The Dynamics of the Racial Wealth Gap

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September 24, 2019

Abstract: This paper shows that the large and persistent racial wealth gap is consistent with the smaller racial earnings gap when viewed through the lens of a dynamic model. We calibrate a general equilibrium heterogeneous agents model to account for racial differences in earnings, wealth, bequests, and returns to savings. Given initial racial wealth inequality in 1962, our model attributes the slow convergence of the racial wealth gap to persistent differences in earnings, with much smaller roles for racial heterogeneity in bequests or returns to savings. Cross-sectional regressions of wealth on income using simulated data produce the same racial gap as documented in the literature. An implication is that one-time wealth transfers closing the racial wealth gap will have transitory effects if they do not eliminate the racial income gap.

Keywords: Racial Inequality, Wealth Dynamics
JEL Classification Codes: D31, D58, E21, E24, J7

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We thank Nick Hoffman for research assistance on this project and Zhigeng Feng, Bill Johnson, Steven Ross, and seminar participants at the Cleveland, Dallas, and Kansas City Feds, CICM, the 2019 North American Meeting of the Econometric Society, the 2018 European Meeting of the Econometric Society, SUNY Albany, the University of Lyon, FAU Nuremberg, and Zhejiang University for helpful comments. The opinions expressed are those of the authors and do not represent views of the Federal Reserve Bank of Cleveland or the Board of Governors of the Federal Reserve System.
1 Introduction

What explains the fact that the racial wealth gap has not closed over the past 50 years? At first glance, the racial earnings gap might appear too small to explain the racial wealth gap. Figure 1a shows that while the average wealth of white families has been 6 times that of black families over recent decades, the average labor income of white families has been about 2 times that of black families.

![Figure 1a: Ratio of Black/White Means](image1.png)

(a) Ratio of Black/White Means

![Figure 1b: Household Wealth Distributions](image2.png)

(b) Household Wealth Distributions

Figure 1: Earnings and Wealth in the Surveys of Consumer Finances (SCFs)
Note: These figures display data from the 1962 Survey of Financial Characteristics of Consumers (SFCC) and the 1983-2016 Survey of Consumer Finances (SCF). Variable definitions are in Section 2.

Focusing on a cross section would seem to confirm that earnings do not explain the racial wealth gap. Figure 1b illustrates the basic reasoning: black-headed families have less wealth conditional on earnings than white-headed families. Even after controlling for variables in addition to income, studies still have difficulty accounting for the wealth gap (Blau and Graham (1990), Altonji and Doraszelski (2005), Thompson and Suarez (2015)). When multivariate regression coefficients estimated from a sample of blacks are used to predict white wealth, most estimates in the literature explain less than one third of the wealth gap (Scholz and Levine (2003)).

We show that the large and persistent racial wealth gap is consistent with the observed racial earnings gap when viewed through the lens of a dynamic model. Our analysis uses a heterogeneous agents dynamic stochastic general equilibrium model in the spirit of Bewley (1986), Aiyagari (1994), and Huggett (1993) to study the key mechanisms thought to drive the racial wealth gap. Agents in the model (i) have a life cycle in which they work, retire, and face increasing mortality risk. (ii) receive idiosyncratic income shocks; (iii) may give or receive intergenerational transfers; and (iv) are heterogeneous in terms of wealth, earnings, expected bequest size, and returns on capital.

We use this model to investigate what it would take to close the racial wealth gap. The model

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1This figure is a version of the main figure in Barsky et al. (2002) using the 2016 wave of the Survey of Consumer Finances.
is initialized to the earnings and wealth distributions for black and white households in the 1962 Survey of Consumer Finances (SCF), with these differences in earnings and wealth translating into large differences in bequests across race.\footnote{We use “the 1962 SCF” to refer to the 1962 Survey of Financial Characteristics of Consumers (SFCC) and the 1963 Survey of Changes in Family Finances (SCFF) since these surveys were the most direct predecessors of the SCF.} We then solve for a transition path to a long-run steady state in which racial differences are absent. Along the transition, we assume the earnings gap stays constant at 42 percent through 2016 (as observed in the 1962-2016 waves of the SCF), begins to slowly close starting in 2016, and closes a long time in the future. We find that it will take a very long time for the gap in average wealth between black and white households to equalize. Only 9 percent of the racial wealth gap will be closed after 100 years, meaning black households’ wealth will still be less than one-fourth that of white households.

We then use the model to decompose the sources of the wealth gap into the earnings gap, the bequest gap, and the return gap. We find that the labor income gap is the primary driver of the persistence of the wealth gap; this factor alone drives the majority of the wealth gap over the transition. We also find that the convergence of the racial wealth gap is highly sensitive to the speed at which the earnings gap closes.

Differences in bequests play a lesser role in maintaining the racial wealth gap. This result may be surprising because there are very large differences in intergenerational transfers across race (Smith (1995), Avery and Rendall (2002)). We incorporate these differences into our model, however they account for only a small fraction of the wealth gap because observed bequests are a small fraction of total wealth (Hendricks (2001)).

Although there is little evidence that returns systematically differ between black and white households either for the same asset (Gittleman and Wolff (2004)) or on portfolios (Wolff (2018)), a return gap would contribute to the wealth gap. We investigate the importance of a return gap by introducing differences in black and white returns to saving. Despite imposing an extremely large gap in returns to capital, different rates of return have no effect on the racial wealth gap as long as there is an earnings gap.\footnote{This result might be surprising because differences in returns likely played a major role in generating the differences in the initial wealth distributions we impose in the model, whether driven by means of violence and direct theft (Coates (2014)), redlining (Aaronson et al. (2017)), or government policies (Rothstein (2018), Hamilton and Logan (2019)).}

Our model results show the same qualitative cross-sectional relationship between earnings and wealth highlighted in the literature. Estimating a conditional expectation function (a la Barsky et al. (2002)) on the model output for the years 2012-2016, we find that black-headed households have lower wealth at all levels of earnings than white-headed households, just as they do in the corresponding 2013 and 2016 SCF data. We interpret this result as demonstrating the importance of controlling for initial wealth.\footnote{Altonji and Doraszelski (2005) take a step in this direction by using sibling fixed effects.} The earnings gap is the reason initial conditions matter, since wealth evolves slowly over time.

Finally, we investigate what these results imply for the effect of reparations. We counterfactually eliminate racial inequality in the initial wealth distributions while holding the level of aggregate
wealth constant. Without changing the racial earnings gap, such one-time wealth transfers are not effective – the racial wealth gap reverts to its initial level within 50 years. These results indicate that efforts to equalize the racial wealth gap will deliver only short-lived benefits if they do not address the racial earnings gap.

There are only two other studies of the black-white wealth gap using a dynamic model of optimizing agents accumulating wealth.\footnote{There is an empirical literature on related topics. In addition to those studies already cited, see also Bleakley and Ferrie (2016), Sacerdote (2005), Collins and Wanamaker (2017), and Ager et al. (2019).} White (2007) studies the labor income and wealth gap jointly using a model with two dynastic households, one black and one white, that face no idiosyncratic risk. The initial distribution of wealth is degenerate, and along the equilibrium path the black household never accumulates positive assets because the relative earnings of black households is rising (which is inconsistent with historical data). Ashman and Neumuller (2019) conducts a complementary partial-equilibrium analysis focused on steady states; they show that the earnings gap is sufficient to generate a large wealth gap even in the long run, consistent with our results that eliminate the initial conditions. Since our model implies that the dynamics of the wealth gap are very persistent (that is, the long run may be very far away), a comparison of steady states is likely to mismeasure the gains and losses associated with redistributive policies in the short run.

The remainder of the paper is organized as follows: Section 2 describes the data we use in our analysis and establishes three facts about income, wealth, and race between 1962-2016. Section 3 presents the model, stating the household’s problem recursively. Section 4 presents our quantitative analysis, first describing how we calibrate the model and then presenting the results of numerical experiments. Section 5 concludes.


2.1 Data

We measure the joint distribution of earnings and wealth using the triennial Survey of Consumer Finances (SCF), which began in 1983 and has been most recently released for 2016. We also use a precursor to the SCF, the 1962 Survey of Financial Characteristics of Consumers (SFCC) and the 1963 Survey of Changes in Family Finances (SCFF), which we refer to as the 1962 SCF.\footnote{In the Historical SCF constructed by Kuhn et al. (2018), the racial differences in the 1962 SCF are not outliers relative to nearby years. We discuss our argument for not using the Historical SCF below.}

Our sample consists of families with heads or respondents who are (i) either black or white and (ii) aged 20-100. It is not entirely obvious how to define “black” in the 1962 SCF. In the 1962 SCF the surveyor assigned household heads to one of three mutually-exclusive categories: “White”, “Nonwhite”, or “Not Ascertained.” We interpret the “Nonwhite” group in the 1962 SCF as being identical to the “Black or African American” group under the 1997 US Office of Management and Budget Classification Standards (OMB (1997)) for defining race. Appendix A describes this interpretation in detail.
We measure wealth using the family net worth variable in the years 1983 and after. For 1962 we define net worth as total assets minus total debts, where we construct total assets and total debts from the list of component variables according to the SCF definitions to match the net worth programs for 1983 onwards.\(^7\) We measure labor income as total family income from wages and salaries. The sources of labor income are the head, wife, and other family members in 1962; the respondent and spouse in 1983-1986; and anyone in the family in 1989-2016. All income and wealth variables are converted to 2016 dollars using the St. Louis Fed’s GDP Implicit Price Deflator.

We measure the racial wealth gap as

\[
1 - \frac{\text{Mean Black Wealth}}{\text{Mean White Wealth}}
\]

and measure the racial income gap analogously. Appendix Figure 17a shows that when measured in terms of medians rather than means, the wealth gap is even larger.

### 2.2 Facts

We document three relevant facts from data over the past half century. First, the ratio of black earnings to white earnings has been statistically constant. Second, the ratio of black net worth to white net worth has also been statistically constant and lower than the earnings gap. Third, expected wealth as a function of labor income is lower for black households than for white households.

**Fact 1**: The earnings gap has been about 40 percent with no trend over the past 50 years.

**Fact 2**: The wealth gap has been about 80 percent with no trend over the past 50 years.

Figure 2 plots the wealth and labor income gaps from 1962 to 2016. Over this period, mean black wealth averaged 18 percent of mean white wealth, resulting in a gap of 0.82. Over this same time period mean black labor income averaged 58 percent of white labor income, resulting in a gap of 0.42. Appendix B shows that these data are consistent with the findings in several other studies using several other data sets, and also plots the ratios obtained from defining the gaps in terms of medians rather than means.

Both gaps have been stubbornly persistent. Table 1 reports the coefficients from regressing the income and wealth gaps on year via OLS. None of the coefficients are statistically different from zero. This relationship is not driven by either the Great Recession or noise in the early waves of the survey. The flat earnings and wealth ratios are also consistent with Kuhn et al. (2018)’s analysis of the racial income and wealth gaps in the Historical SCF (HSCF) going back to 1947.\(^8\)

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\(^7\)The list is available at [https://www.federalreserve.gov/econres/files/Networth%20Flowchart.pdf](https://www.federalreserve.gov/econres/files/Networth%20Flowchart.pdf).

\(^8\)The HSCF is a series of annual SCF datasets from 1947-1971, along with 1977, currently available from ICPSR. We do not use the HSCF because it is a distinct survey from the SCF, raising a separate set of concerns relative to the 1962 and 1983-2016 SCF surveys, with the 1962 SCF being the most direct precursor to the subsequent 1983-2016 SCF surveys. For example, coverage of HSCF variables is incomplete across survey years, with some variables reported in bins or not at all in some years. Moreover, the analysis in Kuhn et al. (2018), as well as our own replication, confirms that our inference from the 1962 data is consistent with the HSCF.
### Table 1: Coefficient on Time

<table>
<thead>
<tr>
<th>Time</th>
<th>Income Gap</th>
<th>Wealth Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>$\beta$</td>
<td>$\beta \times 50$</td>
</tr>
<tr>
<td>1962-2016</td>
<td>1.5e-3</td>
<td>0.08</td>
</tr>
<tr>
<td>1962-2007</td>
<td>7.1e-4</td>
<td>0.04</td>
</tr>
<tr>
<td>1986-2007</td>
<td>-2.6e-3</td>
<td>-0.13</td>
</tr>
</tbody>
</table>

Note: The dependent variable in the linear OLS regressions reported above is either one minus the ratio of black to white mean income or one minus the ratio of black to white mean wealth. The independent variable is year, where each regression is restricted to a particular subset of years.

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Figure 2: Labor Income and Wealth Gaps

**Fact 3**: In all decades, there is a large gap in the black and white Conditional Expectation Functions (CEFs) of wealth conditional on earnings.

Figure 3 shows the Conditional Expectation Functions (CEFs) of wealth conditional on labor income separately for black and white households.\(^9\) Whatever decade of data we look at in the SCF, we see a large gap between the black and white CEFs. Table 2 reports these gaps as the coefficients on having a black household head in OLS quadratic regressions of wealth on income under the restriction of households having positive income and being below the 95th percentile of race-specific labor income distribution in the decade in question. Over all of the years currently available in the SCF, 1962-2016, this coefficient is −$388,000 in 2016 dollars.

\(^9\)As shown in Figure 1b and Barsky et al. (2002), estimating these CEFs under a quadratic OLS specification generates very similar results as semi-parametric local regressions.
Figure 3: Conditional Expectation Functions in
the Surveys of Consumer Finances (SCFs)
Note: This figure shows quadratic OLS estimates with race-
specific coefficients and an indicator for race. Because of the
scarcity of very high income black families in the data, we follow
Barsky et al. (2002) restrict the sample to families with incomes
below the 95th percentile of the race-specific income distribution
during the decade in question.

### Table 2: Coeff. on Black Household Head

<table>
<thead>
<tr>
<th>Time Period</th>
<th>( \beta )</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>1962</td>
<td>(-1.48e3)</td>
<td>0.14</td>
</tr>
<tr>
<td>1980s</td>
<td>(-2.52e3)</td>
<td>0.00</td>
</tr>
<tr>
<td>1990s</td>
<td>(-4.16e3)</td>
<td>0.00</td>
</tr>
<tr>
<td>2000s</td>
<td>(-5.35e3)</td>
<td>0.00</td>
</tr>
<tr>
<td>2010s</td>
<td>(-3.93e3)</td>
<td>0.00</td>
</tr>
<tr>
<td>All</td>
<td>(-3.88e3)</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Note: The dependent variable in the linear OLS regressions
reported above is household net worth. The independent
variables are income, income\(^2\), black, black*income, and
black*income\(^2\). The coefficients reported above are the co-
efficients on black for all waves of the SCF occurring during
the years in question.

### 2.3 Literature and Interpretation

The literature has taken two approaches for understanding the relationship between the racial earnings and wealth gaps. An early literature focused on understanding the change in wealth at a given level of income and point in time. Studies characterizing differences in consumption and savings across race found that black families had higher savings conditional on income (Mendershausen (1940), Klein and Mooney (1953)), and Friedman (1957) interpreted this finding as a natural implication of the permanent income hypothesis.\(^{10}\)

A more recent literature has focused on characterizing the level of wealth given earnings. Studies tend to find that the level of income at a given moment in time cannot entirely account for racial differences in the level of wealth at that same point in time. When multivariate regression coefficients estimated from a sample of blacks are used to predict white wealth, most estimates in the literature explain less than one-third of the wealth gap (Scholz and Levine (2003)). That is, the gap in the Conditional Expectation Functions (CEFs) documented in Fact 3 remains even after relaxing assumptions about functional form and common support (Barsky et al. (2002)); improving the separate measurement of permanent and transitory income (Altonji and Doraszelski (2005), Blau and Graham (1990)); and including additional factors (Thompson and Suarez (2015)). If one focuses on the level of wealth for a given income and point in time, the wealth gap is simply too large to be explained by the earnings gap.\(^{11}\)

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\(^{10}\)The finding of different savings levels was not without debate (Galenson (1972)). It is also worth noting that lower and higher consumption for African Americans have been explained by, respectively, smaller within-group variation in income (Duesenberry (1952)) and larger cross-group variation in income (Charles et al. (2009)).

\(^{11}\)See, for example, the discussions in Menchik and Jianakoplos (1997) and Barsky et al. (2002).
Our analysis builds on the older literature looking at consumption and savings to understand the relationship between the racial earnings and wealth gaps; the key contribution is that we explicitly connect the static differences in savings to the dynamic differences in wealth levels. Differences in wealth today are the result of past differences in many factors, such as savings rates, labor income streams, returns on assets, and intergenerational transfers, as well as initial differences in wealth itself (Gittleman and Wolff (2004)). To measure these forces, we construct a dynamic model of wealth accumulation in which we can look at wealth both at a point in time and over long horizons.

3 Model

In our model there is a unit continuum of households divided between black and white in time-invariant fractions \( s \) and \( 1 - s \). Each household is endowed with one unit of discretionary time to divide between leisure \( \ell \) and working \( h \). All households value consumption and leisure according to the period utility function \( u(c, \ell) \). Households also value leaving wealth bequests according to the function \( z(\cdot) \).

Households have a life cycle: They are born at age \( a_b \), face increasing mortality risk as they age, and die with certainty by age \( \bar{a} \). At age \( a_b \) households receive a bequest \( b \), and retire at age \( a_r \) where \( a_b < a_r < \bar{a} \). Households differ by labor productivity \( \Phi(a) \varepsilon \), where \( \Phi(a) \) is a deterministic age-earnings profile and \( \varepsilon \) follows an AR(1) process in logs:

\[
\log(\varepsilon') = \rho \log(\varepsilon) + \eta' \quad \eta \sim N(0, \sigma^2_\eta).
\]

Households cannot insure against labor productivity shocks but can save in an asset that returns \( 1 + R \) units of consumption tomorrow. Because households cannot perfectly insure against labor income risk, they use their personal savings to smooth consumption. This behavior generates a distribution of wealth as households experience different labor income histories. During retirement households receive a benefit \( \Omega \) which is indexed to the household’s labor productivity in its last period of working life and is funded by a tax \( \tau \) on labor earnings.

A stand-in firm purchases effective labor and rents capital from households. The firm pays white workers their marginal product of labor but pays black workers only a fraction \( \varphi(B) < 1 \) of theirs. As a result, the firm earns profits equal to \( [1 - \varphi(B)] w N_B \) (Appendix C discusses this in detail). These profits are rebated to white households through a dividend that is proportional to their wealth, \( D(k) \). The firm produces output according to a Cobb-Douglas production function

\[
Y = AK^{\alpha}N^{1-\alpha}
\]

where aggregate capital is \( K = K_B + K_W \) and aggregate effective labor is defined analogously. Capital depreciates at constant rate \( \delta \), and the return on capital paid by the firm is the marginal product of capital minus the depreciation rate, \( R = r - \delta \).

To formalize the household’s problem recursively, define the state vector as wealth \( k \), labor market productivity \( \varepsilon \), age \( a \), and race \( j \in \{B, W\} \). Households value consumption \( c \), leisure \( 1 - h \),
and leaving bequests. The Bellman equation is

\[ V(k, \varepsilon, a, j) = \max_{c, k', h} \left\{ u(c, h) + \psi_a \beta \mathbb{E} \left[ V(k', \varepsilon', a + 1, j) \right] + (1 - \psi_a) z(k') \right\} \]

subject to

\[
\begin{align*}
& c + k' \leq (1 - \tau) \phi(j) \varepsilon \text{wh} + (1 + R) k + D(k) \quad \text{when } a < a_r \text{ and } a \neq a_b; \\
& c + k' \leq (1 - \tau) \phi(j) \varepsilon \text{wh} + (1 + R) k + D(k) + b \quad \text{when } a = a_b; \\
& c + k' \leq \Omega + (1 + R) k + D(k) \quad \text{when } a \geq a_r
\end{align*}
\]

where \( \psi_a \) is the conditional probability of surviving from age \( a \) to \( a + 1 \).

4 Quantitative Analysis

Our primary exercise is to explore transitions from a wealth distribution calibrated to 1962 to a racial-equality stationary equilibrium, in which there is no gap in labor income, no difference in bequest schedules by race, and no difference in the conditional wealth distributions across race. Each experiment will explore how the various features of the model contribute to the path for the wealth gap.

4.1 Calibrating to the Racial Equality Steady State

We begin by calibrating the model to a long-run steady state in which all processes are independent of race. Parameters of the model are either estimated from the data (by ourselves or by others in the literature); calibrated using simulated method of moments; or set to standard values from the literature.

We estimate the age-earnings profile, \( \phi(a) \), using the sample of white families in the 2007 SCF. We regress family earnings on a quadratic function of age, measured at an annual frequency, for white households between ages 20 and 64 with positive household incomes.\(^{12}\) Then for the middle age in each of our 5-year age bins we generate the predicted mean. The age profile \( \phi(a) \) is this predicted mean divided by the overall mean. The profile rises quickly over the working ages, peaks in the 50’s, and declines just before retirement.

\(^{12}\)Including African-Americans in the regression sample does not change the estimated age-earnings profile; once the average difference in earnings is controlled for, earnings grow by age at roughly the same rate for black and white households.
The survival probabilities $\psi(a)$ are estimated using data on all-gender survival probabilities for whites in Table 20 of Arias et al. (2016). Appendix D shows a robustness check in which we run an experiment where the mortality schedule for black households begins at the one measured in the data and then convergences slowly to the white schedule. Differences between the implied wealth gap path from this exercise and the one in the baseline are inconsequential.

Estimates taken from the literature include the parameters $\rho_\eta$ and $\sigma_\eta$. These parameters of the stochastic component of labor productivity are set to correspond to the estimates from Floden and Lindé (2001) in 5-year rates. The parameters governing the transfer of wealth across generations are set to capture the stylized facts in Hendricks (2001). When a household dies, there is an inter-vivos transfer of $(1 - \nu)k$ to a newborn household of the same race. The remaining $\nu k$ is pooled with other deceased households of the same race and then distributed to living same-race households of bequest age ($a = a_b$). Bequests $b$ to middle age households take one of three values: 70 percent receive no bequest, 28 percent receive a small amount $b_2$, and 2 percent receive a large amount $b_3$, where $b_3$ is set to 70 percent of the middle age inheritance pool. These features are also broadly consistent with the size distribution of inheritances documented in more recent data in Feiveson and Sabelhaus (2018).

We specify preferences over consumption, leisure, and the warm-glow from bequests as

$$u(c, h) = \frac{c^{1-\gamma_c}}{1-\gamma_c} + \theta_h \log (1 - h) \quad \text{and} \quad z(k') = \theta_b \frac{(1 + k')^{1-\gamma_b}}{1-\gamma_b}.$$  

We set $\gamma_c = 2$ and $\gamma_b = 1.5$ to ensure that bequests are a luxury good (De Nardi and Yang (2014), Hendricks (2007)). We set the capital share of production $\alpha = 0.36$, and the depreciation rate of capital $\delta = 0.25$, which implies that investment is 15 percent of annual output.

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13Arias et al. (2016) is a Center for Disease Control National Vital Statistics Report and we use estimates representing age-specific 2012 survival probabilities.

14We have repeated the estimation exercises from Floden and Lindé (2001) separately for whites and blacks and have found, like Floden and Lindé (2001), economically and statistically identical estimates for both the $\rho_\eta$ and the $\sigma_\eta$. 

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Figure 4: The Age-Earnings Profile
We finish by jointly calibrating the remaining parameters so that the model matches moments from the data. Targeting a capital/output ratio of 0.6 (equivalent to a capital/output ratio at an annual rate of 3.0) generates a model discount factor of $\beta = 0.77$. The total factor productivity (TFP) parameter $A$ being 2.68 generates a five-year output of 1, which serves to normalize units. For the labor supply choice, $\theta_h$ is calibrated to 1.52 in order to match an average hours of work equal to 30 percent of discretionary time. We calibrate $\theta_b$ to match a ratio of bequested wealth to total output equal to 1.7 percent. This value falls between the range 1.2 and 2.0 percent of annual GNP documented in Hendricks (2001). The calibrated value is $\theta_b = 0.10$. The tax rate on labor income is set to clear the government budget constraint when the benefit to retirees $\Omega$ is, on average, 40 percent of their labor earnings in their last period of work. The labor income tax rate required to fund this transfer is $\tau = 0.15$.

Table 3: Calibration for Baseline Steady State

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
<th>Chosen Parameter</th>
<th>Chosen Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_\eta$</td>
<td>0.66 Floden and Lindé (2001)</td>
<td>$\gamma_b$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.33 Floden and Lindé (2001)</td>
<td>$\gamma_c$</td>
<td>2</td>
</tr>
<tr>
<td>$Pr(b_1)$</td>
<td>0.70 Hendricks (2001)</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>$Pr(b_2)$</td>
<td>0.28 Hendricks (2001)</td>
<td>$\delta$</td>
<td>0.25</td>
</tr>
<tr>
<td>$Pr(b_3)$</td>
<td>0.02 Hendricks (2001)</td>
<td>$\nu$</td>
<td>0.50</td>
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</table>

<table>
<thead>
<tr>
<th>Moment</th>
<th>Targeted Value</th>
<th>Calibrated Value</th>
<th>Model Parameter</th>
<th>Calibrated Value</th>
</tr>
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<tbody>
<tr>
<td>$\bar{K}/Y$</td>
<td>0.60</td>
<td>0.60</td>
<td>$\beta$</td>
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<tr>
<td>$Y$</td>
<td>1.0</td>
<td>1.0</td>
<td>$A$</td>
<td>2.68</td>
</tr>
<tr>
<td>$H$</td>
<td>0.30</td>
<td>0.31</td>
<td>$\theta_h$</td>
<td>1.52</td>
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<tr>
<td>$B/Y$</td>
<td>0.017</td>
<td>0.020</td>
<td>$\theta_b$</td>
<td>0.10</td>
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<tr>
<td>$\Omega - \tau wN$</td>
<td>0</td>
<td>0</td>
<td>$\tau$</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Note: $\Omega$ is the total government spending on social security, which is driven by paying retirees on average 40 percent of their income from the last period in which they work. $B$ are aggregate bequests. The estimated age-earnings schedule $\Phi(a)$ is shown in Figure 4, and the estimated mortality schedule $\psi(a)$ is taken from Arias et al. (2016).

4.2 Initial Conditions: 1962 Wealth Distribution

Initiating the model requires that we have a starting wealth distribution over productivity, age, and race. We estimate this distribution using kernel density techniques to smooth the initial distributions of wealth within race and age groups in the 1962 SCF. Age groups are chosen to maximize sample size while grouping similar distributions. We chose age groups based on mean wealth by age and race, which are displayed in Figure 5. We chose the age groups 20-44 and 45-94.
for black-headed families, and for white-headed households we chose the age groups 20-24; 25-29; 30-34; 35-39; 40-44; 45-55; 55-79; and 80-94.

Figure 5: Wealth by Age and Race in the 1962 SCF

Figures 6a and 6b show the best fitting kernel density approximations to the raw data for some of these groups. What is immediately apparent is that African Americans had much lower wealth in 1962 than their white counterparts. While it is relatively rare for white families to possess model wealth over 1, essentially no black families have this level of wealth. The comparison is even more unequal if we look at wealth levels like 0.5.

To make this initial distribution consistent with our long-run steady state, we combine the invariant distributions of the productivity processes and of age with the estimated conditional distributions of wealth by age and race. Finally, we rescale wealth for black and white households.

---

15The purpose of assuming independence to combine these distributions is only to generate a smooth initial joint distribution. In the model, income and age will be correlated with wealth through the endogenous savings decisions.
separately so that the ratio of mean black to white wealth is equal to the ratio observed in the 1962 SCF.

4.3 Baseline Transition Path

In our baseline exercise, we use the calibrated model to address our primary question: “How long should one expect the wealth gap to persist, given large initial wealth differences and a persistent gap between black and white labor incomes?”

To address this question, we feed the model a sequence $\{\varphi_t\}_{t=1}^T$ of exogenous labor income gaps. We begin the sequence with $\varphi_t = 0.42$ for $t = 1, \ldots, 12$, corresponding to the years 1962 to 2017. This implies that a black household earns 58 percent of an equivalent age/productivity white household, the average ratio in the SCF from 1962-2016. After 2017, we must impose a path that closes in order for the economy to reach the racial equality steady state. Given the past 60 years of labor income data, in which the gap remained constant, there is little reason to suppose that the closure should be fast. On the other hand, support for achieving racial equality has risen over the same time period so one could be optimistic that the gap will actually shrink quickly going forward. For our baseline, we assume that the labor income gap closes at an exponential rate of 0.27 percent per year so that the gap closes completely in 200 years. Later, we consider alternative scenarios where the labor income gap closes in 50 years and in 400 years.

Our baseline experiment generates a wealth gap that is very slow to close. Figure 7 plots the wealth gap in the baseline experiment along with the assumed path of the labor income gap. By the year 2016, the wealth gap is 0.90 as compared to 0.84 in the data. To get a sense of how slowly the wealth gap closes, we note that after 100 years it is still 0.82. Even in the year 2217, when the income gap disappears, the wealth gap is still 19 percent.

Nearly all of the discrepancy between the data and the model in 2016 is due to the immediate increase in the wealth gap in the first period of transition from 0.82 to 0.91. This increase is due to the fact that the wealth gap is lower than it would be in a stationary distribution with the observed earnings gap, combined with a strong gap in savings behavior driven by the luxury-good warm-glow function; as a result, white households accumulate wealth very strongly at the beginning of the transition when the earnings gap has not changed significantly, pushing the interest rate down and leading to reductions in black wealth.

---

16 Notice that we do not take a stand on the factors behind the labor income gap. They could range from persistent differences in human capital, lower employment, higher incarceration rates, or racial bias. We discuss interpreting the labor income gap in the conclusion.
4.4 What Factors are Most Important for Maintaining the Racial Wealth Gap?

Our baseline contains differences by race in initial wealth, labor income, and bequests. We now decompose the contributions of these factors by running counterfactual experiments in which some of these elements are removed. In addition, we explore the sensitivity of our findings to differential rates of return on saving by race.

4.4.1 The Labor Income Gap

How important is the labor income gap for perpetuating the wealth gap? To answer this question, we run a counterfactual transition exercise in which the labor income gap closes immediately in 1962. Relative to the baseline, the remaining model ingredients that contribute to the wealth gap are initial racial wealth inequality and smaller bequests for black households due to lower initial black wealth.

Figure 8 contrasts the labor income and wealth gap paths from this experiment to those from the baseline. In the absence of a labor income gap, the wealth gap in the model closes much faster than in the baseline. The wealth gap reaches 10 percent by 2007 instead of 2232. This result shows that the labor income gap between black and white households plays a central role in maintaining the wealth gap.
4.4.2 The Bequest Gap

Differences in inheritances between black and white households has been identified in the literature as a potential source of the persistent wealth gap. Black households are less likely to receive an inheritance. In the Health and Retirement Study (HRS) in 1992, Smith (1995) finds that 34 percent of older white households had received an inheritance compared to 11 percent of older black households. In their study using the 1989 SCF, Menchik and Jianakoplos (1997) found that 26 percent of households headed by whites report receiving inheritances, versus 10 percent of households headed by blacks.

Moreover, black households that do receive an inheritance tend to receive a smaller sum. Smith (1995) finds a mean gift for white recipient households that is almost double that for black households ($149,000 vs $86,000, respectively). Avery and Rendall (2002) find a similar ratio using the 1989 SCF.

In our model, black households face a much lower expected bequest size conditional on receiving an inheritance. To avoid transitory complications in the bequest paths arising from heterogeneous inheritance probabilities, all households have the same probabilities of receiving one of three inheritance sizes; however, because black wealth is so much lower than white wealth, white households in the model get much larger bequests throughout most of the baseline transition.

During the first 100 years of the transition, black inheritance averages just 12 percent of white inheritance. Table 4 reports the 1962 bequest size in the model as a share of average wealth. Regardless of low or high bequest type, $b_2$ or $b_3$, black bequests are a small share of white bequests. Figure 9 shows how these bequests evolve over time, plotting the transition path of bequests to middle-aged households by race. For much of the transition period, white households of either the high bequest or the low bequest type receive substantially larger inheritances than their black counterparts. Although both black and white households have the same structural preferences
for leaving bequests, the unequal nature of the initial wealth distribution produces long-lasting differences in bequests for two reasons. First, because mean wealth is higher for white households, the pool of bequests will be larger for them. Second, because leaving bequests is a luxury good, white households on average have a stronger incentive to accumulate wealth.

Table 4: Model Bequests in 1962 As a Share of Average Wealth

<table>
<thead>
<tr>
<th></th>
<th>Black</th>
<th>White</th>
<th>Prob</th>
</tr>
</thead>
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<tr>
<td>$b_1$</td>
<td>0</td>
<td>0</td>
<td>0.70</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.11</td>
<td>0.66</td>
<td>0.28</td>
</tr>
<tr>
<td>$b_3$</td>
<td>3.62</td>
<td>21.50</td>
<td>0.02</td>
</tr>
<tr>
<td>$\text{Avg}(b_2, b_3)$</td>
<td>0.35</td>
<td>2.05</td>
<td>–</td>
</tr>
</tbody>
</table>

Figure 9: The Transition Path for Bequests under the Baseline

To study how our baseline results are affected by bequests, we consider an alternative case in which estates left by deceased black and white households are pooled jointly, and middle-aged households of either race draw from the same pool. As one would expect, removing racial inequality in bequests speeds the rate of convergence of the wealth ratio; however, the closure is not nearly as fast as when the labor income gap is immediately closed.
4.4.3 Return Gap

A racial gap in the returns to capital is thought to be one of the key factors that helped to generate the 1962 racial wealth inequality used in our analysis. Historical examples that contributed to a return gap include limiting credit in black areas (Aaronson et al. (2017)), subsidizing housing in white areas (Rothstein (2018)), and simple theft (Coates (2014)).

In more recent decades, a return gap would most likely have arisen from differences in the risk/return composition of portfolios rather than different returns on the same types of assets. Even this return gap, however, is hard to find in the data. The most compelling evidence indicates that rates of return have been similar for black and white households over recent decades (Wolff (2018), Gittleman and Wolff (2004)).

Given the empirical evidence, we assume that there is no gap between black and white returns on savings in our baseline experiment (except for the relatively small dividend confiscated from black households and distributed to white households). We are nevertheless interested in understanding the importance of a hypothetical return gap, both at the basic level of understanding mechanisms and because our current measurements might somehow miss the existence of a return gap. To investigate the importance of a gap in rates of return, we run three counterfactuals under which black households experience a gap in the return on their capital. We set the sequence of return gaps to either 50 percent, 100 percent, and 200 percent of the baseline labor income gap sequence. Remembering that under the baseline, white households earn 58 cents on the dollar for the first 55 years of the transition, it is clear that we are imposing very large return gaps. During the initial
phase of the transition, the smallest return gap considered is 21 percent while the largest 83 percent. Converting the latter scenario into an annualized return would imply that white households receive a 143 percent higher return on savings per year.

In the presence of a persistent labor income gap, a return gap has no effect on the convergence of the wealth gap. Figure 11 plots the wealth gap when lower returns for black households are added to the baseline. Even under the most severe return gap, the wealth ratio only requires an additional 25 years over the baseline to reach 0.10.

These findings do not imply that return gaps have no bearing on wealth ratios, but they do imply that the labor income gap is substantially more important. To illustrate, we revisit the counterfactual in which the labor income gap is immediately closed and conduct the same return gap experiments in that environment. The results are shown in 12. Notice that regardless of the size of the return gap, the wealth closes rapidly in the first 50 years. This is a consequence of removing the labor income gap. After that, however, the wealth gap closes much more slowly, and its magnitude is greatly affected by the size of the return gap.
Taken together, the results from our experiments with a return gap indicate that this mechanism could play some role in maintaining the wealth gap. However, the model assigns the income gap a much greater role in maintaining the racial wealth gap.

4.5 What Does the Labor Income Gap Imply for the Wealth Gap Going Forward?

Having shown that the labor income gap is the primary driver of the persistent racial wealth gap, we explore alternative scenarios for the wealth gap given four different assumptions about the rate at which the labor income gap closes. We consider three cases in which the labor income gap is constant until model year 2017, after which it closes at the exponential rate required to reach zero $T$ years in the future. As a reminder, the baseline scenario sets $T = 200$. The “fast close” case assumes that the factors underlying the labor income gap are addressed and their effects rapidly diminish in 50 years. In contrast, the “slow close” case takes more signal from past experience, assuming that the frictions holding black labor income back will take a very long time to address. In this case, $T = 400$. Finally, in the fourth “never close” scenario we suppose that the income gap remains constant forever, so $T = \infty$.

Figure 13 plots the baseline transition paths along with those for the fast, slow, and never close cases. The results reinforce the point that closing the wealth cannot be achieved without addressing the labor income gap. The speed with which the wealth gap closes tracks the speed with which the income gap closes. Moreover, the “never close” experiment highlights that should the income gap not close, there will be little change in the wealth gap. We note that this also serves as a model validation exercise, as the prediction that the wealth gap will remain constant given a constant income gap is precisely what we have observed in the data since 1962.
4.6 Equalizing Wealth but Not Labor Income

We conduct a final experiment to investigate the inference that the wealth gap will not close without closing the income gap. We again suppose that the labor income gap remains constant forever, the $T = \infty$ case. Now we compare what would happen in two extreme cases of racial wealth inequality. We then suppose that the wealth distribution of black households is equalized to that of white households by a system of transfers that keep aggregate wealth constant.17

Equalizing wealth without equalizing labor income has no long-term effect on the wealth gap: within 50 years the wealth gap has all but returned to its initial level. We note again that the steady-state wealth gap is much larger than the steady-state earnings gap, which implies that one should use caution when using the cross-sectional CEFs to infer the causes of the wealth gap; our model says they should not in fact be equal. Our results again emphasize the importance of policies aimed at reducing the income gap in addition to the wealth gap if one wants to have lasting effects.

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17Our results are meant to be illustrative of the potential effects of programs like reparations. We do not attempt to model the various costs and benefits associated with actual reparations plans, including any political or social obstacles to implementation.
4.7 The Cross-Sectional Relationship between Income and Wealth

We have provided evidence that the labor income gap is the principal factor maintaining the racial wealth gap. Moreover, our model predicts a stationary racial wealth gap that is more than twice the stationary earnings gap. Is it possible to reconcile these results with the cross-sectional evidence in the literature, which broadly finds that earnings differences are too small to explain racial differences in wealth? Our answer here is yes.

A cross-sectional look at our dynamic results reconciles our findings with the evidence documented in the cross-sectional literature. Figure 15 reproduces the key figure in Barsky et al. (2002) with SCF data from 2013 and 2016 along with data from the baseline experiment. In both the data and the model, black households have lower expected wealth than white households at the same level of earnings.
Two points are worth emphasizing here. First, since earnings and wealth evolve dynamically over time, inferences from a snapshot can be misleading. Our model implies that in fact the earnings gap, combined with the initial distribution of wealth, is basically all that is needed to account for the current wealth gap. Second, the model also implies that the stationary wealth gap is over twice as large as the earnings gap. That is, we will see differences in the CEFs even in the long run. When viewed through the lens of our dynamic model, there is no need to search beyond initial wealth controls for additional variables to explain the racial wealth gap.

5 Conclusion

This paper used a heterogeneous agents dynamic stochastic general equilibrium model to study mechanisms that can generate the type of persistent black-white wealth inequality documented in the data. Our model was able to explain key features of the racial wealth gap across both time and individuals given the tremendous wealth inequality at the start of the period we examine, 1962, as well as differences in bequests, earnings, and returns to savings. Our model attributed the slow convergence of the racial wealth gap to persistent differences in earnings.

Our results underscore the importance of understanding the sources of continued differences in earnings between black and white households. We take our findings as evidence that policies aimed at reducing the earnings gap would be most effective at eliminating the racial wealth gap. While we do not address the sources of earnings differences in this paper, likely suspects include differences in human capital through education or neighborhood externalities (including peer effects) and market discrimination (whether statistical or taste-based). One set of policies along these lines
would aim to equalize opportunity within public education, since pre-market factors influenced by education are strong predictors of wages and employment (Neal and Johnson (1996), Keane and Wolpin (1997)). Additional policies likely to improve the relative labor market outcomes of African Americans would address incarceration (Neal and Rick (2014)), discrimination in the labor market (Bertrand and Mullainathan (2004), Nunley et al. (2015)), labor market attachment (Ritter and Taylor (2011), Daly et al. (2019)), and persistent residential segregation (Aliprantis and Richter (2019), Chetty et al. (2016), Bayer et al. (2008)).

In contrast, our model predicts that equalizing wealth without equalizing earnings will produce only short-lived improvements to the racial wealth gap. For example, consider two recent proposals in the public forum; (i) encouraging homeownership among black households and (ii) having the government issue “baby bonds.” Regarding the homeownership proposal, there are two questions – would it be better (in terms of the racial wealth gap) if black households held more of their wealth in terms of housing, or should society transfer housing to black households? The first answer is likely no, as housing does not outperform other savings vehicles over long horizons (the appreciation in the Case-Shiller index is only 0.3 percent compared to roughly 6 percent for stocks; even correcting for risk that gap is quite large). The second answer is that, from our model’s perspective, the transfer will not have long-lasting effects without dealing with the labor income gap. This argument applies to baby bonds as well.

However, there may be feedback from wealth to earnings. In a companion paper, Aliprantis et al. (2019), we investigate the effect of wealth for neighborhood selection. We find that wealth does not drive racial differences in earnings through residential sorting. However, wealth could affect labor income through other channels including borrowing constraints that limit the ability to obtain a college degree (Denning et al. (2019), Stinebrickner and Stinebrickner (2008), Cameron and Heckman (2001)), to start or to expand one’s business (Doorley and Pestel (2016)), or to take risk in the labor market over the course of one’s career (Algan et al. (2003), Bloemen and Stancanelli (2001)).

References


Gratton, B. and M. Gutmann. Hispanics in the U.

Gratton, B. and M. Gutmann. Hispanics in the U.


25


A Racial Categories in the 1962 SCF

Mapping previous racial and ethnic categories in the US census to the currently-used categories is a convoluted process (Pratt et al. (2015)). Aside from the issues this fact raises about how we interpret racial statistics (Zuberi (2001)), this fact also raises a measurement issue for our analysis.

Mapping race from the 1983-2016 waves of the SCF to current racial categories is not straightforward because the surveys convolute race and ethnicity. We assign race to families based on the race of the survey respondent, who must choose one mutually-exclusive choice. In 2016, for example, respondents are asked which category best describes them among “white, black or African-American, Hispanic or Latino, Asian, American Indian or Alaska Native, Hawaiian Native or other Pacific Islander, or another race.”

Mapping race in the 1962 survey to current racial categories is less straightforward than choosing the mapping for later waves. Race was determined in the SCF by the surveyor in 1962, and the 1962 SCF labels the family head as being one of three mutually-exclusive categories: “White”, “Nonwhite”, or “Not Ascertained.” In our analysis we interpret these categories in terms of the current US Census Bureau categories established by the 1997 US Office of Management and Budget Classification Standards (OMB (1997)).

We interpret “nonwhite” in the 1962 SCF as meaning “black” in today’s terms. The share of “nonwhite” respondents in the 1962 SCF corresponds closely with the share of black individuals in the US population at the time. Of the weighted sample in the 1962 SCF, white, nonwhite, and not ascertained respondents make up, respectively, 79.5, 9.5, and 11.0 percent of the sample. In the 1960 census, white and black individuals are, respectively, 88.6 and 10.5 percent (US Census (1961)).

We interpret “not ascertained” in the 1962 SCF as meaning “white” in today’s terms. Our interpretation of “nonwhite” as meaning black in today’s terms is based on a view of whiteness being defined primarily in contrast with blackness. However, there are also white groups by today’s terms that were historically viewed as being white in a marginal or inferior way, such as Jews, Greeks, Italians, and Irish (Painter (2015)). We think survey enumerators would have applied the “not ascertained” label to respondents from these marginally white groups.

The numbers would be reasonable if marginally white groups combined with Hispanics to form the 11.0 percent of “not ascertained” family heads in the 1962 SCF. Nearly all of the US population was either white or black in the 1960 Census. Of the nonwhite population in the 1960 census, 92.1 percent were “Negro” (US Census (1961)). Although the Hispanic origin question was first introduced in the 1970 census, Gratton and Gutmann (Gratton and Gutmann) have used other variables, like birthplace, maternal birthplace, mother tongue, and having a Spanish last name, to impute how respondents to censuses before 1970 would have responded to the hispanic origin question had it been posed in those earlier censuses. Figure 16a shows the results of their analysis;

---

18 Race became self-identified starting in the 1989 SCF.
19 The 1960 Census questionnaire asked if each person was “White, Negro, American Indian, Japanese, Chinese, Filipino, Hawaiian, Part Hawaiian, Aleut, Eskimo, (etc.)?”
it is likely that about 3 percent of the US population was Hispanic in 1960.

Evidence on educational attainment also supports our choices for mapping the racial categories in the 1962 SCF to today’s racial categories. BA attainment would be higher than expected if “not ascertained” family heads were mapped to “black”. This can be seen by looking at levels (Figure 16b) or ratios (Figure 16c).

Focusing on ratios, our interpretation of “nonwhite” respondents in the 1962 SCF as being black, and only those respondents, implies trends of educational attainment that are consistent with the trends in other data sets that more precisely and consistently defined “black” as a category. In contrast, if we were to interpret respondents in both the “nonwhite” and the “not ascertained” groups as being black, the 1962 SCF would imply unrealistic rates of educational attainment for blacks in 1962. To see this formally, we estimate a regression where the dependent variable is the ratio of black to white BA attainment, the independent variable is year, and we use both the CPS and SCF data from 1963 onward. In terms of the errors from this regression, the error for the 1962 SCF prediction would have a z-score of -0.03 or 10.12 if we measured black as, respectively, either “nonwhite” or “nonwhite + not-ascertained.”

(a) Hispanic Population in the US over Time  (b) Educational Attainment in the SCF  (c) Educational Attainment in the SCF and CPS

Figure 16: Descriptive Statistics Related to Defining Race in the 1962 SCF
B More Data on Facts 1 and 2

B.1 Means Versus Medians

Figure 17a shows that when measured in terms of medians rather than means, the wealth gap is larger and the income gap has a higher variance. Measured in terms of medians, the wealth gap hovers closer to 0.9. Declines in the income gap from 2001-2010 would appear to be driven less by improvements in the incomes of black households and more by declines in the incomes of white households.

(a) Ratio of Means v. Ratio of Medians

(b) Black Ratio v. Hispanic Ratio

Figure 17: Measures of the Wealth and Income Gaps

B.2 The Hispanic-White Wealth Gap

Figure 17b shows that the Hispanic-white wealth gap is not much smaller than the black-white wealth gap. While the Hispanic-white income gap appears to be slightly lower than the black-white income gap, this difference is not economically large in most years.

B.3 Additional Data Sets

Facts 1 and 2 are consistent with many studies using many data sets. Table 5 shows the analogous gaps found in other studies in the literature. The wealth gap is stable across studies. The income gap is more sensitive than the wealth gap to the unit of observation, the time period over which it is observed, and as shown above, whether it is measured via medians rather than means.
Table 5: Facts 1 and 2 in the Literature

<table>
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<th>Unit of Observation</th>
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<th>Wealth Gap</th>
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<td>This Paper</td>
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<tr>
<td><strong>Total Income</strong></td>
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<td>Wolff (2018)</td>
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<td>HHs</td>
<td>0.52</td>
<td>0.82</td>
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<tr>
<td>Blau and Graham (1990)</td>
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<td>Families or Individuals 24-34</td>
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<td>0.82</td>
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<td>Terrell (1971)</td>
<td>1967 SEO</td>
<td>Families</td>
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<td>0.81</td>
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C The Firm’s Problem

The firm maximizes profits by choosing capital, aggregate labor input, and aggregate labor input from black households. It takes the prices, \( r - \delta \) and \( w \), and the labor income gap, \( \varphi(B) \) for black households as given.

\[
\max_{K,N_B,N_W} AK^\alpha (N_B + N_W)^{1-\alpha} - (r - \delta) K - \varphi(B)wN_B - wN_W
\]

\[
st\ N_B + N_W = N.
\]

Substituting in the aggregate labor input constraint yields

\[
\max_{K,N_B,N} AK^\alpha (N)^{1-\alpha} - (r - \delta) K - \varphi(B)wN_B - w(N - N_B)
\]

and the following first-order conditions:

\[
\alpha AK^{\alpha-1}N^{1-\alpha} = r - \delta
\]

\[
(1-\alpha) AK^{\alpha-1}N^{1-\alpha} = w
\]

\[
[1 - \varphi(B)] w \geq 0
\]

The last condition reflects that all else equal the firm would always prefer to substitute an effective unit of black household labor for a unit of white household labor because it always pays a strictly lower price for it. Because the firm takes the wage \( w^* \) and income gap \( \varphi(B) \) as given, in any equilibrium with \( \varphi(B) \in (0,1) \), the constraint imposed by black household labor supply will be binding and will generate positive profits \([1 - \varphi(B)]w\).

![Figure 18: Profits in the Firm’s Problem](image_url)
D Mortality

The survival probabilities $\psi(a)$ used in all of the numerical exercises in the text are estimated using data on all-gender age-specific 2012 survival probabilities for whites in Table 20 of Arias et al. (2016). Figure 19a shows these survival probabilities along with those for black individuals. We can see that survival probabilities are heterogeneous across race, especially past age 50.

Figure 19b shows that these differences in survival probabilities do not change the transition path of the racial wealth gap in our baseline transition. DISCUSS transition path of mortality rates.

Figure 19: Heterogeneous Survival Probabilities and Implications for the Transition Path