A Quantitative Theory of Information and Unsecured Credit

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Abstract

Over the past three decades six striking features of aggregates in the unsecured credit market have been documented: (1) rising personal bankruptcy rates, (2) rising dispersion in unsecured interest rates across borrowing households, (3) the emergence

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of a discount for borrowers with good credit ratings, (4) a reduction in average interest rates paid by borrowers, (5) an increase in aggregate debt relative to income, and (6) an increase in the amount of debt discharged in bankruptcy relative to income. The main contribution of this paper is to suggest that improvements in the ability of lenders to observe borrower characteristics can help account for a substantial proportion of most, though not all, of these observations. A central aspect of our findings is that the power of signaling is likely to be weaker when it is non-pecuniary costs, rather than the persistent component of income, that are unobservable. From a welfare perspective, our main finding is that model agents prefer to inhabit settings with more information, ex-ante, even though better information can rule out outcomes that some groups might find beneficial ex post.

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1 Introduction

For most of the postwar period the unsecured market for credit has been small. Direct evidence from the Survey of Consumer Finances (SCF), as well as other sources (e.g. Ellis (1998)) shows that unsecured credit did not appear in any significant amount in the US until the late 1960s. However, over the past three decades there have been dramatic changes in this market. First, and perhaps the most well-known attribute of the unsecured credit market in the period we consider has been the large increase in personal bankruptcy rates, from less than 0.1 percent of households filing annually in the 1970s to more than 1 percent annually since 2002. Sullivan, Warren, and Westbrook (2000) also note that not only are bankruptcies more common now than before, they are also larger; as measured by the ratio of median net worth to median US household income, the size of bankruptcies grew from 0.19 in 1981 to approximately 0.26 by 1997. More generally, the use of unsecured credit has intensified; it has nearly tripled, as measured by the ratio of aggregate negative net worth to aggregate income, from 0.30 percent in 1983, to 0.67 percent in 2001, to 0.80 percent in 2004 (when measured in the SCF).

Recent empirical work on consumer credit markets has also documented striking changes in the pricing of unsecured credit and sensitivity of credit terms to borrower characteristics. Perhaps most dramatically, in the SCF the distribution of interest rates for unsecured credit was highly concentrated in 1983 and very diffuse by 2004. Measured in terms of the variance of interest rates paid by those who report rolling over credit card debt, we find that as of 1983 the variance was 7.90 percentage points, but by 2004 this number had more than tripled to 26.63 percentage points. Interestingly, the change in the variance of rates has not been accompanied by large changes in the mean spread on unsecured credit relative to the risk-free rate (measured by the annualized 3-month t-bill rate): these spreads have fallen by
only slightly more than 1 percentage point.\footnote{More discussion of the empirical facts can be obtained upon request.}

The change in dispersion of interest rates appears to be a consequence of more explicit pricing for borrower default risk. A variety of financial contracts, ranging from credit card lines to auto loans to insurance, began to exhibit terms that depended nontrivially on regularly updated measures of default risk, particularly a household’s credit score and whether the household had a delinquent account.\footnote{At least two related findings stand out from the literature. First, the sensitivity of credit card loan rates to the conditional bankruptcy probability grew substantially after the mid-1990s (Edelberg 2006). Second, credit scores themselves became more informative: Furletti (2003), for example, finds that the spread between the rates paid by the highest and lowest risk classifications grew from zero in 1992 to 800 basis points by 2002.} Comparing data from 1983 and 2004, we find that the distribution of interest rates for delinquent households shifted significantly to the right of that of non-delinquent households, with the means of those with past delinquency being over 200 basis points greater than those with no such events on their records.\footnote{All interest rate data is taken from the Survey of Consumer Finances. Specifically, the SCF question regarding late payments is “Now thinking of all the various types of debts, were all the payments made the way they were scheduled during the last year, or were payments on any of the loans sometimes made later or missed?” The variable for SCF1983 is “V930” where a value of “1” means “all paid as scheduled” and a value of “5” means”sometimes got behind or missed payments.” For SCF2004 the variable is “X3004” where the values of “1” and “5” are the same as for 1983.}

The purpose of this paper is to measure the extent to which these changes may reflect improvements in the information directly available on borrowers’ default risk. Unsecured credit markets are a particularly likely place for information to be relevant. Perfectly collateralized lending is, by definition, immune to changes in information: private information is simply irrelevant. By contrast, once collateral is imperfect, or, in the case of unsecured debt, where formal collateral is totally absent, changes in the asymmetry of information will likely have consequences for prices and allocations. We emphasize that we do not think that
information is the only thing at work over this period. Certainly, many other things germane to unsecured credit market statistics have changed over the past three decades (see Livshits, MacGee, and Tertilt 2010 for a detailed discussion of the various factors). Our aim is not to argue that all the observed changes have arisen from changes in information, but rather to isolate the role played by improved information by keeping the environment fixed in all other ways.

We develop a life-cycle model of consumption and savings in which lenders cannot always directly observe the state of a borrowing household. Nonetheless, in general agents’ borrowing decisions convey information about aspects of their condition that are otherwise private information. As a result, the gap between allocations under asymmetric information from their complete- and symmetric-information counterparts may be closed to some extent. The question is: by how much?

Our paper is most closely related to Sánchez (2010) and Livshits, MacGee, and Tertilt (2008). Sánchez (2010) posits that the increased size of the credit market can be attributed to declines in the cost of offering contracts that separate households by risk characteristics. Specifically, the era with contractual homogeneity and low levels of unsecured risky credit is characterized by prohibitively expensive costs of offering screening contracts. Relatedly, Livshits, MacGee, and Tertilt (2008) argue that in the past, high “overhead” costs severely limited lenders from offering a wide menu of risky contracts. These papers therefore both suggest that screening could not, and did not, play an important role in allowing lenders to overcome the effect of asymmetric information on consumer default risk.5

The absence of screening as a route to escape the classical “lemons” problem, however, does not rule out the separation of borrowers whose debts imply high default risk from

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5Our work is also related to Drozd and Nosal (2007), which offers a theory of increased differentiation of borrowers based on declining contracting costs, and to Narajabad (2007) who uses improvements in the quality of symmetric information about borrowers to induce changes in the credit market. Both papers assume strong ex post commitment on the part of lenders which are hard to square with current practice.
those whose debts do not. In particular, credit market signaling by borrowers, who are on
the informed side of the market, still remains an option. And in a setting that precludes
effective screening, borrowers can and will transmit information via the size of the loan they
request, if they find it in their interest to do so. The equilibrium level of activity in the
consumer credit market then depends on the ability of relatively low-risk borrowers to use
debt as a signal to separate themselves from those who pose higher default risk. And in fact,
we will show that such separation is indeed possible, making asymmetric damaging, but not
fatal, to the functioning of the unsecured credit market.

To understand how improvements in information affect outcomes in the credit market,
we compare two equilibria. First, we allow lenders to observe all relevant aspects of the state
vector necessary to predict default risk. We intend this “Full Information” environment
to represent the one currently prevalent (with outcomes compared against the 2004 SCF,
among other sources). Second, we compare the preceding allocation to one where lenders
are no longer able to observe all of these variables. We intend these “Partial Information”
settings to be representative of periods prior to the mid-1980s (and here, compare outcomes
against data from the earliest (1983) wave of the SCF, among other sources). The difference
across these allocations is a quantitative measure of the effect of improved information about
shocks in unsecured credit markets.

Our results suggest that better information is qualitatively consistent with most, but
not all, of the changes observed during the period of interest. Quantitatively, we are able to
account for much of the change observed, depending on the variable in question. Specifically,
equilibrium levels of debt, default, and debt-in-default all fall substantially as lenders are
deprived of information relevant to forecasting default, while the sensitivity of credit terms
to default risk and discount associated with a clean credit record both virtually disappear.
A general lesson of our work is that information frictions may cause nontrivial reductions
in trade in unsecured consumer credit markets, but that ‘signaling’ in particular may still
provide some participants an effective route to overcome the problems created by asymmetric information. Nonetheless, overall, we conclude that improved information is likely to have still played an important role in transforming the unsecured credit market from one in which heterogeneous borrowers were treated homogeneously to one in which heterogeneity became more explicitly priced.

Our results may also have relevance for another welfare-related question: is more information in the credit market better? A view often asserted in policymaking circles is that better information in the credit market would harm disadvantaged groups, such as racial minorities, that benefit from the possibility that they might be able to “pool” themselves with lower-risk groups and obtain better credit terms. Such views have generated strong legislative actions. It turns out that all agent-types in our model are better off, ex ante, under full information, as every individual can borrow more at lower rates. We also show that better information will lead to both “democratization” and “intensification” of credit—that is, we obtain increases in both the extensive and intensive margins of the unsecured credit market. In terms of welfare, the intensification is quantitatively more significant: high school educated agents benefit less than college educated agents under full information, mainly because they are less constrained by low information since their desire to borrow is limited by relatively-flat expected age-income profiles. Thus, in our model, a move to full information does not redistribute credit from bad to good borrowers, it expands credit for everyone (as in the classic lemons problem).

In what follows, we describe our model and parametrization scheme and then present results. The final sections provide more detail on related work and then present some con-

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6Specifically, Section 215 of FACT Act directs the Federal Reserve Board, the Federal Trade Commission, and the Office of Fair Housing and Equal Opportunity (a department of HUD) to study whether “the consideration or lack of consideration of certain factors...could result in negative treatment of protected classes under the Equal Credit Opportunity Act.” “Report to Congress on Credit Scoring and Its Effects on the Availability and Affordability of Credit,” Board of Governors of the Federal Reserve System, August 2007.
2 The Model

The model is a small open economy, in which all agents take a risk-free rate as given, and where this rate is invariant to household decisions. There is a continuum of *ex ante* identical households, who each live for a maximum of $J < \infty$ periods, work until they retire at age $j^* < J$, value only consumption, and differ in their human capital type, $y$. A household’s human capital type governs the mean of income at each age over the life-cycle. A household of age $j$ and human capital type $y$ has a probability $\psi_{j,y} < 1$ of surviving to age $j+1$ and has a pure time discount factor $\beta < 1$.

As will be detailed further below, there exists a competitive market of intermediaries who offer one-period debt contracts and utilize available information to offer individualized credit pricing. Specifically, households have access to a market in which they may save in a one-period risk-free bond, but also may borrow in a defaultable (via bankruptcy) one-period debt instrument. Bankruptcy is costly, and has both a pecuniary cost $\Delta$, and a non-pecuniary component, denoted by the term $\lambda_j < 1$. The explicit resource costs of bankruptcy represent legal fees, court costs, and other direct expenses associated with filing. The existence of non-pecuniary costs of bankruptcy, represented in $\lambda$, is strongly suggested by a range of recent work. First, Fay, Hurst, and White (1998), who find that a large measure of households would have “financially benefited” from filing for bankruptcy but did not. Second, Gross and Souleles (2002) and Fay, Hurst, and White (1998) document significant unexplained variability in the probability of default across households, even after controlling for a large number of observables. These results imply the presence of implicit collateral that may or may not be observable, and is heterogeneous across households; $\lambda$ reflects any such collateral, including (but not limited to) any stigma associated with bankruptcy. However,
it also reflects a large number of other costs that are not explicitly pecuniary in nature (as
in Athreya 2002), such as the added search costs associated with renting apartments when
one does not have clean credit. It will be modeled as a multiplicative factor that alters the
value of consumption in the period in which a household files for bankruptcy.

Households also vary over their life-cycle in size. Let \( n_j \) denote the number of adult-
equivalent members present when the head of the household is age-\( j \). Consumption per
person at age-\( j \), \( c_j \), is a purely private good and therefore lowered as the number of household
members grows. Household preferences are represented by the expected utility function

\[
\sum_{j=1}^{J} \sum_{s^j} \left( \prod_{i=0}^{j} \beta \psi_{i,j} \right) \Pi(s^j) \left[ \frac{n_j}{1 - \sigma} \left( \frac{I_B(\lambda_j) c_j}{n_j} \right)^{1-\sigma} \right]
\]

(1)

where, letting the current period bankruptcy decision be denoted by \( B \in \{0, 1\} \), we define
\( I_B(\lambda_j) = 1 \) if the household does not choose bankruptcy \((B = 0)\), and \( I_B(\lambda_j) = \lambda_j < 1 \)
otherwise. Notice that the higher the value of \( \lambda \), the lower the cost of bankruptcy, as
effective consumption is larger when \( \lambda \) takes a relatively high value. \( \Pi(s^j) \) is the probability
of a given history of events \( s^j \), and \( \sigma \geq 0 \) is the Arrow-Pratt coefficient of relative risk
aversion.

One point we emphasize is that the improvements in information we suspect to be rel-
vant for generating the changes described above may have come from two places. First,
they may have come from improvements in lenders’ ability to forecast the future income of
borrowers. Second, they may have come from improvements in the ability of lenders to assess
factors orthogonal to predicted income. In our model, we will therefore consider not only
improvements in the ability of lenders to forecast borrower income, but also in their ability
to better assess heterogeneity amongst borrowers with respect to the latter’s “willingness”
to repay debt at any given point in time, even if such heterogeneity is being driven by a
variety of underlying “deep” factors. The now common use of detailed expenditure patterns
and general “data-mining” by lenders suggests that they are indeed interested in gleaning differences in default risk amongst groups whose characteristics, perhaps including income, have been forecasted as accurately by lenders as by the borrower.\(^7\) Our model will suggest, in fact, that improvements in such “softer” forms of information are likely to have played an important role in generating the changes seen in unsecured credit markets.

Given that we will study settings with partial information, we will also allow creditors to track the history of default via a binary marker \(m \in \{0, 1\}\), where \(m = 1\) indicates the presence of bankruptcy in a borrower’s past and \(m = 0\) implies no record of past default. This marker will be removed probabilistically to capture the effect of current regulations requiring that bankruptcy filings disappear from one’s credit score after 10 years and agents are prohibited from declaring Chapter 7 bankruptcy more than once every 7 years. We denote by the parameter \(\xi \in (0, 1)\) the likelihood of the bad credit market flag disappearing tomorrow; having \(m = 1\) does not prohibit the household from borrowing. This approach means that some households in our model will be able to declare bankruptcy more than once every 7 years; however, since households in the US economy also have the option to declare Chapter 13 (once every 9 months) and typically have few non-exempt assets (the primary difference between Chapters 7 and 13 is the dispensation of assets), this abstraction is reasonable and avoids the need for a cumbersome state variable tracking “the number of periods since a filing.” Under partial information, the price charged to a household for issuing debt will generally depend on \(m\), so that households with recent defaults will receive

\(^7\)See “What Does Your Credit Card Company Know About You?” (http://www.nytimes.com/2009/05/17/magazine/17credit-t.html?_r=1), for an interesting account of recent practices in credit cards. Among others, the article describes the change in assessment of default risk by lenders in response to bills indicating that a cardholder has utilized marriage counseling, or whether cardholders were “logging in at 1 am (as it might indicate sleeplessness arising from anxiety)”, etc. Similarly, a Canadian company, Canadian Tire, found that those visiting the dentist more frequently were far more likely to repay than those who went to bars, all else equal. Lenders have also found that those who purchased carbon monoxide indicators for their homes were far less likely to miss payments than those who purchased “chrome-skull” car accessories; with the latter likely to be a luxury good. Finally, anecdotal evidence exists of credit terms being adjusted when households made purchases that are indicative of financial trouble, such as used tires.
different credit terms than households with ‘clean’ credit. When information is symmetric, this flag is useless, though it will, in general, be negatively correlated with debt (i.e., those with a documented past history of bankruptcy \((m = 1)\), will borrow less on average, though they are not facing “punishment” per se). This point also illustrates that inferring the extent to which bankruptcy affects future credit access is not clear cut.

Default risk arises from a combination of indebtedness and the risk associated with future income. Under asymmetric information, we make an anonymous markets assumption: no past information about an individual (other than their current credit market status \(m\)) can be used to price credit. This assumption rules out the creation of a credit score that encodes past default behavior through a history of observed debt levels; since income shocks are persistent, past borrowing would convey useful information, although it is an open question how much. Given the difficulties encountered by other researchers in dealing with dynamic credit scoring, we think it useful to consider an environment for which we can compute equilibria.

The timing of decisions within a period is as follows. Agents first draw shocks to their current period income. Log labor income is the sum of five terms: the aggregate wage index \(W\), a permanent shock \(y\) realized prior to entry into the labor market, a deterministic age term \(\omega_{j,y}\), a persistent shock \(e\) that evolves as an AR(1)

\[
\log(e_j) = \rho \log(e_{j-1}) + \epsilon_j, \tag{2}
\]

and a purely transitory shock \(\log(\nu)\). Both \(\epsilon\) and \(\log(\nu)\) are independent mean zero normal random variables with variances \((\sigma^2_\epsilon\) and \(\sigma^2_\nu)\) that are \(y\)-dependent.

All households then face a purely i.i.d. shock to their expenditures, denoted \(\chi_j\), and is meant to capture the effect of sudden changes in obligations that the household may not actively “choose,” but nonetheless acquires. Examples include facing lawsuits, large out-of-
pocket health risk, and unexpected changes to the economies of scale or the legal assignment of debts within the household, such as those arising from divorce (see Chatterjee et al. 2007 or Livshits, MacGee, and Tertilt 2010). Lastly, households are required to pay a proportional tax on labor earnings in each period, \( \tau \), to fund pension payments to retirees. This is the only purpose of government in this model.

2.1 Consumption and Borrowing

After receiving income and expenditure shocks, and paying their taxes, the second step within a period is for the household to make a decision on whether or not file for bankruptcy. Denote by \( b_j \) the face value of debt \( (b_j < 0) \) or savings \( (b_j > 0) \) that is maturing today. If the household chooses bankruptcy, their debts are removed, as is the expense stemming from the expenditure shock.

After the bankruptcy decision, a household’s income and asset position for the current period are fully determined. Given this vector, households choose current consumption \( c_j \), and savings or borrowing, \( b_{j+1} \). If the household chose bankruptcy at the beginning of the current period, they are prohibited for this period only from borrowing or saving.\(^8\) Given the asset structure and timing described above, and using \( B \in \{0, 1\} \) (defined earlier) to indicate whether an agent elected to file for bankruptcy in the current period or not, the household budget constraint during working age and prior to the bankruptcy decision is given by

\[
c_j + q(b_{j+1}, I) b_{j+1} (1 - B) + \Delta B \leq (1 - B) b_j + (1 - \tau) W_{\omega_j, y_j e_j, \nu_j} + (1 - B) \chi_j, \tag{3}
\]

where \( q(\cdot) \) is an individual-specific bond pricing function that depends on bond issuance \( b_{j+1} \) and a vector of individual characteristics \( I \) that are directly observable (i.e. objects that do

\(^8\)This follows the literature (e.g. Livshits, MacGee, and Tertilt 2007), and reflects the legal practice of debtors facing judgements for fraudulent bankruptcies. Unlike this literature, we do not impose exclusion in any subsequent period.
not need to be inferred from behavior). The budget constraint during retirement is

\[ c_j + q (b_{j+1}, I) b_{j+1} (1 - B) + \Delta B \leq (1 - B) b_j + \theta W \omega_{j-1,y} y e_{j-1} \nu_{j-1} + \Theta W + (1 - B) \chi_j, \]  

where for simplicity we assume that pension benefits are composed of a fraction \( \theta \in (0, 1) \) of income in the last period of working life plus a fraction \( \Theta \) of average income (which is normalized 1). Because bankruptcy is not a retiree phenomenon, we deliberately keep the specification of retirees simple. There are no markets for insurance against income, expense, or survival risk.

The survival probabilities \( \psi_{j,y} \) and the deterministic age-income terms \( \omega_{j,y} \) differ according to the realization of the permanent shock. We interpret \( y \) as differentiating between non-high school, high school, and college education levels, as in Hubbard, Skinner, and Zeldes (1994), and the differences in these life-cycle parameters will generate different incentives to borrow across types. In particular, college workers will have higher survival rates and a steeper hump in earnings; the second is critically important as it generates a strong desire to borrow early in the life cycle, exactly when default is highest.

2.1.1 Recursive Formulation

In describing the household’s problem recursively, we denote all “next-period” variables by primes, and current period variables without primes. In addition to the permanent component of human capital (\( y \)), and the deterministically evolving state variable of age (\( j \)), the household’s state vector is composed of (i) the current component of the persistent income shock (\( e \)), (ii) the current component of the persistent bankruptcy cost (\( \lambda \)), (iii) total wealth net of the expenditure shock (i.e. \( b + \chi \)), and (iv) total labor income. We define the state vector to include persistent and all transitory shocks. This is more than is necessary, strictly speaking, but is clearly sufficient as a state vector, and makes it easy to view the
household’s problem. Lastly, under Partial Information, the indicator of a past bankruptcy \( (m) \) will also partially define the constraints, and hence enter the state vector as well.

To make the current-period bankruptcy decision, a household of age \( j \) with current period savings or debts \( b \), current period income shocks \( (e, v) \), expenditure shock realization \( \chi \), current bankruptcy cost \( \lambda \), and “credit record” \( m \), solves the dynamic programming problem below by comparing the value of filing for bankruptcy \( (B = 1) \), with the value of not filing for bankruptcy \( (B = 0) \). Following the bankruptcy decision, the household’s resources available for consumption and savings are fully determined. The household then chooses current consumption \( c \) and current debt issuance \( b' \), and the period ends. The choice of \( b' \) directly defines the set of income, expenditure, and bankruptcy cost realizations \( (e', v', \chi', \lambda') \) which will make bankruptcy optimal next period, which determines the price of debt issued today.

If the household chose at the beginning of the current period to go bankrupt, they are restricted from financial markets in the current period, which makes their current consumption equal to their labor income and next period assets \( b' = 0 \). Next period their bad credit rating \( m = 1 \) disappears with probability \( \xi \). The value functions on the right-hand side of expressions (5) and (6) therefore yield the expected indirect utility function \( V^{B=1}(\cdot) \):

\[
V^{B=1}(b, y, e, \nu, \chi, \lambda, j, m) = \left\{ \frac{n_j}{1-\sigma} \left( \frac{\lambda((1-\tau)W_{j,y}yv)}{n_j} \right)^{1-\sigma} + \beta \psi_{j,y} \sum e', \nu', \chi', \lambda' \pi_{\chi}(\chi') \pi_{\nu}(\nu') \pi_{\lambda}(\lambda'|\lambda) \right\} \times \left[ \xi V(b' = 0, y, e', \nu', \chi', \lambda', j + 1, m' = 0) + (1 - \xi) V(b' = 0, y, e', \nu' \chi', \lambda', j + 1, m' = 1) \right]
\]

If a household chose at the beginning of the period not to file for bankruptcy \( (B = 0) \), it retains access to financial markets in the current period, and realizes expected indirect utility according to \( V^{B=0}(\cdot) \):
In either case, all choices following the bankruptcy decision are restricted by a single budget constraint

\[ c + q(b', I)b'(1 - B) + \Delta B \leq (1 - B)b + (1 - \tau)W\omega y\nu + (1 - B)\chi. \] (7)

Given that the household chooses bankruptcy optimally, the expected indirect utility prior to making the bankruptcy decision, \( V(\cdot) \), will satisfy:

\[ V(b, y, e, \nu, \chi, \lambda, j, m) = \max \{ V^{B=0}(b, y, e, \nu, \chi, \lambda, j, m), V^{B=1}(b, y, e, \nu, \chi, \lambda, j, m) \}. \]

### 2.2 Loan Pricing

We now detail the construction of the price function \( q(b', I) \). Under both symmetric and asymmetric information, we focus on competitive lending arrangements in which lenders must have zero profit opportunities. Recall that \( I \) denotes the information directly observable to a lender. In the full information case, \( I \) includes all components of the household state vector: \( I = (y, e, \nu, \chi, \lambda, j, m) \), while only a subset of these variables are directly observed under asymmetric information.

With full information, a variety of pricing arrangements will lead to the same price function. Full information is also the case previously studied in the literature (see Chatterjee...
et al. 2007 or Livshits, MacGee, and Tertilt 2010), and is a special case of our model. By contrast, under asymmetric information, it is well known (Hellwig 1990) that outcomes often depend on the particular “microstructure” being used to model the interaction of lenders and borrowers. Specifically, since we have modeled households as issuing debt to the credit market, we must take into account the fact that the size of any debt issuance itself conveys information about the household’s current state. In other words, we study a signaling game in which loan size $b'$ is the signal. As we will detail further below, the lender’s task is to form estimates of the current realizations of the two persistent shocks $(e, \lambda)$, given this signal. Given an estimate and knowledge of household decision-making, lenders can then compute the conditional expectation of profits obtaining from any loan price $q$ they may ask of the borrower.

Lenders and borrowers play a two-stage game. In the first stage, borrowers name a level of debt $b'$ that they wish to issue in the current period. Second, a continuum of lenders compete in an auction where they simultaneously post a price for the desired debt issuance of the household and are committed to delivering the amount $b'$ in the event their “bid” is accepted; that is, the lenders are engaging in Bertrand competition for borrowers. In equilibrium borrowers choose the lender who posts the lowest interest rate (highest $q$) for the desired amount of borrowing. Thus, households view the pricing functions as schedules and understand how changes in their desired borrowing will alter the terms of credit (that is, they know $D_{b'I}(b',I)$) because they compute the locus of Nash equilibria under price competition.

Once the estimate of default risk is updated to reflect the signal sent by households, the Bertrand competition spelled out above leads to equilibrium prices that must let lenders do no better than break even. Let $\tilde{\pi}': b'_I \to [0, 1]$ denote the function that provides the best estimate of the probability of default, conditional on surviving, to a loan of size $b'$ under

16
information regime $I$. The break-even pricing function must satisfy

\[
q(b', I) = \begin{cases} 
\frac{1}{1+r} \left(1 - \hat{\pi}'\right) \psi_j & \text{if } b \geq 0 \\
\frac{1}{1+r+\phi} & \text{if } b < 0
\end{cases}
\]  

(8)
given $\hat{\pi}'$. Since default is irrelevant for savers, $\hat{\pi}'$ is identically zero for positive levels of net worth. By contrast, $\hat{\pi}'$ is equal to 1 for all debt levels exceeding some sufficiently large threshold. Given the exogenous risk-free saving rate $r$, let $\phi$ denote the proportional transaction cost associated with lending, so that $r + \phi$ is the risk-free borrowing rate; the pricing function takes into account the automatic default by those households that die at the end of the period (and we implicitly assume any positive accidental bequests are used to finance some wasteful government spending).

2.2.1 Full Information

As mentioned earlier, in the full information setting, given debt issuance $b'$ and knowledge of the two persistent shocks $e$ and $\lambda$, the lender does not actually need to know the current realizations of the transitory shock and expenditure shock as they will not help forecast next period’s realization of the household’s state. We include them here to maintain consistent notation with the partial information setting to be detailed further below.

Zero profit for the intermediary requires that the probability of default used to price debt must be consistent with that observed in the stationary equilibrium, implying that

\[
\hat{\pi}' = \sum_{e', \nu', \chi', \lambda'} \pi_\chi (\chi') \pi_e (e'|e) \pi_\nu (\nu') \pi_\lambda (\lambda'|\lambda) d(b', e', \nu', \chi', \lambda').
\]  

(9)

Since $d(b', e', \nu', \chi', \lambda')$ is the probability that the agent will default in state $(e', \nu', \chi', \lambda')$ tomorrow given current loan request $b'$, integrating over all such events tomorrow is the relevant default risk. This expression also makes clear that knowledge of the persistent
components $e$ and $\lambda$ are critical for predicting default probabilities; the more persistent they are, the more useful knowledge of their current values becomes in assessing default risk.

We now turn to the loan pricing under partial information. As we show, in this case, forecasting default risk will require integrating with respect to probability distributions over even the realizations of current period random variables that are purely *transitory*

### 2.2.2 Partial Information

The main innovation of this paper is to take a first step in evaluating the consequences of changes in the information available for predicting default risk on one-period debt, when borrowers may signal their type through loan size. In addition to being complementary to screening-based models, our approach avoids the well known problems of (i) non-existence and (ii) excessive sensitivity to the number and “order” of moves associated with screening games described in Hellwig (1990).

Partial information in our economies will manifest itself through limits on observability of the stochastic elements of the household state vector, in particular the pair of persistent shocks $(e, \lambda)$, as well as total income.$^{9}$ Some items, such as age and education, are slow-moving components of the individual state vector and can likely be relatively easily inferred. Therefore, in every partial information setting we consider, age and education are assumed observable while total income and the current expenditure shock are not. Our assumptions help us avoid overstating the power of adverse selection to unravel credit markets.

The lender’s problem is to infer all current household state variables useful for forecasting the bankruptcy decision, one period hence, of a household who has requested a loan of $b'$.

The two objects sufficient for this purpose are the two persistent components of a household’s

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$^{9}$In our setting, if total income $\nu$ were observable, $e$ would be observable. Therefore, partial information on income requires restricting income observability the way we do. An alternative would be to allow total income to be observed, but not its components. This endows lenders with more information than we allow them, but as we will show, income observability is actually not very powerful in altering allocations even in our setup, and so would be even less powerful under this alternative.

18
current state: $e$ and $\lambda$. This is because knowledge of the particular realization of these two random variables would allow lenders to construct the exactly correct conditional probability distribution for a borrowing household’s state next period. In other words, no estimates of the household’s current state would be needed.

If lenders cannot directly observe these items, they must resort to constructing an estimate of the value of the pair $(e, \lambda)$ received by a household requesting $b'$. Importantly, the loan size request will itself be influenced by the particular mix of transitory and persistent shocks received by a household, and so will be a signal of $(e, \lambda)$. For example, two households, each with a different realization of the persistent component of income $e$, may ask for the same debt because they have received two different realizations of $\nu$, or two different realizations of $\lambda$, or both. But not all pairs of realizations of the transitory shocks and bankruptcy costs are equally likely to describe the current state of the household, given the loan request $b'$. Therefore, since lenders know the relative likelihoods of these realizations under equilibrium decision-making, a given loan request will be “inverted” by lenders into an assessment of the pair $(e, \lambda)$ describing the persistent components received by a given household.

To make the best possible inference of $(e, \lambda)$, lenders will also use any knowledge they have of the overall distribution of households over the state-vector. For this purpose, we will restrict attention throughout to allocations in which the distribution of households over the state vector is invariant under optimal decision making. Let the invariant joint distribution (probability mass function) governing the proportion of households with any given state vector be denoted by $\Gamma (b, y, e, \nu, \chi, \lambda, j, m)$. Let $b' = g (b, y, e, \nu, \chi, \lambda, j, m)$ describe optimal household borrowing as a function of its state. Let $f(\cdot)$ be the inverse of $g$ with respect to the first argument wherever $\Gamma (b, y, e, \nu, \chi, \lambda, j, m) > 0$. Therefore, $b = f (b', y, e, \nu, \chi, \lambda, j, m)$ is the set of initial net worth levels that could, along with the remaining state variables $(y, e, \nu, \chi, \lambda, j, m)$, give rise to a loan request of $b'$ units. $\Pr (e, \nu, \chi, \lambda | b, y, j, m)$ assigns a
probability to each of these types based on knowledge of the decision rules of agents.

With respect to notation, in what follows, we use the abbreviation “PI(·)” to denote the partial information settings, and “FI” to denote the setting with full information. More specifically, we denote the case where neither the current $e$ nor $\lambda$ are visible by the label “PI($e, \lambda$).” When the partial information is such that only $e$ is not observable, we denote the case “PI($e$)” and lastly, when only $\lambda$ is not visible, “PI($\lambda$).”

In the partial information environment the calculation of $\Pr (e, \nu, \chi, \lambda | b, y, j, m)$ is nontrivial, because it involves the distribution of endogenous variables. In a stationary equilibrium, the joint conditional probability density over shocks $(e, \nu, \chi, \lambda)$ must be given by

$$\Pr (e, \nu, \chi, \lambda | b', y, j, m) = \int_{b \in f(b', y, e, \nu, \chi, \lambda, j, m)} \Gamma (b, y, e, \nu, \chi, \lambda, j, m) db$$

(10)

Thus, the decision rule of the household under a given pricing scheme is inverted to infer the state conditional on borrowing. Using this function the intermediary then integrates over the stationary distribution of net worth, conditional on observables, and uses this density to formulate beliefs.

The first concern for solving the partial information economy is that lenders must hold beliefs over the probability of an individual being in a particular state $(e, \nu, \chi, \lambda)$ given whatever is observed, knowing also that what is observed is a function of lenders’ a priori beliefs; that is, beliefs must satisfy a fixed point condition. We will restrict attention to discrete valued random variables for $\lambda$, $e$, and $\nu$. Let $\Pr (e, \nu, \chi, \lambda | b', y, j, m)$ denote the probability that an individual’s current shock vector in any period takes a given value $(e, \nu, \chi, \lambda)$, conditional on observing the size of borrowing, the permanent shock, age, and credit status. Given this
assessment, the lender can compute the likelihood of default on a loan of size $b'$:

$$
\hat{\pi}' = \sum_{e,\nu,\lambda} \left[ \sum_{e',\nu',\lambda'} \pi_{\chi}(\chi') \pi_{\nu}(e') \pi_{\nu'}(\nu') d\left(b', e', \nu, \lambda \right) \Pr(e, \nu, \lambda|b', y, j, m) \right].
$$

(11)

In a stable environment with an efficient technology for information sharing, competitive intermediaries must form beliefs that incorporate everything they either know or can infer from observables; competitors who exploit this information may be able to ‘cream-skim’ the best borrowers away from those who form beliefs in any other way.

### 2.2.3 Off-Equilibrium Beliefs

In addition to ensuring that pricing reflects equilibrium information transmission, the second key task under asymmetric information is to assign beliefs about a household’s state for values of the state not observed in equilibrium. This is essential because a household’s decision on the equilibrium path depends on its understanding of lender loan pricing at all feasible points in the state space, including those that they, or any other household, may ultimately not choose in the equilibrium, i.e. levels of $b$ where $\Gamma(b, y, e, \nu, \lambda, j, m) = 0$. For example, what interest rate can a household expect if it chooses to borrow more than anyone has actually chosen to do? The answer depends on what (borrowers believe that) lenders will believe regarding repayment probabilities by these households.

The off-equilibrium beliefs used in the model are specifically set to make allocations robust against overstating the role of private information in lowering credit availability, and directly inform the algorithm we use to compute model outcomes. Our algorithm is iterative – we guess pricing functions, compute implied default rates, recompute pricing functions based on the new default rates, and iterate to convergence. The critical choice of the algorithm is therefore the initial pricing function and the rule for updating. We assign the initial off-equilibrium beliefs in order to minimize the effects on equilibrium outcomes.
First, we restrict attention to weakly monotone pricing functions. Intuitively, this requirement merely says that conditional on a given (assessment of) household states, borrowing more never lowers the likelihood of subsequent bankruptcy. Next, since our algorithm will generate a monotone mapping over pricing functions, it is imperative that we begin with a pricing function that does not unnecessarily restrict borrowing and then conclude that “asymmetric information is very powerful.” We therefore begin by guessing a pricing function $q^0$ with the following properties: it is constant at the risk-free borrowing rate $\frac{1}{1+r+\sigma}$ over the range $[0, b_{\text{min}})$, where $b_{\text{min}}$ is a debt level such that no agent could prevent default in any state if he borrowed that much, and then drops to 0 discontinuously. Notice that this is the most optimistic view that a lender could have about the borrower: it literally ignores the possibility of default until it becomes a certainty. The implied beliefs for the intermediary are such that default never occurs except when it must in every state of the world; clearly no equilibrium pricing function could permit more borrowing. As a result, credit availability, defined in terms of the interest rate and maximal loan size that one can obtain, is maximized under this specification of $q^0$.\(^{10}\)

Given $q^0$, we solve households’ decision problems and locate the associated invariant distribution of households over the state space prevailing under this pricing function. At this point, when faced with a debt request $b'$, a lender will construct an estimate of the borrowing household’s type, knowing the set of current state vectors that would lead to such a request. Of course, to do this calculation (using Bayes’ rule), it must be true that a positive measure of households has chosen this particular debt level. In such cases, one can compute an expected default rate (where the expectation averages over all the states that could have led to that debt level), and hence can compute an actuarially fair price for debt issued in

\(^{10}\)It is useful to compare our initial pricing function with the natural borrowing limit, the limit implied by requiring consumption to be positive with probability 1 in the absence of default. Our initial debt limit is larger than the natural borrowing limit, as agents can use default to keep consumption positive in some states of the world; we only require that they not need to do this in every state of the world. This point is also made in Chatterjee et al. (2007).
those amounts.

However, the problem that concerns us arises because there will be levels of debt that no household chooses to acquire. Moreover, such levels of debt may well be surrounded by debt levels both higher and lower that are chosen by a positive mass of households. In other words, the invariant distribution of households over debt levels may well have “holes.” As noted above, from the household’s perspective it is essential that they know the price they will face at any debt level they might contemplate; it is only then that they can solve a well-posed optimization problem. So what is the lender to infer about a household who chooses a debt level in one of these holes?

Our approach is to allow lenders to be maximally optimistic about such households. Specifically, we specify that for debt levels that lie in any range not chosen by any households in the invariant distribution arising from a conjectured price function, lenders assign them the default probability that corresponds the nearest debt level that is lower than the one under consideration, but that attracts a strictly positive measure of agents. In other words, lenders’ beliefs create a step-function for \( q \) which jumps downward (reflecting higher default risk) only at each point in the support of debt levels at which a positive measure of borrowers is located. The algorithm we construct to locate equilibria is detailed in the Appendix.

2.2.4 Equilibrium

We now formally define equilibrium. We denote the state space for households by \( \Omega = \Theta \times \mathcal{Y} \times \mathcal{E} \times \mathcal{V} \times \mathcal{J} \times \{0, 1\} \subset \mathcal{R}^4 \times \mathcal{Z}_{++} \times \{0, 1\} \) and space of information as \( \mathcal{I} \subset \mathcal{Y} \times \mathcal{E} \times \mathcal{V} \times \mathcal{L} \times \mathcal{J} \times \{0, 1\} \).

**Definition** A Perfect Bayesian Equilibrium for the credit market consists of (i) household strategies for borrowing \( b^* : \Omega \to \mathcal{R} \) and default \( d^* : \Omega \times \mathcal{E} \times \mathcal{V} \to \{0, 1\} \) and intermediary strategies for loan pricing \( q^* : \mathcal{R} \times \mathcal{I} \to [0, \frac{1}{1+r}] \) such that \( q^* \) is weakly decreasing in \( b \), and
(ii) beliefs about the borrower state $\Omega$ given borrowing $\mu^* (\Omega | b)$, that satisfy the following:

1. **Households optimize**: Given prices $q^*(b, I)$, $b^*$ solves the household problem.

2. **Lenders optimize**: Given beliefs $\mu^* (\Omega | b)$, $q^*$ is the pure-strategy Nash equilibrium under one-shot simultaneous-offer loan-price competition.

3. **Beliefs are consistent with Bayes’ rule**: The joint density of $\Omega$ and $b$, $\Gamma_{\mu^*} (\Omega, b)$, that is induced by (i) lender beliefs and the resultant optimal pricing, (ii) household optimal borrowing strategies, and (iii) the exogenous process for earnings shocks and mortality, is such that the associated conditional distribution of $\Omega$ given $b$, denoted $\Gamma^b_{\mu^*} (b)$, is $\mu^* (\Omega | b)$.

4. **Off-Equilibrium Beliefs**: For any given debt request $b$ such that $\mu^* (\Omega | b) = 0$, define $b^- < b$ to be the nearest higher debt level relative to $b$ at which $\mu^* (\Omega | b) > 0$. Let $q^*(b^-, I)$ denote the actuarially fair price at $b^-$. Next, define $b^+ > b$ be the nearest lower debt level relative to $b$ at which $\mu^* (\Omega | b) > 0$. Let $q^*(b^+, I)$ denote the actuarially fair price at $b^+$. Next, let $q^*(b, I) = q^*(b^+, I) \forall b \in (b^-, b^+]$. Lastly, if an agent requests a debt level that exceeds all debt levels observed in equilibrium, lenders assign them default probability one.

Given the definition of equilibrium for the game, the equilibrium for the overall model is standard: we simply require that the distribution of households $\Gamma(\cdot)$ over the state-space is invariant under the optimal decision-making described above, and that the tax rate $\tau$ allows the government to meet its budget constraint. The government budget constraint is defined as follows.
2.2.5 Government Budget Constraint

As stated earlier, the only purpose of government in this model is to fund pension payments to retirees using a proportional tax on labor earnings, $\tau$, on all households. The government budget constraint is

$$
\tau W \left( \sum_{\omega, e, y, \chi, \lambda, j, m} \int_b y \omega j y e \nu \Gamma (b, y, e, \nu, \chi, \lambda, j < j^*, m) \, db \right) = W \sum_{\omega, e, y, \chi, \lambda, j, m} \int_b (\theta \omega j_{-1, y} y e_j \nu_j \nu_{j-1} + \Theta) \Gamma (b, y, e, \nu, \chi, \lambda, j \geq j^*, m) \, db. \tag{12}
$$

3 Calibration

Our calibration assigns numerical values to model parameters under the maintained assumption that the current setting is one of full information (no asymmetric information). This requires explanation; it is clear that there is still asymmetric information between unsecured borrowers and lenders. Our approach is driven primarily by two considerations. First, it is computationally intractable to calibrate our asymmetric information model.\textsuperscript{11} Second, it turns out that the FI outcomes are actually close to PI outcomes when the only information that is not directly observable is the current persistent component of income: FI and PI($e$) outcomes are “close.” In this sense, one could consider PI($\lambda$) as a benchmark. Anticipating the results, this finding of our model suggests that it is improvements in the ability of lenders to forecast or assess household characteristics relevant to bankruptcy decisions that are not summarized by income alone that have been important in driving the changes seen in unsecured credit markets.

Our targets are (1) the ratio of median debt discharged in bankruptcy to median US

\textsuperscript{11}Iterations on this model simply take too long (10 days even when efficiently parallelized). Moreover, all steps to make the convergence more rapid generally lead to equilibria in which credit is nearly shut down. To the extent that these are counterfactual outcomes, they do not seem worth considering.
household income, (2) the total fraction of US households with negative net worth, (3) the conditional mean of debt for those who hold debt of each of the three educational groups, and (4) the personal bankruptcy rate for each of the three educational groups. We therefore have a total of eight targets, so we calibrate eight model parameters. These are (1) the discount factor $\beta$, (2) two values for the non-pecuniary cost of bankruptcy for each educational level (6 parameters--$\lambda_{\text{loNHS}}, \lambda_{\text{hiNHS}}, \lambda_{\text{loHS}}, \lambda_{\text{hiHS}}, \lambda_{\text{loCOLL}}, \lambda_{\text{hiNHS}}$), and (3) the (common) persistence parameter for this cost, $\rho_\lambda$.

The statistics on default rates are based on the measures from Sullivan, Warren, and Westbrook (2000). The debt targets are all from the SCF (2004), and use exactly the same definitions employed by Chatterjee et al. (2007) and Sánchez (2010), presented in Table 1; the maintained assumption within the model is that households have a single asset with which to smooth consumption. Given our life-cycle setting, moreover, this assumption is not likely to be crucial for the question we pose. As noted above, in the data (as well as in the model), defaults are largely the province of the young (Sullivan, Warren, and Westbrook 2000); young households also have few gross assets, implying that negative net worth and unsecured debt largely coincide (see also Table 2.4 in Sullivan, Warren, and Westbrook 2000). As in Chatterjee et al. (2007) and Sánchez (2010), we identify debt with negative net worth, and the specific values we target for aggregate-debt-to-income in the FI setting across education levels are given in Table 1. For the measure of borrowers, we choose a target of 12.5 percent, as it lies in middle of the interval defined by the estimate of 6.7 percent in Chatterjee et al. (2007) and 17.6 percent in Wolff (2006).

To parameterize “expense” shocks, $\chi$, we follow Livshits, MacGee, and Tertilt (2007), and specify an i.i.d random variable that is allowed to take three values $\{0, \chi_l, \chi_h\}$. The shock $\chi_l$ is set to be 7 percent of mean income, and $\chi_h$ is set at 27 percent of mean income. The relative likelihoods are $\Pr(\chi = 0)=0.9244$, $\Pr(\chi = \chi_h)=0.0710$, and $\Pr(\chi = \chi_l)=0.0046$. As a result, only a minority of household receive shocks, and of these, most do not receive
a catastrophic one. Expense shocks create involuntary creditors that allow households to suddenly acquire very large debts with no corresponding change in measured consumption. Our calibration target for the model’s bankruptcy rate is 1.2 percent, in line with the average over the period 2000-2006. The median discharge to median US household income ratio that we use takes the ratio of median debt discharged in Chapter 7 bankruptcy (taken from Sullivan, Warren, and Westbrook (2000), Table 2.4) and compares this relative to median US household income from the Census bureau, and is therefore set to 0.27.

For earnings risk, we do not calibrate any parameters. We instead use values consistent with those previously obtained in the literature. We set $\theta = 0.35$ at an exogenous retirement (model) age of 45 and $\Theta = 0.2$, yielding an overall replacement rate for retirement earnings of approximately 55 percent. The income process is of the “Restricted Income Profiles (RIP)” type and is taken from Hubbard, Skinner, and Zeldes (1994), which estimates separate processes for non-high school, high school, and college-educated workers for the period 1982-1986. Figure 5 in the Online Appendix displays the relative incomes over the life-cycle for all three human capital groups.

In Athreya, Tam, and Young (2009) we study the effect of the rise in the volatility of labor income in the US and find it to be quantitatively unimportant. Therefore, we use the process estimated on the early data even though we compute the FI case assuming it applies to 2004. The estimated process for $\omega_{j,y}$ displays a more pronounced hump for college types than the others; details are available in the Online Appendix. The shocks are discretized with 15 points for $\epsilon$ and 3 points for $\nu$. The resulting processes have a common $\rho = 0.95$, with $\sigma_\epsilon^2 = (0.033, 0.025, 0.016)$ and $\sigma_\nu^2 = (0.04, 0.021, 0.014)$ for non-high-school, high school, and college agents; the measures of the three groups are 16, 59, and 25 percent, respectively. The risk-free rate on savings is set to 1 percent to reflect the assumption that households have access to a risk-free and liquid savings instrument. For lending costs, we set $\phi = 0.03$ to generate a 3 percent spread between rates of return on broader measures.
of capital and the risk-free borrowing rate, consistent with transactions costs measured by Evans and Schmalensee (1999). \( \Delta \) is set equal to 0.03; if one unit of model output is interpreted as $40,000 – roughly median income in the US – then the filing cost is equal to $1200, and is an estimate inclusive of filing fees, lawyer costs, and the value of time. Finally, \( \xi = 0.2589 \) implies that 95 percent of households who do not file for bankruptcy again will have clean credit after 10 years.

The calibration generates a stochastic process for the non-pecuniary cost \( \lambda \) with the following properties. We specify that it follows a Markov chain, and the calibration assigns this process very high persistence. As a result, it appears that such costs are likely partly in the nature of a household-level “type,” as they will remain generally unchanged for any given household over time. However, the variance of the cost is non-trivial, and as a result, households will differ substantially from each other in their willingness to repay debt, all else fixed. Lastly, we set risk aversion/ inverse elasticity of intertemporal substitution at \( \sigma = 2 \) for all households. The calibrated parameter values are listed in Table 2.

4 Results

We evaluate the implications of changes in information on six aggregates of interest. These are (1) the overall bankruptcy rate, measured by dividing the American Bankruptcy Institute’s aggregate Chapter 7 filing rate by the number of US households in 2004 (FI case), and in 1983 (PI cases), (2) the variance of interest rates on unsecured debt in the 2004 SCF (FI case) and 1983 SCF (PI case), (3) the difference in average credit-card-to-risk-free rate spreads paid by those with a past bankruptcy on their record relative to those without one (as measured as the difference between the average credit card interest rate on which balances were paid and the ex-post real (using PCE deflator) annualized 3-month t-bill rate), (4) the mean interest rate spread paid by borrowers under PI and FI, (5) the average debt
in the economy, as measured by the ratio of total negative net worth to total economywide income (from the SCF 1983 and 2004), and (6) the median unsecured debt (as measured by Sullivan, Warren, and Westbrook (2000), Table 2.4) at the time of bankruptcy relative to US median household income from US Census data from 2004 (under FI) and 1983 (under PI).

We now address our main question: how do the six aggregates above change when information is systematically withdrawn from lenders? This gives us our measure of the role potentially played by better information. We compare the allocations and prices obtaining under Full Information against three “Partial Information” environments. These are, respectively, the case with (i) only the current persistent shock is unobserved, (ii) only the non-pecuniary cost of bankruptcy is unobserved, and (iii) when both current persistent income risk and current non-pecuniary costs of bankruptcy are unobserved.

4.1 Full Information

In the full information case, lenders are assumed to observe all elements of the household’s state vector, and thereby form forecasts of default risk that coincide with the borrowing household’s own conditional expectation. For brevity, we omit any detailed discussion of the Full Information setting here. It is the standard model, and is discussed in a variety of existing work, including Athreya, Tam, and Young (2008) and Livshits, et al. (2010). The interested reader is directed to the Online Appendix for the details of this case. As a brief summary, Table 1 presents the aggregate unsecured credit market statistics from

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12 We use the SCF sample weights to obtain a representative measure. Also, alternative debt-to-income targets could have been debt-to-income ratios for households where a member files. However, while debt at the time of filing is well measured, household income is not. Instead, for the large fraction of filings, only the filer’s income, and not the household’s income, is recorded. We therefore normalize by US household income instead. While we do not report it here, the results are very similar when we target these other ratios. In particular, as we will show, the model does well along a number of dimensions, but fails to capture the size of bankruptcies seen in the data. To measure the variance of interest rates, we use only those accounts on which interest is paid, and we do not weight observations by debt levels.
the model under full information, and the Online Appendix contains more detail on loan pricing for this case (see Figure 6). As seen from there, the calibration procedure is quite successful; the benchmark model matches closely the targets we set in terms of the fraction of those with negative net worth, the overall level of unsecured debt relative to income for each education group, and the bankruptcy rate for each education group. However, the benchmark model is not able to fully capture the ratio of median discharge to median income. Nonetheless, part of the difference between the model and the data is that the latter employ a substantially broader measure by including the large debts associated with small business failures (see Sullivan, Warren, and Westbrook 2000). Unfortunately, the data that clearly separate discharge according to the reason for filing do not exist.

4.2 The Effect of Asymmetric Information

The central goal of our paper is to assess the extent to which improvements in information held by creditors can help account for the large changes seen in unsecured credit aggregates over the past three decades. Having calibrated the model to current observations under full information, we can now address this question. Our results suggest that along several, but not all, dimensions, the answer is “a substantial amount.” We focus our attention on the behavior of the six unsecured credit-market aggregates laid out earlier, and measure the extent to which moving from the FI case to a partial information setting generates changes in these aggregates. In our model, there are two places where one might locate private information: the persistent component of income, and the non-pecuniary cost of bankruptcy. We take the benchmark partial information environment to be the one where both non-pecuniary costs and the persistent component of household income are not directly observable.
4.2.1 Information and Aggregates

Table 3 shows the results. The first two columns of the Table compare the levels of each of the six items of interest to us in the data as of 1983 against the predictions of the PI($e, \lambda$) model, while the third and fourth column represent recent (2004) data and the predictions of the model under FI. Given that we have calibrated the model to match as closely as possible the 2004 data, what is relevant is the extent to which the change in information accounts for differences seen between 2004 to 1983. For ease of exposition, the bottom half of the Table displays changes in the levels of the variables of interest, and the fraction of these changes generated by our model when we move from an FI setting to the PI($e, \lambda$) environment.

In terms of bankruptcy, our model suggests that relative to the current period, information by itself can account for approximately 46 percent of the total change in bankruptcy seen in the data. In terms of interest rate dispersion, our model suggests that the change in information captures 77 percent of the change in the variance of interest rates paid by households, and a similar amount (73 percent) of the change in the “good borrower” discount. Even more striking is that the model does very well in capturing the observed change in the spread of mean unsecured interest rates relative to the risk-free rate (97 percent). In terms of the change in overall indebtedness, the model predicts that information by itself would have led to an even greater change in borrowing than what was observed, at 147 percent. Finally, we turn to the size of bankruptcies. As stated earlier, we measure the model’s predictions for the ratio of median debt discharged in bankruptcy to median US household income. Given this normalization, it is clear that bankruptcies have grown larger over time. As seen in Table 1, however, the benchmark calibration is unable to match the size of the median bankruptcy, and instead generates bankruptcies of approximately half the size. Part of this difference can certainly be regarded as a shortcoming of the benchmark model with respect to its ability to generate bankruptcies as large as in the data. However, it should also be noted that the
data on the size of bankruptcy targeted here does not distinguish between bankruptcy of wage-earners and the self-employed. The latter are suspected (see Sullivan, Warren, and Westbrook 2000) of contributing disproportionately to relatively large bankruptcies. The model suggests that improvements in information are unlikely to be behind the changes in the size of defaults seen over time. The model predicts rather that improved information by itself should have left the size of equilibrium debt-discharge essentially time invariant.

There are two other asymmetric information economies one could consider within the framework of our model. These are, respectively, the cases where only persistent income shocks were not directly observed, and the one in which only non-pecuniary costs were not observed. Denote these two cases, respectively, by PI(ε) and PI(λ). Table 4 displays the results. Comparing outcomes from these two cases is instructive. First, the ability to observe income is actually not vital to allocations. Comparing the first two columns of Table 4 shows that allocations are largely similar across these two information regimes. One reason for this can be seen in Figure 1. Notice that the household with the median persistent income shock (under both high and low λ) faces a pricing function with FI that is quite close to the single price function that all households face under PI. Second, and by contrast, the ability to assess a household’s cost of bankruptcy is important. Table 4 shows that when λ is the only unobservable, allocations change more substantially relative to FI. While the size of bankruptcies and the measure of borrowers remains stable, we see that debt/income ratios and bankruptcy rates fall very close to the benchmark private information setting. As a result, the model suggests that not all asymmetric information is equally important, and in particular that asymmetry of information on income by itself does not lead to large reductions in credit supply. Our findings also are consistent with the fact that credit card lenders rarely, if ever, follow up on cardholder income over time, and typically ask only for self-reported income at the time of application.13

13As reported on a credit card industry website, current practice is to use credit history and
4.2.2 Information and Credit Supply

We turn now to evaluating the role that asymmetric information plays on the supply-side of the unsecured credit market. This aspect is captured by comparing the pricing of unsecured credit arising from a given asymmetric information setting with others, or the most natural counterpart from the Full Information setting. Figure 1 contains at least three messages. First, quite naturally, private information of every variety considered here constrains credit supply: ceteris paribus, loan prices are high ($q$ is lower) for a given borrowing level under private information. For example, as we move from FI to PI(e), notice that $q$ under either value for the non-pecuniary cost of bankruptcy shifts credit supply inward. Second, under all information regimes, unsecured debt in excess if about 5 percent of economywide average income begins to carry default risk. This is seen in the fact that the loan pricing function begins at this point to fall below the risk-free rate, net of transactions costs. Third, as emphasized already, the model suggests that private information about bankruptcy costs is more important than private information about income, for a relatively natural reason. In an environment where only the persistent shock was unobservable, agents have a way of separating themselves: an agent with a poor current realization will be willing to pay more for a large loan to smooth consumption than will one who has received a good shock. As a result, those asking for large loans in this case will be “correctly” identified as relatively high risk borrowers, who, because of currently low income, will be willing to pay a price that is roughly actuarially fair. As a result, aggregate allocations under PI(e) are relatively close to those under FI.\(^\text{14}\)

In sharp contrast to the ability of the unsecured market to function relatively smoothly in the absence of direct information on the current component of persistent income risk, when self-reported income at the time of application to estimate income, without verification of income. (http://www.creditcards.com/credit-card-news/credit-card-application-income-check-1282.php)

\(^{14}\text{We are indebted to an anonymous referee for clarifying this mechanism to us.}\)
non-pecuniary costs are unobservable, outcomes are more sharply restricted relative to FI. Looking in Figure 1 again, and holding fixed the current persistent shock at its median value, we see that a household faces a much higher interest rate for any given debt label they might choose, and when this is further compounded by unobservable income risk, credit supply shrinks substantially. In the latter case, this is because an interaction between the desire to borrow when faced with a particularly bad persistent shock and a low non-pecuniary cost of default makes it very attractive to borrow, if possible. In this case, borrowing does not carry a strong “single-crossing” property to separate borrowers. The strength of adverse selection incentives can be seen by looking at the FI case. Here, we can see that the difference in loan pricing for those with high and low-\(\lambda\) is very substantial, which further explains why, when combined with asymmetric information on income \(e\), credit is relatively restricted.

### 4.2.3 Information, Interest Rates, and Dispersion

A primary reason for our focus on improved information as a candidate explanation for the facts is the increase in dispersion of credit terms observed (and so paid in equilibrium). Figure 2 displays the model’s implications for the evolution of the variance of equilibrium borrowing rates over the life-cycle across the two main information regimes we consider. These rates are not weighted by the amount of debt, they are direct measures of dispersion computed identically to what we measure from the SCF. The dispersion in interest rates is fairly flat over the life-cycle, and the variance is systematically higher for the higher educated groups, since those groups are the ones who borrow and default on the equilibrium path. In the model, agents are willing to pay fair premiums for the option to default and do so. Furthermore, since all pricing is actuarially-fair with respect to default risk, agents who pose higher risk will pay higher prices to borrow. Both observations are consistent with the relatively high dispersion and sensitivity found in recent data; we show below that the partial information setting will display neither characteristic.
It is also of interest to examine the role of information on the *average* interest rates paid on the equilibrium path. Figure 3 shows the results. First, and perhaps naturally, the less well educated pay higher interest rates throughout life than their better educated counterparts. Given the earnings process, this is not surprising. However, what is interesting here is that the model suggests that partial information may lead to *lower* average interest rates, or alternatively, that improvements in information may well coincide with the observation of more households borrowing at higher interest rates. The reason is intuitive, and reflects the pricing functions displayed earlier. In essence, loan interest rates under partial information are frequently high enough to discourage borrowing. However, as will be detailed shortly, household welfare will be higher under Full Information than any partial information setting we consider. This is seen in Figure 3 by holding human capital fixed and looking across information regimes. The highest interest rates are paid under the FI regime, by the NHS households. This is consistent with the figures we showed earlier on the dispersion of interest rates: better information means more high and low-interest rate borrowing.

Our model makes predictions for the effect of improved information on the mean and variance of interest rates paid by borrowers who have an indicator of past bankruptcy (i.e. \( m = 1 \)) on their record. As documented in Edelberg (2006) and Furletti (2003), past default appears substantially more correlated with credit terms now than in past decades. In particular, the positive correlation of interest rates with past defaults may be seen as a form of “punitive” sanctions imposed by creditors. However, under competitive lending and full information, such penalties will not be viable. Nonetheless, given the persistence of shocks, the income events that trigger default may well persist, and therefore justify risk premia on lending. Indeed, we will argue that this is a plausible interpretation of the data.

Note first that even under FI, there will generally be a non-zero correlation between the terms of credit and the “flag” of past bankruptcy. This is simply because, even though the flag is irrelevant under FI, it turns out that many who file do so when they have been hit by
a bad persistent shock. As a result the pricing they face under FI is worse than that faced by the average household. Table 5 shows how the flag is related to credit pricing under two PI cases relative to FI. The most useful thing to note is that in either case, the entries for \( Var(r|m = 1) \), and \( Var(r|m = 0) \) show that the model does fairly well at replicating the level of the dispersion under FI, and also the fact that dispersion is higher for those with the flag \( m = 1 \). However, we also see that the reduction in information to either the PI(\( \lambda,e \)) case or the PI(\( \lambda \)) case leads to a substantially smaller reduction in the variance than occurred in the data. For instance, in Table 5, we see that the reduction in variance predicted by the model (taking those with \( m = 1 \)) for example, is from 34.55 to 17.07, while in the data, the reduction was almost twice as large, going from 33.88 to 8.68. A related prediction of the model also suggests that information is not the whole story: the model fails to explain why the difference between the mean interest rates paid by those with and without a bad flag is close to zero (−22 basis points), and predicts instead that the difference should be a positive spread of 121 basis points.\(^{15}\)

The model suggests that information can help explain the difference in dispersion amongst those with a flag of \( m = 1 \) and those without it (\( m = 0 \)) quite well. The successes of the model here are of interest because interest rate dispersion was never targeted. We take this as additional evidence that changes in information lie at the heart of observed changes in unsecured credit markets. Crucially, we deliver this result without resorting to exclusion. Exclusion is \textit{ex post} inefficient for lenders under full information, so it cannot be justified on theoretical grounds, and the data available only show that households borrow less and face worse terms after bankruptcy, not whether they were unable to borrow at all.

\(^{15}\)Table 10 in the Online Appendix contains additional results on the relationship between mean interest rates paid by borrowers by education-type when \( m = 1 \) and \( m = 0 \).
4.2.4 Information, Consumption Smoothing, and Welfare

We now turn to some of the consequences of improved information for consumption. We find it helpful to decompose volatility into two components:

\[
\text{Var} \left( \log(\text{c}) \right) = E \left[ \text{Var} \left( \log(\text{c}) | j \right) \right] + \text{Var} \left( E \left[ \log(\text{c}) | j \right] \right); \quad (13)
\]

the total variance of log consumption \( \text{Var} \left( \log(\text{c}) \right) \) is the mean of the variances of log consumption conditional on being age \( j \) plus the variance of mean log consumption conditional on being age \( j \). The first term yields a measure of intratemporal smoothing, while the second term measures intertemporal smoothing by capturing the extent to which mean consumption varies over the life-cycle.\(^{16}\) In Table 6, we present the aggregates for these two measures.

The basic lesson is that in all the environments we consider, consumption smoothing is not radically affected by changes in information. Second, the most skilled are able to most effectively shield the consumption, as seen in the average variance of consumption at various ages ("E(Var(log(c)|age)"). The variance of consumption is high early in the life-cycle because households are restricted in their ability to borrow when young, inhibiting consumption smoothing. Consumption smoothing becomes less effective for most ages under partial information because borrowing is essentially impossible; thus, the young in particular experience significant consumption fluctuations. However, households that borrow early in life must repay debts as they age, leaving them exposed to income risk later in life. Because the partial information economy does not permit borrowing, these older households are able to smooth their consumption effectively using buffer stocks of savings accumulated earlier

\(^{16}\)The Online Appendix contains further detail. In particular, in Figure 7, we plot the mean of log consumption over the life-cycle under the two assumptions about information. By approximately middle-age, mean consumption is lower under partial information relative to full information, after which it is higher. This is a direct reflection of the credit supply being restricted under PI, as discussed earlier and shown in Figure 1. When agents cannot borrow substantial amounts early in life, they do not acquire debts that must be serviced in middle-age to prepare for retirement. Lower transactions costs under PI also play a role in supporting mean consumption, although a minor one.
in life; of course, the fact that they can smooth their consumption effectively when old does not mean that they are better off *ex ante*. We discuss the welfare implications of limited information further below.

Having found that changes in information may have been important in explaining changes in the behavior of the U.S. unsecured credit market, we now address the normative question of whether these changes are likely to have improved household welfare. The reason that it is not *a priori* obvious that welfare will rise with more precise information is that some households may have received a subsidy from others under partial information. That, if the equilibrium allocation and loan-pricing under partial information allowed high-risk households to face better-than-actuarially-fair prices for their debt, simply because lenders presumed that some of the borrowers of those debt levels were very low-risk households, an improvement in information will not help the former group. The preceding is not merely a theoretical curiosity; it is the reasoning behind recent changes in statutory restrictions on the information that lenders may explicitly use, such as the Equal Credit Opportunity Act.

Given our model specification, there are five ways in which the information held by lenders can improve. Starting from the PI(\(\lambda, e\)) case, lenders can move to one of three information settings: PI(\(\lambda\)), PI(\(e\)), and FI. And starting from PI(\(\lambda\)) and PI(\(e\)), lenders’ information can improve to FI. In addition, because the economy is one with a fixed risk-free rate, a useful payoff is that the welfare gains are additive: there are no GE spillovers across agents as a result of the differential credit constraints they face across informational regimes. For example, the welfare gain from moving from the PI(\(\lambda, e\)) case to FI is the sum of three items: the welfare gain/loss from a move from PI(\(\lambda, e\)) to PI(\(\lambda\)) plus the gain/loss from a move from PI(\(\lambda\)) to PI(\(e\)), and lastly from PI(\(e\)) to FI. For brevity, therefore, we present only the minimal number of cases needed to examine all five welfare improvements. The results are given in Table 7. The entries in the Table ask how much welfare would change if we moved according to each specific improvement in information. Given the additivity,
one can directly add the row entries in any given column to see the gains from particular improvement in information.

The most striking finding is that all newborn households are always better off under full information. Our model does not lend support to the view that suppressing information will lead to cross-subsidization that will benefit any group enough, even those that could be described as relatively quite disadvantaged (NHS households), to make it ex ante desirable for them. In fact, summing the entries in the ‘NHS’ column of Table 7, we find that these households gain the most from improvements in information, while the college educated gain the least. Part of the reason that the welfare gains are not huge, and benefit the more skilled by less is that the PI environments still allow for considerable amounts of credit, and the relative contraction in credit supply (see Figure 4) is less important for the skilled than the unskilled. We see that in a move from FI to PI(\(\lambda, e\)), college educated households face lower default premia than their NHS counterparts.

Next, we assess the welfare role played by the non-observability of non-pecuniary costs \(\lambda\) alone, by comparing welfare in the move from PI(\(\lambda, e\)) to PI(\(e\)) and FI (for the particular value of \(\lambda\) under consideration). Here, we focus on College educated households (the results are analogous for other educational types), and see from Figure 1 that when bankruptcy costs are high (i.e. \(\lambda\) is low), credit is far more easily available than when bankruptcy costs are low (i.e. \(\lambda\) is high). Conditional on knowing \(\lambda\), information about the current component of persistent income risk is not vital, as seen by the relative proximity of credit supply in the cases where \(e\) is observable and not-observable. Again, this is evidence that income risk per se can be handled relatively effectively via signaling. Lastly, the fact that the welfare gains for most of the population are very similar could perhaps have been expected from the fact that the importance of \(\lambda\) was already seen in the similarity of aggregates between PI(\(\lambda, e\)) and PI(\(\lambda\)).

The punchline to the preceding results is this: the gains in household welfare from im-
Improvements in information are positive but small. The model suggests that this is due in large part to the ability of signaling to be fairly effective in generating access to credit, and also because the window for which truly unsecured credit is vital for intertemporal consumption smoothing is narrow (young to early-middle age), and this market is most valuable only for those with a significant “hump” in average life-cycle earnings. Nonetheless, for the latter group, average income is also relatively high, making the marginal value of additional consumption smoothing low. By contrast, for the low-skilled households, the intertemporal need to borrow is muted, leaving mainly the intratemporal motivation, i.e. smoothing for those who are unlucky ex-post. By definition, such a group cannot be the majority of households.

5 Concluding Remarks

In this paper, we have developed a model in which improved information held by unsecured creditors on factors relevant for the prediction of default can account for a nontrivial portion of the changes seen in U.S. unsecured credit markets over the past several decades. Our model suggests that borrower signaling can be effective at separating borrowers by their risk characteristics. Quantitatively, a central aspect of our findings is that the power of signaling is likely to be weaker when it is non-pecuniary costs, rather than the persistent component of income, that are unobservable. In terms of welfare, in the model, more information in the unsecured credit market is better for all types, thus providing no support for regulations intended to restrict the use of individual characteristics in credit markets. A technical contribution of our work is an algorithm to compute signaling equilibria with individualized pricing and asymmetric information.

A feature of recent work on consumer default, including the present paper, is that it imposes a type of debt product that may not mimic all the features of a standard unsecured contract offered by real-world credit markets. In our model, individuals issue one-period
bonds in the credit market. As a result, any bad outcome is immediately reflected in the terms of credit, making consumption smoothing in response to bad shocks difficult (credit tightens exactly during the period in which it is most needed); this arrangement would seem to artificially increase the incentive for default. In our defense, credit contracts explicitly permit repricing by the lender at will, but an open question regards the extent to which they use this option.\footnote{In an interview with Frontline in 2004, Edward Yingling, at the time the incoming president of the American Bankers Association, notes that the contract with the credit card company is that “We have a line of credit with you, but we do have the right at any time to say we’re not going to extend that credit to you anymore,” which, by the way, also includes, “We have the alternative to say you are now a riskier customer than we had when we opened the agreement; we have the right to increase the interest rate, because you now have become a riskier customer.” Thus, credit card companies do appear to use this option on occasion.}

Given that credit conditions are typically only adjusted by two events – default or entering the market to either purchase more credit or to retire existing lines – a model of credit lines seems useful to have. This issue is tackled in symmetric information settings by Mateos-Planas (2007) and Tam (2009).

Lastly, the “RIP” income process we use is standard, but future work on bankruptcy will likely benefit from allowing for learning about income. Both consumption dynamics and the usefulness of bankruptcy ultimately hinge on the persistence of risk faced by households. Using the “HIP” specification of Guvenen (2007), where estimated persistence is lower, is therefore a natural next step in assessing the role of consumer default and bankruptcy, and the role of information in altering unsecured credit use.

References


Table 1: Calibration Targets and Model Performance

<table>
<thead>
<tr>
<th>Target</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>Med(Discharge)/Med(US HH Income)</td>
<td>0.1329</td>
<td>0.2688</td>
<td>Sullivan et al. (2000)</td>
</tr>
<tr>
<td>Fraction(Net Worth &lt; 0)</td>
<td>0.1720</td>
<td>0.1250</td>
<td>Chatterjee et al. (2007) &amp; Wolff (2006)</td>
</tr>
<tr>
<td>Agg. NW(NW &lt; 0)/Agg. Income</td>
<td>NHS</td>
<td>0.1432</td>
<td>0.0800</td>
</tr>
<tr>
<td>Agg. NW(NW &lt; 0)/Agg. Income</td>
<td>HS</td>
<td>0.1229</td>
<td>0.1100</td>
</tr>
<tr>
<td>Agg. NW(NW &lt; 0)/Agg. Income</td>
<td>COLL</td>
<td>0.0966</td>
<td>0.1500</td>
</tr>
<tr>
<td>Default Rate</td>
<td>NHS</td>
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<td>1.228%</td>
</tr>
<tr>
<td>Default Rate</td>
<td>HS</td>
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<td>1.314%</td>
</tr>
<tr>
<td>Default Rate</td>
<td>Coll</td>
<td>0.769%</td>
<td>0.819%</td>
</tr>
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Table 2: Calibrated Parameters

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<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
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</thead>
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<tr>
<td>$\beta$</td>
<td>discount rate</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>persist. of non-pec. cost</td>
<td>0.9636</td>
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<tr>
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<td>low val. of $\lambda$ for Non-High-School</td>
<td>0.7675</td>
</tr>
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<td>$\lambda_{NHS}^{hi}$</td>
<td>high val. of $\lambda$ for Non-High-School</td>
<td>0.9088</td>
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<td>$\lambda_{HS}^{lo}$</td>
<td>Low val. of $\lambda$ for High-School</td>
<td>0.7310</td>
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<td>$\lambda_{HS}^{hi}$</td>
<td>high val. of $\lambda$ for High-School</td>
<td>0.9320</td>
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<tr>
<td>$\lambda_{NHS}^{lo}$</td>
<td>low val. of $\lambda$ for College</td>
<td>0.7831</td>
</tr>
<tr>
<td>$\lambda_{NHS}^{hi}$</td>
<td>high val. of $\lambda$ for College</td>
<td>0.9071</td>
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Table 3: Model Performance on Aggregates of Interest

<table>
<thead>
<tr>
<th>Levels</th>
<th>1983</th>
<th>2004</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Data Model</td>
<td>Data Model</td>
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<tr>
<td>BK Rate</td>
<td>0.20%</td>
<td>0.72%</td>
</tr>
<tr>
<td>Rate Variation</td>
<td>7.90%</td>
<td>15.53%</td>
</tr>
<tr>
<td>Good Borrower Discount</td>
<td>-0.22%</td>
<td>1.21%</td>
</tr>
<tr>
<td>Mean Rate Spread</td>
<td>10.08%</td>
<td>8.21%</td>
</tr>
<tr>
<td>Agg. NW(NW &lt; 0)</td>
<td>0.40%</td>
<td>0.42%</td>
</tr>
<tr>
<td>Med(Discharge)/Med(US HH Income)</td>
<td>0.192</td>
<td>0.134</td>
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</table>

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BK Rate</td>
<td>-1.01%</td>
<td>45.54%</td>
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<tr>
<td>Rate Variation</td>
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<td>Good Borrower Discount</td>
<td>-2.45%</td>
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<td>Mean Rate Spread</td>
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<td>96.97%</td>
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<td>Agg. NW(NW &lt; 0)</td>
<td>-0.40%</td>
<td>147.62%</td>
</tr>
<tr>
<td>Med(Discharge)/Med(US HH Income)</td>
<td>-0.077</td>
<td>-1.35%</td>
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Table 4: Unsecured Credit Market Aggregates

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<tr>
<th>Aggregate</th>
<th>FI</th>
<th>PI(ε)</th>
<th>PI(λ)</th>
<th>PI(λ, ε)</th>
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<tr>
<td>Aggregate Debt</td>
<td>0.0104</td>
<td>0.0094</td>
<td>0.0053</td>
<td>0.0042</td>
</tr>
<tr>
<td>Med(Discharge)/Med(US HH Income)</td>
<td>0.1329</td>
<td>0.1351</td>
<td>0.1371</td>
<td>0.1342</td>
</tr>
<tr>
<td>Fraction of Borrowers</td>
<td>0.1720</td>
<td>0.1709</td>
<td>0.1711</td>
<td>0.1493</td>
</tr>
<tr>
<td>Debt/Income Ratio</td>
<td>NHS</td>
<td>0.1432</td>
<td>0.1339</td>
<td>0.1206</td>
</tr>
<tr>
<td>Debt/Income Ratio</td>
<td>HS</td>
<td>0.1229</td>
<td>0.1182</td>
<td>0.0964</td>
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<td>Debt/Income Ratio</td>
<td>COLL</td>
<td>0.0966</td>
<td>0.0944</td>
<td>0.0863</td>
</tr>
<tr>
<td>Default Rate</td>
<td>NHS</td>
<td>1.237%</td>
<td>1.018%</td>
<td>0.809%</td>
</tr>
<tr>
<td>Default Rate</td>
<td>HS</td>
<td>1.301%</td>
<td>1.197%</td>
<td>0.819%</td>
</tr>
<tr>
<td>Default Rate</td>
<td>COLL</td>
<td>0.769%</td>
<td>0.728%</td>
<td>0.638%</td>
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Table 5: Dispersion and Credit Sensitivity: PI and FI

<table>
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<tr>
<th>Levels</th>
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<tbody>
<tr>
<td></td>
<td>Data</td>
<td>PI(λ, e)</td>
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<tr>
<td>$E(r - r^f)$</td>
<td>10.08</td>
<td>8.21</td>
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<tr>
<td>$E(r - r^f</td>
<td>m = 1)$</td>
<td>9.86</td>
</tr>
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<td>$E(r - r^f</td>
<td>m = 0)$</td>
<td>10.08</td>
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<tr>
<td>$Var(r)$</td>
<td>7.90</td>
<td>15.53</td>
</tr>
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<td>$Var(r</td>
<td>m = 1)$</td>
<td>8.68</td>
</tr>
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<td>$Var(r</td>
<td>m = 0)$</td>
<td>7.53</td>
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<table>
<thead>
<tr>
<th>Changes</th>
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<tr>
<td></td>
<td>Data</td>
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</tr>
<tr>
<td>$E(r - r^f</td>
<td>1983)-E(r - r^f</td>
<td>2004)$</td>
</tr>
<tr>
<td>$E(r</td>
<td>m = 1)-E(r</td>
<td>m = 0)$</td>
</tr>
<tr>
<td>$Var(r</td>
<td>m = 1)-Var(r</td>
<td>m = 0)$</td>
</tr>
<tr>
<td>$Var(r</td>
<td>1983)-Var(r</td>
<td>2004)$</td>
</tr>
<tr>
<td>$Var(r</td>
<td>m = 1, 1983)-Var(r</td>
<td>m = 1, 2004)$</td>
</tr>
<tr>
<td>$Var(r</td>
<td>m = 0, 1983)-Var(r</td>
<td>m = 0, 2004)$</td>
</tr>
</tbody>
</table>

Table 6: Distribution of Consumption

|          | E($c$) | Var(log($c$)) | E(Var(log($c$)|age)) | Var(E(log($c$)|age)) |
|----------|--------|---------------|----------------------|----------------------|
| College  |        |               |                      |                      |
| FI       | 1.0255 | 0.1483        | 0.1070               | 0.0413               |
| PI($e$)  | 1.0362 | 0.1546        | 0.1119               | 0.0427               |
| PI($λ$)  | 1.0662 | 0.1545        | 0.1123               | 0.0422               |
| PI($λ, e$) | 1.0730 | 0.1550        | 0.1122               | 0.0428               |
| High School |        |               |                      |                      |
| FI       | 0.7426 | 0.1999        | 0.1683               | 0.0315               |
| PI($e$)  | 0.7532 | 0.2073        | 0.1763               | 0.0310               |
| PI($λ$)  | 0.7684 | 0.2071        | 0.1766               | 0.0305               |
| PI($λ, e$) | 0.7698 | 0.2064        | 0.1756               | 0.0308               |
| Non-High School |        |               |                      |                      |
| FI       | 0.6477 | 0.2603        | 0.2374               | 0.0230               |
| PI($e$)  | 0.6540 | 0.2649        | 0.2422               | 0.0227               |
| PI($λ$)  | 0.6582 | 0.2664        | 0.2429               | 0.0224               |
| PI($λ, e$) | 0.6591 | 0.2644        | 0.2417               | 0.0227               |
Table 7: Information and Ex-Ante Welfare

<table>
<thead>
<tr>
<th></th>
<th>COLL</th>
<th>HS</th>
<th>NHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{PI} (\lambda, e) \rightarrow \text{PI} (\lambda)$</td>
<td>0.029%</td>
<td>0.052%</td>
<td>0.097%</td>
</tr>
<tr>
<td>$\text{PI} (\lambda) \rightarrow \text{PI} (e)$</td>
<td>0.023%</td>
<td>0.030%</td>
<td>0.047%</td>
</tr>
<tr>
<td>$\text{PI} (e) \rightarrow \text{FI}$</td>
<td>0.034%</td>
<td>0.035%</td>
<td>0.065%</td>
</tr>
<tr>
<td>$\text{FI} \rightarrow \text{NBK} (\text{if } \chi = 0)$</td>
<td>1.403%</td>
<td>1.205%</td>
<td>1.103%</td>
</tr>
<tr>
<td>$\text{NBK} (\text{if } \chi = 0) \rightarrow \text{NBK} (\text{if } c &gt; 0)$</td>
<td>3.242%</td>
<td>3.432%</td>
<td>5.045%</td>
</tr>
</tbody>
</table>

Note: $\chi = 0$ represents no adverse expenditure shock.

NBK (if $\chi = 0$): Can only file for bankruptcy if $\chi < 0$
NBK (if $c > 0$): Can only file for bankruptcy if $c \leq 0$.

Figure 1: Information and Credit Supply
Figure 2: Information and the Variance of Loan Interest Rates over the Life-Cycle

Figure 3: Information and the Mean of Loan Interest Rates over the Life-Cycle
Figure 4: Information, Education, and Credit Supply
6 Appendix: Not For Publication

6.1 Computing Partial Information Equilibria

The imposition of conditions on beliefs off-the-equilibrium path makes the computational algorithm we employ relevant for outcomes, and in this section we therefore discuss in some detail our algorithm for computing partial information competitive equilibria. The computation of the full information equilibrium is straightforward using backward induction; since the default probabilities are determined by the value function in the next period, we can solve for the entire equilibrium, including pricing functions, with one pass. The partial information equilibrium is not as simple, since the lender beliefs regarding the state of borrowers influence decisions and are in turn determined by them; an iterative approach is therefore needed.

1. Guess the initial function \( q^0 (b, y, j, m) \)

2. Solve household problem to obtain \( g (b, y, e, \nu, \chi, \lambda, j, m) \), \( f (b', y, e, \nu, \chi, \lambda, j, m) \), and \( d (b', e', \nu', \chi', \lambda') \);

3. Compute \( \Gamma (b, y, e, \nu, \chi, \lambda, j, m) \) and \( \Pr (b, y, e, \nu, \chi, \lambda, j, m) = \Gamma (f (b', y, e, \nu, \chi, \lambda, j, m), y, e, \nu, \chi, \lambda, j, m) \);

4. Locate all debt levels that have zero measure of agents, and locate \( b_{\text{min}} (y, e, \nu, \chi, \lambda, j, m) \), the minimum level of debt observed conditional on the other components of the state vector;

5. Assign off-equilibrium beliefs as described earlier, and for debts exceeding the maximum requested by agents, set \( q^* (b \leq b_{\text{min}}, y, j, m) = 0 \) (that is, borrowing that exceeds any observed triggers default with probability 1);
7. Compute

\[ \pi^{b'} (b', y, e, \nu, \chi, \lambda, j, m) = \sum_{e'} \sum_{\nu'} \sum_{\lambda'} \sum_{\chi'} \pi_{\chi'} (\chi) \pi_{e'} (e | e) \pi_{\nu'} (\nu | \nu) \pi_{\lambda} (\lambda | \lambda) \ d (b', e', \nu', \chi', \lambda'); \]  

(14)

8. Compute \( \Pr (e, \nu, \chi, \lambda | b', y, j, m) \) from \( P (b', y, e, \nu, \chi, \lambda, j, m) \) for each \((b', y, j, m)\), the probability that an individual is in \((e, \nu, \chi, \lambda)\) given observed \((b', y, j, m)\);

9. Compute

\[ \hat{\pi}^b (b', y, j, m) = \sum_{e} \sum_{\nu} \sum_{\lambda} \sum_{\chi'} \pi_{\chi'} (\chi) \pi^{b'} (b', y, e, \nu, \chi, \lambda, j, m) \Pr (e, \nu, \chi, \lambda | b', y, j, m), \]  

(15)

the expected probability of default for an individual in observed state \((b', y, j, m)\);

10. Fill in the “holes” in \( \hat{\pi}^{b'} \) by applying the off-equilibrium beliefs;

11. Compute an “intermediate” price function \( q^*_{\text{int}} \) for all \( b \), that is actuarially fair (competitive) given the preceding estimate \( \hat{\pi}^b \):

\[ q^*_{\text{int}} (b', y, j, m) = \frac{(1 - \hat{\pi}^{b'} (b', y, j, m)) \psi_j}{1 + r + \phi} \]  

for all \( b \geq b_{\text{min}} (b, y, j, m) \);  

(16)

12. Set

\[ q^1 (b, y, j, m) = \Xi q^0 (b, y, j, m) + (1 - \Xi) q^*_{\text{int}} (b, y, j, m) \]

where \( \Xi \) is set very close to 1, return to Step 1, and repeat until the pricing function converges.

Because the household value function is continuous but not differentiable or concave, we solve the household problem on a finite grid for \( a \), using linear interpolation to evaluate it at points off the grid. Similarly, we use linear interpolation to evaluate \( q \) at points off the
grid for $b$. To compute the optimal savings behavior we use golden section search (see Press et al. 1993 for details of the golden section algorithm) after bracketing with a coarse grid search; we occasionally adjust the brackets of the golden section search to avoid the local maximum generated by the nonconcave region of the value function. To calibrate the model we use a derivative-free method to minimize the sum of squared deviations from the targets; the entire program is implemented using OpenMPI instructions over 16 processors.

We now describe the algorithm formally. Let $Q$ denote the compact range of the pricing function $q$; as noted in the main body of the paper, $Q = [0, \frac{1}{1+r}]$. Our iterative procedure maps weakly-monotone functions over $Q$ back into themselves (weakly) monotonically. Pricing functions can never increase in our procedure, so the contraction property will not hold. As a result, both the initial condition and the updating scheme could matter for outcomes (since the equilibrium pricing function cannot be proven to be unique). We have detailed above our approach for selecting the initial condition and the updating procedure; we then set $\Xi$ close enough to 1 that the iterative procedure defines a monotone mapping, ensuring the existence of at least one fixed point. Notice that $q = 0$ is also an equilibrium under certain restrictions on lender beliefs, and is a fixed point of our iterative procedure; if no agent receives any current consumption for issuing debt and lenders' off-equilibrium beliefs are that any debt will be defaulted upon with probability 1, no debt is issued, and intermediaries make zero profit while acting optimally. The key advantage of our initial condition is that it guarantees convergence to the competitive equilibrium which supports the largest amount of borrowing – formally, it generates a descending Kleene chain (Kleene 1952). Importantly, the setting of $\Xi$ close to one ensures that in addition to keeping lenders’ optimistic about those households borrowing off-equilibrium amounts, we also do not allow the effect of any single invariant distribution to alter beliefs significantly in one iteration. In other words, the initial guess (which allowed for maximal debt and the lowest possible interest rate) will continue to retain influence after many iterations. This safeguards against
inevitably reaching the “no-debt” equilibrium.

Our interest in the equilibrium which permits the most borrowing at the lowest rates arises from the fact that such an equilibrium Pareto-dominates all the others (conditional on the monotonicity restriction). In our economy all pricing is individualized and \( r \) is exogenous, meaning that the decisions of one type do not impose pecuniary externalities on any other type (\( \tau \) is dependent only on the invariant and exogenous age and productivity distribution). Thus, we can analyze the efficiency of an allocation individual-by-individual (in an \textit{ex ante} sense). For any individual, the outcome under \( q^0 \) dominates any other, whether they exercise the default option or not, because it maximizes the amount of consumption-smoothing that an individual could achieve. Since \( q \) is a monotone-decreasing function of \( b \), it follows that any allocation which generates higher \( q \) for each \( b \) dominates one with lower \( q \); that is, \( q_1 \succeq q_2 \) (with the natural ordering for functions) implies that allocation 1 Pareto-dominates allocation 2.

**6.2 Calibration: The Roles of \( \lambda \) and \( \Delta \) Under Full Information**

The non-pecuniary cost of bankruptcy, \( \lambda \), plays an important role in allocations. It is meant to capture various aspects of deadweight costs borne by households in bankruptcy. There are two key aspects to this process – the coefficient of variation and the persistence. If one forces \( \lambda \) to remain constant and uniform across households when it is chosen to match the observed filing rate, the model produces a too-small discharge-income ratio and can no longer capture the heterogeneity in default costs implied by the estimates of Fay, Hurst, and White (1998). The calibrated value of \( \lambda \) in this case is also too small, in the sense that the model generates counterfactually-small bankruptcies, and as a result will understate the welfare costs of frequent default. If \( \lambda \) differs across households but is i.i.d, discharge rates remain too low as the average \( \lambda \) needs to be small. Without persistence, no household’s
implicit collateral is expected to be particularly valuable in the next period and thus cannot support large debts. Thus, our calibration allows for the high cross-sectional dispersion and high persistence in \( \lambda \) needed in order to jointly support (i) large risky debts on which default premia are paid, (ii) frequent default, and (iii) relatively large discharges. If we change the average \( \lambda \) in the economy, the effect is to move default rates and discharge levels in opposite directions (see Athreya 2004). Furthermore, changing merely the average \( \lambda \) delivers little change in the dispersion in the terms. Thus, changes in stigma can be dismissed as the force driving all of the changes in the unsecured credit market.

Lastly, we discuss the roles played by the two main transactions costs, \( \Delta \) and \( \phi \). As noted in Livshits, MacGee, and Tertilt (2007) and Athreya (2004), dropping transactions costs can potentially deliver the trends in the default rates and debts observed in the data, so these changes are worth examining as competitor stories. No household in the model would default on any debt less than this cost (i.e., when \( b > -\Delta \)), so higher values of \( \Delta \) can support larger debts in general. Changes in \( \Delta \) only alter the length of the initial flat segment where risk-free borrowing is sustained (Figure 8 plots price \( q \) as a function of borrowing \( b \)). Changes in \( \phi \) only shift the pricing functions up and down (see Figure 8), altering the cost of issuing any given amount of debt. Thus, neither change will alter the variance of interest rates that agents receive, as they affect all agents symmetrically. To get a change in the distribution of interest rates, one needs to generate changes in the slope of the pricing functions. As a result, stories that place falling transactions costs at the heart of the changes in the unsecured credit market cannot account for the homogeneity observed in the earlier period. More details on experiments with \((\lambda, \Delta, \phi)\) are available upon request.

Having given a flavor of how pricing works in the model, we turn now to credit availability under full information; the ‘supply side’ of the credit market is seen most clearly in the pricing of debt facing households in varying states of income. For questions regarding unsecured credit, the young are the most relevant population, and we therefore focus on their access to
credit. Figure 1 displays the pricing functions for college types at age 29 given both the low and high value of $\lambda$; as would be expected the higher the realization of $e$ the more credit is available (at any given interest rate). For low realizations of $e$ the pricing functions look like credit lines – borrowing can occur at a fixed rate (in this case, the risk-free rate) up to some specified level of debt, after which the interest rate goes to $\infty$. For higher realizations the increase in the interest rate is more gradual, meaning that some risky borrowing will occur in equilibrium; for some borrowers, the marginal gain from issuing debt is sufficiently high that they are willing to pay a default premium to do it. The pricing functions for noncollege types look similar but involve higher interest rates at any given level of debt. Similar pictures arise for older agents – they are weakly decreasing in debt with more gradual increases in interest rates for luckier agents. Middle-aged agents (say, age 45) can borrow significantly more than their younger counterparts, although they choose not to do so in equilibrium because they are saving for retirement. Given a low value of $\lambda$ agents can borrow a lot more (as would be expected).

### 6.2.1 Information and the ‘ Causes’ of Bankruptcy

Here, we detail the joint role played by expenditure shocks and non-pecuniary costs of bankruptcy. The results are given in Table 8. Each cell in which the information regime is held fixed presents the joint distribution of expenditure shocks and non-pecuniary costs of those who have filed for bankruptcy. What is clear is that under all information regimes the bulk of filers have low non-pecuniary costs (high $\lambda$). More interestingly, defaulting households typically have not received the largest expenditure shock. Specifically, the median expenditure shock is zero, and yet households with this realization account for more than half of all filers. Perhaps the most interesting finding here is that as that information becomes more limited, we see that the fraction of households in bankruptcy who have a low non-pecuniary cost of filing and have received no expenditure shock grow dramatically. This
result suggests that “strategic bankruptcy” is certainly a real possibility under PI regimes in a way that is more severely restricted under FI. In particular, while the latter group accounts for 46 percent under FI, they are 76 percent of all filers under PI(\(\lambda,e\)). With respect to income, Figure 9 shows that most defaulting households have not experienced extremely bad transitory shocks.

Despite the possibility that the joint distribution of expenditure shocks and non-pecuniary costs suggests some strategic filing, what is still true is that the high expenditure shock is rare, and so may not be seen often among filers for that reason alone. Nonetheless, such a shock may well “push” someone into bankruptcy when it occurs. Table 9 presents the conditional probability of bankruptcy, given a particular constellation of the two variables of interest. It is clear here that the high shock does indeed “cause” a disproportionate amount of bankruptcy, relative to its unconditional likelihood. Moreover, the Table makes clear the power of this shock: conditional on getting this bad expenditure shock, the probability of bankruptcy is not highly sensitive to either the non-pecuniary cost or the information regime that prevails. By contrast, filing probabilities vary much more substantially across information and non-pecuniary costs when the expenditure shocks are smaller.

6.3 Tables

Table 8: Expenditure Shocks and Non-Pecuniary Costs Among Filers

<table>
<thead>
<tr>
<th></th>
<th>FI</th>
<th>PI ((e))</th>
<th>PI ((\lambda))</th>
<th>PI ((\lambda,e))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (\chi)</td>
<td>0.3776 0.0118</td>
<td>0.1664 0.0065</td>
<td>0.0405 0.0000</td>
<td>0.0496 0.0000</td>
</tr>
<tr>
<td>Median (\chi)</td>
<td>0.4621 0.0623</td>
<td>0.6299 0.0755</td>
<td>0.7793 0.0393</td>
<td>0.7608 0.0476</td>
</tr>
<tr>
<td>High (\chi)</td>
<td>0.0515 0.0346</td>
<td>0.0725 0.0492</td>
<td>0.0849 0.0561</td>
<td>0.0853 0.0567</td>
</tr>
</tbody>
</table>

6.4 Figures
Table 9: Expenditure Shocks and Non-Pecuniary Costs as Triggers

<table>
<thead>
<tr>
<th></th>
<th>FI</th>
<th>PI (e)</th>
<th>PI (λ)</th>
<th>PI (λ, e)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High λ</td>
<td>Low λ</td>
<td>High λ</td>
<td>Low λ</td>
</tr>
<tr>
<td>Low χ</td>
<td>0.964%</td>
<td>0.030%</td>
<td>0.385%</td>
<td>0.015%</td>
</tr>
<tr>
<td>Median χ</td>
<td>15.380%</td>
<td>2.071%</td>
<td>18.986%</td>
<td>2.276%</td>
</tr>
<tr>
<td>High χ</td>
<td>26.422%</td>
<td>17.751%</td>
<td>33.728%</td>
<td>22.889%</td>
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</table>

Table 10: Credit Sensitivity

<table>
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<th>b &lt; 0, m = 0</th>
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<tbody>
<tr>
<td></td>
<td>b</td>
<td>r</td>
</tr>
<tr>
<td>Mean</td>
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<td>0.0718</td>
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<td></td>
<td>HS</td>
<td>0.0402</td>
</tr>
<tr>
<td></td>
<td>NHS</td>
<td>0.0310</td>
</tr>
</tbody>
</table>

Figure 5: Labor Productivity over the Life-cycle
Figure 6: Credit Supply Under Full Information

Figure 7: Information and Consumption over the Life-Cycle
Figure 8: The Role of Transactions Costs

Figure 9: The Income Shocks of Defaulters