multiplied by appropriate stochastic discount factors, equal to zero.

## 2.7 Market Clearing Conditions

There are three market clearing conditions: for Home equity, Foreign equity and bonds. Denote $z_{jk,t} = \int_0^1 z^i_{jk,t}di$ for $j = H, F$ and $k = H, F$. Similarly, aggregate Home and Foreign wealth is $W_{H,t} = \int_0^1 W^i_{H,t}di$ and $W_{F,t} = \int_0^1 W^i_{F,t}di$. There is an aggregation issue in that asset demand involves the product of wealth and portfolio shares. Specifically, $\beta\int_0^1 z^i_{jk,t}W^i_{j,t}di$ is the total demand for country $k$ equity by agents from country $j$. We have $\int_0^1 z^i_{jk,t}W^i_{j,t}di = z_{jk,t}W_{j,t} + cov(z^i_{jk,t}, W^i_{j,t})$, where the latter is a cross-sectional covariance term.[13] We ignore the covariance term. This turns out to be numerically very accurate, with the correlation between $\int_0^1 z^i_{jk,t} W^i_{j,t} di$ and $z_{jk,t} W_{j,t}$ above 0.9997 for all $j, k$ based on the solution of the model with the parameterization in Table 2 that we discuss in Section 5. The accuracy was checked by simulating the solution over 100,000 months, keeping track of the wealth and portfolio shares of 100 million agents as an approximation of the continuum of agents in the model.

Market clearing conditions will then be

$$z_{HH,t} W_{H,t} + z_{FH,t} W_{F,t} = Q_{H,t}/\beta \tag{27}$$

$$z_{HF,t} W_{H,t} + z_{FF,t} W_{F,t} = Q_{F,t}/\beta \tag{28}$$

$$(1 - z_{HH,t} - z_{HF,t}) W_{H,t} + (1 - z_{FH,t} - z_{FF,t}) W_{F,t} = 0. \tag{29}$$

## 2.8 Control and State Variables

The control and state variables are respectively

$$cv_t = \left\{ q_{H,t}, q_{F,t}, r_t, \tilde{z}_{HH,t}, \tilde{z}_{HF,t}, \tilde{z}_{FF,t}, \tilde{z}_{FH,t}, f^n_{H,t}, f^n_{F,t}, cv_{H,t}, cv_{F,t} \right\}' \tag{30}$$

$$sv_t = \{ S_t, s_{H,t}, s_{F,t} \}' \tag{31}$$

where $q_{H,t}$, $q_{F,t}$ and $r_t$ are the log equity prices and interest rate, and

$$cv_{H,t} = \left\{ f^{o,t-1}_{H,t}, \lambda^{t-1}_{HH,t}, \lambda^{t-1}_{HF,t} \right\}$$
$$cv_{F,t} = \left\{ f^{o,t-1}_{F,t}, \lambda^{t-1}_{FH,t}, \lambda^{t-1}_{FF,t} \right\}$$
$$S_t = \left\{ d_{H,t}, d_{F,t}, \tau_{H,t}, \tau_{F,t}, w^D_t, w^D_{t-1}, z^A_{H,t-1}, z^D_{H,t-1}, z^D_{F,t-1} \right\}$$
$$s_{H,t} = \{ \tau_{H,t-1}, \tilde{z}_{HH,t-1}, \tilde{z}_{HF,t-1} \}$$
$$s_{F,t} = \{ \tau_{F,t-1}, \tilde{z}_{FH,t-1}, \tilde{z}_{FF,t-1} \}.$$

The last five state variables in $S_t$ are relative log wealth $w^D_t = ln(W_{H,t}) - ln(W_{F,t})$, $w^D_{t-1}$, $z^A_{H,t-1} = \omega_{t-1} z_{HH,t-1} + (1 - \omega_{t-1}) z_{FH,t-1}$, $z^D_{H,t-1} = z_{HH,t-1} - z_{FH,t-1}$ and

[13] In theory the covariance term may not be exactly zero. Agents are picked at random when chosen with probability $p$ to make new portfolio decisions, and independent of their wealth they all make the same portfolio decision. But $z^i_{HH,t}$ and $W^i_{H,t}$ could be cross sectionally correlated as a result of wealth accumulation after the most recent portfolio decision. For example, agents with a large portfolio share in the Home country will have seen their wealth rise a lot if Home equity returns have recently been relatively high.

$z^D_{F,t-1} = z_{FF,t-1} - z_{HF,t-1}$. Here $\omega_t = W_{Ht}/(W_{Ht} + W_{Ft})$ is the relative wealth of the Home country.[14]

The vector $S_t$ of nine state variables determines asset prices, the interest rate, the new time $t$ portfolio shares and the Bellman variables $f^n_{H,t}$ and $f^n_{F,t}$. The additional Home control variables $cv_{H,t}$ depend on both $S_t$ and the additional state variables $s_{H,t}$. Similarly, the additional Foreign control variables $cv_{F,t}$ depend on both $S_t$ and the additional state variables $s_{F,t}$. Although the control variables $cv_{H,t}$ and $cv_{F,t}$ are not of separate interest to us, we need to keep track of them as their values one period later enter the portfolio Euler equations.

Regarding the evolution of the state variables $S_t$, the processes for dividends and taxes are given by (3), (4), (10), and (11). The portfolio share $z_{HH,t}$ evolves according to

$$z_{HH,t} = (1-p)z_{HH,t-1} + p\tilde{z}_{HH,t} \tag{32}$$

with similar equations for $z_{HF,t}$, $z_{FH,t}$, and $z_{FF,t}$. Using (5) and (9), we have[15]

$$\begin{aligned} w^D_{t+1} \;=\;& w^D_t + ln\left(R_t + z_{HH,t}(R_{H,t+1} - R_t) + z_{HF,t}(R_{F,t+1} - R_t)\right) - \\ & ln\left(R_t + z_{FH,t}(R_{H,t+1} - R_t) + z_{FF,t}(R_{F,t+1} - R_t)\right). \end{aligned} \tag{33}$$

These dynamic equations for portfolio shares and wealth also tell us how the last three state variables in $S_t$ evolve.[16]

There are two reasons why only one-period lagged portfolios are in the state space. First, (32) implies that the aggregate portfolio share $z_{HH,t}$ depends on the one-period lagged portfolio share $z_{HH,t-1}$ and the new portfolio share chosen at

[14] Adding up the market clearing conditions, the sum of wealth of both countries is proportional to the sum of their asset prices, so aggregate wealth is not a state variable. The lagged bond market equilibrium condition implies $\omega_{t-1}(z_{HH,t-1}+z_{HF,t-1})+(1-\omega_{t-1})(z_{FH,t-1}+z_{FF,t-1}) = 1$. We therefore cannot use lagged relative wealth and all four lagged portfolio shares as state variables. We also do not use the four lagged portfolio shares as state variables. In a symmetric state, where $\omega_{t-1} = 0.5$, the four lagged portfolio shares are locally in a linear relationship (adding to 2). Also note that $z^A_{F,t-1} = \omega_{t-1}z_{HF,t-1} + (1-\omega_{t-1})z_{FF,t-1} = 1 - z^A_{H,t-1}$ is redundant from the time $t-1$ bond market clearing condition.

[15] This uses the same approximation that we made for the market clearing conditions, that $\int_0^1 z^i_{jk,t}W^i_{j,t}di = z_{jk,t}W_{j,t}$ for $j = H, F$ and $k = H, F$. As discussed, this is numerically extremely accurate.

[16] Relative wealth is stationary in the model. In Appendix D we discuss the logic behind the stationarity and report the ergodic distribution of relative wealth $w^D_t$ for the parameterization in Table 2 that is discussed in Section 5.

time $t$. The lagged portfolio share $z_{HH,t-1}$ aggregates all portfolio shares chosen at $t-1$ and earlier. Second, the new portfolio share $\tilde{z}_{HH,t}$ chosen at time $t$ depends through the portfolio Euler equation on beliefs about $\lambda^{t}_{HH,t+1}$. The same variable one period earlier, $\lambda^{t-1}_{HH,t}$, is one of the control variables. It depends on portfolio choice at $t-1$, but not earlier. We do not need to solve for $\lambda^{t-j}_{HH,t}$ for $j > 1$, which depends on portfolio shares prior to $t-1$.

## 2.9 Definition of Equilibrium

Appendix A lists the Foreign country portfolio Euler equations, Bellman equations and first-order difference equations for $\lambda^{t-1}_{FH,t}$ and $\lambda^{t-1}_{FF,t}$. These are all derived analogously to those for the Home country.

**Definition 1** *An equilibrium consists of* $\left\{q_{H,t}, q_{F,t}, r_t, \tilde{z}_{HH,t}, \tilde{z}_{HF,t}, \tilde{z}_{FF,t}, \tilde{z}_{FH,t}, f^n_{H,t}, f^n_{F,t}\right\}$ *as functions of* $S_t$, $\left\{f^{o,t-1}_{H,t}, \lambda^{t-1}_{HH,t}, \lambda^{t-1}_{HF,t}\right\}$ *as functions of* $S_t$ *and* $s_{H,t}$, *and* $\left\{f^{o,t-1}_{F,t}, \lambda^{t-1}_{FH,t}, \lambda^{t-1}_{FF,t}\right\}$ *as functions of* $S_t$ *and* $s_{F,t}$ *such that the following are satisfied: (i) The Home portfolio Euler equations (21)-(22), (ii) the Home Bellman equations (23)-(24), (iii) the Home* $\lambda$ *difference equations (25)-(26), (iv) the Foreign country analogues of (21)-(26) shown in Appendix A, and (v) the market clearing conditions (27)-(29).*

# 3 Solution Method

The model is solved with a global solution method. The large number of state and control variables (a total of 15 each) makes this challenging when using standard projection methods. It runs into a dimensionality problem both when control variables are approximated as step functions on a rectangular grid of state variables and when they are approximated as polynomial functions that minimize average equation errors on a large number of points of the state space. Even quadratic polynomial functions for the control variables applied to the entire ergodic space (which may not be precise enough) would involve 2040 parameters with 15 state and control variables.[17] We therefore instead follow the Taylor projection method

[17] The actual number in our case is 1041 as not all control variables depend on all state variables, but it would still be prohibitive.

developed in Levintal (2018).[18] This involves approximating the solution locally at various nodes of the state space, and then combining these local solutions to form the global solution. To this end, it is sufficient for us to use a local linear approximation of the control variables as a function of state variables. This involves far fewer parameters, although it needs to be repeated at many points of the state space. We now describe the various steps involved.

The aim is to find a solution

$$cv_t = g(sv_t) \tag{34}$$

Given a particular node in the state space, Taylor projection locally approximates $g(sv_t)$ as a polynomial, which in our case will be linear. For a particular node $sv^i$ in the state space, this takes the form

$$cv_t = cv^i + M^i(sv_t - sv^i) \tag{35}$$

where $M^i$ is a matrix with a non-zero value in element $(j, k)$ if state variable $k$ affects control variable $j$. As discussed, not all control variables depend on all state variables. There are a total of 153 non-zero coefficients in $M^i$, plus 15 constants in the vector $cv^i$, for a total of 168 coefficients.

The model can be written in the form

$$E_t F(cv_t, cv_{t+1}, sv_t, sv_{t+1}) = 0 \tag{36}$$

$$sv_{t+1} = G(sv_t, cv_t, \epsilon_{t+1}) \tag{37}$$

where $\epsilon_{t+1} = (\epsilon^d_{H,t}, \epsilon^d_{F,t}, \epsilon^\tau_{H,t}, \epsilon^\tau_{F,t})'$ is the vector of shocks. $F$ consists of the 15 equations listed in Definition 1. Equation (37) describes the evolution of state variables and is discussed in Appendix B1. Using (35) at both $t$ and $t+1$, together with (37), we can write (36) in the form

$$\mathcal{H}(sv_t) = E_t H(sv_t, \epsilon_{t+1}) = 0. \tag{38}$$

We compute expectations using an order-5 monomial method (Judd, 1998) with 33 integration nodes for the shocks. $\mathcal{H}(sv_t)$ represents the errors of the equations. At the node $sv^i$, the "Taylor" part of "Taylor projection" involves setting both

[18] Den Haan et al. (2016) develop an analogous method. The method is applied in Fernandez-Villaverde and Levintal (2017) and Barro et al. (2018) to solve models with rare disasters.

the level of $\mathcal{H}$ and its derivatives with respect to the state variables equal to zero: $\mathcal{H}(sv^i) = 0$ and $\partial\mathcal{H}/\partial sv_t(sv^i) = 0$. These give respectively 15 and 153 constraints on the 168 parameters $\{cv^i, M^i\}$.[19] We compute numerical derivatives using two-sided finite-differences (using two or five-point stencils makes no difference). We then solve the 168 parameters $\{cv^i, M^i\}$ from the 168 equations.

We first obtain the local solution at the deterministic steady state.[20] The other nodes $sv^i$ are obtained as follows. Since a rectangular grid is infeasible in such a high-dimensional problem, we use the approach from Maliar and Maliar (2015). We generate a long simulation using the linear solution at the symmetric state (10 million periods). We sample every 1000 points to eliminate autocorrelation. From this sample we construct a set of 150 points using Ward's clustering algorithm. We were able to obtain solutions for $\{cv^i, M^i\}$ at 120 of these points outside of the deterministic steady state. We then use symmetry to obtain the solution at another 120 points. So we have a solution at 241 points. We find that these points cover the ergodic set sufficiently well.[21]

To construct the global solution, we use the modified Shepard's inverse-weighting interpolation. Define the weights

$$w_i(sv^i, sv_t) = \frac{\tilde{w}_i(sv^i, sv_t)}{\sum_{j=1}^{241} \tilde{w}_j(sv^j, sv_t)} \tag{39}$$

where

$$\tilde{w}_i(sv^i, sv_t) = \left( \frac{max\{0, k - \|sv_i - sv_t\|\}}{k\|sv_i - sv_t\|} \right)^2 .$$

$\|sv_i - sv_t\|$ is the Euclidean distance and $k$ is set to 4.[22] Then

$$cv(sv_t) = \sum_{i=1}^{241} w_i(sv^i, sv_t) \left( cv^i + M^i(sv_t - sv^i) \right) \tag{40}$$

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[19] Specifically, all 15 equations depend on $S_t$ (9 state variables), which gives 135 derivatives. In addition, the Bellman equation for $f_{Ht}^{o,t-1}$ and the difference equations for $\lambda_{HH,t}^{t-1}$ and $\lambda_{HF,t}^{t-1}$ also depend on $sv_{H,t}$ (3 state variables). This gives an additional 9 derivatives and an analogous 9 derivatives for the Foreign country. This gives a total of 153 derivatives.

[20] Variables other than portfolio shares are equal to their deterministic steady states. The portfolio shares are set at $z_{HH} = z_{FF} = \bar{z}$, $z_{HF} = z_{FH} = 1 - \bar{z}$, where $\bar{z}$ is set at an empirically realistic value (see Section 4). The value for $\tau$ is set to make sure that also $\tilde{z}_{HH} = \tilde{z}_{FF} = \bar{z}$, $\tilde{z}_{HF} = \tilde{z}_{FH} = 1 - \bar{z}$ at this symmetric node of the state space.

[21] If we create a new set of points by simulating the resulting global solution, the new set of points is very similar.

[22] Setting $k$ lower than 4 raises Euler equation errors, while setting it higher makes little difference to the weights.

Further technical details on the solution can be found in Appendix B.

We also solve the model in the frictionless case where $p = 1$. The same solution method is followed, but the solution is significantly faster as there are far fewer state and control variables. The set of state variables consists of the exogenous state variables $d_{H,t}$, $d_{F,t}$, $\tau_{H,t}$, $\tau_{F,t}$ and relative wealth $w_t^D$. The other four state variables in $S_t$, related to lagged relative wealth and portfolio shares, as well as $s_{H,t}$ and $s_{F,t}$, are no longer state variables. The additional control variables $cv_{H,t}$ and $cv_{F,t}$ also disappear. Overall, the number of state variables is reduced from 15 to 5 and the number of control variables is reduced from 15 to 9.

# 4 Approximate Portfolio Expression

In this section we discuss an approximate portfolio expression in order to develop intuition about what is driving portfolio allocation in the model.

## 4.1 Notation

Since our data in the next section applies to equity portfolio shares, we focus on the equity portfolio. For agents who make new portfolio decisions, the share of the equity portfolio that is invested in Home equity by respectively Home and Foreign agents is denoted

$$\tilde{z}^e_{HH,t} = \frac{\tilde{z}_{HH,t}}{\tilde{z}_{HH,t} + \tilde{z}_{HF,t}}$$
$$\tilde{z}^e_{FH,t} = \frac{\tilde{z}_{FH,t}}{\tilde{z}_{FH,t} + \tilde{z}_{FF,t}}.$$

We are particularly interested in the average portfolio share invested in Home equity, $\tilde{z}^{e,A}_t = 0.5(\tilde{z}^e_{HH,t} + \tilde{z}^e_{FH,t})$. The main effect of infrequent portfolio decisions relates to the way investors respond to changes in expected excess returns. Expected excess returns affect the average portfolio share $\tilde{z}^{e,A}_t$, but not the difference in portfolio shares, $\tilde{z}^{e,D}_t = \tilde{z}^e_{HH,t} - \tilde{z}^e_{FH,t}$, which is a measure of equity home bias.[23]

Some notation regarding asset returns is in order as well. Asset returns will be denoted in logs, so $r_{Ht} = log(R_{Ht})$, $r_{Ft} = log(R_{Ft})$ and $r_t = log(R_t)$. The world

[23]Moreover, at least up to the time of the Great Recession, there has been a trend decrease in home bias for reasons that have little to do with gradual portfolio adjustment.

equity return is $r^A_{t+1} = 0.5(r_{H,t+1} + r_{F,t+1})$. We define the excess returns of stocks over bonds in the two countries as $er_{H,t+1} = r_{H,t+1} - r_t$ and $er_{F,t+1} = r_{F,t+1} - r_t$. The excess return of Home equity over Foreign equity is denoted $er_{t+1} = r_{H,t+1} - r_{F,t+1}$ and we denote $er_{t+1,t+i} = er_{t+1} + ... + er_{t+i}$ as the cumulative excess return of Home equity over Foreign equity over the next $i$ periods. Other cumulative returns are denoted analogously. We also denote $er^{sum}_{t+i} = er_{H,t+i} + er_{F,t+i}$.

## 4.2 Approximated Portfolio

To derive approximate portfolio shares, we follow a methodology similar to Campbell and Viceira (1999), although the portfolio problem is considerably more complicated here. After a significant amount of algebra described in the Online Appendix,[24] we find the following approximate expression for the average Home equity portfolio share:

$$\tilde{z}^{e,A}_t = 0.5 + \frac{1}{D}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{t+i} + \frac{\mu}{(1-\theta)D}\tau^D_t + h_t \tag{41}$$

where $\mu = 0.5 + (\bar{z} - 0.5)D/d$ and

$$D = \sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma \bar{var}_t(er_{t+i}) + 2(\gamma-1)\bar{cov}_t(er_{t+i}, er_{t+1,t+i-1})\right) \tag{42}$$

$$d = \sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma \bar{var}_t(er^{sum}_{t+i}) + 2(\gamma-1)\bar{cov}_t(er^{sum}_{t+i}, er^{sum}_{t+1,t+i-1})\right) \tag{43}$$

Moments with a bar refer to the mean of these moments.

The optimal portfolio depends on three terms. The first is a present discounted value of expected future excess returns (international equity return differentials). The second is proportional to $\tau^D_t = \tau_{Ht} - \tau_{Ft}$. This term is associated with financial shocks. $\tau^D_t$ rises when the cost of investment abroad rises for Home agents relative to Foreign agents. This leads to an exogenous portfolio shift toward Home equity.

[24] We start by deriving expressions for $\tilde{z}_{HH,t}$, $\tilde{z}_{HF,t}$, $\tilde{z}_{FH,t}$ and $\tilde{z}_{FF,t}$ using portfolio Euler equations, Bellman equations, and $\lambda$ difference equations. We log-linearize portfolio returns, though we treat the new time $t$ portfolio shares as unknown parameters that need to be solved and do not linearize around these variables. Most expectations take the form of $E_t e^x$, where $x$ includes log asset returns and the Bellman variables. Assuming log normality, these expectations are approximated as $e^{Ex+0.5var(x)}$, where $Ex$ and $var(x)$ are moments that vanish to zero in the deterministic steady state. We then approximate this as $1 + Ex + 0.5var(x)$.

The third term is $h_t$, which will be discussed further below. It involves various hedge terms associated with time-varying expectations of future risk.

### 4.3 Comparison to Frictionless Portfolio

It is instructive to compare (41) to what it would be when $p = 1$:

$$z_t^{e,A} = \frac{E_t er_{t+1}}{\gamma v\bar{a}r_t(er_{t+1})} + \frac{\mu}{\gamma v\bar{a}r_t(er_{t+1})}\tau_t^D + h_t \tag{44}$$

We will focus here on the expected excess return term. When $p < 1$ the average share invested in Home equity depends on the present discounted value of all expected future excess returns of Home equity over Foreign equity, as opposed to just the expected excess return over the next period as in (44). The effective horizon that investors have is longer as they do not know when they will make a portfolio decision again. The discount rate is $\theta = \beta(1-p)$. A lower value of $p$ therefore implies a longer effective horizon when decisions are made and a relatively higher weight on expected excess returns further into the future. There is a close analogy between this optimal portfolio and the optimal price that a firm sets under Calvo price setting. The latter assumes that there is a probability $p$ of firms setting a new price each period. When a firm sets a new price, the expression for the optimal price (e.g. page 45 of Gali, 2008) depends on a weighted average of expected future marginal costs, with the weight declining at the same rate $\beta(1-p)$ as in the optimal portfolio expression (41).

A lower $p$ changes not just the relative weights of expected excess returns at different horizons, but also the absolute weight. It implies that investors are less responsive to expected excess returns in the near future. To see this, consider the portfolio response to a change in $E_t er_{t+1}$, which has a coefficient $1/D$. When $p = 1$, $D = \gamma v\bar{a}r_t(er_{t+1})$, as seen in (44). When $p < 1$, the expression for $D$ is more complicated. The term in $D$ multiplying $\theta^{i-1}$ is approximately equal to $\gamma$ times the component of the variance of $er_{t+1,t+i}$ that is associated with the time $t+i$ excess return, $cov(er_{t+i}, er_{t+1,t+i})$. $D$ therefore depends on long run excess return risk, with a longer effective horizon when $p$ is smaller. This higher risk implies a weaker portfolio response to the expected excess return next period. If, for illustrative purposes, for $p < 1$ we simplify the expression for $D$ by ignoring the second term in (42), which depends on autocorrelations of excess returns, and assume the same variance of all future excess returns, we have $D = \gamma var_t(er_{t+1})/(1-\theta)$.

The portfolio response to a change in $E_t er_{t+1}$ is therefore a fraction $1-\theta$ of the portfolio response when $p=1$. The smaller the $p$, the weaker the response.

There is a second reason why investors respond less to expected excess returns when $p<1$, which is simply that only a limited fraction of investors make a new portfolio decision at any time. Analogous to $\tilde{z}_t^{e,A}$, we define the overall portfolio share $z_t^{e,A}$ as the average of $z_{HH,t}/(z_{HH,t}+z_{HF,t})$ and $z_{FH,t}/(z_{FH,t}+z_{FF,t})$. Linearization implies that it evolves according to

$$z_t^{e,A}=(1-p)z_{t-1}^{e,A}+p\tilde{z}_t^{e,A} \tag{45}$$

For a given response of $\tilde{z}_t^{e,A}$ to changes in expected excess returns, this implies a weaker and more gradual response of the overall portfolio share $z_t^{e,A}$.

The weaker portfolio response to expected excess returns under the portfolio friction is a key aspect of the model. It implies that larger equilibrium changes in expected excess returns are needed to clear the market when there are financial shocks associated with an exogenous change in $\tau_t^D$. Larger changes in expected excess returns imply larger changes in the relative asset price, which in turn also implies larger changes in equilibrium relative asset supplies and portfolios.

## 4.4 Hedge Terms

The last term in (41) is

$$\begin{aligned}
h_t = {} & \frac{1-\gamma}{D}\sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{t+i},r^A_{t+1,t+i}) \\
& \frac{1-\gamma}{D}\sum_{i=1}^{\infty}\theta^{i-1}\left(cov_t(er_{H,t+i},(1-\mu)f^n_{H,t+i}+\mu f^n_{F,t+i})-cov_t(er_{F,t+i},\mu f^n_{H,t+i}+(1-\mu)f^n_{F,t+i})\right) \\
& +\frac{(1-2\bar{z})^2}{d}\sum_{i=1}^{\infty}\theta^{i-1}cov_t(r^A_{t+i}-r_{t+i-1},(\gamma-1)er_{t+1,t+i-1}+\gamma er_{t+i}).
\end{aligned}$$

The terms in $h_t$ are hedge terms associated with time-varying risk. These involve the variance and covariance of asset return variables and the Bellman variables $f^n_{H,t+s}$, $f^n_{F,t+s}$. Analogous to expected asset returns, it is not just uncertainty about asset returns and Bellman variables over the next period that affects portfolios, but rather perceived risk at all future dates, with discount rate $\theta$. In what follows, assume that $\gamma>1$. We will focus on the first and second terms. The last hedge term is less intuitive and is discussed in Appendix C.

The first term in $h_t$ implies that the average share invested in Home equity is higher when Home equity has a relatively high payoff (compared to Foreign equity) in bad future states where the world equity return has been low. Home equity is then an attractive hedge against such bad states. The second term of $h_t$ captures a hedge against future changes in expected portfolio returns. The approximated solution for $f^n_{H,t}$ is

$$f^n_{H,t} = E_t \sum_{i=1}^{\infty} \beta^i \bar{r}^{p,H}_{t+i} \tag{46}$$

where $\bar{r}^{p,H}_{t+i} = r_{t+i-1} + \bar{z}er_{H,t+i} + (1-\bar{z})er_{F,t+i}$ is the Home portfolio return evaluated at the mean of portfolio shares. An analogous solution applies to $f^n_{F,t}$. The second term of (46) then says that the Home portfolio share is high when Home equity returns are relatively high in bad future states where subsequent future expected portfolio returns are low.

# 5 Numerical Illustration

In this section we provide a numerical illustration of the impact of infrequent portfolio adjustment on asset prices, excess returns and portfolios. In this illustration one period will equal one month. We first discuss the calibration of model parameters. Next we show that the global solution for $\tilde{z}^{e,A}_t$ is almost identical to the sum of the first two terms in (41), the expected excess return term and the financial shock term. The hedge term $h_t$ is not important quantitatively. After discussing two parameterizations where $p = 1$, we compare these frictionless cases to the case of infrequent portfolio decisions, focusing on asset prices, excess returns and portfolios. We discuss both impulse responses to dividend and financial shocks and various moments involving these variables based on model simulation, which are compared to the data.

## 5.1 Calibration

The numerical solution when $p < 1$ is very time consuming. We therefore consider just one set of parameters. The calibration is shown in Table 2. It involves the parameters of the dividend and financial shock processes, as well as $p$, $\gamma$ and $\beta$.

The most important parameter is clearly $p$. We set it at 0.04, so that agents on average change their portfolio once in two years (25 months). The Investment

Company Institute reports that only 40 percent of US investors change their stock or mutual fund portfolio during any particular year. In the year 2001, 61 percent made no change. In 2007, 57 percent made no change.[25] $p = 0.04$ implies that 61 percent of agents will make no portfolio change in any particular year.[26] We can also draw a comparison to BTW for US mutual funds. They regress US mutual fund portfolio shares in foreign countries on last month's portfolio share and the present discounted value of expected future excess returns. The coefficient on last month's portfolio share is an estimate of $1 - p$ in our framework. Their results imply $p = 0.07$. This gives equal weight to all mutual funds. For large mutual funds, which matter more in the aggregate, their estimates imply $p = 0.05$.

We set $\gamma = 10$ and $\beta = 0.99668$. Risk aversion of 10 is simply adopted from Bacchetta and van Wincoop (2010), who use their model of infrequent portfolio adjustment to account for the forward discount puzzle. They provide a variety of motivations for this choice. A time discount rate of 0.99668 implies a risk-free rate that is about 4 percent annualized in the risky steady state.

The parameters of the dividend process are calibrated to the United States (Home) and the rest of the world (Foreign). The latter, also referred to as ROW, consists of an aggregate of 44 foreign countries. We use 230 months of MSCI data, from November 1995 to December 2014. Data on earnings are used as opposed to dividends as the latter do not include share repurchases, which have become the preferred method of shareholder payments.[27] Defining $d_t^D = d_{Ht} - d_{Ft}$ and $d_t^A = 0.5(d_{Ht} + d_{Ft})$ as the relative and average log dividend, we have $d_t^D = \rho_d d_{t-1}^D + \epsilon_t^{d,D}$ and $d_t^A = \rho_d d_{t-1}^A + \epsilon_t^{d,A}$. We set $\rho_d = 0.9767$ as the autocorrelation of $d_t^D$. We then compute $\epsilon_t^{d,D}$ and $\epsilon_t^{d,A}$ and use their standard deviations, which are reported in Table 2. We set $d = (1 - \beta)/\beta$, which implies an annualized dividend yield of 4

[25] The 2001 number is from Equity Ownership of America, 2002, while the 2007 number is from Equity and Bond Ownership in America, 2008.

[26] Even less frequent portfolio changes apply to retirement accounts. Ameriks and Zeldes (2004) find that over a 10-year period, 44 percent of households made no changes at all to their TIAA-CREF portfolio allocations. This corresponds to $p = 0.007$. Similarly, Mitchell et al. (2006) find that 80 percent of 1.2 million workers with 401(k) plans initiated no trades over a two year period. This corresponds to $p = 0.01$.

[27] The MSCI earnings data is a 12-month trailing average. Companies do not report monthly dividends. The measure is reasonable if dividends plus repurchases keep up with the 12-month trailing average of earnings. The correlation between $d_{Ht} - d_{Ft}$ computed based on relative earnings and relative dividends is 0.81.

percent in steady state.

There are analogously also four financial shock parameters. We set $\tau$ equal to 0.0002, which implies realistic equity home bias in the risky steady state in the sense of matching the average of the fractions that US and ROW investors invest in respectively the US and ROW. Over the November 1995 to December 2014 sample this average is 0.7634. The portfolio data are discussed in Section 5.5. Analogous to dividends, we define the difference and average of $\tau_{Ht}$ and $\tau_{Ft}$ as $\tau_t^D$ and $\tau_t^A$. These follow AR processes with AR coefficients of $\rho_\tau$ and innovations of $\epsilon_t^{\tau,D}$ and $\epsilon_t^{\tau,A}$. We set $\rho_\tau$ and the standard deviation of these two innovations at the values shown in Table 2. As we will see in Section 5.5, this implies properties of the excess returns that are reasonably close to the data. This includes the standard deviation and autocorrelation of $er_{t+1}$ and the correlation between $er_{H,t+1}$ and $er_{F,t+1}$.

## 5.2 Approximated Solution

Equation (41) gives a linear approximation of the solution of $\tilde{z}_t^{e,A}$ as the sum of three terms. We will show that the global solution is very close to just the sum of the first two terms, so ignoring the hedge term. This approximated solution is

$$\tilde{z}_t^{e,A,approximate} = 0.5 + \frac{1}{D}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{t+i} + \frac{\mu}{(1-\theta)D}\tau_t^D. \tag{47}$$

To show that this is close to $\tilde{z}_t^{e,A}$ from the global solution, we simulate the model over 230 months, the sample length used for calibration and to compute data moments in Section 5.5. During each month the present discounted value of expected excess returns is computed by generating 100,000 different futures of 150 months.[28] The parameters $D$ and $\mu$ are computed using the mean over the 230 months of the present discounted value of the moments in the expressions for $D$ and $d$, again using 100,000 futures of 150 months to compute the present discounted value of the moments.

Figure 1 shows both the global solution for $\tilde{z}_t^{e,A}$ and the approximated solution (47). The two lines are extremely close, with a correlation of 0.964. Sometimes they are indistinguishable and overlap. Any deviation that is left is caused either by the approximation itself used to derive (41) or the hedge term $h_t$. We have

[28] Truncating after 150 months is sufficient as $\theta^{150}$ is equal to 0.0013, so that expected excess returns further into the future get virtually no weight.

not been able to numerically approximate the time varying hedge term accurately enough as it would require an even much larger number of futures, but clearly it does not play a significant role.

This result implies that we can focus on the expected excess return term and the financial shock term to understand portfolio behavior. The intuition about the impact of expected excess returns discussed in the previous section, and the comparison to the frictionless case $p = 1$, is important to the solution. The approximation also helps quantify the financial shock in terms of a portfolio shock. We will define the size of the financial shock as the instantaneous change in $z_t^{e,A}$ due to a one standard deviation innovation in $\epsilon_t^{\tau,D}$. Using the approximation (47), this is equal to

$$p\frac{\mu}{(1-\theta)D}\sigma_{\epsilon^{\tau,D}}. \tag{48}$$

Measuring the financial shock as a portfolio shock this way allows for easier comparison to $p = 1$, as we will discuss below.

### 5.3 Two Frictionless Cases

In comparing the solution with $p = 0.04$ to a frictionless world where $p = 1$, we consider two cases, labeled Case 1 and Case 2. These differ only in the magnitude of the financial shocks. Since all investors face the financial shock when $p = 1$, while only 4 percent of investors experience the shock when $p = 0.04$, the same size shock to $\tau_t^D$ has a much larger portfolio impact when $p = 1$. In Case 1 the standard deviations of $\epsilon_t^{\tau,D}$ and $\epsilon_t^{\tau,A}$ are proportionately reduced such that the size of financial shocks, as measured by (48), remains the same as when $p = 0.04$.[29] In that case a one standard deviation innovation $\epsilon_t^{\tau,D}$ raises $z_t^{e,A}$ by 0.0103. In addition $\tau$ is reduced to 0.000083 to keep the fraction invested in domestic equity at 0.763 in the risky steady state.

In Case 2 all parameters remain the same as in Table 2 for $p = 0.04$, except that $\tau = 0.00013$ is set to match home bias. In this case a one standard deviation innovation $\epsilon_t^{\tau,D}$ raises $z_t^{e,A}$ by 0.34. This is a huge financial shock, where exogenously (before general equilibrium effects set in) a two standard deviation financial shock would raise the average share invested in Home equity from 50% to

[29] Specifically, the standard deviations of $\epsilon_t^{\tau,D}$ and $\epsilon_t^{\tau,A}$ are both reduced by a factor 232, to respectively $6.47E-6$ and $2.16E-7$.

117% in just one month. The financial shock in Case 2 is so large mainly because the fraction of investors that face the same shock to $\tau_t^D$ is now 25 times as large as when $p = 0.04$. While a comparison of $p = 0.04$ to Case 1 is therefore more appropriate, Case 2 is nonetheless useful for illustrative purposes.

## 5.4 Impulse Response Functions

Figure 2 shows the impulse response of the relative log asset price $q_t^D$ and the average equity share $z_t^{e,A}$ in the Home country for both relative dividend shocks (top two charts) and financial shocks (bottom two charts). The impulse response functions are shown for $p = 0.04$ and the two frictionless cases. The relative dividend shock is a one standard deviation increase in $\epsilon_t^{d,D}$, while the financial shock is a one standard deviation increase in $\epsilon_t^{\tau,D}$.[30] The reported impulse response functions are an average of 10,000 impulse responses that are computed at different points in the state space after first simulating the model over 10,000 months. For financial shocks only, Figure 3 reports expected excess returns based on the same exercise. It shows the expectation at the time of the shock of future expected excess returns $E_t er_{t+i}$ and expected cumulative excess returns $E_t er_{t+1,t+i}$, both as a function of $i$.[31] Since expected excess returns are dominated by financial shocks, we do not show the much smaller expected excess returns for relative dividend shocks.

Panels A and B of Figure 2 show that the effect of dividend shocks on the relative price and average portfolio is quite similar in the three scenarios. There is some slight delayed overshooting when $p = 0.04$, but it does not markedly affect the equilibrium. Setting $p$ even smaller would make a difference in that the immediate impact of the dividend shocks on the relative price and portfolio would be smaller and there would be more delayed overshooting. But only having four percent of investors actively changing their portfolio in response to a change in dividends is sufficient to generate a price and portfolio impact that is similar to the frictionless case.

[30] We do not show the impulse response to average shocks as these have no effect on our variables of interest. They do not operate through expected excess returns, where portfolio frictions critically come into play.

[31] These are not impulse response functions, but they are computed from impulse response functions by assuming that excess returns after the shock at time $t$ are equal to expected excess returns at time $t$ due to the shock. Numerically we find this to be very accurate.

The biggest impact of infrequent portfolio choice is with regards to financial shocks, shown in panels C and D. An increase in $\tau_t^D$ leads to a portfolio shift to the Home country, which raises the relative Home price $q_t^D$ and the average portfolio share $z_t^{e,A}$ in the Home country. We first consider a comparison of $p = 0.04$ to $p = 1$ for Case 1, where the financial shock is equal in size. Panels C and D show that in the frictionless case both the relative asset price and average portfolio remain virtually unchanged. Exogenous portfolio shifts are easily absorbed by investors in the frictionless case, without requiring much of a change in the relative price (Figure 2, panel C) and expected excess return (Figure 3). The reason is that the portfolio share is very sensitive to the expected excess return.

In Section 4.3 we discussed two reasons for the stronger sensitivity of the portfolio share to expected excess returns in the frictionless case. First, the fraction of investors that actively responds to a change in the expected excess return is 25 times higher when $p = 1$. Second, within the group of investors who choose a new portfolio, the response to a change in the expected excess return is much smaller when $p = 0.04$ because investors have a longer effective horizon and therefore respond less to expected excess returns in the near future. From (47) the change in $\tilde{z}_t^{e,A}$ is $1/D$ times the change in $E_t er_{t+1}$. $1/D$ is 14 when $p = 0.04$ versus 3290 when $p = 1$ (Case 1).[32] The strong sensitivity of the portfolio to changes in expected excess returns when $p = 1$ implies that a very small drop in the expected excess return on Home equity is sufficient to absorb the exogenous shift toward Home equity due to the financial shock. The overall portfolio $z^{e,A}$ therefore changes very little.

It is however possible to generate a large asset price and portfolio response even in the frictionless case, as is shown for $p = 1$, Case 2. As discussed, the financial shocks in this case are 34 times as large as when $p = 0.04$. Even with portfolios being very sensitive to the expected excess return, an extreme enough financial shock will still require a substantial change in the relative asset price and expected excess return, as shown in Figure 2, panel C, and Figure 3. Notably though, even with the financial shock 34 times as high in the frictionless case, the relative asset

[32] Part of the reason that the response is so extreme when $p = 1$ is that the excess return is less volatile (see Section 5.5), which is itself the result of the small effect of financial shocks on the relative price. But even when we set the standard deviation of the excess return equal to that of the $p = 0.04$ case, we would get $1/D = 1/[\gamma var_t(er_{t+1})] = 189$, which is still over 13 times as large as when $p = 0.04$.

price still changes considerably less than with $p = 0.04$, and the expected excess return is also smaller in size. The initial portfolio response is larger when $p = 1$ than $p = 0.04$, but much less persistent. When $p = 0.04$ the average portfolio continues to rise for about 10 months before slowly going down.

## 5.5 Data versus Model Moments

Tables 3 and 4 report various model moments and compare them to data moments. Table 3 reports regression coefficients of 1 month, 3 month, 12 month and 48 month equity excess returns $er_{t+1,t+i}$ on the current relative log dividend yield $d_t^D - q_t^D$. These are population moments, obtained from a simulation of the model over one million months. The corresponding data moments in the first column are from Table 1. These are not exactly comparable as the model has only two countries, while the data moments are based on a panel regression of US minus foreign equity returns for 73 foreign countries. But it is exactly the panel aspect that allows for good empirical precision.

Table 4 reports model and data moments for excess returns and portfolio shares. In the data these are based on 230 months from November 1995 to December 2014. The model moments are based on 100,000 simulations of 230 months, showing both the average moments and standard errors. The portfolio data used to compute the data moments are obtained from US external equity assets and liabilities from Bertaut and Tryon (2007) and Bertaut and Judson (2014), together with US and ROW market capitalization data. The US equity portfolio share by respectively US and ROW investors is computed as

$$z^e_{HH,t} = \frac{z_{HH,t}}{z_{HH,t} + z_{HF,t}} = \frac{US\ market\ cap - US\ ext\ liab}{US\ market\ cap - US\ ext\ liab + US\ ext\ assets}$$
$$z^e_{FH,t} = \frac{z_{FH,t}}{z_{FH,t} + z_{FF,t}} = \frac{US\ ext\ liab}{ROW\ market\ cap + US\ ext\ liab - US\ ext\ assets}$$

where US ext liab and US ext assets refer to US external equity liabilities and assets. Average and relative portfolios are $z_t^{e,A} = 0.5(z^e_{HH,t} + z^e_{FH,t})$ and $z_t^{e,D} = z^e_{HH,t} - z^e_{FH,t}$. We are mainly interested in $z_t^{e,A}$, which depends on expected excess returns. $z_t^{e,D}$ is a home bias variable that is mainly driven by exogenous changes in $\tau_t^A$.[33]

[33] We only report the volatility of its monthly change as home bias trends upward in the data.

Table 3 shows that there is significant excess return predictability by the relative dividend yield when $p = 0.04$, which increases with the horizon as is the case in the data. Overall the extent of predictability is comparable to that seen in the data, a little higher at the 3 month horizon and lower at the 48 month horizon. Not surprisingly, in Case 1 of the frictionless world there is virtually no excess return predictability. This is consistent with Figure 3. The strong sensitivity of portfolios to expected excess returns implies very small equilibrium expected excess returns. There is naturally more excess return predictability in Case 2 of the frictionless world where financial shocks are 34 times larger. But even with such large financial shocks the excess return predictability is a factor 3 to 5 smaller than when $p = 0.04$.

Table 4 shows various moments involving the volatility of excess returns and portfolios, their autocorrelations, as well as contemporaneous correlations with changes in relative dividends. When $p = 0.04$ all the moments are reasonably close to those in the data. This is not the case in the two frictionless cases. This is particularly evident in Case 1. The volatility of the excess return $er_t$ and portfolio $z_t^{e,A}$ is much smaller than in the data. Moreover, both $er_t$ and $\Delta z_t^{e,A}$ are almost perfectly correlated with the change in relative dividends. We have seen that financial shocks have little effect on asset prices and portfolios in that case, so that they are mainly affected by dividend shocks.

Case 2 of the frictionless world matches the data better as financial shocks matter due their extreme size. But even in this case the deviation from the data is larger than with $p = 0.04$. The excess return $er_t$ is not sufficiently volatile, while $\Delta z_t^{e,A}$ is more than twice as volatile as in the data and negatively autocorrelated (opposite to the data).

# 6 Conclusion

We have introduced a Calvo type portfolio friction in a two-country DSGE model for the global equity market. There is extensive micro evidence that investors make infrequent portfolio decisions and recently Giglio et al. (2019) have shown that the Calvo type infrequent trading friction is particularly relevant. In addition, BTW have documented the importance of portfolio frictions for international portfolio choice by focusing on mutual funds.

A significant contribution of this paper was to develop and solve such a model. Introducing an infrequent random portfolio decision making friction is more complex than the more familiar Calvo price setting. Portfolio Euler equations are complex and involve new control variables that capture the expected product of excess returns and stochastic discount factors beyond the next period. Through these variables, optimal portfolios depend not just on expected excess returns and risk over the next period, but over the infinite future. The effective horizons of investors is increased as they do not know when they will make a portfolio decision again. Although there are a large number of state and control variables due to the portfolio friction, we have been able to solve the model using a Taylor projection method, combined with the modified Shepard's inverse-weighting interpolation.

We have provided intuition by developing an approximated portfolio expression using a methodology similar to Campbell and Viceira (1999). The approximation shows that the optimal portfolio is the sum of three terms: a term that depends on the present discounted value of expected excess returns, a term that depends on exogenous portfolio shocks (financial shocks) and a hedge term that depends on the present discounted value of risk associated with future asset returns. We find numerically that the portfolio solution based on the global solution method is very close to the sum of the first two terms of the approximated expression.

We have provided a numerical illustration by comparing the model with the portfolio friction to two frictionless cases. The portfolio friction primarily affects equilibrium asset prices, expected excess returns and portfolios through the impact of financial shocks. The same size financial shock has a much smaller impact on these variables in the frictionless world. The model with the calibrated friction does a much better job in accounting for excess return predictability and other moments involving excess returns and portfolios. Under portfolio frictions, portfolios respond much less to expected excess returns and more gradually, which improves the empirical fit to the data.

# Appendix

# A Foreign Country Equations

First, define:

$$m_{F,t+1}^{n,t-j} = \left[R_{t+1}^{p,F,t-j}\right]^{-\gamma} e^{(1-\gamma)f_{F,t+1}^{n}}$$
$$m_{F,t+1}^{o,t-j} = \left[R_{t+1}^{p,F,t-j}\right]^{-\gamma} e^{(1-\gamma)f_{F,t+1}^{o,t-j}}$$

where the portfolio return is defined as

$$R_{t+1}^{p,F,t-j} = R_t + \tilde{z}_{FH,t-j}(R_{H,t+1} - R_t) + \tilde{z}_{FF,t-j}(R_{F,t+1} - R_t) \tag{A.1}$$

Also define $m_{F,t+1}^{t-j} = pm_{F,t+1}^{n,t-j} + (1-p)m_{F,t+1}^{o,t-j}$.

The Foreign country portfolio Euler equations are

$$E_t m_{F,t+1}^{t}(R_{F,t+1} - R_t) + (1-p)E_t m_{F,t+1}^{o,t} R_{t+1}^{p,F,t} \lambda_{FF,t+1}^{t} = 0 \tag{A.2}$$
$$E_t m_{F,t+1}^{t}(e^{-\tau_{F,t}} R_{H,t+1} - R_t) + (1-p)E_t m_{F,t+1}^{o,t} R_{t+1}^{p,F,t} \lambda_{FH,t+1}^{t} = 0 \tag{A.3}$$

The Foreign country Bellman equations are

$$e^{(1-\gamma)f_{F,t}^{n}/\beta} = \alpha E_t m_{F,t+1}^{t} R_{t+1}^{p,F,t} \tag{A.4}$$
$$e^{(1-\gamma)f_{F,t}^{o,t-1}/\beta} = \alpha E_t m_{F,t+1}^{t-1} R_{t+1}^{p,F,t-1} \tag{A.5}$$

The Foreign country $\lambda$ difference equations are

$$E_t R_{t+1}^{p,F,t-1}\left(m_{F,t+1}^{t-1}\lambda_{FH,t}^{t-1} - \theta m_{F,t+1}^{o,t-1}\lambda_{FH,t+1}^{t-1}\right) = \beta E_t m_{F,t+1}^{t-1}(e^{-\tau_{F,t-1}} R_{H,t+1} - R_t) \tag{A.6}$$
$$E_t R_{t+1}^{p,F,t-1}\left(m_{F,t+1}^{t-1}\lambda_{FF,t}^{t-1} - \theta m_{F,t+1}^{o,t-1}\lambda_{FF,t+1}^{t-1}\right) = \beta E_t m_{F,t+1}^{t-1}(R_{F,t+1} - R_t) \tag{A.7}$$

# B Further Details on Solution Method

## B.1 State Variables at $t+1$

(37) writes the evolution of the state variables as $sv_{t+1} = G(sv_t, cv_t, \epsilon_{t+1})$. To see this, first consider the last 6 state variables at $t+1$: $sv_{H,t+1} = (\tau_{Ht}, \tilde{z}_{HH,t}, \tilde{z}_{HF,t})'$ and $sv_{F,t+1} = (\tau_{Ft}, \tilde{z}_{FH,t}, \tilde{z}_{FF,t})'$. Clearly, these are elements of $sv_t$ (the tax rates) and $cv_t$ (the portfolio shares). Next consider the first 9 state variables: $S_{t+1} =$

$(d_{H,t+1}, d_{F,t+1}, \tau_{H,t+1}, \tau_{F,t+1}, w^D_{t+1}, w^D_t, z^A_{Ht}, z^D_{Ht}, z^D_{Ft})'$. The Home and Foreign dividends and tax rates at $t+1$ depend on the Home and Foreign tax rates at time $t$ (part of $sv_t$) and the shocks $\epsilon_{t+1}$. Skip over $w^D_{t+1}$ for a moment. $w^D_t$ is part of $sv_t$. $z^A_{Ht}$, $z^D_{Ht}$ and $z^D_{Ft}$ depend on $\omega_t$, $z_{HH,t}$, $z_{HF,t}$, $z_{FH,t}$ and $z_{FF,t}$. $\omega_t$ depends on $w^D_t$, which is part of $sv_t$. We can write $z_{HH,t} = (1-p)z_{HH,t-1} + p\tilde{z}_{HH,t} = (1-p)z^A_{H,t-1} + (1-p)(1-\omega_{t-1})z^D_{H,t-1} + p\tilde{z}_{HH,t}$, where $\omega_{t-1}$ depends on $w^D_{t-1}$. So $z_{HH,t}$ can be written as a function of state variables at time $t$ and control variables at time $t$. The same is the case for the other portfolio shares.

Some more discussion is warranted regarding $w^D_{t+1}$. Denote all state variables at $t+1$ other than $w^D_{t+1}$ as $\tilde{sv}_{t+1}$. It follows from the discussion above that $\tilde{sv}_{t+1} = G_s(sv_t, cv_t, \epsilon_{t+1})$ for a known function $G_s$. From (33), and the return expressions (1) and (2), as well as the discussion above, it follows that we can write $w^D_{t+1} = G_w(sv_t, cv_t, \epsilon_{t+1}, q_{H,t+1}, q_{F,t+1})$ for a known function $G_w$. At this point we substitute the linear projection (35) at a particular node $sv^i$, applied to $t+1$: $cv_{t+1} = cv^i + M^i(sv_{t+1} - sv^i)$. For a given $cv^i$ and $M^i$ (first and second row), this gives $q_{H,t+1}$ and $q_{F,t+1}$ as linear functions of $sv_{t+1}$, which in turn implies a linear function in $\tilde{sv}_{t+1}$ and $w^D_{t+1}$. Write these as $q_{i,t+1} = G_i(\tilde{sv}_{t+1}, w^D_{t+1})$. Then we have

$$w^D_{t+1} = G_w(sv_t, cv_t, \epsilon_{t+1}, G_H(G_s(sv_t, cv_t, \epsilon_{t+1}), w^D_{t+1}), G_F(G_s(sv_t, cv_t, \epsilon_{t+1}), w^D_{t+1})) \tag{B.8}$$

We linearize the right hand side around $w^D_{t+1} = w^D_t$, where $w^D_t$ is the fifth element of the node $sv^i$, to solve for $w^D_{t+1}$ as a function of $sv_t$, $cv_t$ and $\epsilon_{t+1}$.

## B.2 Other Details

We start the solution of the 168 parameters either at the deterministic steady state or at the nearest node in the state space for which we have solved the local solution. We go in steps of 0.001 times the distance towards the new node in the state space, each time resolving the parameters, until we have reach the new node. We normalize the variables $f^n_{H,t+1}$, $f^{o,t}_{H,t+1}$, $f^n_{F,t+1}$, $f^{o,t}_{F,t+1}$ by $f^{o,t-1}_{Ht}$ and $f^{o,t-1}_{Ft}$ to avoid overflows, given the large steady state values of the $f$ variables. We use a dampened quasi-Newton method to solve the parameters before switching to hybrid-Powell once the largest absolute value of the elements of $\mathcal{H}(sv_t)$ is less than $10^{-4}$. All codes are written in Fortran95 and compiled with the Intel Compiler, except for Ward's clustering algorithm, which is written in Matlab. All codes are available on request. No proprietary software is needed.

# C Last Hedge Term

The last hedge term in (46) is less intuitive. Here we attempt a brief intuition. This term is multiplied by $(1-2\bar{z})^2$ and therefore only applies when the mean $\bar{z}$ of the fraction invested domestically differs from 0.5. Assume that $\bar{z}$ is well above 0.5. The average share invested in Home equity is then affected mostly by $\tilde{z}_{HF}$ and $\tilde{z}_{FH}$ (as opposed to domestic portfolio shares). It also means that a particularly bad state for Home (Foreign) investors is a low Home (Foreign) equity return. The average Home equity share is larger when the covariance in the last line of (46) is positive. This means that in a bad state for the Home (Foreign) country, world equity returns tend to be low (high) relative to bond interest rates. To hedge against such bad states, it is attractive for Home agents to lower $\tilde{z}_{HF}$ and for Foreign agents to raise $\tilde{z}_{FH}$. Both raise the average Home equity share.

# D Ergodic Distribution Relative Wealth

We compute the ergodic distribution of relative wealth by simulating the model over one million months. This is done for the parameterization in Table 2. The result is shown in Figure A1. Ninety five percent of the distribution is between plus and minus 0.52. The logic behind the stationarity of relative wealth is as follows. Assume that a shock leads to an increase in the relative wealth of the Home country. As a result of home bias (which is matched in the parameterization), this leads to an increase in the relative demand for Home equity. This raises the relative Home equity price and therefore lowers the expected return on Home equity relative to Foreign equity. This lowers the expected portfolio return of Home agents relative to Foreign agents, which in turn reduces the relative wealth of the Home country.

# References


[1] Abel, Andrew B., Janice C. Eberly and Stavros Panageas (2007), "Optimal Inattention in the Stock Market," *American Economic Review Papers and Proceedings*, May 2007, 244-249.

[2] Ameriks, John and Stephen Zeldes (2004), "How do Household Portfolio Shares vary with Age," working paper.

[3] Bacchetta, Philippe, Simon Tièche and Eric van Wincoop (2020), "International Portfolio Choice with Frictions: Evidence from Mutual Funds" working paper.

[4] Bacchetta, Philippe and Eric van Wincoop (2006), "Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle?" *American Economic Review*, 96(3), 552-576.

[5] Bacchetta, Philippe and Eric van Wincoop (2010), "Infrequent Portfolio Decisions: A Solution to the Forwardc Discount Puzzle," *American Economic Review* 100, 837-869.

[6] Bacchetta, Philippe and Eric van Wincoop (2019), "Puzzling Exchange Rate Dynamics and Delayed Portfolio Adjustement," NBER Working Paper No. 26259.

[7] Barro, Robert, Jesus Fernandez-Villaverde, Oren Levintal and Andrew Mollerus (2018), "Safe Assets," Safe Assets, working paper.

[8] Benhima, Kenza and Rachel Cordonier (2020), "News, Sentiment and Capital Flows," working paper, University of Lausanne.

[9] Bertaut, Carol C. and Ruth Judson (2014), "Estimating U.S. Cross-Border Securities Positions: New Data and New Methods," Board of Governors of the Federal Reserve System (U.S.), International Finance Discussion Paper 1113.

[10] Bertaut, Carol C. and Ralph W. Tryon (2007), "Monthly Estimates of U.S. Cross-Border Securities Positions, " Board of Governors of the Federal Reserve System (U.S.), International Finance Discussion Paper 910.

[11] Bilias, Yannis, Dimitris Georgarakos and Michael Haliassos (2010), "Portfolio Inertia and Stock Market Fluctuations" *Journal of Money, Credit and Banking* 42: 715-742.

[12] Blanchard, Olivier, Irineu de Carvalho Filho, and Gustavo Adler (2015), "Can Sterilized Foreign Exchange Intervention Stem Exchange Rate Pressures from the Global Financial Cycle?,"IMF Working Paper 2015-159.

[13] Bogousslavsky, Vincent (2016), "Infrequent Rebalancing, Return Autocorrelation, and Seasonality," *Journal of Finance* LXXI (6), 2967-3006.

[14] Brunnermeier, Markus and Stefan Nagel (2008), "Do Wealth Fluctuations Generate Time-Varying Risk-Aversion? Micro-Evidence on Individuals' Asset Allocation," *American Economic Review* 98, 713-736.

[15] Campbell, John Y. and Luis M. Viceira (1999), "Consumption and Portfolio Decisions when Expected Returns are Time Varying," *Quarterly Journal of Economics* 114(2): 433-495.

[16] Campbell, John Y., Andrew W. Lo and Archie C. MacKinlay (1997), The Econometrics of Financial Markets, Princeton University Press, New York.

[17] Chien, Yili, Harold Cole, and Hanno Lustig (2012), "Is the Volatility of the Market Price of Risk Due to Intermittent Portfolio Rebalancing," *American Economic Review* 102, 2859-2896.

[18] Cochrane, John H. (2008), "The Dog That Did Not Bark: A Defense of Return Predictability," *The Review of Financial Studies* 21(4) 1533–1575.

[19] Davis, J. Scott and Eric van Wincoop (2018),"Globalization and the Increasing Correlation between Capital Inflows and Outflows," *Journal of Monetary Economics* 100, 83-100.

[20] Den Haan, Wouter J., Michal L. Kobielarz and Pontus Rendahl (2016), "Exact-Present Solution with Consistent-Future Approximation: A gridless Algorithm to Solve Stochastic Dynamic Models," working paper.

[21] Devereux, Michael B. and Alan Sutherland (2010), "Country Portfolio Dynamics," *Journal of Economic Dynamics and Control*, 34, 1325-1342.

[22] Devereux, Michael B. and Alan Sutherland (2007), "Monetary Policy and Portfolio Choice in an Open Economy Macro Model," *Journal of the European Economic Association* 5, 491-499.

[23] Didier, Tatiana and Alexandre Lowenkron (2012), "The Current Account as a Dynamic Portfolio Choice Problem," *Journal of The Japanese and International Economies* 26, 518-541.

[24] Dow, James and Gary Gorton (1995), "Profitable Informed Trading in a Simple General Equilibrium Model of Asset Pricing," *Journal of Economic Theory* 67, pp. 327-369.

[25] Duffie, Darrell (2010), "Asset Price Dynamics with Slow-Moving Capital," *Journal of Finance* LXV, 1237-1267.

[26] Evans, Martin D.D. and Viktoria V. Hnatkovska (2012), "A Method for Solving General Equilibrium Models with Incomplete Markets and Many Financial Assets," *Journal of Economic Dynamics and Control* 36(12), 1909–1930.

[27] Evans, Martin D.D. and Hnatkovska, Viktoria V. (2014), "International Capital Flows, Returns and World Financial Integration," *Journal of International Economics* 92(1), 14-33.

[28] Fernandez-Villaverde, Jesus and Oren Levintal (2018), "Solution Methods for Models with Rare Disasters," *Quantitative Economics* 9, 903-944

[29] Gabaix, Xavier and David Laibson (2002), "The 6D Bias and the Equity Premium Puzzle,"in Ben Bernanke and Kenneth Rogoff, eds, *NBER Macroeconomics Annual,* 16, 257-312.

[30] Gabaix, Xavier and Matteo Maggiori (2015), "International Liquidity and Exchange Rate Dynamics," *Quarterly Journal of Economics* 130(3), 1369-1420.

[31] Gali, Jordi (2008), Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications, Princeton University Press.

[32] Gârleanu, Nicolae, and Lasse Heje Pedersen (2013), "Dynamic Portfolio Choice with Frictions," *Journal of Finance* LXVIII, No. 6, 2309-2340.

[33] Giglio, Stefano, Matteo Maggiori, Johannes Stroebel and Stephen Utkus (2019), "Five Facts about Beliefs and Portfolios," NBER Working Paper 25744.

[34] Greenwood, Robin, Samuel G. Hanson, and Gordon Y. Liao (2018), "Asset Price Dynamics in Partially Segmented Markets," *Review of Financial Studies* 31(9), 3307-3343.

[35] Gromb, Denis and Dimitri Vayanos (2010), "Limits of Arbitrage," *Annual Review of Financial Economics* 2, 251–75.

[36] Hau, Harald, Massimo Massa, and Joel Peress (2010), "Do Demand Curves for Currencies Slope Down? Evidence from the MSCI Global Index Change," *Review of Financial Studies* 23, 1681–1717.

[37] He, Hua, and Jiang Wang (1995), "Differential Information and DynamicBehavior of Stock Trading Volume," *Review of Financial Studies* 8, 919-972.

[38] Hendershott, Terrence, Sunny X. Li, Albert J. Mankfeld, Mark S. Seasholes (2013), "Asset Price Dynamics with Limited Attention," mimeo.

[39] Hnatkovska, Viktoria V. (2010), "Home Bias and High Turnover: Dynamic Portfolio Choice with Incomplete Markets," *Journal of International Economics* 80(1), 113-128.

[40] Itskhoki, Oleg and Dmitry Mukhin (2019), "Exchange Rate Disconnect in General Equilibrium," working paper, Princeton University.

[41] Judd, Kenneth (1998), Numerical Methods in Economics, MIT Press Books, The MIT Press.

[42] Levintal, Oren (2018), "Taylor Projection: A New Solution Method for Dynamic General Equilibrium Models," *International Economic Review* 59, 1345-1373,

[43] Lynch, Anthony W. (1996), "Decision Frequency and Synchronization Across Agents: Implications for Aggregate Consumption and Equity Return," *The Journal of Finance* 51(4), 1479-1497.

[44] Maliar, Lilia and Serguei Maliar (2015), "Merging Simulation and Projection Approaches to Solve High-Dimensional Problems with an Application to a New Keynesian model," *Quantitative Economics* 6, 1-47.

[45] Mitchell, Olivia S., Gary R. Mottola, Stephen P. Utkus and Takeshi Yamaguchi (2006), "The Inattentive Participant: Portfolio Trading Behavior in 401(k) plans," working paper.

[46] Rey, Hélène (2013), "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence," *Jackson Hole Conference Proceedings*, Kansas City Fed.

[47] Spiegel, Matthew and Avanidhar Subrahmanyam (1992), "Informed Speculation and Hedging in a Noncompetitive Securities Market," *The Review of Financial Studies* 5, 307-329.

[48] Tille, Cédric and Eric van Wincoop (2010a), "International Capital Flows," *Journal of International Economics* 80(2), 157-175.

[49] Tille, Cédric and Eric van Wincoop (2010b), "A New Perspective on the "New Rule" for the Current Account," *Journal of International Economics* 80(1), 89-99.

[50] Tille, Cédric and Eric van Wincoop (2014), "International Capital Flows under Dispersed Private Information," *Journal of International Economics* 93(1), 31-49.

[51] Vayanos, Dimitri and Paul Woolley (2012), "An Institutional Theory of Momentum and Reversal," *Review of Financial Studies* 6, 1087-1145.

[52] Wang, Jiang (1994), "A Model of Competitive Stock Trading Volume," *Journal of Political Economy* 102, 127-168.

## Table 1 Predictability International Equity Return Differentials

| | 1 month | 3 months | 12 months | 48 months |
|---|---|---|---|---|
| Earning-Price | 0.00660*** | 0.0185*** | 0.0779*** | 0.0490*** |
| | (0.00197) | (0.00383) | (0.00895) | (0.00444) |
| Constant | -0.00177 | -0.00366 | -0.00849 | -0.0283 |
| | (0.00149) | (0.00294) | (0.00765) | (0.0315) |
| Observations | 22033 | 21889 | 21241 | 18643 |
| $R^2$ | 0.006 | 0.015 | 0.044 | 0.128 |

$N$otes: Standard errors in parenthesis. $*p < 0.10$, $**p < 0.05$, $***p < 0.01$. Results are based on panel regressions of equity returns in 73 countries minus the equity return in the US on the log relative earnings-price ratio over the period 1970:01-2019:02. All regressions include country fixed effects and standard errors are clstered at the monthly level.

## Table 2 Calibrated Parameters

| Parameter | Description |
|---|---|
| $p = 0.04$ | frequency of portfolio adjustment |
| $\gamma = 10$ | rate of relative risk-aversion |
| $\beta = 0.99668$ | time discount rate |
| $\rho_d = 0.9767$ | autoregressive coefficient dividend process |
| $\sigma_{\epsilon^{dD}} = 0.0447$ | standard deviation relative dividend innovation $\epsilon^d_{Ht} - \epsilon^d_{Ft}$ |
| $\sigma_{\epsilon^{dA}} = 0.0325$ | standard deviation average dividend innovation $0.5(\epsilon^d_{Ht} + \epsilon^d_{Ft})$ |
| $\rho_\tau = 0.95$ | autoregressive coefficient financial shock process |
| $\tau = 0.0002$ | average tax on foreign returns |
| $\sigma_{\epsilon^{\tau D}} = 0.0015$ | standard deviation relative financial shock $\epsilon^\tau_{Ht} - \epsilon^\tau_{Ft}$ |
| $\sigma_{\epsilon^{\tau A}} = 0.00005$ | standard deviation average financial shock $0.5(\epsilon^\tau_{Ht} + \epsilon^\tau_{Ft})$ |

## Table 3 Predictability International Equity Return Differentials

| HORIZON | DATA | MODEL | | |
|---|---|---|---|---|
| | | $p = 0.04$ | $p = 1$ Case 1 | $p = 1$ Case 2 |
| 1 month | 0.0066 | 0.0101 | 0.00002 | 0.0033 |
| 3 months | 0.0185 | 0.0385 | 0.00005 | 0.0086 |
| 12 months | 0.0779 | 0.1076 | 0.00011 | 0.0255 |
| 48 months | 0.490 | 0.2127 | 0.00036 | 0.0490 |

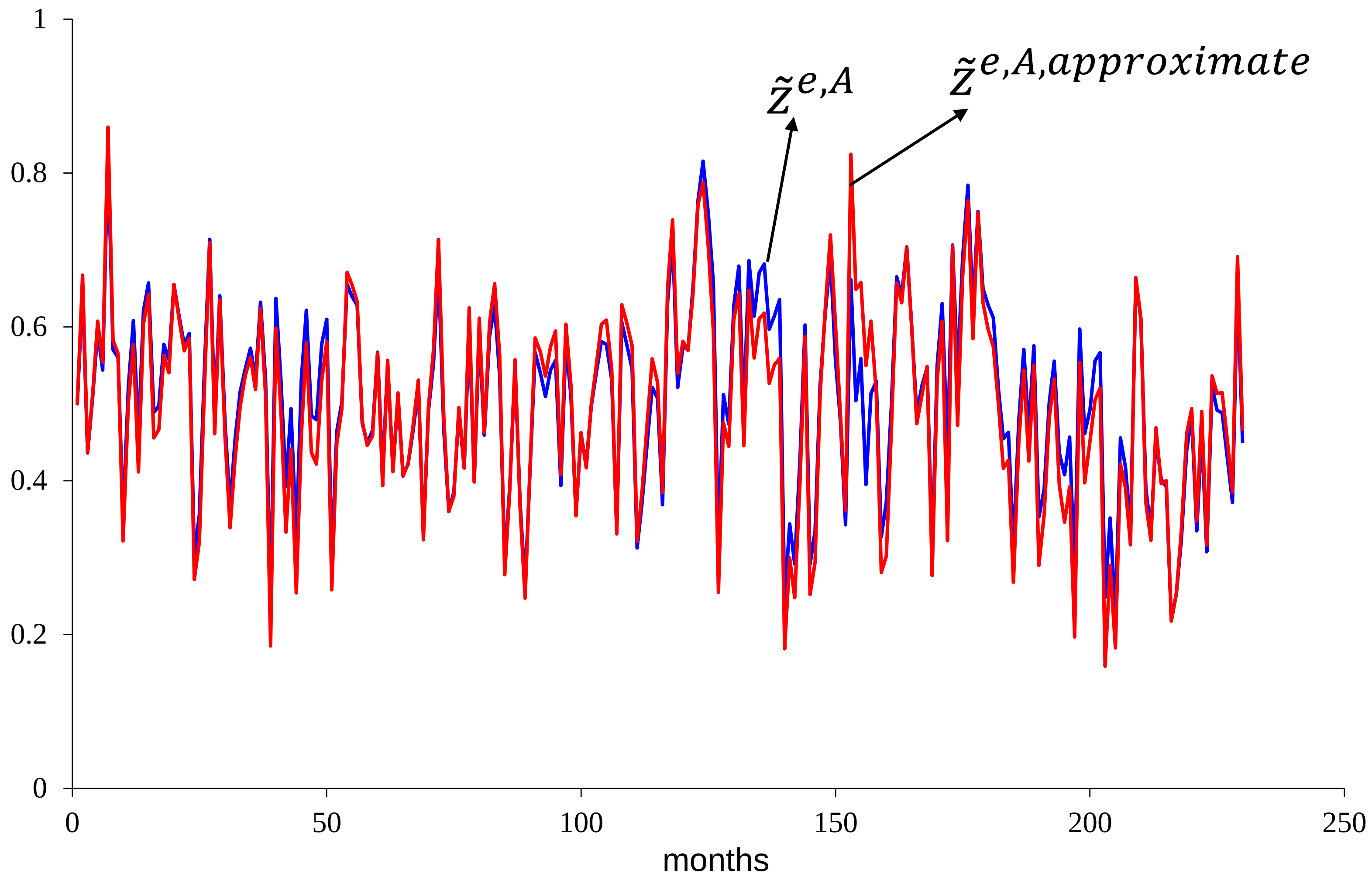
*N*otes: The data moments correspond to Table 1, representing the coefficients of a panel regression of international equity return differentials (foreign countries minus US) on the log relative earnings-price ratio. Model moments represent the model population moments when regressing the Home minus Foreign equity return on the Home minus Foreign log dividend yield. They are based on one simulation of the model over one million months. Results are shown for excess returns over 1, 3, 12 and 48 months.

Table 4 Data and Model Moments with Gradual Portfolio Adjustment

| | DATA | MODEL | | |
|---|---|---|---|---|
| | | $p = 0.04$ | $p = 1$ Case 1 | $p = 1$ Case 2 |
| **STANDARD DEVIATIONS** | | | | |
| $er_{H,t}$ | 0.045 | 0.034 (0.0017) | 0.033 (0.0015) | 0.033 (0.0016) |
| $er_t$ | 0.027 | 0.023 (0.0012) | 0.0055 (0.0003) | 0.016 (0.0011) |
| $z_t^{e,A}$ | 0.026 | 0.028 (0.0082) | 0.0039 (0.0011) | 0.029 (0.0068) |
| $\Delta z_t^{e,A}$ | 0.0045 | 0.0046 (0.0003) | 0.0010 (0.0001) | 0.0110 (0.0012) |
| $\Delta z_t^{e,D}$ | 0.0044 | 0.0029 (0.0004) | 0.0023 (0.0003) | 0.0701 (0.0134) |
| **AUTOCORRELATIONS** | | | | |
| $er_{H,t}$ | 0.135 | 0.017 (0.067) | -0.005 (0.066) | -0.008 (0.067) |
| $er_t$ | 0.086 | 0.212 (0.064) | 0.003 (0.066) | -0.062 (0.070) |
| $z_t^{e,A}$ | 0.976 | 0.982 (0.012) | 0.958 (0.024) | 0.922 (0.030) |
| $\Delta z_t^{e,A}$ | 0.155 | 0.270 (0.069) | -0.013 (0.066) | -0.053 (0.073) |
| **CONTEMPORANEOUS CORRELATIONS** | | | | |
| $corr(\Delta d_t^D, er_t)$ | 0.177 | 0.193 (0.064) | 0.995 (0.002) | 0.364 (0.067) |
| $corr(\Delta d_t^D, \Delta z_t^{e,A})$ | 0.249 | 0.171 (0.064) | 0.997 (0.001) | 0.121 (0.070) |
| $corr(\Delta z_t^{e,A}, er_t)$ | 0.922 | 0.967 (0.014) | 0.989 (0.005) | 0.844 (0.054) |
| $corr(er_{H,t}, er_{F,t})$ | 0.84 | 0.772 (0.028) | 0.986 (0.002) | 0.882 (0.018) |

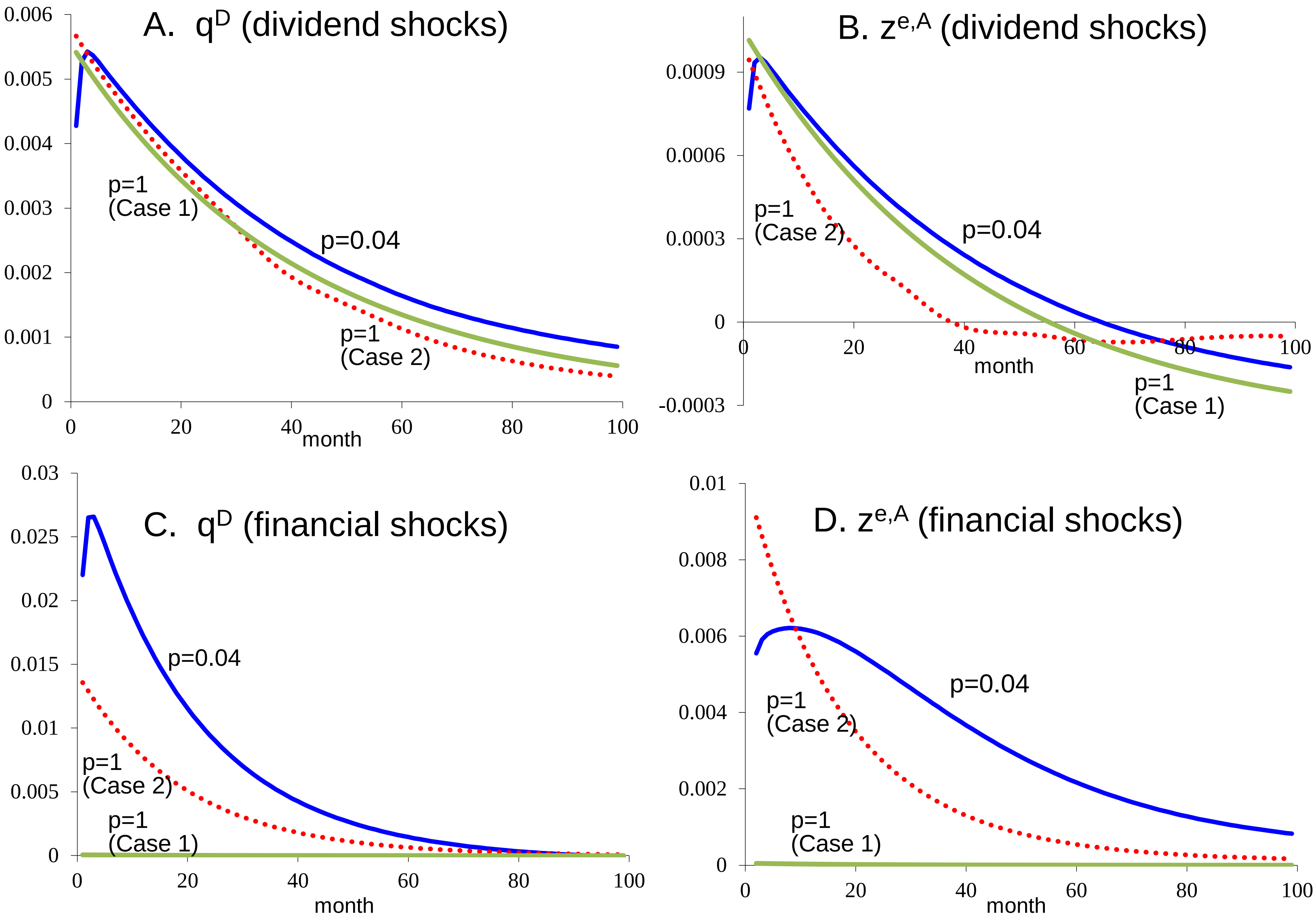
*Notes*: Model moments and associated standard errors (in parentheses) are based on 100,000 simulations of a 230 month period. Results are shown for three model parameterizations. The $p = 0.04$ parameterization is shown in Table 2. The $p = 1$, Case 1, parameterization sets $\tau = 0.000083$ and sets $\sigma_{\epsilon^{\tau D}} = 6.47 * (10^-6)$ and $\sigma_{\epsilon^{\tau A}} = 2.16 * (10^-7)$. The $p = 1$, Case 2, parameterization is the same as Table 2, except that $\tau$ is set at 0.0013.

# Figure 1 Approximation Optimal Portfolio*



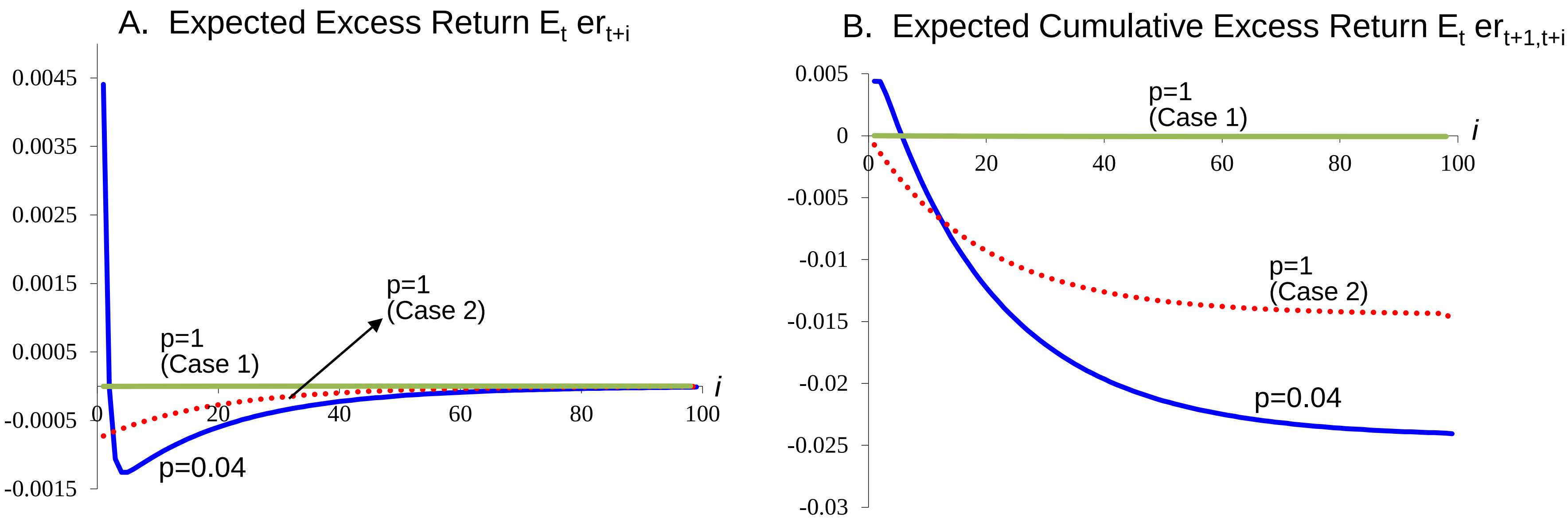

*The chart shows the global solution of the optimal portfolio based on a simulation over 230 months, as well as the linear approximation (47), including the expected present discounted value of excess returns and the financial shock that is proportional in $\tau^D$, but not the hedge term $h_t$. The correlation between the global solution and the approximation is 0.964.

# Figure 2 Impulse Response Functions



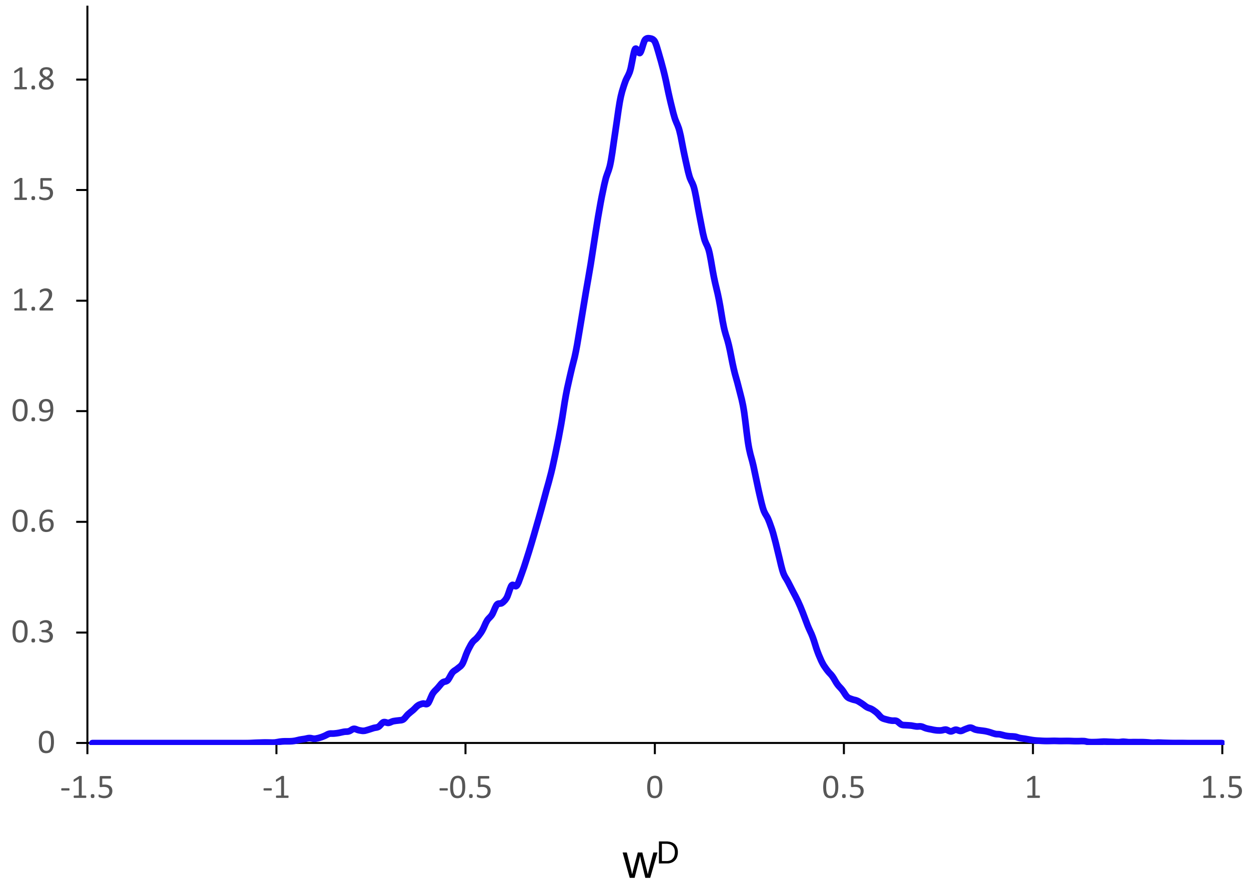

*Notes*: The impulse response functions represent an average of 10,000 impulse response functions, starting from states generated by simulating the model for 10,000 months. Dividend shocks one standard deviation innovation in $\varepsilon^{d,D}$. Financial shocks are one standard deviation innovation in $\varepsilon^{\tau,D}$.

# Figure 3 Expected Excess Returns—Financial Shocks




*Notes*: Expected excess returns are computed from impulse response functions. Excess returns in the periods subsequent to the shock are expected excess returns. Chart A shows the expectation at the time of a one standard deviation financial shock innovation $\varepsilon^{\tau,D}$ of the excess return *i* months later. Chart B shows the cumulative excess returns, which represent the total excess returns from the period after the shock until *i* periods later.

# Figure A1 Ergodic Distribution Relative Wealth




*Notes*: The ergodic distribution is obtained by simulating the model over one million months.

# Online Appendix

Infrequent Random Portfolio Decisions in an Open Economy Model

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June 2020

This Appendix derives approximated portfolio expressions using an approach similar to Campbell and Viceira (1999). The Appendix focuses on the Home country. The derivations for the Foreign country are analogous and shown after the derivation for the Home country. The aim is to obtain approximated expressions for $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$. While at times we linearize some expressions, we never linearize around these two portfolio variables. They are treated as parameters that need to be solved. We ultimately derive an expression for $\tilde{z}_t^{e,A}$.

# 1 Home Equations

We start from the portfolio Euler equations in the text of the paper. Substituting the stochastic discount factors, we have

$$E_t\left[R_{t+1}^{p,H,t}\right]^{-\gamma}\left((1-p)e^{(1-\gamma)f_{H,t+1}^{o,t}}+pe^{(1-\gamma)f_{H,t+1}^{n}}\right)(R_{H,t+1}-R_t)$$
$$+(1-p)E_t\left[R_{t+1}^{p,H,t}\right]^{1-\gamma}e^{(1-\gamma)f_{H,t+1}^{o,t}}\lambda_{HH,t+1}^{t}=0 \quad (1)$$
$$E_t\left[R_{t+1}^{p,H,t}\right]^{-\gamma}\left((1-p)e^{(1-\gamma)f_{H,t+1}^{o,t}}+pe^{(1-\gamma)f_{H,t+1}^{n}}\right)(e^{-\tau_{Ht}}R_{F,t+1}-R_t)$$
$$+(1-p)E_t\left[R_{t+1}^{p,H,t}\right]^{1-\gamma}e^{(1-\gamma)f_{H,t+1}^{o,t}}\lambda_{HF,t+1}^{t}=0 \quad (2)$$

where

$$g_{Ht}\lambda_{HH,t}^{t-1} = \theta E_te^{(1-\gamma)f_{H,t+1}^{o,t-1}}\left(R_{t+1}^{p,H,t-1}\right)^{1-\gamma}\lambda_{HH,t+1}^{t-1}+ \quad (3)$$
$$\beta E_t\left(pe^{(1-\gamma)f_{H,t+1}^{n}}+(1-p)e^{(1-\gamma)f_{H,t+1}^{o,t-1}}\right)\left(R_{t+1}^{p,H,t-1}\right)^{-\gamma}(R_{H,t+1}-R_t)$$
$$g_{Ht}\lambda_{HF,t}^{t-1} = \theta E_te^{(1-\gamma)f_{H,t+1}^{o,t-1}}\left(R_{t+1}^{p,H,t-1}\right)^{1-\gamma}\lambda_{HF,t+1}^{t-1}+ \quad (4)$$
$$\beta E_t\left(pe^{(1-\gamma)f_{H,t+1}^{n}}+(1-p)e^{(1-\gamma)f_{H,t+1}^{o,t-1}}\right)\left(R_{t+1}^{p,H,t-1}\right)^{-\gamma}(R_{F,t+1}e^{-\tau_{H,t-1}}-R_t)$$

with

$$g_{Ht}=E_t\left(pe^{(1-\gamma)f_{H,t+1}^{n}}+(1-p)e^{(1-\gamma)f_{H,t+1}^{o,t-1}}\right)\left(R_{t+1}^{p,H,t-1}\right)^{1-\gamma}$$

Using the Bellman equation for $f_{Ht}^{o,t-1}$, we can also write

$$g_{Ht} = (1/\alpha)e^{(1-\gamma)f_{Ht}^{o,t-1}/\beta}$$

Finally, the Bellman equations are

$$e^{(1-\gamma)f_{Ht}^{n}/\beta} = E_t\left(pe^{(1-\gamma)f_{H,t+1}^{n}} + (1-p)e^{(1-\gamma)f_{H,t+1}^{o,t}}\right)\left(R_{t+1}^{p,H,t}\right)^{1-\gamma} \tag{5}$$

$$e^{(1-\gamma)f_{Ht}^{o,t-1}/\beta} = \alpha E_t\left(pe^{(1-\gamma)f_{H,t+1}^{n}} + (1-p)e^{(1-\gamma)f_{H,t+1}^{o,t-1}}\right)\left(R_{t+1}^{p,H,t-1}\right)^{1-\gamma} \tag{6}$$

We will first transform some of these variables. In the deterministic steady state of the model all gross asset returns are $1/\beta$ and the Bellman variables $f_{Ht}^{n}$ and $f_{Ht}^{o,t-1}$ are equal to $ln(1-\beta)$. From hereon we will denote all asset returns in logs with a lower case $r$ and substract $ln(1/\beta)$. The resulting log returns in deviation from their deterministic steady states are denoted with a hat: $\hat{r}$. Analogously, after substracting $ln(1-\beta)$ from the Bellman variables we also denote them with a hat, e.g. $\hat{f}_{Ht}^{n}$ and $\hat{f}_{Ht}^{o,t-1}$. Finally, $\lambda_{HH,t}^{t-1}$ and $\lambda_{HF,t}^{t-1}$ are redefined as their previous values times

$$(1-p)e^{(1-\gamma)\hat{f}_{Ht}^{o,t-1}}e^{(1-\gamma)\hat{r}_{t}^{p,H,t-1}}$$

The Home portfolio Euler equation for Home equity is then

$$\begin{aligned}&(1-p)E_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{o,t}+\hat{r}_{H,t+1}} - (1-p)E_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{o,t}+\hat{r}_{t}} + \\ &pE_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{n}+\hat{r}_{H,t+1}} - pE_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{n}+\hat{r}_{t}} + E_t\lambda_{HH,t+1}^{t} = 0\end{aligned} \tag{7}$$

The Home portfolio Euler equation for Foreign equity is

$$\begin{aligned}&(1-p)E_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{o,t}+\hat{r}_{F,t+1}-\tau_{Ht}} - (1-p)E_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{o,t}+\hat{r}_{t}} + \\ &pE_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{n}+\hat{r}_{F,t+1}-\tau_{Ht}} - pE_te^{-\gamma\hat{r}_{t+1}^{p,H,t}+(1-\gamma)\hat{f}_{H,t+1}^{n}+\hat{r}_{t}} + E_t\lambda_{HF,t+1}^{t} = 0\end{aligned} \tag{8}$$

The new difference equations for $\lambda_{HH,t}^{t-1}$ and $\lambda_{HF,t}^{t-1}$ are

$$\begin{aligned}&\lambda_{HH,t}^{t-1} = \theta E_te^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}_{Ht}^{o,t-1}}e^{(1-\gamma)\hat{r}_{t}^{p,H,t-1}}\lambda_{HH,t+1}^{t-1} + \\ &\theta e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}_{Ht}^{o,t-1}}E_t\left((1-p)e^{(1-\gamma)\hat{f}_{H,t+1}^{o,t-1}} + pe^{(1-\gamma)\hat{f}_{H,t+1}^{n}}\right)e^{(1-\gamma)\hat{r}_{t}^{p,H,t-1}-\gamma\hat{r}_{t+1}^{p,H,t-1}}\left(e^{\hat{r}_{H,t+1}} - e^{\hat{r}_{t}}\right)\end{aligned} \tag{9}$$

$$\begin{aligned}&\lambda_{HF,t}^{t-1} = \theta E_te^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}_{Ht}^{o,t-1}}e^{(1-\gamma)\hat{r}_{t}^{p,H,t-1}}\lambda_{HF,t+1}^{t-1} + \\ &e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}_{Ht}^{o,t-1}}\theta E_t\left((1-p)e^{(1-\gamma)\hat{f}_{H,t+1}^{o,t-1}} + pe^{(1-\gamma)\hat{f}_{H,t+1}^{n}}\right)e^{(1-\gamma)\hat{r}_{t}^{p,H,t-1}-\gamma\hat{r}_{t+1}^{p,H,t-1}}\left(e^{\hat{r}_{F,t+1}-\tau_{H,t-1}} - e^{\hat{r}_{t}}\right)\end{aligned} \tag{10}$$

The Bellman equations are

$$e^{(1-\gamma)\hat{f}_{Ht}^{n}/\beta} = E_te^{(1-\gamma)\hat{r}_{t+1}^{p,H,t}}\left(pe^{(1-\gamma)\hat{f}_{H,t+1}^{n}} + (1-p)e^{(1-\gamma)\hat{f}_{H,t+1}^{o,t}}\right) \tag{11}$$

$$e^{(1-\gamma)\hat{f}_{Ht}^{o,t-1}/\beta} = E_te^{(1-\gamma)\hat{r}_{t+1}^{p,H,t-1}}\left(pe^{(1-\gamma)\hat{f}_{H,t+1}^{n}} + (1-p)e^{(1-\gamma)\hat{f}_{H,t+1}^{o,t-1}}\right) \tag{12}$$

## 2 Solving the Bellman equations

In what follows we will need expressions for $f^n_{H,t+s}$ and $f^{o,t}_{H,t+s}$ for $s \geq 1$. In analogy to (11)-(12), we have

$$e^{(1-\gamma)\hat{f}^n_{H,t+s}/\beta} = E_{t+s} e^{(1-\gamma)\hat{r}^{p,H,t+s}_{t+s+1}} \left( p e^{(1-\gamma)\hat{f}^n_{H,t+s+1}} + (1-p) e^{(1-\gamma)\hat{f}^{o,t+s}_{H,t+s+1}} \right) \quad (13)$$

$$e^{(1-\gamma)\hat{f}^{o,t+j}_{H,t+s}/\beta} = E_{t+s} e^{(1-\gamma)\hat{r}^{p,H,t+j}_{t+s+1}} \left( p e^{(1-\gamma)\hat{f}^n_{H,t+s+1}} + (1-p) e^{(1-\gamma)\hat{f}^{o,t+j}_{H,t+s+1}} \right) \quad (14)$$

Linearizing around zero values of these variables, we have

$$\hat{f}^n_{H,t+s} = \beta E_{t+s} \left( \hat{r}^{p,H,t+s}_{t+s+1} + p \hat{f}^n_{H,t+s+1} + (1-p) \hat{f}^{o,t+s}_{H,t+s+1} \right) \quad (15)$$

$$\hat{f}^{o,t+j}_{H,t+s} = \beta E_{t+s} \left( \hat{r}^{p,H,t+j}_{t+s+1} + p \hat{f}^n_{H,t+s+1} + (1-p) \hat{f}^{o,t+j}_{H,t+s+1} \right) \quad (16)$$

where the portfolio returns are

$$e^{\hat{r}^{p,H,t+j}_{t+s+1}} = e^{\hat{r}_{t+s}} + \tilde{z}_{HH,t+j} \left( e^{\hat{r}_{H,t+s+1}} - e^{\hat{r}_{t+s}} \right) + \tilde{z}_{HF,t+j} \left( e^{\hat{r}_{F,t+s+1}} - e^{\hat{r}_{t+s}} \right)$$

In differentiating the log portfolio returns, we differentiate around values of the portfolio shares invested in Home and Foreign equity equal to their ergodic means, which are denoted $\bar{z}_{HH}$ and $\bar{z}_{HF}$. The only exception are the portfolio shares at time $t$, $\tilde{z}_{HH,t}$ and $\tilde{z}_{HF,t}$. These are the ones that we are trying to solve. We treat them as parameters that we do not linearize around. This means that for $j \geq 1$ the linearized portfolio returns are

$$\hat{r}^{p,H,t+j}_{t+s+1} = \hat{r}_{t+s} + \bar{z}_{HH} er_{H,t+s+1} + \bar{z}_{HF} er_{F,t+s+1} \quad (17)$$

Here $er_{H,t+s+1} = r_{H,t+s+1} - r_{t+s}$ and $er_{F,t+s+1} = r_{F,t+s+1} - r_{t+s}$. For $j = 0$ we have

$$\hat{r}^{p,H,t}_{t+s+1} = \hat{r}_{t+s} + \tilde{z}_{HH,t} er_{H,t+s+1} + \tilde{z}_{HF,t} er_{F,t+s+1} \quad (18)$$

It then follows from (15)-(16) that for $j \geq 1$ and $s \geq 1$

$$\hat{f}^n_{H,t+s} = \hat{f}^{o,t+j}_{H,t+s} = E_{t+s} \sum_{i=1}^{\infty} \beta^i \bar{r}^{p,H}_{t+s+i} \quad (19)$$

where

$$\bar{r}^{p,H}_{t+s+i} = \hat{r}_{t+s+i-1} + \bar{z}_{HH} er_{H,t+s+i} + \bar{z}_{HF} er_{F,t+s+i} \quad (20)$$

We also have

$$\hat{f}^{o,t}_{H,t+s} = \beta E_{t+s} \left( \hat{r}_{t+s} + \tilde{z}_{HH,t} er_{H,t+s+1} + \tilde{z}_{HF,t} er_{F,t+s+1} + p \hat{f}^n_{H,t+s+1} + (1-p) \hat{f}^{o,t}_{H,t+s+1} \right) \quad (21)$$

Substracting

$$\hat{f}^n_{H,t+s} = \beta E_{t+s}\left(\hat{r}^{p,H,t+s}_{t+s+1} + \hat{f}^n_{H,t+s+1}\right) \tag{22}$$

we have

$$\begin{aligned}\hat{f}^{o,t}_{H,t+s} - \hat{f}^n_{H,t+s} &= \beta(\tilde{z}_{HH,t} - \bar{z}_{HH})E_{t+s}er_{H,t+s+1} + \beta(\tilde{z}_{HF,t} - \bar{z}_{HF})E_{t+s}er_{F,t+s+1} + \\ &\quad \theta E_t\left(\hat{f}^{o,t}_{H,t+s+1} - \hat{f}^n_{H,t+s+1}\right) \end{aligned} \tag{23}$$

Integrating, we have

$$\hat{f}^{o,t}_{H,t+s} = \hat{f}^n_{H,t+s} + (\tilde{z}_{HH,t} - \bar{z}_{HH})\beta\sum_{i=1}^{\infty}\theta^{i-1}E_{t+s}er_{H,t+s+i} + (\tilde{z}_{HF,t} - \bar{z}_{HF})\beta\sum_{i=1}^{\infty}\theta^{i-1}E_{t+s}er_{F,t+s+i} \tag{24}$$

# 3 Solving Expected Lambdas

The solution to (9)-(10) is

$$\lambda^{t-1}_{HH,t} = E_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t-1}_{H,t,t+i-1}}\left((1-p)e^{(1-\gamma)\hat{f}^{o,t-1}_{H,t+i}} + pe^{(1-\gamma)\hat{f}^n_{H,t+i}}\right)e^{(1-\gamma)\hat{r}^{p,H,t-1}_{t,t+i-1}}e^{-\gamma\hat{r}^{p,H,t-1}_{t+i}}\left(e^{\hat{r}_{H,t+i}} - e^{\hat{r}_{t+i-1}}\right)$$

$$\lambda^{t-1}_{HF,t} = E_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t-1}_{H,t,t+i-1}}\left((1-p)e^{(1-\gamma)\hat{f}^{o,t-1}_{H,t+i}} + pe^{(1-\gamma)\hat{f}^n_{H,t+i}}\right)e^{(1-\gamma)\hat{r}^{p,H,t-1}_{t,t+i-1}}e^{-\gamma\hat{r}^{p,H,t-1}_{t+i}}\left(e^{\hat{r}_{F,t+i}-\tau_{H,t-1}} - e^{\hat{r}_{t+i-1}}\right)$$

where

$$\hat{f}^{o,t-1}_{H,t,t+i-1} = \hat{f}^{o,t-1}_{Ht} + ... + \hat{f}^{o,t-1}_{H,t+i-1} \tag{25}$$

$$\hat{r}^{p,H,t-1}_{t,t+i-1} = \hat{r}^{p,H,t-1}_{t} + ... + \hat{r}^{p,H,t-1}_{t+i-1} \tag{26}$$

It follows that

$$\begin{aligned} &E_t\lambda^t_{HH,t+1} = \\ &(1-p)E_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i} + (1-\gamma)\hat{f}^{o,t}_{H,t+i+1} + (1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i} - \gamma\hat{r}^{p,H,t}_{t+i+1} + \hat{r}_{H,t+i+1}} \\ &-(1-p)E_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i} + (1-\gamma)\hat{f}^{o,t}_{H,t+i+1} + (1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i} - \gamma\hat{r}^{p,H,t}_{t+i+1} + \hat{r}_{t+i}} \\ &pE_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i} + (1-\gamma)\hat{f}^{n}_{H,t+i+1} + (1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i} - \gamma\hat{r}^{p,H,t}_{t+i+1} + \hat{r}_{H,t+i+1}} \\ &-pE_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i} + (1-\gamma)\hat{f}^{n}_{H,t+i+1} + (1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i} - \gamma\hat{r}^{p,H,t}_{t+i+1} + \hat{r}_{t+i}} \end{aligned} \tag{27}$$

and

$$E_t\lambda^t_{HF,t+1} = \tag{28}$$
$$(1-p)E_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i}+(1-\gamma)\hat{f}^{o,t}_{H,t+i+1}+(1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i}-\gamma\hat{r}^{p,H,t}_{t+i+1}+\hat{r}_{F,t+i+1}-\tau_{Ht}}$$
$$-(1-p)E_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i}+(1-\gamma)\hat{f}^{o,t}_{H,t+i+1}+(1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i}-\gamma\hat{r}^{p,H,t}_{t+i+1}+\hat{r}_{t+i}}$$
$$pE_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i}+(1-\gamma)\hat{f}^{n}_{H,t+i+1}+(1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i}-\gamma\hat{r}^{p,H,t}_{t+i+1}+\hat{r}_{F,t+i+1}-\tau_{Ht}}$$
$$-pE_t\sum_{i=1}^{\infty}\theta^i e^{(1-\gamma)\frac{\beta-1}{\beta}\hat{f}^{o,t}_{H,t+1,t+i}+(1-\gamma)\hat{f}^{n}_{H,t+i+1}+(1-\gamma)\hat{r}^{p,H,t}_{t+1,t+i}-\gamma\hat{r}^{p,H,t}_{t+i+1}+\hat{r}_{t+i}}$$

In approximating, write the expectations of exponents as $Ee^x = e^{E(x)+0.5var(x)}$ and then approximate this as $1 + E(x) + 0.5var(x)$. Applying this to the expression for $E_t\lambda^t_{HH,t+1}$ gives

$$E_t\lambda^t_{HH,t+1} = \sum_{i=1}^{\infty}\theta^i E_t er_{H,t+i+1} + 0.5\sum_{i=1}^{\infty}\theta^i\left(var_t(r_{H,t+i+1}) - var_t(r_{t+i})\right) +$$
$$\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1}, (1-\gamma)\frac{\beta-1}{\beta}f^{o,t}_{H,t+1,t+i} + (1-\gamma)r^{p,H,t}_{t+1,t+i} - \gamma r^{p,H,t}_{t+i+1}) +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1}, (1-p)f^{o,t}_{H,t+i+1} + pf^{n}_{H,t+i+1}) \tag{29}$$

Hats are now removed from variables as they just subtract a constant and therefore do not affect variances and covariances.

Now use the linearized portfolio expressions:

$$r^{p,H,t}_{t+i+1} = r_{t+i} + \tilde{z}_{HH,t}er_{H,t+i+1} + \tilde{z}_{HF,t}er_{F,t+i+1} \tag{30}$$
$$r^{p,H,t}_{t+1,t+i} = r^{p,H,t}_{t+1} + ... + r^{p,H,t}_{t+i} = r_{t,t+i-1} + \tilde{z}_{HH,t}er_{H,t+1,t+i} + \tilde{z}_{HF,t}er_{F,t+1,t+i}$$

where

$$r_{t,t+i-1} = r_t + ... + r_{t+i-1} \tag{31}$$
$$er_{H,t+1,t+i} = er_{H,t+1} + ... + er_{H,t+i} \tag{32}$$

We then have

$$E_t\lambda^t_{HH,t+1} = \sum_{i=1}^{\infty}\theta^i E_t er_{H,t+i+1} + 0.5\sum_{i=1}^{\infty}\theta^i\left(var_t(r_{H,t+i+1}) - var_t(r_{t+i})\right) +$$

$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},\frac{\beta-1}{\beta}f^{o,t}_{H,t+1,t+i})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},r_{t,t+i-1})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{H,t+1,t+i})\tilde{z}_{HH,t}+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{F,t+1,t+i})\tilde{z}_{HF,t}-$$
$$\gamma\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},r_{t+i})-$$
$$\gamma\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{H,t+i+1})\tilde{z}_{HH,t}-$$
$$\gamma\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{F,t+i+1})\tilde{z}_{HF,t}+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},(1-p)f^{o,t}_{H,t+i+1}+pf^{n}_{H,t+i+1}) \tag{33}$$

Substituting (24) in the last line, we have

$$E_t\lambda^t_{HH,t+1}=\sum_{i=1}^{\infty}\theta^i E_t er_{H,t+i+1}+0.5\sum_{i=1}^{\infty}\theta^i\left(var_t(r_{H,t+i+1})-var_t(r_{t+i})\right)+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},\frac{\beta-1}{\beta}f^{o,t}_{H,t+1,t+i})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},r_{t,t+i-1})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{H,t+1,t+i})\tilde{z}_{HH,t}+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{F,t+1,t+i})\tilde{z}_{HF,t}-$$
$$\gamma\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},r_{t+i})-$$
$$\gamma\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{H,t+i+1})\tilde{z}_{HH,t}-$$
$$\gamma\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1},er_{F,t+i+1})\tilde{z}_{HF,t}+$$

$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j}cov_t(er_{H,t+i+1},er_{H,t+i+1+j})(\tilde{z}_{HH,t}-\bar{z}_{HH})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j}cov_t(er_{H,t+i+1},er_{F,t+i+1+j})(\tilde{z}_{HF,t}-\bar{z}_{HF})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{H,t+i+1},f^{n}_{H,t+i+1}) \tag{34}$$

Analogously

$$E_t\lambda^t_{HF,t+1}=\sum_{i=1}^{\infty}\theta^{i}E_t er_{F,t+i+1}-\frac{\theta}{1-\theta}\tau_{Ht}+0.5\sum_{i=1}^{\infty}\theta^{i}\left(var_t(r_{F,t+i+1})-var_t(r_{t+i})\right)+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},\frac{\beta-1}{\beta}f^{o,t}_{H,t+1,t+i})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},r_{t,t+i-1})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},er_{H,t+1,t+i})\tilde{z}_{HH,t}+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},er_{F,t+1,t+i})\tilde{z}_{HF,t}-$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},r_{t+i})-$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},er_{H,t+i+1})\tilde{z}_{HH,t}-$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},er_{F,t+i+1})\tilde{z}_{HF,t}+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j}cov_t(er_{F,t+i+1},er_{H,t+i+1+j})(\tilde{z}_{HH,t}-\bar{z}_{HH})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j}cov_t(er_{F,t+i+1},er_{F,t+i+1+j})(\tilde{z}_{HF,t}-\bar{z}_{HF})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},f^{n}_{H,t+i+1}) \tag{35}$$

# 4 Solving Portfolio Shares from Portfolio Eulers

Now go back to the portfolio Euler equations. The Home equity Euler equation (7) can be written as

$$\begin{aligned}
&(1-p)E_t e^{-\gamma \hat{r}^{p,H,t}_{t+1}+(1-\gamma)\hat{f}^{o,t}_{H,t+1}+\hat{r}_{H,t+1}} - \\
&(1-p)E_t e^{-\gamma \hat{r}^{p,H,t}_{t+1}+(1-\gamma)\hat{f}^{o,t}_{H,t+1}+\hat{r}_t} + \\
&pE_t e^{-\gamma \hat{r}^{p,H,t}_{t+1}+(1-\gamma)\hat{f}^{n}_{H,t+1}+\hat{r}_{H,t+1}} - \\
&pE_t e^{-\gamma \hat{r}^{p,H,t}_{t+1}+(1-\gamma)\hat{f}^{n}_{H,t+1}+\hat{r}_t} + \\
&E_t\lambda^t_{HH,t+1} = 0 \qquad (36)
\end{aligned}$$

Using $Ee^x = e^{E(x)+0.5var(x)}$, and again approximating this as $1 + E(x) + 0.5var(x)$, we have

$$\begin{aligned}
&E_t er_{H,t+1} + 0.5var_t(r_{H,t+1}) - \gamma var_t(er_{H,t+1})\tilde{z}_{HH,t} - \gamma cov_t(er_{H,t+1}, er_{F,t+1})\tilde{z}_{HF,t} + \\
&(1-\gamma)cov_t(er_{H,t+1}, (1-p)f^{o,t}_{H,t+1} + pf^n_{H,t+1}) + E_t\lambda^t_{HH,t+1} = 0 \qquad (37)
\end{aligned}$$

This uses that $\hat{r}^{p,H,t}_{t+1} = \hat{r}_t + \tilde{z}_{HH,t}er_{H,t+1} + \tilde{z}_{HF,t}er_{F,t+1}$. Substituting (24) with $s = 1$, this becomes

$$\begin{aligned}
&E_t er_{H,t+1} + 0.5var_t(r_{H,t+1}) - \gamma var_t(er_{H,t+1})\tilde{z}_{HH,t} - \gamma cov_t(er_{H,t+1}, er_{F,t+1})\tilde{z}_{HF,t} + \\
&(1-\gamma)\sum_{j=1}^{\infty}\theta^j cov_t(er_{H,t+1}, er_{H,t+1+j})(\tilde{z}_{HH,t} - \bar{z}_{HH}) + \\
&(1-\gamma)\sum_{j=1}^{\infty}\theta^j cov_t(er_{H,t+1}, er_{F,t+1+j})(\tilde{z}_{HF,t} - \bar{z}_{HF}) + \\
&(1-\gamma)cov_t(er_{H,t+1}, f^n_{H,t+1}) + E_t\lambda^t_{HH,t+1} = 0 \qquad (38)
\end{aligned}$$

Substituting the expression for $E_t\lambda^t_{HH,t+1}$, we have

$$\begin{aligned}
&\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{H,t+i} + 0.5\sum_{i=1}^{\infty}\theta^{i-1}\left(var_t(r_{H,t+i}) - var_t(r_{t+i-1})\right) + \\
&(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1}, \frac{\beta-1}{\beta}f^{o,t}_{H,t+1,t+i}) + \\
&(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1}, r_{t,t+i-1}) + \\
&(1-\gamma)\sum_{i=1}^{\infty}\theta^i cov_t(er_{H,t+i+1}, er_{H,t+1,t+i})\tilde{z}_{HH,t} +
\end{aligned}$$

$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{H,t+i+1},er_{F,t+1,t+i})\tilde{z}_{HF,t} -$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{H,t+i+1},r_{t+i}) -$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i-1}var_t(er_{H,t+i})\tilde{z}_{HH,t} -$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{H,t+i},er_{F,t+i})\tilde{z}_{HF,t} +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j-1}cov_t(er_{H,t+i},er_{H,t+i+j})(\tilde{z}_{HH,t}-\bar{z}_{HH}) +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j-1}cov_t(er_{H,t+i},er_{F,t+i+j})(\tilde{z}_{HF,t}-\bar{z}_{HF}) +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{H,t+i},f^{n}_{H,t+i}) = 0 \tag{39}$$

Analogously, the first-order condition for Foreign equity becomes

$$\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{F,t+i} - \frac{1}{1-\theta}\tau_{Ht} + 0.5\sum_{i=1}^{\infty}\theta^{i-1}\left(var_t(r_{F,t+i}) - var_t(r_{t+i-1})\right) +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},\frac{\beta-1}{\beta}f^{o,t}_{H,t+1,t+i}) +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},r_{t,t+i-1}) +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},er_{H,t+1,t+i})\tilde{z}_{HH,t} +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},er_{F,t+1,t+i})\tilde{z}_{HF,t} -$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},r_{t+i}) -$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{F,t+i},er_{H,t+i})\tilde{z}_{HH,t} -$$
$$\gamma\sum_{i=1}^{\infty}\theta^{i-1}var_t(er_{F,t+i})\tilde{z}_{HF,t} +$$
$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j-1}cov_t(er_{F,t+i},er_{H,t+i+j})(\tilde{z}_{HH,t}-\bar{z}_{HH}) +$$

$$(1-\gamma)\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\theta^{i+j-1}cov_t(er_{F,t+i},er_{F,t+i+j})(\tilde{z}_{HF,t}-\bar{z}_{HF})+$$
$$(1-\gamma)\sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{F,t+i},f^n_{H,t+i})=0 \tag{40}$$

Define the matrix $D_t$ as

$$D_t=\sum_{i=1}^{\infty}\theta^{i-1}\Sigma_{i,t} \tag{41}$$

where $\Sigma_{i,t}$ is a symmetric matrix with

$$\Sigma_{i,t}(1,1)=\gamma var_t(er_{H,t+i})+2(\gamma-1)cov_t(er_{H,t+i},er_{H,t+1,t+i-1})$$
$$\Sigma_{i,t}(2,2)=\gamma var_t(er_{F,t+i})+2(\gamma-1)cov_t(er_{F,t+i},er_{F,t+1,t+i-1})$$
$$\Sigma_{i,t}(1,2)=\gamma cov_t(er_{H,t+i},er_{F,t+i})+(\gamma-1)cov_t(er_{H,t+i},er_{F,t+1,t+i-1})+(\gamma-1)cov_t(er_{F,t+i},er_{H,t+1,t+i-1})$$

Note that when $i=1$ the terms multiplying $(\gamma-1)$ are zero as $er_{H,t+1,t}$ is not defined or zero.

Also define the matrix $P_t$ as

$$P_t=(\gamma-1)\sum_{i=1}^{\infty}\theta^{i-1}\Omega_{i,t} \tag{42}$$

where

$$\Omega_{i,t}(1,1)=cov_t(er_{H,t+i},er_{H,t+1,t+i-1})$$
$$\Omega_{i,t}(2,2)=cov_t(er_{F,t+i},er_{F,t+1,t+i-1})$$
$$\Omega_{i,t}(1,2)=cov_t(er_{F,t+i},er_{H,t+1,t+i-1})$$
$$\Omega_{i,t}(2,1)=cov_t(er_{H,t+i},er_{F,t+1,t+i-1})$$

In a symmetric state this is a symmetric matrix, but in an asymmetric state the matrix will generally be asymmetric. The portfolio solution is then

$$D_t\begin{pmatrix}\tilde{z}_{HH,t}\\ \tilde{z}_{HF,t}\end{pmatrix}=\begin{pmatrix}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{H,t+i}\\ \sum_{i=1}^{\infty}\theta^{i-1}E_t er_{F,t+i}\end{pmatrix}+h_{Ht} \tag{43}$$

where

$$h_{Ht} \;=\; P_t\begin{pmatrix}\bar{z}_{HH}\\ \bar{z}_{HF}\end{pmatrix}-\begin{pmatrix}0\\ \frac{1}{1-\theta}\end{pmatrix}\tau_{Ht}+0.5\begin{pmatrix}\sum_{i=1}^{\infty}\theta^{i-1}\left(var_t(r_{H,t+i})-var_t(r_{t+i-1})\right)\\ \sum_{i=1}^{\infty}\theta^{i-1}\left(var_t(r_{F,t+i})-var_t(r_{t+i-1})\right)\end{pmatrix}+$$

$$\left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i}cov_t(er_{H,t+i+1},(1-\gamma)r_{t,t+i-1}-\gamma r_{t+i}) \\ \sum_{i=1}^{\infty}\theta^{i}cov_t(er_{F,t+i+1},(1-\gamma)r_{t,t+i-1}-\gamma r_{t+1}) \end{array}\right)+$$
$$(1-\gamma)\left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{H,t+i},f^{n}_{H,t+i}) \\ \sum_{i=1}^{\infty}\theta^{i-1}cov_t(er_{F,t+i},f^{n}_{H,t+i}) \end{array}\right) \tag{44}$$

Note that there is one additional term in $h_{Ht}$, involving the covariance between $[(1-\beta)/\beta]f^{o,t}_{H,t+1,t+i}$ and either $er_{H,t+i+1}$ or $er_{F,t+i+1}$. This term will be omitted as it is negligible in size since $\beta$ is close to 1.

Now consider the left hand side of the portfolio expression. Expand this around the erogodic mean of the matrix $D_t$ and the portfolio shares. It then becomes (with a bar referring to the ergodic mean)

$$D_t\left(\begin{array}{c} \bar{z}_{HH} \\ \bar{z}_{HF} \end{array}\right)+\bar{D}\left(\begin{array}{c} \tilde{z}_{HH,t}-\bar{z}_{HH} \\ \tilde{z}_{HF,t}-\bar{z}_{HF} \end{array}\right) \tag{45}$$

Now substract the ergodic mean from both sides of the portfolio expression and refer to deviations from the ergodic mean with a hat. This gives

$$\left(\begin{array}{c} \tilde{z}_{HH,t} \\ \tilde{z}_{HF,t} \end{array}\right)=\left(\begin{array}{c} \bar{z}_{HH} \\ \bar{z}_{HF} \end{array}\right)+\bar{D}^{-1}\left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i-1}E_t\hat{er}_{H,t+i} \\ \sum_{i=1}^{\infty}\theta^{i-1}E_t\hat{er}_{F,t+i} \end{array}\right)+\bar{D}^{-1}\hat{h}_{Ht} \tag{46}$$

where

$$\begin{aligned}
\hat{h}_{Ht} \;=\;& (\hat{P}_t-\hat{D}_t)\left(\begin{array}{c} \bar{z}_{HH} \\ \bar{z}_{HF} \end{array}\right)-\left(\begin{array}{c} 0 \\ \frac{1}{1-\theta} \end{array}\right)(\tau_{Ht}-\tau) \\
& +0.5\left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i-1}\left(v\hat{a}r_t(r_{H,t+i})-v\hat{a}r_t(r_{t+i-1})\right) \\ \sum_{i=1}^{\infty}\theta^{i-1}\left(v\hat{a}r_t(r_{F,t+i})-v\hat{a}r_t(r_{t+i-1})\right) \end{array}\right)+ \\
& \left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i}c\hat{o}v_t(er_{H,t+i+1},(1-\gamma)r_{t,t+i-1}-\gamma r_{t+i}) \\ \sum_{i=1}^{\infty}\theta^{i}c\hat{o}v_t(er_{F,t+i+1},(1-\gamma)r_{t,t+i-1}-\gamma r_{t+1}) \end{array}\right)+ \\
& (1-\gamma)\left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v_t(er_{H,t+i},f^{n}_{H,t+i}) \\ \sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v_t(er_{F,t+i},f^{n}_{H,t+i}) \end{array}\right)
\end{aligned} \tag{47}$$

The optimal portfolio for the Foreign country is analogous:

$$\left(\begin{array}{c} \tilde{z}_{FH,t} \\ \tilde{z}_{FF,t} \end{array}\right)=\left(\begin{array}{c} \bar{z}_{FH} \\ \bar{z}_{FF} \end{array}\right)+\bar{D}^{-1}\left(\begin{array}{c} \sum_{i=1}^{\infty}\theta^{i-1}E_t\hat{er}_{H,t+i} \\ \sum_{i=1}^{\infty}\theta^{i-1}E_t\hat{er}_{F,t+i} \end{array}\right)+\bar{D}^{-1}\hat{h}_{Ft} \tag{48}$$

where

$$\begin{aligned}
\hat{h}_{Ft} &= (\hat{P}_t - \hat{D}_t)\begin{pmatrix} \bar{z}_{FH} \\ \bar{z}_{FF} \end{pmatrix} - \begin{pmatrix} \frac{1}{1-\theta} \\ 0 \end{pmatrix}(\tau_{Ft} - \tau) \\
&+0.5\begin{pmatrix} \sum_{i=1}^{\infty}\theta^{i-1}\left(v\hat{a}r_t(r_{H,t+i}) - v\hat{a}r_t(r_{t+i-1})\right) \\ \sum_{i=1}^{\infty}\theta^{i-1}\left(v\hat{a}r_t(r_{F,t+i}) - v\hat{a}r_t(r_{t+i-1})\right) \end{pmatrix} + \\
&\begin{pmatrix} \sum_{i=1}^{\infty}\theta^{i}c\hat{o}v_t(er_{H,t+i+1}, (1-\gamma)r_{t,t+i-1} - \gamma r_{t+i}) \\ \sum_{i=1}^{\infty}\theta^{i}c\hat{o}v_t(er_{F,t+i+1}, (1-\gamma)r_{t,t+i-1} - \gamma r_{t+1}) \end{pmatrix} + \\
&(1-\gamma)\begin{pmatrix} \sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v_t(er_{H,t+i}, f^n_{F,t+i}) \\ \sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v_t(er_{F,t+i}, f^n_{F,t+i}) \end{pmatrix} \qquad (49)
\end{aligned}$$

# 5 Average Equity Portfolio Share

Now consider the equity portfolio shares. We have

$$\tilde{z}^e_{HH,t} = \frac{\tilde{z}_{HH,t}}{\tilde{z}_{HH,t} + \tilde{z}_{HF,t}} \qquad (50)$$

$$\tilde{z}^e_{FH,t} = \frac{\tilde{z}_{FH,t}}{\tilde{z}_{FH,t} + \tilde{z}_{FF,t}} \qquad (51)$$

Linearizing gives

$$\tilde{z}^e_{HH,t} = \bar{z} + (1-\bar{z})\tilde{z}_{HH,t} - \bar{z}\tilde{z}_{HF,t} \qquad (52)$$

$$\tilde{z}^e_{FH,t} = 1 - \bar{z} + \bar{z}\tilde{z}_{FH,t} - (1-\bar{z})\tilde{z}_{FF,t} \qquad (53)$$

Define

$$\tilde{z}^{e,A}_t = 0.5(\tilde{z}^e_{HH,t} + \tilde{z}^e_{FH,t}) \qquad (54)$$

$$\tilde{z}^{e,D}_t = \tilde{z}^e_{HH,t} - \tilde{z}^e_{FH,t} \qquad (55)$$

We have

$$\tilde{z}^{e,A}_t = 0.5 + 0.5\left((1-\bar{z})\tilde{z}_{HH,t} - \bar{z}\tilde{z}_{HF,t} + \bar{z}\tilde{z}_{FH,t} - (1-\bar{z})\tilde{z}_{FF,t}\right) \qquad (56)$$

Define $\hat{A}(i,j) = \hat{D}(i,j) - \hat{P}(i,j)$. Then using the results from the previous section we have

$$\tilde{z}^{e,A}_t = 0.5 + \frac{0.5}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{t+i} + \frac{0.5}{1-\theta}\frac{\mu}{D_1 - D_2}\tau^D_t$$

$$
\begin{aligned}
&-\frac{\bar{z}(1-\bar{z})}{D_1+D_2}\left(\hat{A}(1,1)-\hat{A}(1,2)+\hat{A}(2,1)-\hat{A}(2,2)\right) \\
&-\frac{0.5}{D_1^2-D_2^2}\left(D_1(\hat{A}(1,2)-\hat{A}(2,1))+D_2(\hat{A}(1,1)-\hat{A}(2,2))\right) \\
&+\frac{0.25}{D_1-D_2}\sum_{i=1}^{\infty}\theta^{i-1}(\hat{var}_t(r_{H,t+i})-\hat{var}_t(r_{F,t+i})) \qquad (57) \\
&+\frac{0.5}{D_1-D_2}\sum_{i=1}^{\infty}\theta^{i}\hat{cov}_t(er_{t+i+1},(1-\gamma)r_{t,t+i-1}-\gamma r_{t+i}) \\
&+\frac{0.5(1-\gamma)}{D_1-D_2}\sum_{i=1}^{\infty}\theta^{i-1}\left(\hat{cov}(er_{H,t+i},(1-\mu)f^n_{H,t+i}+\mu f^n_{F,t+i})-\hat{cov}(er_{F,t+i},\mu f^n_{H,t+i}+(1-\mu)f^n_{F,t+i})\right)
\end{aligned}
$$

where

$$\mu=\frac{\bar{z}D_1+(1-\bar{z})D_2}{D_1+D_2}$$

and $er_{t+i}=r_{H,t+i}-r_{F,t+i}=er_{H,t+i}-er_{F,t+i}$.

We have

$$
\begin{aligned}
\hat{A}(1,1)&=\sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma\hat{var}_t(er_{H,t+i})+(\gamma-1)\hat{cov}_t(er_{H,t+i},er_{H,t+1,t+i-1})\right) \\
\hat{A}(2,2)&=\sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma\hat{var}_t(er_{F,t+i})+(\gamma-1)\hat{cov}_t(er_{F,t+i},er_{F,t+1,t+i-1})\right) \\
\hat{A}(2,1)-\hat{A}(1,2)&=(\gamma-1)\sum_{i=1}^{\infty}\theta^{i-1}\left(\hat{cov}_t(er_{F,t+i},er_{H,t+1,t+i-1})-\hat{cov}_t(er_{H,t+i},er_{F,t+1,t+i-1})\right)
\end{aligned}
$$

Collecting the $\hat{A}$ terms in the expression for $\tilde{z}_t^{e,A}$ and using $D_1=0.5(D_1+D_2)+0.5(D_1-D_2)$ and $D_2=0.5(D_1+D_2)-0.5(D_1-D_2)$, we can write the sum of these terms as

$$
\begin{aligned}
&-\frac{0.25}{D_1-D_2}\left(\hat{A}(1,2)-\hat{A}(2,1))+\hat{A}(1,1)-\hat{A}(2,2)\right) \\
&+\frac{0.25(1-2\bar{z})^2}{D_1+D_2}\left(\hat{A}(1,1)-\hat{A}(1,2))+\hat{A}(2,1)-\hat{A}(2,2)\right) \qquad (58)
\end{aligned}
$$

Using the expressions above for the $\hat{A}$ terms, this becomes

$$
\begin{aligned}
&-\frac{0.25}{D_1-D_2}\sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma(\hat{var}(er_{H,t+i})-\hat{var}(er_{F,t+i}))+(\gamma-1)\hat{cov}(er_{t+i},er_{H,t+1,t+i-1}+er_{F,t+1,t+i-1})\right) \\
&+\frac{0.25(1-2\bar{z})^2}{D_1+D_2}\sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma(\hat{var}(er_{H,t+i})-\hat{var}(er_{F,t+i}))+(\gamma-1)\hat{cov}(er_{H,t+i}+er_{F,t+i},er_{t+1,t+i-1})\right)
\end{aligned}
$$

Now take the first line of the last expression and add the fourth and fifth lines of (57):

$$-\frac{0.25}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma(v\hat{a}r(er_{H,t+i}) - v\hat{a}r(er_{F,t+i})) + (\gamma-1)c\hat{o}v(er_{t+i}, er_{H,t+1,t+i-1} + er_{F,t+1,t+i-1})\right)$$
$$+\frac{0.25}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}\left((v\hat{a}r_t(er_{H,t+i}) - v\hat{a}r_t(er_{F,t+i})) + 2c\hat{o}v(er_{t+i}, r_{t+i-1})\right)$$
$$+\frac{0.5}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i}c\hat{o}v_t(er_{t+i+1}, (1-\gamma)r_{t,t+i-1} - \gamma r_{t+i})$$

In the second line we used that $var(r_{H,t+i}) = var(er_{H,t+i}) + var(r_{t+i-1}) + 2cov(er_{H,t+i}, r_{t+i-1})$. Adding these terms, we have

$$-(\gamma-1)\frac{0.5}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v(er_{t+i}, r^A_{t+1,t+i}) \tag{59}$$

Here $r^A_{t+1,t+i} = r^A_{t+1} + ... + r^A_{t+i}$ with $r^A_{t+s} = 0.5(r_{H,t+s} + r_{F,t+s})$.

To summarize, we have

$$\begin{aligned}
\tilde{z}^{e,A}_t &= 0.5 + \frac{0.5}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{t+i} + \frac{0.5}{1-\theta}\frac{\mu}{D_1 - D_2}\tau^D_t \\
&-(\gamma-1)\frac{0.5}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v(er_{t+i}, r^A_{t+1,t+i}) \\
&+\frac{0.25(1-2\bar{z})^2}{D_1 + D_2}\sum_{i=1}^{\infty}\theta^{i-1}c\hat{o}v(er_{H,t+i} + er_{F,t+i}, (\gamma-1)er_{t+1,t+i-1} + \gamma er_{t+i}) \\
&+\frac{0.5(1-\gamma)}{D_1 - D_2}\sum_{i=1}^{\infty}\theta^{i-1}\Big(c\hat{o}v(er_{H,t+i}, (1-\mu)f^n_{H,t+i} + \mu f^n_{F,t+i}) - c\hat{o}v(er_{F,t+i}, \mu f^n_{H,t+i} + (1-\mu)f^n_{F,t+i})\Big)
\end{aligned} \tag{60}$$

We can further rewrite this as follows. Introduce the parameters $D = 2(D_1 - D_2)$ and $d = 2(D_1 + D_2)$. These are

$$D = \sum_{i=1}^{\infty}\theta^{i-1}\left(\gamma v\bar{a}r_t(er_{t+i}) + 2(\gamma-1)c\bar{o}v_t(er_{t+i}, er_{t+1,t+i-1})\right) \tag{61}$$
$$d = \sum_{i=1}^{\infty}\theta^{i-1}\Big(\gamma v\bar{a}r_t(er_{H,t+i} + er_{F,t+i}) + 2(\gamma-1)c\bar{o}v_t(er^{sum}_{t+i}, er^{sum}_{t+1,t+i-1})\Big) \tag{62}$$

where $er^{sum}_{t+i} = er_{H,t+i} + er_{F,t+i}$ and $er^{sum}_{t+1,t+i-1} = er^{sum}_{t+1} + ... + er^{sum}_{t+i-1}$. Define

$$f^{n,1}_{t+i} = (1-\mu)f^n_{H,t+i} + \mu f^n_{F,t+i} \tag{63}$$
$$f^{n,2}_{t+i} = \mu f^n_{H,t+i} + (1-\mu)f^n_{F,t+i} \tag{64}$$

Then we have

$$
\begin{aligned}
&\tilde{z}_t^{e,A} = 0.5 + \frac{1}{D}\sum_{i=1}^{\infty}\theta^{i-1}E_t er_{t+i} + \frac{\mu}{(1-\theta)D}\tau_t^D + \frac{1-\gamma}{D}\sum_{i=1}^{\infty}\theta^{i-1}\hat{cov}_t(er_{t+i}, r_{t+1,t+i}^A) \\
&+\frac{(1-2\bar{z})^2}{d}\sum_{i=1}^{\infty}\theta^{i-1}\hat{cov}_t(r_{t+i}^A - r_{t+i-1}, (\gamma-1)er_{t+1,t+i-1} + \gamma er_{t+i}) \\
&\frac{1-\gamma}{D}\sum_{i=1}^{\infty}\theta^{i-1}\left(\hat{cov}_t(er_{H,t+i}, f_{t+i}^{n,1}) - \hat{cov}_t(er_{F,t+i}, f_{t+i}^{n,2})\right) \qquad (65)
\end{aligned}
$$

We can also write $\mu = 0.5 + (\bar{z} - 0.5)D/d$. Since the mean of the covariance moments is equal to zero, this expression also applies after removing the hats from the covariances, which leads to equation (41) in the paper, with the hedge term as in Section 4.4 of the paper.