Unsecured credit markets are not insurance markets

Kartik Athreya, Xuan S. Tam, Eric R. Young

Research Department, Federal Reserve Bank of Richmond, USA
Department of Economics, University of Virginia, PO Box 400182, Charlottesville, VA 22904, USA

Abstract

We study the extent to which unsecured credit markets have altered the transmission of increased income risk to consumption variability over the past several decades. We find that unsecured credit markets pass through increased income risk to consumption, irrespective of bankruptcy policy and the information possessed by lenders. If risk sharing has indeed improved over this period, the reasons do not therefore lie in the unsecured credit market.

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1. Introduction

Labor earnings have become more dispersed in the cross-section since 1980, a fact that has been established by a number of authors. This increase reflects two changes in the dispersion of earnings. First, the variance of earnings between non-random, clearly delineated groups, such as education, race, and gender, has grown, and second, the variance of growth in earnings within similarly defined groups has increased. It is the second observation that suggests that earnings have become riskier. To this point, Krueger and Perri (2006) argue that the rise in inequality is heavily driven by bigger idiosyncratic shocks experienced after entering the workforce, attributing nearly half of the rise in inequality to an increase in the volatility of innovations to a highly persistent component of the earnings process and one-fourth to an increase in...
the volatility of a purely transitory component.\textsuperscript{2} Primiceri and van Rens (2006) also finds that an increase in the volatility of the highly persistent component is a major contributor to rising earnings inequality.

The idiosyncratic component of income risk dwarfs other components, making the study of its effect on consumption variability a topic of enduring interest. Implicit in the assumption that idiosyncratic income risk may actually matter for consumption is the presumption that markets are incomplete. In particular, ever since Friedman (1957), the canonical “income fluctuation problem” has been that of a household facing idiosyncratic risk that it may smooth only through the use of a non-contingent bond.\textsuperscript{3} Based on life-cycle data on consumption and income, Storesletten et al. (2004) argue that models with only non-contingent debt can account for a variety of features of life-cycle smoothing. In contrast, Krueger and Perri (2006) argue that such models dramatically understate the insurance opportunities available to households; they prefer a model based on a large number of contingent claims with limits to commitment that prevent full risk sharing.

In this paper, we evaluate how unsecured credit markets are likely to have helped households manage recent increases in idiosyncratic income risk. We are particularly interested in the implications of rising risk for consumption variability for US households, holding other conditions fixed. To do this, we modify the “standard” income fluctuation problem in two ways: (1) we allow for default on debt and (2) we model improvements over time in the information held by lenders on the default risk of borrowers. Our paper is therefore part of the research agenda advocated by Deaton and Paxson (1994) and echoed in Blundell et al. (2008): it is a study of a model that lies between the polar extremes of risk-free bond economies and models with complete sets of contingent claims. We are motivated to model bankruptcy and information improvements because of two observations regarding the unsecured loan market: (i) the rapid growth in unsecured debt use and (ii) the rapid growth in default rates on these debts. Specifically, in precisely the period in which income risk appears to have grown, the conditional mean of negative net worth has more than doubled from 0.6% to approximately 1.4% of GDP (Sánchez, 2008), while the annual US Chapter 7 bankruptcy filing rate increased fivefold, from a rate, per thousand households, of less than 1.5 in 1980 to nearly 5 in 2004 (see Fig. 1; filing under Chapter 7 eliminates all unsecured debt and corresponds to the notion of default we use in our model).\textsuperscript{4}

Both the accumulation of debt and the use of bankruptcy reflect the attempts of households to smooth consumption in response to shocks. With respect to debt, Gross and Souleles (2002) and Sullivan (2008) argue that in the data unsecured credit is used to smooth consumption—households whose income prospects decline tend to accumulate more debt.\textsuperscript{5} With respect to bankruptcy, Sullivan et al. (2000) reports that at least 80% of filers cite income interruption—particularly

\textsuperscript{2} The residual increase is attributed to an increase in a permanent shock realized upon entry into the workforce (education, race, or gender). In particular, the rising inequality between educational groups has received a significant amount of attention—see Acemoglu (2002) or Hornstein et al. (2005) for surveys.

\textsuperscript{3} Notable contributions include Schectman (1976), Bewley (1977), and Chamberlain and Wilson (2000).

\textsuperscript{4} The expansion of unsecured debt does not seem to be merely a replacement of other, more collateralized forms of credit; rather, households now appear to be able to borrow more against future income than two decades ago. Specifically, Bird et al. (1997), and Narajabad (2007) both argue that overall access to credit expanded for households, with the former study showing this expansion was largest for the riskiest populations. Similar trends appear in UK data; Attanasio et al. (2002) and Blundell and Preston (1998) document a rise in earnings risk and Power and Young (2007) show a sharp rise in bankruptcy filings. In the US, the rise in filings was large enough that approximately one in six US households filed for bankruptcy in the period 1980–2000.

\textsuperscript{5} Calem et al. (2006) argue that credit card usage has high persistence. This result seems consistent with the use of debt to smooth both life-cycle changes in income, as well as highly persistent income shocks.
employment disruption—as one of their reasons for filing. Holding income risk fixed, one might expect that consumption variability should fall as a result of the greatly expanded use of credit and default. Furthermore, one might expect that the increase in income risk documented at the outset led households to both borrow more and default more frequently. We show that neither prediction is likely to be correct.

We show that in general, increases in income risk will lead to increases in consumption risk, whether or not lenders are fully informed about the default risk posed by a given household. If credit markets possess full information on the default risk posed by borrowers, the supply side will not allow borrowing to increase in the face of poor future income prospects. In particular, the increased incentives to borrow and default created by growth in earnings risk will trigger the repricing of debt, choking off opportunities to use both debt and bankruptcy as insurance mechanisms. On the other hand, if information between borrowers and lenders is asymmetric, adverse selection leads to a severe curtailment of credit supply. As a result, the model essentially collapses to a standard life-cycle model with exogenous, and tight, borrowing limits (Huggett, 1996). Therefore, while the mechanisms are very different, the information regime ultimately does not matter for the relationship between income risk and consumption.

With respect to overall consumption variability, as measured by the variance of log consumption over the life-cycle, we show that surprisingly it is higher under a “full-information” regime than under a partial information one. When information is very limited, equilibrium credit supply contracts severely, whereby young households cannot borrow large amounts. However, even though intertemporal smoothing is hindered by these constraints, as these households enter middle age, few have large debts that they must repay which keeps ex post keeps consumption variability low. By contrast, under full information, lenders price default risk and the only households who borrow are those with relatively good future earnings prospects. Intertemporal smoothing improves for these households; their intertemporal needs are high, and the premium they pay for borrowing is low. Ex post, however, households who are unlucky first allow consumption to fall, and if they are very unlucky finally file for bankruptcy. In the interim, consumption variability between the unlucky and lucky diverges. With respect to income risk and welfare, we find that holding fixed the ability to default, higher income risk lowers welfare, but the welfare loss of increased income risk can be mitigated by harshly penalizing default.

The growth in debt and default, especially in the face of rising income risk, demands an explanation as to why households were able to borrow and default more than ever before. A growing empirical literature—Chandler and Parker (1989), Barron and Staten (2003), Furleretti (2003), and Edelberg (2006), to name but a few—has assembled evidence that the supply side of the unsecured credit market has expanded in response to improvements in the ability of lenders to distinguish between borrowers’ risk characteristics. Athreya et al. (2008a) shows that a quantitative equilibrium model that allows for asymmetric information can indeed generate facts consistent with the view that information held by lenders about borrowers has improved; Narajabad (2007) finds similar results using a model of improved, but symmetric, signal precision.

Our punchline, therefore, is that changes in household use of an a priori plausible set of markets and institutions (unsecured debt and bankruptcy) are not the mechanisms that seem to have decoupled income and consumption volatility. Our work therefore appears to stand in contrast to Krueger and Perri (2006). The reason for the difference in outcomes lies in the way in which income risk matters for limited commitment. Krueger and Perri (2006) assumes that failure to deliver on financial obligations is met with permanent autarky, which immediately means that increased income risk actually mitigates the limited commitment problem. Given that markets are otherwise assumed complete, consumption smoothing improves. By sharp contrast, in our model markets are not assumed to be complete and increases in income risk worsen the limited commitment problem created by non-waivable personal bankruptcy. As a result, the young and the unlucky have difficulty accessing unsecured credit and face greater difficulty smoothing consumption.

Despite the preceding implication, our approach is not inconsistent with the basic mechanism in Krueger and Perri (2006), precisely because we do not allow for improvements in insurance arrangements that may have occurred as a result of heightened income risk. In fact, our findings suggest that one must assign even more importance to changes in the direct insurability of income risk, given that credit markets are not likely to have helped. Speaking to growth in the importance of “pure” insurance markets, Primiceri and van Rens (2006) argue that much of the increase in income risk was anticipated and therefore insurable, while Heathcote et al. (2008) argue that while much of the increase in income volatility represents genuinely increased risk to households it has been in components that are insurable. However, it has not been possible to subject these alternatives to the kind of test we have performed here, forcing the stylized models to confront observable trends in the specific markets that “decentralize” a given story. Thus, a useful aspect of our model that differentiates it from complete-markets approaches is that there is a natural link from the assets purchased and sold by agents in the theoretical model to observable transactions in credit markets and bankruptcy court.

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6 White (2007) argues that these survey data are not reliable; Fay et al. (1998) in fact argue that there is no evidence that bankrupt households have experienced negative economic shocks. However, it is also evident in other data (SCF, PSID) that bankrupt households are generally in poor economic shape; see Budría Rodríguez et al. (2002), Livshits et al. (2006), and Chatterjee et al. (2007).

7 It is legitimate to question whether unsecured debt markets work this way, given the observation that many credit cards appear to offer lines (fixed rates and limits); Mateos-Planas (2007) and Mateos-Planas and Rios-Rull (2007) are two attempts to model lines. Since credit card agreements are explicit about the ability of lenders to reprice at will, we believe that our arrangement captures key features of the data; see also Athreya (2002).

8 See also work by Chatterjee et al. (2006,2008), Livshits et al. (2008), Sánchez (2008), and Drozd and Nosal (2008).
Our final comments in this section are intended to spell out further precisely what we do and do not do here. Our paper is about the consequences of changes in income inequality, not the causes. With respect to these consequences, the innovations in our work are twofold. First, and most importantly, ours is the first detailed analysis of how consumption changes with income risk in the presence of unsecured debt and bankruptcy. Second, we are the first to accommodate the role played by income risk in conjunction with reductions in the asymmetry of information between lenders and borrowers suggested by recent empirical work.

Lastly, while our model makes predictions for a variety of conditional moments of consumption and for changes in these moments in response to changes in income risk, we do not attempt to directly confront these predictions with the data. Our reason for this decision is that the evolution of consumption inequality in the data is at present in dispute. Krueger and Perri (2004, 2006) argue that the rising risk in earnings has not been fully transmitted to consumption, leading to a much smaller increase in consumption volatility. Davis (2004) argues against the characterization of consumption inequality of Krueger and Perri (2004, 2006). Specifically, Davis (2004) notes that Slesnick (1992) and Battistin (2003) find serious discrepancies between aggregate measures constructed from the CEX and from the National Income and Product Accounts, while Attanasio (2002) and Battistin (2003) find discrepancies between the Interview Survey and the Diary Survey of the CEX; the Diary Survey, which is a more comprehensive survey intended to mitigate measurement error, shows rising consumption inequality. Fisher and Johnson (2006) note that a comprehensive measure of consumption expenditures from the PSID also displays more inequality. More recently, Attanasio et al. (2007) finds that non-durable consumption risk did not remain flat, but rather rose non-trivially in the 1990s. Based on discussions with a number of authors, we conclude that the consensus view is that a small increase in consumption risk over the period in question is reasonable.

The paper is organized as follows. We first lay out a simple two-period model of default to highlight some key links between default and consumption risk. We then present our main models, both with symmetric and asymmetric information. The results section assesses the model’s implications for the impact of increased income risk, both for consumption under complete markets is labeled CEX; the Diary Survey, which is a more comprehensive survey intended to mitigate measurement error, shows rising discrepancies between aggregate measures constructed from the CEX and from the National Income and Product Accounts, while Attanasio (2002) and Battistin (2003) find discrepancies between the Interview Survey and the Diary Survey of the CEX; the Diary Survey, which is a more comprehensive survey intended to mitigate measurement error, shows rising consumption inequality. Fisher and Johnson (2006) note that a comprehensive measure of consumption expenditures from the PSID also displays more inequality. More recently, Attanasio et al. (2007) finds that non-durable consumption risk did not remain flat, but rather rose non-trivially in the 1990s. Based on discussions with a number of authors, we conclude that the consensus view is that a small increase in consumption risk over the period in question is reasonable.

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2. Smoothing with default: a two-period model

In this section we present two examples that clarify the role that bankruptcy plays in determining the extent of risk-sharing and intertemporal smoothing. Consider a model with two periods, $t = 1, 2$. In the first period, labor income $w_1$ is certain. Income in the second period is stochastic and may take two values $\{w'_{2H}, w'_{2L}\}$ with $w'_{2H} > w'_{2L}$; the probabilities of these two events are $\pi_{1H}$ and $1 - \pi_{1H}$, respectively. The household maximizes the expectation of a time-separable utility function and discounts the future at rate $\beta$. We consider two cases. In the first case, the income process is set such that households benefit in terms of \textit{ex ante} welfare from the option to default, while in the second case we consider the opposite situation. Preferences will remain fixed across the two cases; as in Zame (1993) and Dubey et al. (2005), the cost of default is a loss of utility (a complete description is contained in the Appendix).

Without bankruptcy households can obtain the risk-free rate on debt, since they must repay debt in all income states in period 2. Consider next a model in which default is a costly procedure whereby borrowers can decide not to repay debts acquired in period 1 but must promise to pay more in non-default states at the time of borrowing. The statement of the household’s problem in each of these cases is presented fully in the Appendix; in what follows, we present the results of two examples designed to highlight the role bankruptcy plays in both consumption smoothing over time and across states.

In the first case, the parameter values are set to capture three features: (i) to have a low-income state that is bad enough that the level of debt that can be feasibly repaid in that state is very low, (ii) to have the agent default only if the low income is realized, and (iii) preferences and income are such that the household would borrow under complete markets. In Fig. 2, consumption under complete markets is labeled C and is given by the pair $c_{1C}^M, c_{2C}^M$. Turning next to a “standard incomplete-markets model” in which only non-defaultable borrowing is allowed, the optimal choice is given by first-period consumption $c_1$, and because full insurance is no longer possible, period 2 consumption is a state-contingent pair $c_{2L}^D, c_{2H}^D$. In this case, the household’s inability to insure against uncertain second-period income actually leads it to save, even though under complete markets it would have borrowed; this outcome is the usual precautionary savings effect found in models of uninsurable idiosyncratic risk (our two-period model is essentially Aiyagari, 1994). The preceding two cases are polar, in the sense that the first offers complete contingency of assets and the second none. An impetus for allowing bankruptcy and default is to aid risk sharing by making repayment contingent on circumstances that prevail in the future. The consumption allocation that results from permitting default is given by $c_{1D}^D, c_{2L}^D, c_{2H}^D$. The household again borrows in the first period, as it did under complete markets, but borrows an amount that it will repay only if $w_2 = w_{2H}^H$, i.e. only in the good period 2 income state. The fact that it will default in one state is what makes the household’s period 2 consumption uncertain, but as seen in the figure the \textit{ex post} utility attained in each state is higher than that under standard incomplete markets. \textit{Ex ante} expected utility is therefore also higher when default is allowed.

The key intuition for this case is that when households face an income process that is likely to generate higher future income, but features some rare and bad states, they will in general wish to borrow but the maximal risk-free borrowing they can access will be small. In our two-period example, the household cannot guarantee non-negative second-period consumption unless it borrows less than $w_{2L}^L$; if $w_{2L}^L$ is small, this limitation will hamper purely intertemporal smoothing.
The reason that bankruptcy (subject to penalties) is able to improve on this outcome is that the household may now borrow more, knowing that it can default and pay the penalty if the income realization is $w^L_2$. It is here that the probability of low income comes into play; if $1 - \pi_H$ is low, then the premium that must be paid on risky loans will remain small and essentially allow the household to borrow against future income. In doing so, they achieve an allocation closer to complete markets than is possible with uncontingent debt. Thus, for households in situations where rare bad states are possible, default can not only help share risk ex post but also can facilitate borrowing to improve intertemporal smoothing.

We turn next to a case where the non-waivable right to bankruptcy actually harms both intertemporal and intratemporal smoothing (see Fig. 3). The key feature of this example is an income process that is expected to grow between the first and second periods: both period 2 income states are higher than period 1 income. As before, consumption
under complete markets is labeled C and is given by the pair \((c_1^{CM}, c_2^{CM})\). When non-defaultable borrowing is allowed, the optimal choice is the triple \((c_1, \{c_0^d, c_2^d\}, c_{2H})\), and the consumption allocation when default is permitted is given by \((c_1^d, \{c_0^d, c_2^d\})\). The example is set so that the household will default in both states of income for any level debt taken in period 1. Therefore, \(c_{2H} = w_2^*\). Thus, the household now is no longer able to smooth intertemporally at all—the outcome is unambiguously worse than the outcome with pure non-contingent debt. The intuition for the harm done by the bankruptcy option is that households cannot commit to repaying debt and therefore cannot borrow. But given that they face very little utility risk in the second period—endowments in both states are higher than current income and are fairly close together—bankruptcy limits intertemporal smoothing without helping risk sharing.

In summary, the relationship between bankruptcy, consumption smoothing, and welfare is not obvious and is likely to vary non-trivially across households that differ in income prospects. As is well known, there is substantial heterogeneity in the slope and variability of household income across educational attainment; young, poorly educated households in the US face relatively flat earnings profiles with substantial risk, while their college-educated counterparts face much steeper earnings growth when young and much smaller risks for much of their working lives. Therefore, the trade-offs created by bankruptcy in the first example correspond to those facing less-educated households, while the trade-offs faced by college-educated groups may be better described by the second example. However, these statements must also be conditioned on age: older households who are close to their peak in earnings may look more like the first example than the second. By modeling precisely this source of heterogeneity, we hope to provide reasonable conclusions on the way defaultable debt helps households manage changes in income and income risk.

3. A quantitative model of unsecured debt

In order to provide a quantitatively serious evaluation of the roles played by bankruptcy and unsecured borrowing in modifying the effect of income risk on consumption, we use the framework laid out in Athreya et al. (2008a). The salient features of that setting are: (1) households live for a maximum of \(j < \infty\) periods; (2) households save and borrow using defaultable debt; and (3) households belong to two classes differing in terms of (i) access to financial intermediation and (ii) exposure to idiosyncratic risk.

3.1. Normal households

If alive at age \(j\), a normal household faces probability \(\psi_j < 1\) of surviving to age \(j + 1\). Households value consumption, discount the future at rate \(\beta < 1\), and attach a negative value \(\lambda_j\) (in utility terms) to all non-pecuniary costs of bankruptcy; we assume \(\lambda\) evolves stochastically.\(^9\) Heterogeneous default costs appear relevant, as argued in Fay et al. (1998), and permit us to match the data on both borrowing and default.\(^10\) Households maximize expected utility:

\[
\sum_{j=1}^{\infty} \sum_{s} \left( \prod_{i=0}^{j-1} \beta \psi_{j+s} \right) \Pi(s) \left[ \frac{n_j}{1 - \sigma} \left( \frac{c_j}{n_j} \right)^{1-\sigma} - \lambda_j 1(d_j = 1) \right].
\]

Households may differ in their “credit” status (described in detail further below) denoted by the state variable \(m_j\); \(d_j = 1\) denotes a default in the current period. Let \(\Pi(s)\) be the probability of a given history of events \(s\) through age \(j\). \(\sigma \geq 0\) is the coefficient of relative risk aversion. Households retire exogenously at age \(j^* < j\), and \(n_j\) is the number of household members at age \(j\).\(^11\)

In each period a typical normal household makes a consumption-savings decision \((c, b)\), where \(b\) is the amount of borrowing/saving and \(c\) is consumption. The household also selects a rule for default on any accumulated debt as a function of next period’s shock. Three costs of default are paid only in the period in which the default occurs: a non-pecuniary cost \(\lambda_j\), a pecuniary cost \(A\), and exclusion from the credit market; going forward the household carries a marker of “bad” credit, \(m_j = 1\). The parameter \(\xi \in (0, 1)\) is the probability of the bad marker disappearing. Under partial information, the price charged to a household for issuing debt will in general depend on \(m\), so that households with recent defaults will receive different credit terms than households with “clean” credit. The probabilistic removal of this marker represents current regulations forcing the removal of bankruptcy from one’s credit score after a fixed period (10 years in the US).

During working age, the household’s budget constraint is given by

\[
c_j + q(b_j, l)b_j + A1(d_j = 1) \leq o_j + (1 - \tau)W_{\lambda j}(\psi_{j+}\psi_{j+}ve).
\]

\(^9\) This cost is intended to be a parsimonious method for capturing all of the factors and complications associated with bad credit other than ones associated with the direct filing cost or the terms of credit, as in Athreya (2002). As Gross and Souleles (2002) document, there is significant unexplained variability in the probability of default across households, even after controlling for a large variety of observables.

\(^10\) Chatterjee et al. (2007) permits shocks to discount factors; these shocks are troublesome because they induce simultaneously a desire to borrow and to default. In contrast, shocks to stigma costs only alter the incentive to default (directly).

\(^11\) We assume that labor is supplied inelastically. Many studies of rising inequality assert that changes in labor hours are a critical source of insurance (Storesletten et al., 2004; Heathcote et al., 2008) and abstracting away from it will bias the outcomes in the model. While we do not disagree with the idea that agents can vary their labor supply in response to shocks, we suspect that the frictionless divisible labor model overstates the extent to which this variation is feasible. Since our interest is not in providing a complete accounting of risk sharing, we abstract from this issue.
The function \( q(\cdot) \) denotes an individual-specific bond price that depends on bond issuance \( b_j \) and a vector of individual characteristics \( l, a \) is net worth, \( \lambda \) is the pecuniary cost of filing for bankruptcy, and the last term is after-tax current income. Log labor income is the sum of five terms: the aggregate wage index \( W \), a permanent shock \( y \) realized prior to entry into the labor market, a deterministic age term \( \omega_{j,y} \), a persistent shock \( e \) that evolves as an AR(1)

\[
\log(e') = \rho \log(e) + \varepsilon',
\]

and a purely transitory shock \( \log(v) \). The shocks to labor earnings, \( e \) and \( \log(v) \), are both independent mean zero normal random variables with variances that are \( y \)-dependent. The budget constraint during retirement is

\[
c_j + q(b_j, l)b_j \leq a + \theta W\omega_{j-1,y}y\epsilon_{j-1} + \theta W,
\]

where for simplicity we assume that pension benefits are composed of a fraction \( \theta \in (0, 1) \) of income in the last period of working life plus a fraction \( \Theta \) of average income (which is normalized (1)). Because bankruptcy is not a retiree phenomenon, we deliberately keep the specification of retirees simple. There do not exist markets for insurance against income or survival risk and we abstract from any sources of long-run growth.

The permanent shock in the model is intended to capture the large differences in lifetime income associated with different human capital levels; \( y \) allows us to differentiate between non-high school, high school, and college education levels, as in Hubbard et al. (1994). We abstract away from any connection between \( y \) and family size, but we permit the survival probabilities \( \psi_{j,y} \) and the deterministic age-income terms \( \omega_{j,y} \) to differ according to the realization of the permanent shock.\(^{12}\) The differences in these life-cycle parameters will generate different incentives to borrow across types; in particular, college workers will have a steeper hump in earnings, which is critically important as it generates empirically relevant demand for borrowing early in the life cycle when observed default rates are highest.

We model the non-pecuniary cost process \( \lambda \) as a two-state Markov chain with realizations \( \{\lambda_1, \lambda_2\} \) and transition matrix \( \{p_{ij}\}_{i,j=1} \). We will discipline these costs by requiring that the benchmark model replicates several moments summarizing the behavior of unsecured credit markets.

### 3.1. Recursive formulation

Let \( d(b, e', v', \lambda') \) denote the (indicator) decision rule governing bankruptcy if the household declares bankruptcy in the event that next period’s shocks are \( (e', v', \lambda') \) and 0 otherwise. For the convenience of the reader, let \( X = (e, v, \lambda) \) denote the exogenous components of the household state vector. In recursive terms, a household of age \( j \) with good credit \( m = 0 \) solves

\[
v(a, y, X, j, m = 0) = \max_{b, d(X) \in (0, 1)} \left\{ \frac{n_j}{1 - \sigma} \left( \frac{c_j}{n_j} \right)^{1 - \sigma} + \beta \psi_{j,y}E^X \left[ (1 - d(b,X'))v(b, y, X', j + 1, m' = 0) + d(b,X')v(0, y, X', j + 1, m' = 1) \right] \right\},
\]

where

\[
v(0, y, X', j + 1, m' = 1) = \left\{ \frac{n_j}{1 - \sigma} \left( \frac{c_j}{n_j} \right)^{1 - \sigma} - \lambda + \beta \psi_{j,y}E^X \left[ \xi v(a', y, X', j + 1, m' = 0) + (1 - \xi) v(a', y, X', j + 1, m' = 1) \right] \right\}
\]

denotes the value from defaulting on previously accumulated debts; notice that households cannot borrow or save during the period in which they declare bankruptcy.\(^{13}\) Households who currently have a bad credit indicator (i.e. \( m = 1 \)) solve

\[
v(a, y, X, j, m = 1) = \max_{b, d(X) \in (0, 1)} \left\{ \frac{n_j}{1 - \sigma} \left( \frac{c_j}{n_j} \right)^{1 - \sigma} + \beta \psi_{j,y}E^X \left[ \xi v(b, y, X, j + 1, m' = 0) + d(b,X')v(0, y, X', j + 1, m' = 1) \right] + (1 - \xi) \left\{ (1 - d(b,X'))v(b, y, X', j + 1, m' = 1) \right\} \right\},
\]

### 3.2. Symmetric information

We focus throughout on competitive lending. There exists a competitive market of intermediaries who offer one-period debt contracts and utilize available information to offer individualized credit pricing. Let \( l \) denote the information set for a lender and \( \pi^b: b_j \rightarrow [0, 1] \) denote the function that assigns a probability of default to a loan of size \( b \), given information \( l \). \( \pi^b \) is identically zero for positive levels of net worth and is equal to 1 for some sufficiently large debt level. The break-even

\(^{12}\) Stochastic mortality and age-dependent family size do not play an important role in our model, but we keep them so as not to bias our estimates of the parameters of the model in systematic ways.

\(^{13}\) Under current US bankruptcy code, a filing is likely to be judged fraudulent if it is immediately followed by significant asset accumulation; since filings normally take several months to resolve, it seems that a short period of exclusion is imposed as a punishment by law. Longer periods of exclusion that are imposed by the market are not consistent with competitive behavior.
pricing function must satisfy

\[
q(b, I) = \begin{cases} 
\frac{1}{1 + r} & \text{if } b \geq 0, \\
\frac{(1 - \bar{\pi}^b)p_j}{1 + r + \phi} & \text{if } b < 0
\end{cases}
\]  

(8)
given \(\bar{\pi}^b\). The risk-free saving rate is given by \(r\) and \(\phi\) represents transactions costs for lending, so that \(r + \phi\) is the risk-free borrowing rate.\(^{14}\) With full information, a variety of pricing arrangements will lead to the same price function. However, as is well known (Hellwig, 1990), under asymmetric information, settings outcomes often depend on the particular “microstructure” being used to model the interaction of lenders and borrowers. Under full information, our approach is completely standard (see Chatterjee et al., 2007; Livshits et al., 2006, 2007), as we seek a setting that delivers to households a function \(q(b, y, e, v, \lambda, j, m) : b \rightarrow [0, 1/(1 + r)]\) that they can take parametrically when optimizing; the compactness of the range for \(q\) implies that the household problem has a compact opportunity set and therefore possesses a solution.

The microstructure that underlies our pricing function is modeled as a two-stage signaling game between borrowers and lenders. In the first stage, borrowers name a level of debt \(b\) that they wish to issue. Second, a continuum of lenders compete in an auction where they simultaneously post a price for the desired debt issuance of the household and are committed to delivering the amount \(b\) in the event their “bid” is accepted; that is, the lenders are engaging in price competition for borrowers. Thus, households view the pricing functions as schedules and understand how changes in their desired borrowing will alter the terms of credit (that is, they know \(D_b q(b, I)\)) because they compute the locus of Nash equilibria under price competition. Exactly how the pricing function depends on the components in \(I\) will be specified next.

A formal statement of equilibrium for the full information case is omitted since it is entirely standard.

In the full information case, \(I\) includes all components of the household state vector: \(I = (y, e, v, \lambda, j, m)\). Zero profit for the intermediary requires that the probability of default used to price debt must be consistent with that observed in the stationary equilibrium, implying that

\[
\bar{\pi}^b = \sum_{e', v', \lambda'} \pi_e(e'|e)\pi_v(v'|v)\pi_{\lambda|\lambda}(\lambda'|\lambda) d(b(a, y, e, v, \lambda, j, m), e', v', \lambda').
\]  

(9)
Since \((b, e', v', \lambda')\) is the probability that the agent will default in state \((e', v', \lambda')\) tomorrow at debt level \(b\), integrating over all such events tomorrow is the relevant default risk. With partial information, we will need to integrate over current states as well as future ones, since pieces of the state vector will not be observable. We defer a formal discussion of the partial information case until later in the paper.

3.3. Asymmetric information

As argued at the outset, a variety of empirical work and the quantitative model of Athreya et al. (2008a) suggest that over the past several decades there has been a systematic decline in the asymmetry of information between borrowers and lenders. We now model an increase in income risk when it coincides with an improvement in the information lenders possess about borrowers. To accommodate asymmetric information, we use the signaling model developed in our previous paper. We assume the “earlier period” of the model corresponding to the early 1980s is one where information is asymmetric with respect to the stochastic components of a household’s income (including total income), current stigma cost, and current net worth. We maintain the assumption throughout that age and education are observable.\(^{15}\) Therefore, we have \(I = (y, j, m)\) (with \((a, e, v, \lambda)\) not observed).\(^{16}\)

The first concern for solving the partial information economy is that lenders must form beliefs over the probability of an individual being in a particular state \((e, v, \lambda)\) given whatever is observed, knowing also that what is observed is a function of lenders’ a priori beliefs; that is, beliefs must satisfy a fixed point condition. Let \(\Pr(e, v, \lambda|b, y, j, m)\) denote the probability that an individual’s shock vector in any period takes a given value \((e, v)\), conditional on observing the size of borrowing, the permanent shock, age, and credit status. Given this assessment, the lender can compute the likelihood of default on a loan of size \(b\):

\[
\bar{\pi}^b = \sum_{e, v, \lambda} \sum_{e'} \sum_{v'} \sum_{\lambda'} \left[ \sum_{e, v, \lambda} \pi_e(e'|e)\pi_v(v'|v)\pi_{\lambda|\lambda}(\lambda'|\lambda) d(b(a, y, e, v, \lambda, j, m), e', v', \lambda') \right] \Pr(e, v, \lambda|b, y, j, m).
\]  

(10)
In a stable environment with a small number of creditors, or one with an efficient technology for information sharing, intermediaries must form beliefs that incorporate everything they either know or can infer from observables; competitors

\(^{14}\) The pricing function takes into account the automatic default by those households that die at the end of the period.

\(^{15}\) Regulatory restrictions prohibit the use of age in determining credit terms, at least in the unsecured credit market, along with race and gender. We study the possibility that types are unobserved in a companion paper, which focuses on estimating the costs of such regulations.

\(^{16}\) We separate \(b\) from \(I\) even though \(b\) is observable because the borrower takes the derivative of \(q\) with respect to \(b\) and it is therefore more convenient to make it a separate argument.
who exploit this information may be able to "cream-skim" the best borrowers away from those who form beliefs in any other way. In equilibrium, if this information exists it must be incorporated by all intermediaries into their belief functions; we view this arrangement as a natural analogue to the conditions that prevailed in the early 1980s. $Pr(e, v, \lambda | b, y, j, m)$ assigns a probability to each type that borrows $b$ in equilibrium.

In the partial information environment the calculation of $Pr(e, v, \lambda | b, y, j, m)$ is non-trivial, because it involves the distribution of endogenous variables. First, let the invariant distribution over states be denoted by $f$ on observables, and uses this probability to formulate beliefs. Thus, the decision rule of the household under a given pricing scheme is inverted to infer the state conditional on $y$, depends on its understanding of lender behavior at $G$. However, as described in detail in Athreya et al. (2008a), a central complication to the pure signaling model, which is not only tractable but also consistent with the relative homogeneity of unsecured borrowing, is the difficulty in assigning off-equilibrium beliefs to deal with the proliferation of equilibria typically present in signaling models, without assigning off-equilibrium beliefs to deal with the proliferation of equilibria typically present in signaling models, without assigning off-equilibrium beliefs.

It is possible that intermediaries in the partial information world would find it profitable to offer a menu of contracts to households by $O$ and space for households by $I$. We view this arrangement as a natural analogue to the conditions that prevailed in the early 1980s.

Definition 1. A perfect Bayesian equilibrium for the model consists of (i) household strategies for borrowing $b^*$ : $\Omega \rightarrow R$ and default $d^* : \Omega \times \mathcal{E} \times \mathcal{V} \times \mathcal{L} \rightarrow [0, 1]$ and intermediary strategies for lending $q^* : R \times \mathcal{J} \rightarrow [0, 1/(1 + r)]$ and (ii) beliefs about the borrower state $\Omega$ given borrowing $\mu^*(\Omega | b)$, that satisfy

1. **Lenders optimize**: Given beliefs $\mu^*(\Omega | b)$, $q^*$ is the pure-strategy Nash equilibrium under price competition.
2. **Households optimize**: Given prices $q^*(b, l)$, $b^*$ solves the household problem.
3. **Beliefs are consistent with Bayes’ rule**: The stationary joint density of $\Omega$ and $b$, $\Gamma_{\mu^*}(\Omega, b)$, that is induced by (i) lender beliefs and the resultant optimal pricing, (ii) household optimal borrowing strategies, and (iii) the exogenous process for earnings shocks and mortality is such that the associated conditional distribution of $\Omega$ given $b$, denoted $\Gamma^b_{\mu^*}(b)$, is $\mu^*(\Omega | b)$.
4. **Off-equilibrium beliefs**: $q^*(b, l) = 0$ for such that $\Gamma^b_{\mu^*}(b) = 0$.

---

17 This point is related to the extensive survival literature, which investigates whether agents who form beliefs that deviate from rational expectations can survive in asset markets.

18 Why the intermediaries did not use these contracts in the earlier period is a question beyond the scope of this paper.

19 It turns out that modeling the game as signaling rather than screening is significantly easier. In a screening game the lenders would move first, and then we would need to check deviations in the infinite-dimensional space of alternative pricing functions. Here households move first and we only need to check deviations in the space of borrowing levels, which is implicit in our use of the pricing function as a schedule confronting the borrower. There is a connection between our procedure and the Intuitive Criterion from Cho and Kreps (1987); explicitly checking all the types is prohibitively expensive, however, so we do not claim that our off-equilibrium belief structures implies the Intuitive Criterion is satisfied.
Since our shocks are continuous random variables, the debt levels that get zero weight in the stationary distribution are those above and below any levels that get positive weight \( \Gamma \) has a connected support. Obviously, for default decisions the upper limit is irrelevant; thus, as noted above, we are imposing a belief about the behavior of agents who borrow more than any agent would in equilibrium, no matter what their current unobserved state. Given that \( q \) is weakly decreasing in \( b \), the natural assumption is that this agent intends to default with probability one. Again, we direct the reader to Athreya et al. (2008a) for a complete discussion, including an investigation of alternative off-equilibrium beliefs.

3.4. Special households

As mentioned above (and as discussed in more detail in Athreya et al., 2008a), we posit a measure \( \mu_s \) of households who face neither idiosyncratic risk nor financial market frictions and therefore earn a higher rate of return on savings. These households face a present-value budget constraint

\[
\sum_{j=1}^{J} \left( 1 + \frac{1}{1 + \text{MPK} - \delta} \right)^{j-1} c_j = A_1
\]

with lifetime wealth given by

\[
A_1 = k_1 + \sum_{j=1}^{J} \left( 1 + \frac{1}{1 + \text{MPK} - \delta} \right)^{j-1} (1 - \tau) wy_{j,y} + \left( 1 + \frac{1}{1 + \text{MPK} - \delta} \right)^{j-1} \sum_{j=0}^{J} \left( 1 + \frac{1}{1 + \text{MPK} - \delta} \right)^{j-j-1} (1 - \tau) wy_{j-1,y} + \Theta.
\]

Special households face the mean earnings profiles of the college type, and for convenience we assume logarithmic preferences. As a result, we can obtain closed-form representations for their decisions:

\[
c_1 = \frac{1 - \beta_s}{\beta_s} A_1,
\]

\[
c_j = \beta_s^j (1 + \text{MPK} - \delta)^{j-1} c_1.
\]

Given the decision rule for consumption, household capital stocks evolve as follows:

\[
k_{j+1} = (1 + \text{MPK} - \delta) k_j + (1 - \tau) wy_{j,y} - \frac{1 - \beta_s}{\beta_s} \beta_s^{j-1} (1 + \text{MPK} - \delta)^{j} A_1.
\]

The purpose of the special households is to permit sufficient flexibility to match both the average amount of wealth held in the economy—which determines the marginal product of capital—and the concentration of wealth in the hands of a small minority of households for whom the option to default is irrelevant. An alternative to our approach is used in Chatterjee et al. (2007) and Sánchez (2008)—they calibrate the earnings process to match the US distributions of income and wealth. For our overlapping generations model the burden of this approach is immense as it forces us to extend the range of permissible asset positions out to very high positive levels. Given that we are not interested in the behavior of agents with positive net worth, the cost seems too much to bear.

3.5. Government

Government in this model exists simply to fund pension payments to retirees, which implies the following government budget constraint:

\[
\tau (1 - \mu_s) W \int y_{j,y} \omega_j \Gamma(a, y, e, v, \lambda, j < j^*, m) + \tau \frac{H}{J} W \sum_{j=1}^{J-1} y_{j,y}
\]

\[
= (1 - \mu_s) W \int ((1 - \mu_s) W \sum_{j=1}^{J-1} y_{j,y} \omega_{j-1,y} + \Theta) \Gamma(a, y, e, v, \lambda, j \geq j^*, m) + \frac{H}{J} W \sum_{j=1}^{J-1} ((1 - \mu_s) W \sum_{j=1}^{J-1} y_{j-1,y} + \Theta).
\]

3.6. Market clearing

Loan markets clear when the risk-free saving rate equals the marginal product of capital net of depreciation \( \delta \):

\[
r = zK^{\alpha - 1} N^{1-\alpha} - \delta - \vartheta.
\]


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92

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The parameter $\vartheta$ denotes intermediation costs that apply to both saving and borrowing. Given the resources destroyed in intermediation, total capital solves the equation

$$K = \frac{1 - \mu_s}{1 + 2K^{a-1}}N^{1-\vartheta} - \delta + \vartheta \int b(a, y, e, v, \lambda, j, m) \Gamma(a > 0, y, e, v, \lambda, j, m)$$

$$+ \int (1 - \pi b(a, y, e, v, \lambda, j, m) b(a, y, e, v, \lambda, j, m) \Gamma(a < 0, y, e, v, \lambda, j, m) + \mu_s \sum_{j} k_j. \quad (18)$$

Total labor input is the weighted sum of the labor inputs of the two classes of households, and so is given by

$$N = (1 - \mu_s) \int y \omega_{jy} e \Gamma(a, y, e, v, \lambda, j, m) + \mu_s \sum_{j} y \omega_{jy}, \quad (19)$$

where $\alpha \in (0, 1)$ is the elasticity of output with respect to capital. The labor market clears when the aggregate wage index equals the marginal product of labor:

$$W = (1 - \alpha)K^\alpha N^{-\alpha}. \quad (20)$$

Goods market clearing occurs when total consumption plus total transactions costs equals total output less depreciation:

$$C = (1 - \mu_s) \int c \Gamma(a, y, e, v, \lambda, j, m) + \mu_s \sum_{j} c_j, \quad (21)$$

$$T = (1 - \mu_s) \int [\varphi \mathbf{1}(b < 0) b + A \mathbf{1}(m = 0, m' = 1)] + \vartheta b \Gamma(a, y, e, v, \lambda, j, m), \quad (22)$$

$$C + T = K^\alpha N^{1-\alpha} - \delta K, \quad (23)$$

where $\mathbf{1}(A)$ is the indicator function for set $A$.

4. Calibration

We set $\sigma = 2$. We set $\vartheta = 0.35$ at an exogenous retirement (model) age of 45 and $\Theta = 0.2$, yielding an overall replacement rate around 55%. The income process is taken from Hubbard et al. (1994), which estimates separate processes for non-high school (NHS), high school (HS), and college-educated (Coll) workers for the period 1982–1986. Fig. 4 displays the path for $\omega_{jy}$ for each type; the large hump present in the profile for college-educated workers implies that they will want to borrow early in life to a greater degree than the other types will (despite their effective discount factor being

\[\text{Fig. 4. Efficiency units of labor.}\]
somewhat higher due to higher survival probabilities). We hold the age-dependent component fixed and assume that the persistence parameter is unchanged across periods as well. For each period the process is discretized with 15 points for $e$ and 7 points for $v$. The resulting processes for the low risk period are

$$\log(e') = 0.95 \log(e) + \varepsilon',$$
$$e' \sim N(0, 0.033),$$
$$\log(v) \sim N(0, 0.04)$$

for non-high school agents,

$$\log(e') = 0.95 \log(e) + \varepsilon',$$
$$e' \sim N(0, 0.025),$$
$$\log(v) \sim N(0, 0.021)$$

for high school agents, and

$$\log(e') = 0.95 \log(e) + \varepsilon',$$
$$e' \sim N(0, 0.016),$$
$$\log(v) \sim N(0, 0.014)$$

for college agents. The increases are 15%, 58%, and 22%, for non-high school, high-school, and college-educated households, respectively, while the measure of special agents is $\mu = 0.05$. We set $\phi = 0.03$ to generate a 6% spread between risk-free saving rates received by normal households and the risk-free borrowing rate. A is set equal to 0.03; if one unit of model output is interpreted as $40,000—roughly median income in the US—then the filing cost is equal to $1200. Finally, $\xi = 0.25887$ implies that 95% of households who do not file for bankruptcy again will have clean credit after 10 years.

We introduce changes in the income process consistent with the measurements in Krueger and Perri (2006): the variance of the permanent component, the persistent component, and the transitory component contribute 36%, 40%, and 24% to the overall rise in cross-sectional variance, respectively, with the total unconditional variance of lifetime income assumed to rise by 20%. Under the high-risk regime we calibrate the other parameters $(\beta, \beta_1, l_2, \theta_2, \pi, \sigma, \delta)$ to match seven targets: a median discharge to median income ratio of 1, a bankruptcy filing rate of 1.2%, median discharge to median income ratio of 1, a risk-free saving rate of 1%, a 70% labor income share, and an annual depreciation rate of 10%. Our target for bankruptcy is consistent with a model in which only income is uncertain; that is, there are no shocks to expenses. Expense shocks create involuntary creditors that allow households to suddenly acquire very large debts without a corresponding change in measured consumption. The difficulties in measuring rare “catastrophic” shocks, their true “uninsurability” (for example, Medicare and emergency rooms are always available to deal with medical shocks), and their persistence can lead to a serious mismeasurement of the role of credit use and bankruptcy for managing income risk. We therefore calibrate the model’s bankruptcy target to be net of this measure. Sullivan et al. (2000) reports that one-fifth of filers report that health played some role in their decision to file. Using an overall filing rate of 1.5%, our target becomes 1.2%. Our choice of a target for debt is motivated by two considerations. First, ours is a single-asset model in which the only model-consistent measure of debt is negative net worth. In a life-cycle model, negative net worth and unsecured debt largely coincide for the young. The young are the population group of interest precisely because they account for both a large fraction of unsecured debt holdings and for most defaults. Second, gross measures of revolving debt overstate the amount of truly uncollateralized debt held by households, because in the absence of exemptions for bankruptcy, all assets held by the households can be seized to repay uncollateralized creditors in the event of default. Thus, in this case, the only uncollateralized debt is negative worth. Moreover, exemptions, even when positive, apply overwhelmingly to home equity, an asset rarely held by the young.

We calibrate the high-risk period and then reduce the variances to match the earlier period; we argue that the earlier period is best interpreted as suffering from asymmetric information and that model is too computationally demanding to be calibrated effectively. The resulting calibrated parameter values are listed in Table 1, along with all other parameters for the convenience of the reader; we do reasonably well at matching the targeted moments, but not perfectly.

---

20 Recall that all special agents are college-educated types, so the total measure of college agents is 20%. We normalize units of measurement such that $N = 1$.

21 The spread between saving rates and capital returns is thus equal to 3%, consistent with transactions costs measured by Evans and Schmalensee (1999).

22 This cost is an estimate inclusive of filing fees, lawyer costs, and the value of time.

23 The value for the ratio of negative net worth to GDP is taken from Budría Rodríguez et al. (2002); we exclude the self-employed and consider both credit cards and installment loans as unsecured debt. For the measure of borrowers, we roughly average the numbers from Chatterjee et al. (2007)—6.7%—and Wolff (2004)—17.6% to get a ballpark measure of 12.5%.

24 For example, revolving debt relative to disposable income grew from 1% to nearly 10% between 1980 and 2005; this measure represents an upper bound on the amount of unsecured indebtedness present in the data. The quantitative implications of the model for the question of how income risk is transmitted to consumption volatility are not changed and are not presented here. Results are available on request.
5. Results

We present our results in three steps. First, we investigate the role of bankruptcy in the transmission of income risk to debt accumulation and consumption risk under full (and symmetric) information (FI) under the relatively high risk (H) income process corresponding to recent (2004) data; here, we compare outcomes with and without (NBK) the bankruptcy option. This case represents our baseline calibration and is denoted (FI-H). Given our benchmark, we then isolate the role of income risk by studying allocations obtaining under full information under the relatively lower-risk (L) income processes faced by households in the early 1980s. We denote this case (FI-L). Third, we evaluate the role that information changes play in this transmission. Throughout our analysis we restrict attention to comparisons of steady states.25

To understand how unsecured credit affects unconditional consumption volatility, it is critical to recognize that both intertemporal and intratemporal smoothing will be affected by changes in bankruptcy policy and credit availability (as we demonstrated in the two-period examples). Therefore, in what follows, we will repeatedly focus on a useful decomposition for the variance of (log) consumption:

\[
V(\log(c)) = E[V(\log(c) | j)] + V(E[\log(c) | j]); \tag{24}
\]

the total variance of consumption is the mean of the variances of consumption conditional on being age \(j\) plus the variance of mean consumption conditional on being age \(j\). The first term yields the effectiveness of any given market arrangement on intratemporal smoothing—it is the average dispersion of consumption occurring within households of any given age and so provides a natural measure of “risk sharing.” The second term measures intertemporal smoothing by capturing the extent to which mean consumption evolves over the life-cycle. Once bankruptcy is a possibility and income risk is even partially uninsured, credit supply may tighten and lead to an increase in the contribution of intertemporal movements to overall consumption volatility.26

5.1. What effect does bankruptcy have on consumption smoothing?

Our first step in studying the role that bankruptcy plays in smoothing consumption is to compare the full-information model to an otherwise identical one where bankruptcy is prohibited. Fig. 5 plots the variance of log consumption over the life-cycle for our benchmark model with and without the bankruptcy option (under both income risk regimes).27 Focusing on the solid lines (high income risk), we see that the figure suggests a trade-off with respect to intratemporal smoothing—when bankruptcy is an option, households experience higher consumption variance when young but lower consumption variance when old. The intuition is similar to that discussed in Athreya (2008). Without default, households can borrow at the risk-free rate up to the natural debt limit and can therefore achieve good intertemporal smoothing. 

Ex post, however, any borrowing done early in life must be repaid, leaving unlucky households relatively exposed to bad shocks later in their life-cycle. In contrast, when bankruptcy is permitted, borrowing does not have to be repaid if the household gets bad enough outcomes; thus, intratemporal smoothing is improved later in life.

A second cause of this reduction in variance beyond middle-age is a borrowing rate that moves against unlucky households, leading to a curtailment of borrowing when young and a decline in the ability to respond to shocks early in life. Fig. 6 plots the price functions for an age 29 household over various different values of the shocks (for brevity we plot only

\[\text{Table 1} \]

Parameters/calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model</th>
<th>Target</th>
</tr>
</thead>
<tbody>
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<td>(\sigma = 2.0000)</td>
<td>(\beta = 0.0300)</td>
<td>(\lambda_2 = 0.0358)</td>
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<td>(\beta_l = 1.0024)</td>
<td>(\xi = 0.2589)</td>
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<td>(\phi = 0.0300)</td>
<td>(\mu_s = 0.0500)</td>
<td>(\lambda_1 = 1.6715)</td>
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<table>
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<tr>
<th>Calibration</th>
<th>Model</th>
<th>Target</th>
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</thead>
<tbody>
<tr>
<td>Discharge/income ratio</td>
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<tr>
<td>Fraction of borrowers</td>
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<td>Debt/GDP ratio</td>
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<td>0.0068</td>
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<tr>
<td>Default rate (%)</td>
<td>1.366</td>
<td>1.200</td>
</tr>
</tbody>
</table>

Parameters defined in text.

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25 Krueger and Perri (2006) investigate an infinite-horizon model and are therefore able to explicitly compute transitional dynamics. For our OLG model the memory requirements for even a 10-year transition path exceed the capacity of a 64-bit addressing system.

26 In any infinite horizon setting without aggregate risk—such as Chatterjee et al. (2007)—the second term is zero.

27 See Eq. (24) for notation.
the college type with the low stigma cost; the top panel is for the lowest realizations of $e$, the middle panel is the middle realizations, and the bottom panel is the highest realizations of $e$, and we ignore the transitory shock since it does not alter the pricing functions in a quantitatively important way). With full information about borrowers, any change in circumstances that increases default risk—such as a low realization of $e$—will increase the borrowing rate and therefore inhibit the ability to smooth when young (since only the young borrow it is mainly their consumption smoothing that is affected). Nonetheless, the model is able to generate quantitatively accurate bankruptcy filing rates, as well as the age-profile of bankruptcy filings. In the model, those who default are those who receive shocks that are not disastrous, and therefore can expect higher future income, and in turn pay relatively small premia on their debt. Nonetheless, some of these households will be unlucky, ex post, and will choose to default. Moreover, much of the borrowing by the young is motivated by intertemporal smoothing considerations, which in turn leads to default rates in the model, as in the data,
being highest in the early parts of the life cycle. Fig. 7 displays default rates over the life-cycle (the measure of defaulters divided by the measure of households of a given age). Therefore, the inability to borrow as much in the presence of bankruptcy as in its absence leads to a reduction in the measure of households entering middle-age with large debts. Lastly, even when risk-free debt is available, the eventual need to accumulate wealth for retirement limits the accumulation of debt early in life.

Overall measures of intratemporal smoothing, as seen in the top row of Table 2 are similar with and without bankruptcy. However, the absence of a trade-off in overall intratemporal consumption variability is misleading, as Fig. 5 shows that the introduction of bankruptcy generates a significant deterioration in the ability of the young to share risk across states-of-nature and a significant relative improvement in the ability of the old to smooth across states. In contrast, when bankruptcy is ruled out, intratemporal smoothing is significantly better for young households, but substantially worse for the old. As a result, the fact that overall intratemporal smoothing appears essentially invariant to bankruptcy policy is an artifact of averaging over significant life-cycle heterogeneity. In sum, bankruptcy removes the ability of young households to commit to repay debts at a time when they have not yet been able to accumulate precautionary balances, leaving them vulnerable to income risk.

Unlike intratemporal smoothing, overall intertemporal smoothing is uniformly and substantially better when bankruptcy is ruled out, as seen in the second row of Table 2. Livshits et al. (2006) was the first paper to note that bankruptcy may induce a trade-off whereby intertemporal smoothing is restricted but intratemporal smoothing can be improved. Fig. 8 shows that the prohibition of default generates a much smoother profile of mean consumption over the life-cycle; consumption rises by less from youth to middle-age, and falls by less from middle-age to retirement. Bankruptcy is therefore unambiguously detrimental to intertemporal smoothing. Taken as whole, our results suggest that bankruptcy does not offer policymakers the choice of better intratemporal smoothing at the expense of worsened intertemporal smoothing. Nonetheless, as noted in the two-period setting, there will continue to be trade-offs for households of different ages.
5.2. How does bankruptcy link rising income risk to consumption?

We now discuss how the presence or absence of bankruptcy under full information alters the transmission of rising income risk into consumption. The main result of this section is that the option to declare bankruptcy, by itself, does not mitigate the transmission of increased income risk into increased consumption volatility. Fig. 5 displays the variance of log consumption with and without bankruptcy under high- and low-income risk and full information. The pass-through of increased income risk to consumption volatility in this case does not depend on the option to declare bankruptcy. That is, unsecured credit markets do not operate as insurance markets under full information. An important implication of this finding is that to the extent that our model is a reasonable instrument for measuring the effects of bankruptcy and default, one should conclude that the ability to default is unlikely to have played a major role in any improvements in hedging income risk. Our findings here are also consistent with a similar experiment conducted in Livshits et al. (2006).

Why does bankruptcy fail to mitigate the effect of income risk? First, the increase in income risk modeled here posits growth in the volatility of the persistent component of income, in addition to transitory factors. As a result, the cross-sectional variance of income grows steadily with age and therefore so does the difference between income volatility in the two regimes. Therefore, when unsecured debts are most useful and default is most prevalent, the two income regimes generate relatively small differences in the variance of cross-sectional income. This fact sharply limits the ability of bankruptcy to blunt increases in income risk.

The second factor making bankruptcy a poor form of social insurance against increased income risk is that the conditional mean of future expected income of a household with a bad current shock is even lower than before. This force is especially powerful when the increase is driven by growth in the persistence of income shocks. As a result, under full information the choice facing an indebted young household is between defaulting or rolling over increasingly expensive debt for another period in hopes of a better draw of income. By middle age, expected future income is declining at a rate which makes further borrowing for the unfortunate expensive, as it would trigger default at high rates; the result is that intermediaries do not extend credit to such households. Thus, increased income risk does not markedly change overall borrowing; as seen in Fig. 7 it is the only low-risk college-educated workers who default at appreciable rates beyond very young ages. In summary, increased income risk does not generate enough income dispersion early in life, full information makes high-risk borrowing more expensive among the young, and bankruptcy is irrelevant for older households. Thus, under full information the ability to declare bankruptcy does not meaningfully limit the transmission of default risk to consumption risk.

Table 3 explicitly computes the amount of consumption that newborn households would pay to live under different regimes of income risk and information. The cost of increased income risk is larger when bankruptcy is permitted, even though eliminating bankruptcy results in higher consumption volatility for older households. One must therefore be careful to not interpret changes in consumption volatility as being indicative of changes in welfare.28

\[ E(\log(c|\text{age})) \]

\[ \text{age} \]

28 This point is obviously well known when leisure is valued.
Fully informed credit markets thwart risky borrowing and therefore prevent unsecured credit from operating as insurance; a natural question is whether this result survives when lenders are less informed. Under partial information, income risk is again passed through to consumption volatility, though for entirely different reasons. In contrast to full information, where credit was easily available to those with moderate income shocks, the equilibrium under partial information features a strong “lemons” effect that makes credit very difficult to obtain. In particular, households are typically only able to borrow amounts small enough to never make default worthwhile; the equilibrium prices are step functions which drop from the risk-free borrowing rate to zero at a low level of debt. As a result, under PI the model operates essentially as a standard incomplete market model (Huggett, 1996) with exogenous, and very tight, borrowing limits. As shown in Krueger and Perri (2006), such a setting generates a near one-for-one transmission of income volatility to consumption volatility.

5.3. Consumption smoothing under changing information

We have shown that without changes in information, bankruptcy options do not alter the transmission of increased income risk to consumption. However, there are good reasons to believe that information held by lenders on borrower characteristics has changed over the period in question. In Athreya et al. (2008a), we show that improved information generates rising debt, rising default, and expanding diversity in credit terms. The appropriate comparison for evaluating the consequences of increased income risk is therefore between a low-income risk, partial-information setting and a high-risk, full information one. Table 4 shows how selected credit market aggregates change when the economy moves from a low-risk partial information setting to one with full information. For a complete discussion of the underlying mechanisms we direct the reader to our earlier work.

Our main finding here is that although changes in information are critical in accounting for changes in several major credit aggregates—particularly default rates—improvements in information simply do not matter for the transmission of income risk. Fig. 9 shows the variance of log consumption over the life cycle under the joint change in information and income risk. When it comes to the effect of changes in income risk on consumption smoothing, neither the bankruptcy regime nor the information regime matter. In particular, even if creditors were to observe all relevant factors for assessing default risk, credit markets will not act as insurance markets. In light of the findings of Krueger and Perri (2006) that rising income risk was not accompanied by higher consumption volatility, our results suggest that one must look to improvements in more explicit insurance arrangements.

### Table 3
Change in welfare

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coll (%)</th>
<th>HS (%)</th>
<th>NHS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FI → NBK</td>
<td>2.27</td>
<td>0.86</td>
<td>0.80</td>
</tr>
<tr>
<td>FI → NBK</td>
<td>2.64</td>
<td>1.18</td>
<td>1.06</td>
</tr>
<tr>
<td>FI → FI</td>
<td>−0.50</td>
<td>−0.66</td>
<td>−0.91</td>
</tr>
<tr>
<td>NBK → NBK</td>
<td>−0.15</td>
<td>−0.36</td>
<td>−0.73</td>
</tr>
<tr>
<td>PI → FI</td>
<td>0.91</td>
<td>0.33</td>
<td>0.13</td>
</tr>
<tr>
<td>PI → FI</td>
<td>0.86</td>
<td>0.32</td>
<td>0.13</td>
</tr>
<tr>
<td>PI → PI</td>
<td>−0.46</td>
<td>−0.65</td>
<td>−0.91</td>
</tr>
<tr>
<td>PI → PI</td>
<td>0.40</td>
<td>−0.34</td>
<td>−0.78</td>
</tr>
<tr>
<td>PI → NBK</td>
<td>3.04</td>
<td>0.84</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The welfare measure is the percentage of lifetime consumption required to make newborns indifferent between two economies.

### Table 4
Unsecured credit market aggregates

<table>
<thead>
<tr>
<th>Variable</th>
<th>FI-H</th>
<th>PI-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge/income ratio</td>
<td>0.2760</td>
<td>0.1488</td>
</tr>
<tr>
<td>Fraction of borrowers</td>
<td>0.1259</td>
<td>0.0521</td>
</tr>
<tr>
<td>Debt/GDP ratio</td>
<td>0.0208</td>
<td>0.0013</td>
</tr>
<tr>
<td>Default rate (%)</td>
<td>1.366</td>
<td>0.000</td>
</tr>
</tbody>
</table>

5.4. Empirical comparison

Before proceeding to welfare analysis, we need to convince the reader that our model is a reasonable approximation to the consumption data. Here, we compare Figs. 8 and 9 to two figures from the data—Figure (4.2) from Fernández-Villaverde and Krueger (2006) and Figure (1d) from Heathcote et al. (2005). Relative to the first figure, our model does well at capturing the profile of mean consumption—in our model the ratio of peak consumption to consumption at age 25 is 1.64, very close to the 1.67 ratio found in the data. Furthermore, consumption peaks at the same age in the model and the data. If we disaggregate by education levels, our numbers are also similar to those reported in Figure (4.2) of Gourinchas and Parker (2002)—the peaks are all in the mid-to-late 1940s and the ratios of peak to age 25 consumption are roughly constant across education types.

Turning to consumption inequality within cohorts and the second figure, our model understates the level of inequality by about one-third—the variance of log consumption for 25-year-old households in the data is 0.21 whereas the model only produces 0.14. However, the change in this number over the life-cycle is captured quite well—in the data it rises to 0.25 and in the model it rises to 0.18. As shown above, the effects of bankruptcy do not depend on the level of income risk; that is, the failure of unsecured credit to act as insurance does not depend critically on the amount of risk present in the model. Thus, it seems reasonable to expect that our results would survive modifications aimed solely at matching the level of inequality in consumption.

5.5. Welfare

We turn now to the evaluation of the effect of income risk and information on consumer welfare. Our welfare metric is the percentage increase in consumption in all dates and states that leaves a newborn household (who enters with zero wealth) indifferent between two economies. As seen in our experiments, measured consumption inequality in general rises with improvements in information; however, Table 3 shows that ex ante welfare rises as well. The first section of Table 3 shows that eliminating bankruptcy under full information is desirable for all newborns, regardless of income risk. That is, all newborns value the gains from commitment made possible by the elimination of the bankruptcy option. The gains are largest for college-educated workers since their demand for intertemporal smoothing is largest (see Fig. 4). Conversely, the smallest gains accrue to the non-high-school types because their intertemporal motives are very weak. Rising income risk harms all groups, with the largest costs borne by the non-high-school group. The losses from rising income risk are uniformly larger when bankruptcy is allowed than otherwise. The availability of bankruptcy substantially hinders the ability to borrow, and therefore particularly affects young households. Our measures of welfare should be thought of as giving upper bounds, as they are from the perspectives of those households who value credit markets most (low-wealth newborns).\textsuperscript{29} The information assumption is not critical here; rising income risk always harms all households.

\textsuperscript{29} Note also that none of the changes in income risk and information are likely to involve large changes in the aggregate capital stock. As a result, abstracting from transitional dynamics is does not distort the welfare consequences of income risk and information.
Turning to the welfare implications of better information, we see that improved information benefits all groups. The largest gains accrue to the college types again, for the same reason as above—their ability to borrow intertemporally is substantially improved under full information. Non-high-school workers do not really care which information regime they live in, since they cannot borrow either way. When we combine the changes, college workers are better off under the high risk full information regime than under partial information with low risk, whereas high school workers are worse off (but still better off than if information had not improved); again, the non-high-school workers are essentially indifferent.

The result that some households are worse off has implications for the concerns that motivate regulations intended to ensure access to credit by blocking the use of some information. There is a “second-best” aspect to the idea that one may want to protect some households from increases in income risk by restricting information; our model does not support this concern because the welfare losses are driven by the heightened income risk, not the change in information. In fact, ridding the economy of the bankruptcy option generates large gains in welfare even when income risk is rising; recent bankruptcy reform is a move towards the welfare-maximizing regime of NBK-H.31

6. Concluding remarks

In this paper, we studied the extent to which unsecured credit and personal bankruptcy have altered the transmission of increased income risk to consumption volatility. We find that unsecured credit markets will pass through any increase in income risk to consumption, irrespective of the information regime. However, the mechanisms that prevent households from insuring themselves through credit markets do depend on the prevailing information structure. Under full information, default risk precludes households from borrowing to deal with adverse income shocks, and when income shocks get bigger, credit supply simply contracts further. Under partial information, adverse selection restricts borrowing severely enough to collapse the model to a essentially a version of Huggett (1996) in which borrowing limits are tight. Since changes in credit markets are not likely to have improved risk sharing, any such improvements must have come from other more explicit insurance arrangements, such as those studied in Krueger and Perri (2006).

With respect to the role of changes in information, in the actual economy not all improvements need be due to technological progress. In particular, many countries place a host of regulations that prevent any observed equilibrium from being a full information outcome; as we referenced above, the Equal Credit Opportunity Act (ECOA) bans lenders from conditioning terms on age, gender, and race. Furthermore, the ECOA also bans even the appearance of such conditioning; correlations between proscribed characteristics and lending terms can trigger penalties even if the protected characteristics are not directly used (one such practice is redlining based on zip code data). Preliminary investigation of the costs of banning such information suggests that they may be quite large—particularly, pooling over age is very costly as it limits the ability of young college workers to borrow against their high expected future income. Athreya et al. (2008b) represents ongoing work aimed at a thorough analysis of the welfare and distributional consequences of such statutory limits on information use.

Appendix A. Two-period model of default

In our two-period model of default, filing for bankruptcy in the second period entails a utility penalty (as in Zame, 1993). The household problem can be written as

$$u = \max_{c_1, c_2(w_2), d(w_2) \in [0, 1]} \left\{ \log(c_1) + \beta \int [(1 - d(w_2)) \log(c_2(w_2)) + d(w_2)(\log(w_2) - \lambda)] f(w_2 | w_1) dw_2 \right\}$$

subject to the period budget constraints

$$c_1 \leq a + w_1 - a',$$
$$c_2(w_2) \leq a'(1 + r) + w_2.$$

For the case where default is welfare-improving, we choose $\beta = 0.5$, $w_1 = 1$, $w_2$ is uniform over the two-point set $(0.25, 1.75)$, and $\lambda = 0.25$. $r = 0.1$ for the default-free environment and $r = 0.2$ when the agents can default (the risk-free rate adjusted for the fact that default occurs in 50% of the states in period 2). For the case where default is not welfare-improving, we set $w_1 = 0.1$ and assume $w_2$ is uniform over the two-point set $(1.25, 1.75)$; now the equilibrium interest rate in the default environment is infinite. Clearly, these parameters are rigged to deliver the outcome we desire in each case. The maximization is computed using FFSQP from AEM Design (a feasible sequential quadratic programming approach with both active set and interior point methods to handle the constraints).

30 The Equal Credit Opportunity Act in the US is one such regulation. The conclusion contains some discussion of this act and similar regulations.
31 If bankruptcy is so damaging, why it exists at all is an open question. One study that attempts to integrate bankruptcy into an efficient risk sharing arrangement is Grochulski (2004).
32 The Data Protection Directives in the E.U. and several acts in the UK (the Race Relations Act and Sex Discrimination Act) are similar regulations.
Appendix B. Computation

The computational optimal method for the main model is detailed in Athreya et al. (2008a). We use golden section search to compute the optimal position tomorrow, using a coarse grid search to avoid the local optima, and parameterize the value function using linear splines. Due to the significant burden this model poses, particularly the partial information setting, we implement it using OpenMP interfacing across 8 Pentium 4 processors. To calibrate the model we set up a system of five equations in five unknowns (the calibration targets plus the market clearing condition for capital); we then minimize the sum of squared deviations using a variety of methods (Nelder–Meade, Hooke–Jeeves, and quasi-Newton), giving higher weight to the equilibrium condition for the capital market than the calibration targets. We found that the Nelder–Meade method produced the most robust (albeit slow) convergence to the calibration targets. Overall, the calibration procedure took several days to converge. Each FI equilibrium took less than one day, as it involves only one nonlinear equation in one unknown (the capital market clearing condition); we solve this equation using Brent's method. The PI economy is extremely costly to compute; it took several days to compute an allocation for a given interest rate, and computing equilibria takes weeks. Since the interest rate does not change much relative to FI (the presence of the special households ensures that the interest rate is almost pinned down by agents who do not value the option to default), we chose to fix r in the PI settings, and thereby save dramatically on computation time.

References


