Robust Policymaking in the Face of Sudden Stops*

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Abstract

This paper considers optimal policy without commitment in an emerging market economy that is subject to occasional Sudden Stops – declines in aggregate activity that are magnified by a binding collateral constraint. The key question is whether interventions before a crisis (prudential) or during (mopping up) are appropriate. I find that prudential policymaking is an equilibrium only if (i) the government has a limited set of tools; (ii) the government faces model uncertainty that the private sector does not; and (iii) land is a source of collateral. The first case involves a welfare loss relative to expanding the number of policy instruments and the second involves a welfare loss relative to the competitive equilibrium, so only the third example justifies prudential policymaking.

Keywords: Robustness, Model Uncertainty, Optimal Taxation, Sudden Stops.

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1. Introduction

Emerging market economies frequently experience episodes – called Sudden Stops – in which output falls, the terms of trade deteriorate, and current accounts erode significantly. Figures (1) and (2) show one country – Mexico – that experienced a Sudden Stop in 1995 (the Tequila crisis). The ratio of the current account to GDP went from −5 percent to 4 percent in one quarter, and the trade balance went from −6 percent to 0. At the same time, output and consumption growth both collapsed – both were below −10 percent for the quarter (annualized). Mexico is far from alone in experiencing these kinds of large events, and the recent financial crisis has a similar flavor.

A recent debate has arisen over the nature of optimal policy in economies that face Sudden Stops. Contributions to this debate include Bianchi (2010), Bianchi and Mendoza (2011), Benigno et al. (2011a,b,c,d), and Jeanne and Korinek (2011a,b). There are two primary issues that the literature has raised. First, what are the appropriate tools to combat Sudden Stops? The papers cited above focus on two instruments: capital controls (Tobin taxes on new debt, see Yashiv 1997,1998) and exchange rate interventions (subsidies to nontradable sectors). Second, when should the government intervene? Here, the literature comes to two different conclusions; some papers (in particular Jeanne and Korinek 2011b) argue for ”prudential” interventions that take place before the Sudden Stop, whereas others (Benigno et al. 2011c) argue for interventions only during the crisis (“mopping up” policies). The purpose of this paper is to examine this second question more closely.

Since Sudden Stops are contained in the ergodic set of the economy, policy must be specified for both normal and crisis periods. Naturally, the two are connected – policy in normal times will influence the probability and magnitude of a Sudden Stop, and policy during the Sudden Stop will influence the behavior of the economy in normal times.¹ Benigno et al. (2011c) studies this issue, computing optimal time-consistent policy rules both in and out of a crisis state, in a simple model of an emerging economy subject to TFP shocks in the tradable goods sector, debt denominated in tradable goods, and a constraint that limits borrowing to a multiple of current income (which crucially depends endogenously on the price of nontraded output); the paper considers both capital controls and exchange rate interventions. The key result from that paper is that prudential interventions are only an equilibrium if the number of instruments is limited to one (although it

¹Some papers in the literature consider Sudden Stops as zero probability events, such as Braggion, Christiano, and Roldos (2009), Hevia (2007), and Cúrdia (2009), and investigate the optimal policy once one is in a crisis. Obviously these papers have nothing to say about policies intended to prevent the crisis in the first place.
does not matter which instrument); if the government has both tools it waits until the crisis (defined loosely as a period in which the collateral constraint binds) to intervene.

The first question taken up here is the robustness of the key results in Benigno *et al.* (2011c) to a model with additional shocks that are empirically important for emerging market economies, namely shocks to trend productivity and the real interest rate (see Aguiar and Gopinath 2007 and Neumeyer and Perri 2005 for evidence). These results go through without modification – the optimal time-consistent policy is to subsidize nontradable consumption and tax new debt only when the constraint is binding.\(^2\) However, if the government is restricted to only one instrument, then policy becomes active before the constraint is binding. With one tool the government cannot achieve both goals in the binding region, namely to discourage debt and support the relative price of nontradables. As a result, it uses the one tool it has outside the binding region to reduce the probability that the constraint binds and then uses the tool inside the binding region to correct the relevant distortion.

The second question is whether the model’s predictions are robust to a fear of model mis-specification (model uncertainty), particularly on the part of the government. I assume that the stochastic process generating the shocks to tradable TFP, trend productivity, and the interest rate are not completely trusted by the agents in the economy, and thus they twist their expectations in order to guard against the worst process contained within some specified set (as in Hansen and Sargent 2007). Since crisis events are rare, it seems reasonable to investigate how policy should be set when agents recognize this fact; for example, the model is roughly calibrated to Mexican data so that the crisis probability is estimated using a single event (the Tequila crisis). Obviously, error bounds on such a probability would be rather large. Combining all countries together to calculate a Sudden Stop probability, as in Calvo and Reinhart (2000) or the disaster literature (Barro and Ursúa 2011), obviously carries some potential for problems as well, particularly if the events that drive the crisis are likely to be country-specific or are unlikely to reoccur.

A desire for robust decision rules distorts the intertemporal relationship between the marginal utility of consumption today and tomorrow. As a result, the potential gains from intervening along this dimension – that is, for using the capital control aggressively before the constraint binds – could increase. However, robustness acts as a form of risk aversion (as noted in Tallarini 2000).\(^3\) As a

\(^2\) In a deterministic model it is easy to show that mopping up is the appropriate policy perspective; in a stochastic model precautionary principles could imply prudent interventions, but for the benchmark model they do not.

\(^3\) In fact, Tallarini (2000) shows that robust decision-making implies that an agent has a restricted form of the preferences from Epstein and Zin (1989); the restriction is that the intertemporal elasticity of substitution is unity.
result, the economy may endogenously move away from the region where marginal gains are large, rendering welfare gains small again. The main result from these experiments is that there is still no scope for prudential intervention if both households and the government fear misspecification, but if only the government faces model uncertainty it intervenes aggressively in a prudential manner. The welfare gains from optimal policy are small if both agents want robust decision rules (and actually get smaller as the desire for robustness increases, reflecting the precautionary response of saving), but are large and negative if only the government does. The surprising part of the second result is not the sign, which is to be expected given that the government is not maximizing the subjective utility of the household, but rather the magnitude: if the government desires even modest levels of robustness and households do not, welfare losses are orders of magnitude larger than the gains when both desire the same amount of robustness.

I then do a number of "robustness" checks in this model (the alternative sense of robustness, meaning insensitivity to model modifications). I first investigate the results when the government is required to finance government spending with distortionary taxation (in particular, a tax on labor income). It turns out that the benchmark policy involves almost no lump-sum taxation; that is, the costs incurred by subsidizing the nontraded sector are almost completely covered by the tax on new bonds. As a result, the optimal policy when lump-sum taxes are permitted looks very similar to that when only distortionary finance is available. These results are important because they imply that optimal interventions can be achieved without significant fiscal implications.

Second, I consider environments in which households can pledge their land holdings along with income as collateral, as in Bianchi and Mendoza (2011). Similar to Jeanne and Korinek 2011b, the optimal policy is to intervene before the crisis using the capital control; specifically, to tax new bonds to inhibit debt accumulation. The intuition here is that the pricing equation for land is dynamic, rather than static as with the price of nontradable goods. As a result, constraints that may bind in the future will directly affect borrowing constraints today, giving the government an incentive to intervene whenever the constraint will bind tomorrow with positive probability; specifically, the government will generally tax new debt in this region. When the constraint actually binds today, the optimal policy is to intervene using both tools; this policy can involve subsidizing rather than taxing new debt, however, and does come with modest financing requirements.

There is by now an extensive literature on optimal fiscal policy under model uncertainty; representative examples include Karantounias, Hansen, and Sargent (2009), Carvalho (2005), and Svec (2011a,2011b), who consider problems with commitment, and Luo, Nie, and Young (2011c)
who investigates tax-smoothing in the Barro (1979) sense. There is also a number of papers on monetary policy with model uncertainty, including Adam and Woodford (2011) and Dennis (2010). Finally, the work here connects more tenuously to optimal policy with sovereign debt in open economies, both with model uncertainty (Costa 2009, Pouzo and Presno 2011) and without (Aguir and Amador 2011a,b). While there are more examples than these, I refrain from a tedious listing and apologize to any authors whose work has not been referenced.

2. Model

2.1. Household Decision Problem

The model economy is populated by a continuum of identical households. Each household has identical preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\rho} \left( [\omega c_{T,t}^\kappa + (1-\omega) c_{N,t}^\kappa]^{\frac{1}{\kappa}} - \frac{\Gamma_{t-1} h_t^\delta}{\delta} \right)^{1-\rho}$$

(2.1)

where \( \rho \geq 0 \) is the coefficient of relative risk aversion (the inverse of the intertemporal elasticity of substitution), \( \beta < 1 \) is the pure time discount factor, \( \frac{1}{1+\kappa} \) is the intratemporal elasticity of substitution between traded \( (c_T) \) and nontraded \( (c_N) \) consumption goods, \( \omega \in (0,1) \) is a share parameter, \( h \) is labor effort, \( \frac{1}{\kappa} \geq 1 \) is the Frisch elasticity of labor supply, and \( E_0 \) is an expectation operator to be defined below; for consistency with the numerical section I will assume \( \kappa \) is such that \( c_T \) and \( c_N \) are complements, and of course the utility function used here is known to have zero wealth effect on labor supply.\(^4\) \( \Gamma \) is the trend component of TFP:

$$\Gamma_t = \exp (g_t) \Gamma_{t-1}$$

(2.2)

for random variable \( g_t \); because these preferences do not balance wealth and substitution effects caused by permanent wage increases, the household must "enjoy" leisure more as productivity grows. Normalizing by \( \Gamma_{t-1} \) yields an effective discount factor of

$$\beta^* (g_t) = \beta \exp (g_t (1-\rho)) \cdot \beta^* (g_t (1-\rho))$$

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\(^4\)See Greenwood, Hercowitz, and Huffman (1988). The so-called GHH preferences are used to increase the volatility of labor supply.
Households face a budget constraint of the form

\[ c_{T,t} + (1 + \tau_{N,t}) p_{N,t} c_{N,t} + \frac{(1 + \tau_{B,t}) b_{t+1}}{1 + r_t} \leq b_t + w_t h_t + D_t + T_t \]  

(2.3)

and a collateral constraint of the form

\[ \frac{b_{t+1}}{1 + r_t} \geq -\varphi (w_t h_t + d_t), \]

(2.4)

where \( b \) is the current net foreign asset position of the household, \( D \) is the dividend from owning the firms, \( w \) is the wage, \( p_N \) is the relative price of nontradable goods, \( r \) is the random real world interest rate, \( \tau_i \) (\( i = N, B \)) are flat taxes/subsidies on nontraded consumption spending and new debt respectively, and \( T \) is a lump-sum tax/transfer. Normalizing the constraints by \( \Gamma_{t-1} \) yields

\[ c_{T,t} + (1 + \tau_{N,t}) p_{N,t} c_{N,t} + \frac{\exp (g_t) (1 + \tau_{B,t}) b_{t+1}}{1 + r_t} \leq b_t + w_t h_t + D_t + T_t \]  

(2.5)

and

\[ \frac{\exp (g_t) b_{t+1}}{1 + r_t} \geq -\varphi (w_t h_t + D_t), \]

(2.6)

where all variables going forward are understood to be the normalized quantity. The fact that debt is denominated in tradable goods (foreign currency) implies that the economy faces a "liability dollarization" problem, as discussed in Krugman (1999) and Aghion, Bacchetta, and Banerjee (2004); it is rare that emerging economies can borrow externally in their own currency, so this assumption is consistent with empirical observations, but it may not be entirely innocuous in terms of policy implications.

2.2. Firm Decision Problem

There are two stand-in firms in the domestic economy.\(^5\) One firm produces tradable output, the other produces nontradable output, according to the production technologies

\[ Y_{T,t} = \exp (z_t) H_{T,t}^\alpha, \]

(2.7)

\[ Y_{N,t} = A H_{N,t}^\theta. \]

(2.8)

\(^5\)It is equivalent to think of the model as populated by a single stand-in firm that produces two goods using different technologies.
$z_t$ is a persistent but stationary shock to productivity in the tradable sector, $\alpha < 1$ and $\theta < 1$ are labor’s share of income in each sector, and $A$ is a normalization constant. The dividends for firm ownership paid to the households are

$$D_t = Y_{Tt} - w_t H_{Tt} + p_{Nt} Y_{Nt} - w_t H_{Nt}.$$  

### 2.3. Competitive Equilibrium

Market clearing for the consumption goods requires

$$C_{T,t} = Y_{T,t} + B_t - \frac{\exp (g_t) B_{t+1}}{1 + r_t}$$  \hspace{1cm} (2.9)$$

$$C_{N,t} = Y_{N,t}.$$  \hspace{1cm} (2.10)$$

The efficient allocation of labor across sectors yields

$$w_t = \alpha \exp (z_t) H_{T,t}^{\alpha - 1} = \theta p_{N,t} A H_{N,t}^{\theta - 1}.$$  \hspace{1cm} (2.11)$$

In equilibrium it must be the case that aggregate and individual allocations coincide

$$h_t = H_{T,t} + H_{N,t} \hspace{1cm} (2.12)$$

$$b_t = B_t \hspace{1cm} (2.13)$$

$$c_{T,t} = C_{T,t} \hspace{1cm} (2.14)$$

$$c_{N,t} = C_{N,t}, \hspace{1cm} (2.15)$$

and the government budget constraint

$$T_t = \tau_{N,t} p_{N,t} C_{N,t} + \tau_{B,t} \frac{\exp (g_t) B_{t+1}}{1 + r_t} \hspace{1cm} (2.16)$$

must also be satisfied.

### 2.4. Recursive Formulation with Fear of Misspecification

Following Hansen and Sargent (2007), households may not trust the probability model they use to predict the movements of the random variables in the model ($z$, $g$, and $r$). Here, I will represent this distrust using multiplier preferences; Strzalecki (2009) provides the axiomatic foundations for
multiplier preferences and connects them to a wide range of other representations of uncertainty-averse preferences. A household who desires robustness solves the minmax problem

\[
v(b, B, z, g, r) = \max_{b', c_T, c_N, h} \min_{m'} \left\{ \frac{1}{1-\rho} \left( [\omega c_T^e + (1 - \omega) c_N^e]^{\frac{1}{\rho}} - \frac{1}{2} h^\delta \right)^{1-\rho} + \beta \exp (g (1 - \rho)) E_\pi \left[ m' v (b', B', z', g', r') - \frac{1}{\varsigma} m' \log (m') \right] z, g, r \right\}
\]

where \( m' \) is the increment to the probability distortion \( M \) and

\[
M' = m' M
\]

\[
E [M | z, g, r] = 1.
\]

Note that \( M \) is not a state variable in the recursive problem because the value function is homogeneous of degree 1 in \( M \), meaning it can be dropped.\(^6\) \( \varsigma \leq 0 \) governs the strength of the demand for robustness and \( E_\pi \) is the standard expectation operator with respect to the empirical density \( \pi \). As shown in the appendix (or in Backus, Routledge, and Zin 2005), solving the minimization yields the utility recursion

\[
v(b, B, z, g, r) = \max_{b', c_T, c_N, h} \left\{ \frac{1}{1-\rho} \left( [\omega c_T^e + (1 - \omega) c_N^e]^{\frac{1}{\rho}} - \frac{1}{2} h^\delta \right)^{1-\rho} + \beta \exp (g (1 - \rho)) \log (E_\pi [\exp (\varsigma v (b', B', z', g', r') | z, g, r)]) \left[ m' v (b', B', z', g', r') \right] z, g, r \right\}.
\] (2.17)

### 2.5. Recursive Competitive Equilibrium

For now, suppose that all taxes are state-independent, and denote the state vector by \( S \equiv (B, z, g, r) \). A recursive competitive equilibrium in this model is then characterized as a solu-

\(^6\)This property will not hold for the government’s problem unless both households and the government distrust the model to the same degree, or only the government distrusts the model.
tion to the following system of functional equations:

\[ u_1 (S) = \lambda (S) \]  \hspace{1cm} (2.18)

\[ u_2 (S) = (1 + \tau_N) p_N (S) \lambda (S) \]  \hspace{1cm} (2.19)

\[ u_3 (S) = w (S) \left( \lambda (S) + \varphi \max \{ \mu (S), 0 \} \right) \]  \hspace{1cm} (2.20)

\[ \frac{(1 + \tau_B) \lambda (S) - \max \{ \mu (S), 0 \}^2}{1 + r} = \beta \exp (-g \rho) \frac{E \pi [\exp (\varsigma V (S')) \lambda (S') | S]}{E \pi [\exp (\varsigma V (S' | S)) | S]} \]  \hspace{1cm} (2.21)

\[ \max \{ -\mu (S), 0 \}^2 = \frac{\exp (g) B^t (S)}{1 + r} + \varphi (w (S) (H_T (S) + H_N (S)) + D (S)) \]  \hspace{1cm} (2.22)

\[ w (S) = \alpha \exp (z) H_T (S)^{\alpha - 1} \]  \hspace{1cm} (2.23)

\[ w (S) = \theta p_N (S) A H_N (S)^{\theta - 1} \]  \hspace{1cm} (2.24)

\[ D (S) = (1 - \alpha) \exp (z) H_T (S)^{\alpha} + (1 - \theta) p_N (S) A H_N (S)^{1 - \theta} \]  \hspace{1cm} (2.25)

\[ C_T (S) = B + \exp (z) H_T (S)^{\alpha} - \frac{\exp (g) B^t (S)}{1 + r} \]  \hspace{1cm} (2.26)

\[ C_N (S) = A H_N (S)^{\theta} \]  \hspace{1cm} (2.27)

\[ T (S) = \tau_N p_N (S) C_N (S) + \tau_B \frac{\exp (g) B^t (S)}{1 + r} \]  \hspace{1cm} (2.28)

where \( u_i \) denotes the derivative of the period utility function with respect to its \( i \)th argument, \( \lambda \) is the Lagrange multiplier on the budget constraint, and \( \mu^* = \max \{ \mu, 0 \}^2 \) is the Lagrange multiplier on the collateral constraint. The equations use the trick from Garcia and Zangwill (1981) that converts the complementary slackness conditions into a single nonlinear equation.\(^7\) The equilibrium value function is defined as \( V (B, z, g, r) \equiv v (B, B, z, g, r) \).

The expectation of future marginal utility is distorted by the ratio

\[ \frac{\exp (\varsigma V (B', z', g', r'))}{E \pi [\exp (\varsigma V (B', z', g', r')) | z, g, r]} \]

which means that states in which continuation utility is low are upweighted relative to those in which continuation utility is high. Defining

\[ p (z', g', r' | z, g, r) \equiv \frac{\exp (\varsigma V (B', z', g', r'))}{E \pi [\exp (\varsigma V (B', z', g', r')) | z, g, r]} \pi (z', g', r' | z, g, r) \]

\(^7\)Existence and uniqueness of solutions is an outstanding question. It seems reasonable to expect that the monotone-operator methods from Datta \textit{et al.} (2005) could be adapted to this question, but it lies well beyond my goals here.
the Euler equation can be rewritten as

$$\frac{\lambda(B, z, g, r)}{1 + r} = \beta E_p \left[ \lambda \left( B'(B, z, g, r), z', g', r' \right) \bigg| z, g, r \right] + \frac{\max \{\mu(B, z, g, r), 0\}^2}{1 + r}.$$

As $\varsigma \to 0$ $p(z', g', r'| z, g, r) \to \pi(z', g', r'| z, g, r)$, so that the agent’s probability distribution is not distorted.\(^8\)

### 2.5.1. The Effect of a Binding Constraint

The crucial equations are (2.20) and (2.21), the intratemporal tradeoff between consumption and labor and the intertemporal Euler equation. Consider first the static condition (2.20); if the constraint is not binding ($\mu^* = 0$), then the marginal rate of substitution between tradable consumption and leisure is equated to the wage. If $\mu^* > 0$, however, the marginal utility from consumption will be too low, implying that total labor effort will be excessively high in this region. To see this, let

$$\hat{\mu} \equiv \max \{\mu(S), 0\}^2 \left( \frac{1}{[\omega C_T^\varsigma + (1 - \omega) C_N^\varsigma]^{\frac{1}{\delta}} - \frac{1}{\delta} (H_T + H_N)^\delta} \right)^{-\rho}$$

be the multiplier on the collateral constraint normalized by marginal utility. Then (2.5.1) can be rearranged to obtain

$$H_T + H_N = \left( \frac{\partial C}{\partial C_T} + \hat{\mu} \varphi \right)^{\frac{1}{\delta-1}} w^{\frac{1}{\delta-1}}.$$

The right-hand-side is increasing in $\hat{\mu}$, so that a binding constraint implies high labor effort; with the utility function employed here low leisure means low aggregate consumption.

Now consider the Euler equation (2.21). For notational simplicity ignore model uncertainty, so that the Euler equation (2.21) becomes

$$\frac{(1 + \tau_B) \lambda(S)}{1 + r} = \beta \exp(-g \rho) E_{\pi} \left[ \lambda \left( S'(S) \right) \bigg| S \right] + \frac{\max \{\mu(S), 0\}^2}{1 + r}.$$

A binding constraint implies that the marginal utility of tradable consumption today is too high, so that tradable consumption is low; with complementarity between $C_T$ and $C_N$, nontradable consumption will also be low.

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\(^8\) As $\varsigma \to -\infty$, the recursion approaches the minmax utility functional from Epstein and Schneider (2003).
2.6. Government Problem

The emerging market economy is governed by a sequence of policymakers that cannot commit to future actions; I also assume that reputational mechanisms are completely ineffective and thus confine myself to Markov-perfect equilibria that depend only on the aggregate state vector \((B, z, g, r)\).\(^9\)

The current policymaker solves the problem

\[
W(B, z, g, r) = \max_{\psi_p, \psi_g} \left\{ \frac{1}{1-\rho} \left( \frac{\kappa}{\rho} C_T^\kappa (1-\omega) C_N^\kappa \right)^{\frac{1}{\kappa}} - \frac{1}{\delta} (H_T + H_N)^\delta \right\}^{1-\rho} + \beta \sigma \exp \left( g (1-\rho) \right) \log \left( E \pi \left[ \exp \left( \sigma W(B', z', g', r') \right) | z, g, r \right] \right) \tag{2.29}
\]

subject to the conditions for a competitive equilibrium (2.18)-(2.28), where \(\psi_g = (\tau_N, \tau_B)\) is the vector of policy tools and \(\psi_p = (C_T, C_N, H_T, H_N, \lambda, \mu, B', V, p_N)\) is the vector of competitive equilibrium objects. Note that the government may have a different preference parameter for robustness (\(\sigma\)) than the private sector does (\(\varsigma\)).

In the government problem, if \(\tau_N < 0\) then the government is subsidizing the consumption (and therefore also the production) of nontraded goods. More subtly, if \(\tau_B < 0\) the government is taxing new debt. To see why, consider a lending market in which the domestic household is issuing one new bond. In exchange for the new bond, which is a promise to repay one unit of tradable output tomorrow, the household receives \(\frac{1+\tau_B}{1+r}\) units of tradable output today (ignoring the normalization); therefore, if \(\tau_B < 0\) the household receives fewer resources today in exchange for the same amount of promised repayment. That is, the government is taxing the new issue.

Note that the Euler equation (2.21) contains future control variables (\(\lambda'\) and the continuation value function) which the current government must take as given (as a function of future states); a Markov perfect equilibrium results when the current government chooses the same functions today as are taken as given for tomorrow. Note also that the presence of the occasionally-binding constraint means that, in general, the policy functions are continuous but not differentiable; as a result, the smooth equilibrium of Klein, Krusell, and Ríos-Rull (2009) does not exist, so I look for equilibria in which the functions are only required to be continuous.\(^10\) The appendix contains a discussion of the computational method used to solve the model.

\(^9\)See Klein, Krusell, and Ríos-Rull (2009).

\(^10\)If the equilibrium for a finite horizon is unique, then my method selects the unique infinite horizon equilibrium that is the limit of these finite horizon equilibria. Other equilibria that cannot be reached as the limit of finite horizon equilibria may exist; see Klein, Krusell, and Ríos-Rull (2009) or Young (2006) for related examples of multiplicity. Jeanne and Korinek (2011a) discuss multiplicity of the finite horizon equilibria in a simple, but related, model. Multiple equilibria for a finite horizon can arise in this model if \(\kappa\) is close enough to 1 (meaning \(C_T\) and \(C_N\) are very close substitutes).
A formal definition for the optimal policy equilibrium will now be given; for notational simplicity I use the "compact" definition defined in Krusell (2002). The "noncompact" definition involves indexing the competitive equilibrium by taxes today given tax functions for the future. Hopefully omitting a formal statement of this equilibrium does not leave the reader too confused.

Definition 1. A Markov-perfect equilibrium for the government is (i) a government value function \( W : B \times Z \times G \times R \rightarrow R \), (ii) policy functions \( \psi_g : B \times Z \times G \times R \rightarrow R^2 \), and (iii) equilibrium functions \( \psi_p : B \times Z \times G \times R \rightarrow R^3 \) such that \((W, \psi_g, \psi_p)\) satisfy the recursion (2.23) and the competitive equilibrium conditions (2.18)-(2.28).

Essentially, a Markov-perfect equilibrium requires that the government’s choices for \( \psi_p \) be consistent with an equilibrium when policies are set using \( \psi_g \); because the government takes the future functions \((\lambda, V)(B', z', g', r')\) as fixed, the Markov-perfect requirement bites by requiring that the government find it optimal today to use the same functions it takes as given for tomorrow.

2.7. Welfare

I calculate the welfare change of a given set of policy instruments relative to the competitive equilibrium with zero taxes. The percent increase in tradable consumption that makes an agent in the no-tax competitive equilibrium unwilling to move to the equilibrium of the policy game is given by the solution \( \chi(B, z, g, r) \) to the equation

\[
V(B, z, g, r) = V^*(B, z, g, r; \chi(B, z, g, r))
\]

where

\[
V^*(B, z, g, r; \chi) = u((1 + \chi)C_T, C_N, H) + \frac{\beta}{\varsigma} \exp(g(1 - \rho)) \left[ \log \left( E_\pi \left[ \exp \left( \varsigma V^* (B', z', g', r'; \chi) \right) \right] (z, g, r) \right) \right] ;
\]

the decision rules are evaluated using the competitive equilibrium functions with no taxes. This policy measure therefore takes into account any transient dynamics associated with moving toward the stationary distribution of the optimal tax allocation.

Note that this welfare measure need not deliver positive values for \( \chi \), since the policy functions are the equilibrium of a game and not a single-agent optimization problem; for example, doing nothing may not be an equilibrium even if \( \chi(B, z, g, r) < 0 \) at some point in the state space.
Obviously, if $\sigma \neq \varsigma$ the equilibrium policies chosen by the government may not improve welfare, at least not as measured using the agent’s subjective utility function. Nevertheless, I will generally refer to the optimal policies being chosen, since they are optimal from the perspective of the current government. Average welfare is computed by integrating $\chi$ with respect to the stationary distribution from the no-tax competitive equilibrium.

3. Parametrization

I do not attempt to calibrate the robustness parameters. The detection error probability approach – advocated in Hansen and Sargent (2007) and used, for example, in Luo, Nie, and Young (2011a, 2011b) or Bidder and Smith (2011) – is too computationally costly given the nonlinearities inherent in the model. Therefore, I will roughly calibrate the other parameters of the model and examine how changes in $\varsigma$ and $\sigma$ change the optimal policies; I will also recalibrate the model for given $\varsigma$ and check whether any conclusions change.

I set $\rho = 1$ (logarithmic preferences) and $\delta = 1.75$ (a Frisch elasticity of 1.5). $\alpha = \theta = 0.66$ is used to ensure that labor’s share of income is consistent with Gollin (2002). $\kappa = -0.32$ ensures an elasticity of substitution consistent with Mendoza (2002). I assume $z_t = 0.690z_{t-1} + 0.0083\varepsilon^z_t$

$g_t = 0.524g_{t-1} + 0.0138\varepsilon^g_t$

$\varepsilon^z \sim N(0,1)$

$\varepsilon^g \sim N(0,1)$

and approximate each process as a Markov chain with 7 and 9 states, respectively, using the procedure from Rouwenhorst (1995) and Kopecky and Suen (2010); these processes do a reasonable job matching the volatility of output and consumption growth (although consumption growth is a bit more volatile than in Mexican data). The parameters for these processes, as well as $\beta = 0.965$, $\varphi = 1.6667$, and $\omega = 0.350$, generate a debt/income ratio of $-36$ percent, a frequency of Sudden Stops (defined as a period in which a reduction in debt exceeds two standard deviations and the constraint is strictly binding) of $2.1$ percent, and a ratio of nontradable to tradable consumption

---

11I use more points for $g$ because it generates more nonlinearity in the model. The answers are insensitive to increasing the number of points in each Markov process, but not to approximating the model using a different method, such as that advocated by Flodén (2009).
of 1.66.

The interest rate follows a two-state Markov chain with realizations \( \{r_1, r_2\} \) and transition matrix \( \Pi_r \), where

\[
\begin{align*}
  r_1 &= 1.0548 \\
  r_2 &= 1.144
\end{align*}
\]

(both annualized) and

\[
\Pi_r = \begin{bmatrix}
  0.993 & 0.007 \\
  0.333 & 0.667
\end{bmatrix}.
\]

Interest rate shocks therefore occur roughly 2 percent of the time in the stationary distribution, and last on average 1.5 quarters.\(^{12}\) Table 1 summarizes the parameters for the benchmark model.

The model’s predictions with respect to the main question – the timing and nature of optimal interventions – are insensitive to the parameters that are chosen; for example, I can vary the frequency of Sudden Stops, the size of the nontraded sector relative to the traded sector, the average level of debt (provided it remains everywhere negative), the elasticity of substitution between traded and nontraded goods, and the elasticity of labor supply significantly without changing them qualitatively (or quantitatively to any important degree).

4. Results

The results section is divided into four main parts. First, I summarize the nature of optimal policy when \( \varsigma = \sigma = 0 \); that is, we are in a world of rational expectations where the approximating model is fully trusted (and is assumed also to be the true model). Second, I examine how optimal policy changes when \( \sigma < 0 \), so that the government does not trust the model but the households do. Third, I examine cases where \( \varsigma = \sigma < 0 \), so both the households and the government distrust the model (but do so equally). Finally, I do some sensitivity analysis (robustness in another sense) by (i) replacing the lump-sum tax/transfer with a distortionary tax on labor income and (ii) permitting households to use land holdings as additional collateral.

\(^{12}\) These two values are the average interest rate in Mexico in "normal" times and in the four quarters surrounding the Tequila crisis.
4.1. The RE Model

I begin with the basic model where $\zeta = \sigma = 0$, so that all agents trust their model of the stochastic process. This version of the model is the same as in Benigno et al. (2011c), but extended to include the $r$ and $g$ shocks. To give the reader some intuition about how the model works, Figure (3) shows the process of a Sudden Stop. The solid lines are the equilibrium functions for $B'$ for fixed values of the exogenous states; specifically, for the highest and lowest values of $z$, given the mean value of $g$ and the low value for $r$. The dashed lines are the collateral constraints for these shock values; where the dashed line disappears into the solid line is where the constraint begins to bind. Note first that both decision rules lie below the 45-degree line when the constraint is not binding; because $\beta (1 + r_1) < 1$, the agent would like to bring consumption forward in time and accumulate debt. At low values of $z$ the household wants to borrow more than at high $z$ values, for standard consumption-smoothing purposes. However, the collateral constraint for the low $z$ state is tighter than for the high $z$ state, implying that the household cannot do so indefinitely. Thus, a more indebted economy may actually have higher productivity; indeed, point A corresponds to the highest productivity state and the maximal indebtedness this economy can achieve.

To understand the dynamics of a Sudden Stop, suppose the economy has reached point A (it has received the highest $z$ value for a very long time). Now suppose there is a negative shock; for the ease of presentation, this shock consists of a shift from the highest to the lowest value of $z$, but the intuition holds for smaller changes. The economy shifts to the 'Low $z$' decision rule, forcing debt to contract immediately to point B. If this contraction is large enough – that is, if the decision rules are sufficiently far apart at A – then the economy experiences a crisis event (remember that a sudden stop is defined as a reduction in debt that exceeds two standard deviations). If the economy remains in the low productivity state indefinitely, it will converge to point C.

To give the reader further insight into the effects of a binding constraint, Figure (4) plots the key endogenous variables (tradable and nontradable labor supply, tradable consumption, and the price of nontraded goods). The kinks correspond to the debt level (for the given shock values) at which the constraint begins to bind; as the economy approaches a debt level at which the constraint will bind, labor in the nontraded sector is falling (due to a decline in $p_N$) and labor in the traded sector is rising. But when the constraint binds, labor effort in both sectors rise due to an incentive to shift the constraint outward; the result is a large drop in the relative price of nontraded goods. The drop in $p_N$ enters into the collateral constraint and causes it to become
tighter, setting off a debt-deflation mechanism (because debt is denominated in tradables, the drop in $p_N$ actually raises the value of the existing debt).

Figure (10) graphs the optimal policies when both instruments are available – it is rather clear that policy is inactive until the constraint binds, and then does two things. First, the government subsidizes nontradable consumption (that is, $\tau_N < 0$); second, it taxes new bonds (that is, $\tau_B < 0$). Although not shown, the equilibrium $T$ function is nearly zero everywhere, showing that the optimal policy does not demand unreasonable levels of lump-sum taxation (or foreign reserve accumulation) to finance. The intuition for these two actions are discussed extensively in Benigno et al. (2011c), but I will quickly summarize them here. In general, an efficient allocation involves more debt and less labor supply than obtains in the competitive equilibrium. In order to discourage excess labor supply and to permit more borrowing, the government would like to shift the borrowing constraint to the left; by subsidizing nontraded production, the government raises $p_N$ and permits the household to borrow more without resorting to additional labor supply. However, there will now be too much borrowing in the sense of Bianchi (2010), so $\tau_B$ is employed to reduce debt (see Jeanne and Korinek 2011 for a simple discussion of this point).13

Figure (4) shows how the endogenous variables behave when policy is set optimally. The government defends the real exchange rate via the subsidy to nontraded consumption, resulting in a $p_N$ function that is nearly flat in the binding region. This subsidy prevents the debt-deflation spiral from taking place. The government also prevents the shift of consumption from the nontraded to the traded sector, allowing $C_T$ to fall more than it would otherwise. As discussed in the Appendix, the constrained-efficient allocation involves $p_N$ actually rising (see Benigno et al. 2011c) in the constrained region; the government cannot achieve the constrained efficient allocation given the combination of the commitment problem and the limited set of tools available here, but it does attempt to move in the right direction.

Figure (7) shows the dynamics from Figure (3) under optimal policy. The decision rules for the OP problem are nearly independent of $z$ in the constrained region and equal to those from the CE in the unconstrained region. Thus, there really is no Sudden Stop, because there is no significant difference between the debt levels across $z$ values; policy has eliminated the consequences of the

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13In the Appendix I compare the constrained efficient and conditionally-efficient allocations of the model to those in the competitive equilibrium. In the first case the relative price $p_N$ is determined using the competitive equilibrium equation (the ratio of the marginal utilities of $C_T$ and $C_N$), while in the second it is determined using the competitive equilibrium function ($p_N(B, z, g, r)$).
shock.\textsuperscript{14}

Figures (10) show how the optimal policies are set when only one instrument is available – note that the interventions begin before the kink that denotes a binding constraint. This result is the main one in Benigno et al. (2011c) – prudential interventions are only appropriate if the government is constrained in the number of tools that it possesses. In the case with only one instrument, $T$ takes on both positive and negative values (and is occasionally significantly away from zero in either direction).

Figure (5) shows the endogenous variables under optimal policy with only one tool. It is clear that the exchange rate tool is primarily responsible for preventing the debt-deflation spiral, since $p_N$ falls substantially in the binding region when only $\tau_B$ is available. Because it plays a central role in the dynamics of the Sudden Stop, Figure (6) plots the nontradable price for all four policy equilibria in one panel; without intervention $p_N$ drops significantly further (and therefore the constraint tightens more) than when the government implements the optimal policy. It is clear that $\tau_N$ is the crucial tool for preventing the drop in the relative price; when the government only has the capital control $\tau_B$, it does not moderate the decline significantly. This result is intuitive, since $\tau_N$ directly affects $p_N$:

$$p_N = \frac{1 - \omega}{\omega} \left( \frac{C_N}{C_T} \right)^{\kappa-1} \frac{1}{1 + \tau_N}.$$  

In contrast, $\tau_B$ can only indirectly influence $p_N$ through changes in tradable consumption today versus tomorrow.

The ergodic sets for the four allocations are plotted in Figure (8). Relative to the competitive equilibrium without taxes, the two instrument set is shifted to the right; that is, the optimal policy actually produces more debt in equilibrium, despite the fact that $\tau_B$ is negative. It is interesting to see that $\tau_B$ by itself has only modest effects on the ergodic set, but $\tau_N$ shifts the distribution even further than the two instrument case. This extra shift occurs because the government wants to prop up the nontradable price, but with only the exchange rate tool it must accept a concomitant increase in $C_T$ financed via foreign borrowing; with both instruments it can use $\tau_B$ to blunt this extra debt accumulation by making it expensive.

Finally, Figure (9) plots the welfare gains from implementing the optimal policy – the chosen exogenous shocks are mean $z$ and $g$ and the low $r$ state, but the basic shapes are always the same. The welfare gain rises with the amount of debt, falls with $z$ and $g$, and rises with $r$. Setting $\tau_B$

\textsuperscript{14}The same results hold across different realizations of $r$ and $g$; for readability I have omitted explicitly plotting them in a figure that is already quite complicated.
optimally yields a significantly smaller welfare gain than $\tau_N$ if only one instrument is available; the additional curve is the welfare loss in the two instrument case that arises solely from the $\tau_B$ policy (that is, it is the result of setting $\tau_B$ equal to the function from the two-instrument case but setting $\tau_N = 0$ instead of optimally). And of course the one-instrument policy function is not an equilibrium in the two-instrument environment.

For the benchmark economy, implementing the two-instrument optimal policy yields an average welfare gain of 0.09 percent of tradable consumption, which while small is significantly larger than the typical calculation of the welfare cost of business cycles; as can be seen from Figure (9) there can be gains that are an order of magnitude larger in some states of the world, but because Sudden Stops are rare the optimal policy cannot generate a large average welfare gain. When only the exchange rate policy is permitted, the welfare gain is only 0.05 percent, while with only the capital control the gain is only 0.01 percent; note that the welfare gains are not additive, consistent with the results that the optimal choice for one tax depends critically on whether the other one is chosen optimally as well.\(^{15}\)

To get a sense of how commitment plays a role in the optimal policy choice, Figure (11) shows the optimal policy for the first backward step starting from the competitive equilibrium with no taxes; that is, the 'step one' curves are the policy if the continuation equilibrium has zero taxes. The $\tau_N$ curves lie on top of each other; whether the government tomorrow uses the exchange rate tool has no bearing on whether the government today will. But if the government tomorrow is not going to tax new debt, the current government will tax it more than it would in the equilibrium, using a lump-sum transfer to return the proceeds to the household.

### 4.2. The Government Fears Misspecification

This part of the paper explores the optimal policy allocation when the government fears a misspecification of the approximating model but households do not, so that $\zeta = 0$ and $\sigma < 0$.

The policy function for $\tau_N$ does not change significantly from the case where $\sigma = 0$, so I do not plot it in the interest of conserving space; because $\tau_N$ primarily operates to fix a static distortion (labor is misallocated across sectors), the intertemporal distortion induced by $\sigma < 0$ does not have much effect. However, Figure (12) shows that the capital control tool is not invariant to $\sigma$; now $\tau_B < 0$ before constraint becomes binding even if both tools are present. The amount of the

\(^{15}\)Fixing one tax function at the equilibrium function from the two-instrument case and choosing the other leads to the two-instrument equilibrium.
subsidy increases as the government faces more model uncertainty and it converges to zero as $B$ becomes positive.

Under the optimal policy with $\sigma < 0$, the ergodic set for $B$ is shifted to the right (see Figure (13)). Since $\sigma$ acts like enhanced risk aversion, the government would like the households to save more to protect against Sudden Stop events. In order to achieve this goal, the government uses $\tau_B$ to induce additional saving. Of course, this extra precautionary saving must come from consumption and leisure, so the economy will experience welfare losses as it transitions to the new ergodic set. Combining Figures (12) and (13) we see that, in the stationary equilibrium, a government facing model uncertainty will not actually tax very much; along the transition though it must tax debt quite a bit.

This result is informative about how a government that gets more concerned about model uncertainty would behave. Suppose initially $\sigma = 0$, so that the government trusts the approximating model. If $\sigma$ suddenly increases to $\sigma = 5$, the government would impose capital controls even if the economy were not currently in a crisis. As a result, capital would flow out of the country and the economy would transition to a state with low capital controls and low debt, along with significantly lower consumer welfare.

While it is not surprising that welfare drops (given that the government is not maximizing the welfare of the household as perceived by the household, what may be surprising is the size of the welfare loss associated with the government’s equilibrium policy. The average welfare loss for the two instrument case – computed using the stationary distribution from the competitive equilibrium without robustness – is 2.33 percent; as a comparison, the welfare gain from the optimal policy in the benchmark economy with both instruments is only 0.09 percent.

Given the high cost of having a robust decision rule for the government, it is useful to understand what the government fears – that is, what does the worst-case distribution look like? If $\sigma = -5$ is "unreasonable" in the sense of implying a worst-case process that is far from the approximating model, we should not care whether the costs are high. It turns out that the worst-case distribution involves a negative correlation between $r$ and $z$, a negative correlation between $r$ and $g$, and a positive correlation between $z$ and $g$ (that is, increases in $r$ when either $z$ or $g$ are low, and decreases in $z$ when $g$ is low). In the objective distribution, the correlations are all 0; in contrast, when $\sigma = -5$ the correlations are $-0.006$, $-0.017$, and $0.020$ respectively without optimal policy and $-0.065$, $-0.005$, and $0.008$ when taxes are set optimally. Thus, the stochastic process that the government fears is not that different from the objective process, but it turns out to imply very
different equilibrium policies.\textsuperscript{16}

An alternative way to examine the worst-case distribution is given in Figure (14), which shows the ergodic distribution of debt for $\sigma = -5$ compared to the "worst-case" distribution; the worst-case distribution is the one that would result if the transition probabilities were given by $p$ instead of $\pi$. The worst-case distribution has been shifted to the left relative to the equilibrium one and is more spread out, so the government "fears" a distribution with high debt and higher variance.

If the government is not permitted to use the capital control, it will use $\tau_N$ instead. The welfare loss associated with $\sigma = -5$ is slightly smaller when the government is not permitted to use $\tau_B$ and is slightly higher when the government is not permitted to use $\tau_N$. Neither of the differences are significant, however, so the main conclusion of this section remains unaltered.

4.3. Households and the Government Fear Misspecification

I now turn to the case where households and the government both do not fully trust the approximating model. I will confine myself to cases where the government is maximizing the subjective utility of the household, so that $\varsigma = \sigma$; the results for cases where this equality does not hold tend to behave like the case where only the government has model uncertainty and therefore are not formally reported. To gain some intuition, I first present cases where only these parameters are changed; that is, all other parameters are fixed, implying that the distribution of debt in the no-tax competitive equilibrium will generally differ from the targeted data. It is known in linear-quadratic-Gaussian settings that there are a variety of observational-equivalence results that link rational expectations and models with robustness (see Hansen, Sargent, and Tallarini 1999 or Luo and Young 2010); specifically, for any agent with preference for robustness there exists a rational expectations agent with a lower discount factor whose consumption-savings decisions are identical. The two environments do not have the same implications for asset prices or the welfare costs of either business cycle fluctuations or incomplete information, however, so they are not the same model. With these results in mind, I also examine how the optimal policy functions change if $\beta$ is reduced to keep the average ratio of debt to GDP equal as the desire for robustness increases; that is, I find a $\beta$ that shifts the mean of the ergodic distribution by the amount needed to keep it equal to the case where $\varsigma = \sigma = 0$.

\textsuperscript{16}Although for computational reasons I do not explore detection error probabilities here, it seems safe to assume that with standard data sample lengths the two distributions would be hard to distinguish. That is, $\sigma = -5$ does not seem to imply that the government fears unreasonable models.
Figure (16) plots the tax functions for the two instrument case for various values of robustness, holding fixed $\beta$. It is rather obvious that little changes here – interventions are still confined to the region where the constraint is binding, and quantitatively the values of $\varsigma$ and $\sigma$ do not seem to matter; of course, these values imply only mild concern for model uncertainty, as noted previously, but it is interesting that the large welfare losses obtained when only $\sigma < 0$ are replaced by zero welfare gains.

Figure (17) plots the stationary distributions when $\beta$ is fixed. One thing to note immediately is that the effect of optimal policy on the stationary distribution disappears, because the economy shifts endogenously into a region where Sudden Stops become extremely rare (in fact, with $\sigma = \varsigma = -5$ Sudden Stops do not occur at all). Since the optimal policy becomes completely inactive in the ergodic set, the welfare gain also disappears; the average gain for this parametrization is zero to machine precision; there are gains in some states of the world, but these states get zero weight in the CE stationary distribution.

Obviously, the presence of increased precautionary saving disguises the effect of robustness on optimal policy and welfare. To counteract this effect, I set $\beta = 0.9$ in order to reproduce the same average debt-to-income ratio with $\varsigma = \sigma = -15$ as $\beta = 0.965$ produces with $\varsigma = \sigma = 0$. Figures (18) displays the functions for $\tau_B$ and $\tau_N$ when $\beta$ is adjusted; the only difference is a modest decline in the size of the taxes.

Figure (19) compares the objective and worst-case distributions for the recalibrated economy. With a strong concern for model uncertainty ($\varsigma = \sigma = -15$), the stochastic process for the shocks has correlations of $-0.011$ between $r$ and $z$, $-0.018$ between $r$ and $g$, and $0.030$ between $z$ and $g$, while under the optimal policy these correlations change to $-0.013$, $-0.018$, and $0.036$; again, there is only a modest change in the nature of the stochastic process, and the resulting worst-case ergodic set for debt is not that different from the objective one (and in fact, under OP, there is essentially no difference at all).

4.4. Robustness in the Other Sense

In this part I explore two modifications to the basic model designed to test the limits of the main results. The first modification assumes that even the small lump-sum tax/transfer is not available; instead, the government must use labor income taxation to balance the government budget constraint. Second, I introduce into the borrowing constraint the value of land, which is used in the nontraded sector only.
4.4.1. Distortionary Labor Taxation

Here I abandon the assumption of lump-sum tax/transfers and replace them with distortionary taxation on labor income at rate $\tau_H$. The household budget constraint is now

$$c_T + (1 + \tau_N) p_N c_N + (1 + \tau_B) \frac{\exp(g) b'}{1 + r} \leq b + (1 - \tau_H) w h + D;$$

(4.1)

the lump-sum tax/transfer has been eliminated. The government budget constraint is

$$\tau_H w H + \tau_N p_N c_N + \tau_B \frac{\exp(g) B'}{1 + r} = 0.$$

(4.2)

The presence or absence of wasteful government spending does not affect any of the results, so I ignore it.

There is a good *ex ante* reason to expect that the results will be invariant to this extension – namely, that the benchmark model had almost zero lump-sum taxes. Thus, the benchmark economy should be close to this one, and in fact it is – the required labor income tax rate is zero, as can be seen in Figure (20); the other tax functions are naturally exactly the same as the case with lump-sum taxes.

The welfare gain from this case is 0.03 percent on average, roughly one-third that found for the lump-sum case. Because the welfare gain is already small, even small absolute changes in the tax functions can generate large proportional changes in welfare.

When the government is not permitted to use one of the other two instruments things change. Figure (21) shows the optimal policies when either the capital control or the exchange rate tool are prohibited; the key difference is that now the distortionary labor tax differs significantly from zero, as did the lump-sum tax for these cases. The welfare gains are for the no-capital-control and no-exchange-rate cases are 0.02 percent and 0.004 percent, respectively; again, these gains are a fraction of the gains when lump-sum taxation is permitted.

---

17 An alternative is to eliminate lump-sum taxes but not transfers; that is, the government faces the constraint $T_t \geq 0$ at each date $t$. Because the optimal value of $T_t$ is close to zero, the results for the two cases are similar.
4.4.2. Asset-Based Collateral Constraint

Finally, I consider a collateral constraint of the kind used in Bianchi and Mendoza (2011):

\[
\frac{\exp (g) b'}{1 + r} \geq -\varphi (w h + d) - \varpi Q \exp (g) l'
\]  

(4.3)

where \( Q \) is the market price of land \( l \). The household budget constraint is now given by

\[
c_T + (1 + \tau_N) p_N c_N + \frac{(1 + \tau_B) \exp (g) b'}{1 + r} + Q \exp (g) l' \leq b + (Q + K) l + w h + D + T
\]  

(4.4)

where \( K \) is the rental payment for land. The production function for nontradables only is assumed to depend on land:

\[
Y_{N,t} = A H_{N,t}^\theta L_t^\gamma
\]  

(4.5)

for \( \gamma > 0 \); in equilibrium \( L_t = 1 \) and

\[
K_t = \gamma p_{N,t} A H_{N,t}^\theta.
\]  

(4.6)

I set \( \gamma = 0.05 \) as in Bianchi and Mendoza (2011), \( \varpi = 0.2 \), and set \( \varphi = 0.4 \) in order to maintain the same average debt-to-income ratio as in the benchmark economy (where \( \varsigma = \sigma = 0 \); Table 1 contains the values of the parameters below the double line.\(^{18}\)

This version of the model adds one additional equation to the equilibrium, the pricing equation for land:

\[
\lambda (S) Q (S) = \beta \exp (-\rho g) \frac{E_{\pi} [\exp (\varsigma V' (S')) \lambda (S') (Q (S') + K (S')) | S]]}{E_{\pi} [\exp (\varsigma V' (S') | S)] | S]} + \varpi \max \{\mu (S), 0\}^2 Q (S).
\]  

(4.7)

The first two terms are standard, while the third one is the added benefit from purchasing land today – it serves as collateral that permits additional borrowing. Rearranging this equation yields

\[
Q (S) = \frac{\beta \exp (-\rho g)}{\lambda (S) - \varpi \max \{\mu (S), 0\}^2} \frac{E_{\pi} [\exp (\varsigma V' (S')) \lambda (S') (Q (S') + K (S')) | S]]}{E_{\pi} [\exp (\varsigma V' (S') | S)] | S]}.
\]  

(4.8)

\(^{18}\)High values of \( \varpi \) and low values of \( \varphi \) destabilize the backward induction procedure used to solve the model. Exactly why this happens is unclear, but as a result I do not consider cases where the asset part of the collateral constraint is too large relative to the current income part.
When the constraint is binding today, the price for land is higher than it would be if the constraint were absent. But unlike the income constraint, the possibility that the constraint might bind tomorrow also plays an important role. The rental payment for land \( K \) depends on \( p_N \), and \( p_N \) declines when the constraint is binding; therefore, a binding constraint tomorrow (in some state of the world that has positive probability) has the effect of tightening the constraint today through (4.8). By iterating (4.8) forward, as in Bianchi and Mendoza (2011), one sees immediately that a binding constraint at any future state reduces the value of land today.

As noted in Hansen, Sargent, and Tallarini (1999), among others, model uncertainty tends to drive up the return on risky assets. Land here is a risky asset; since the distorted probability measure places more weight on the states in which land rental payments will be low, all other things equal \( Q \) will decrease with \( \varsigma \). Since the optimal policy counteracts the decline in \( p_N \) that binding constraints cause, the taxes also tend to prop up the price of land.

Figure (22) plots the optimal policy without model uncertainty. It is clear that the \( \tau_B \) function goes negative before the constraint begins to bind, but the range of "prudential" intervention is rather small. Once the constraint begins to bind, \( \tau_N \) turns negative and \( \tau_B \) begins to rise and becomes positive (rather than negative as when \( \varpi = 0 \)). Thus, as in Jeanne and Korinek (2011b), the government will use both \textit{ex ante} and \textit{ex post} tools whenever there is a positive probability of a binding constraint tomorrow.\(^{19}\)

What are the financing consequences of these taxes? Without land, the optimal policy mix was basically revenue-neutral, as the tax on new debt raised almost exactly the amount of revenue needed to cover the subsidy to nontraded consumption. Given that \( \tau_B \) is non-zero in a range where \( \tau_N \) is still zero – namely, where there is a positive probability of a binding constraint tomorrow – this property obviously cannot hold everywhere, so it is of interest to determine the size of the lump-sum tax/transfer needed to finance the optimal policy. It turns out that \( T_t < 0 \) in the region where the constraint binds today and \( T_t > 0 \) where the constraint does not bind today but might tomorrow (that is, where \( \tau_N < 0 \) and \( \tau_N = 0 \), respectively); quantitatively, these numbers are not large. Thus, there are fiscal considerations that were not present before; if lump-sum taxes are not available, there will be additional distortions associated with raising sufficient revenue during the runup to a crisis.

\(^{19}\)To obtain this result Jeanne and Korinek (2011b) appeal to a utility cost of distorting asset prices (they permit households to counterfeit land holdings subject to a convex cost function and assume the government cannot distinguish between counterfeit and real land holdings but international lenders can, an assumption which seems a bit backwards). The results here show that this cost is unnecessary to obtain prudential interventions.
5. Conclusion

This paper explored optimal policymaking in models with Sudden Stops – declines in aggregate activity exacerbated by a binding collateral constraint – in an emerging market economy governed by policymakers who cannot commit to future policy. The key result is that prudential intervention – intervention that attempts to limit the probability of entering a region with a binding constraint – is appropriate in one of three circumstances. First, if the government is limited in the set of tools it is permitted to use. Second, if the government does not trust the approximating model but households do. Or third, if households can use land holdings as collateral in addition to income. The first two results imply welfare losses relative to the case in which the government is not constrained and maximizes the subjective utility of the households (whether they trust the model or not is irrelevant), so they should not be used to justify prudential intervention. The third result would call for prudential intervention, but it is not clear whether that case applies empirically; exactly what lenders will ”accept” as collateral for international lending needs to be investigated.20

Obviously, this paper is just a first step and is unsuited for making actual policy advice. A number of extensions would be natural, including the introduction of monetary factors (including nominal rigidities), physical capital, and sovereign default, although each poses significant computational challenges. Deriving the collateral constraint from formal microeconomic principles and endogenizing the interest rate also seem like important directions to proceed, perhaps by formally modeling the ”rest of the world” lender.21

To conclude I will simply point out some challenges associated with these extensions. Benigno et al. (2011d) investigates the role of optimal monetary policy in a model similar to the one used here, but uses a three-period environment for tractability reasons. The primary obstacle for adding either nominal rigidities or capital is that the two-dimensional state space makes the optimal policy problem exceptionally difficult; preserving the shape of the value function is not easy to do in more than one dimension, and the algorithm relies critically on that shape (see Cai and Judd 2011 for a promising approach). Sovereign default introduces nonconvexities, rendering

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20It is not obvious why international lenders would accept land as collateral, given that it would likely be quite difficult for a foreign bank to seize land in an emerging economy.

21Preliminary calculations using a patient, large country that simply imposes the income-based collateral constraint (2.4) imply counterfactual behavior of the interest rate when the constraint binds; specifically, the real interest rate collapses during a crisis. Adding an additional asset with a guaranteed non-zero return would presumably eliminate this issue, but substantially increases the computational burden.
the computation of the competitive equilibrium more challenging; with the exception of Mendoza and Yue (2008), most sovereign default models assume output is exogenous and thereby completely ignore the private sector’s behavior.

Exploring the role of commitment is also important, although extremely difficult from a computational perspective. Benigno et al. (2011c) contains a brief discussion of how commitment matters in this context. Computationally, an extension using the multiplier method of Marcell and Marimon (2011) seems straightforward, but preliminary investigations have revealed that the domain of the multiplier state is difficult to characterize.

6. Appendix

6.1. The Utility Recursion

In this appendix I provide a proof that the utility recursion (2.17) represents the decision problem of an agent with multiplier preferences; this proof is nearly identical to that in Backus, Routledge, and Zin (2005) and is presented for the convenience of the reader. The continuation value of an agent who desires robust decision rules with respect to random variable $z'$ is

$$
\hat{V}(z) = \min_{\{p(z')\} \in [0,1]^Z} \left\{ \sum_{z'} p(z') V(z') - \frac{1}{\varsigma} \left( \sum_{z'} p(z') \log \left( \frac{p(z')}{\pi(z'|z)} \right) \right) + \Omega \left( \sum_{z'} p(z') - 1 \right) \right\};
$$

Note that the value depends on $z$ through the conditional probability under the approximating model $\pi(z'|z)$. The problem is convex in $p(z')$, since $x \log(x)$ is a convex function and the other terms are linear; the constraint set is convex and compact as it is a finite product of intervals. The necessary and sufficient first-order condition with respect to $p(z')$ is

$$
V(z') - \frac{1}{\varsigma} \log \left( \frac{p(z')}{\pi(z'|z)} \right) + \frac{1}{\varsigma} + \Omega = 0.
$$

Now multiply by $p(z')$ and sum over $z'$ to obtain

$$
\sum_{z'} p(z') V(z') - \frac{1}{\varsigma} \left( \sum_{z'} p(z') \log \left( \frac{p(z')}{\pi(z'|z)} \right) \right) + \frac{1}{\varsigma} + \Omega = 0,
$$

written more simply as

$$
\hat{V}(z) - \frac{1}{\varsigma} + \Omega = 0
$$
so that

\[ \Omega = -\hat{V}(z) + \frac{1}{\zeta}. \]

Substituting this expression into the first-order condition yields

\[ p(z') = \pi(z'|z) \exp \left( -\frac{V(z') - \frac{1}{\zeta} + \Omega}{\frac{1}{\zeta}} \right) \]

or

\[ p(z') = \exp \left( \zeta \hat{V}(z) \right) \pi(z'|z) \exp (\zeta V(z')). \]

Now sum over \( z' \) to obtain

\[ 1 = \exp \left( -\zeta \hat{V}(z) \right) \sum_{z'} \pi(z'|z) \exp (\zeta V(z')). \]

Taking logs and rearranging yields

\[ \hat{V}(z) = \frac{1}{\zeta} \log \left( \sum_{z'} \pi(z'|z) \exp (\zeta V(z')) \right). \]
6.2. Computational Method

Consider the system of functional equations that determine a recursive competitive equilibrium:

\[ u_1(S) = \lambda(S) \]  
(6.1)

\[ u_2(S) = (1 + \tau_N) p_N(S) \lambda(S) \]  
(6.2)

\[ u_3(S) = w(S) \left( \lambda(S) + \varphi \max \{ \mu(S), 0 \} \right)^2 \]  
(6.3)

\[ \frac{(1 + \tau_B) \lambda(S) - \max \{ \mu(S), 0 \}^2}{1 + r(S)} = \beta \exp \left( -g(S) \rho \right) E_p \left[ \lambda(S' | S) \right] \]  
(6.4)

\[ \max \{ -\mu(S), 0 \}^2 = \frac{B'(S)}{1 + r(S)} + \varphi \left( w(S) (H_T(S) + H_N(S)) + D(S) \right) \]  
(6.5)

\[ w(S) = \alpha \exp (z(S)) H_T(S)^{\alpha-1} \]  
(6.6)

\[ D(S) = (1 - \alpha) \exp (z(S)) H_T(S)^{\alpha} + (1 - \vartheta) p_N(S) A H_N(S)^{\vartheta-1} \]  
(6.7)

\[ C_T(S) = B(S) + \exp (z(S)) H_T(S)^{\alpha} - \frac{\exp (g(S)) B'(S)}{1 + r(S)} \]  
(6.8)

\[ C_N(S) = A H_N(S)^{\vartheta} \]  
(6.9)

\[ T(S) = \tau_N p_N(S) C_N(S) + \tau_B \frac{\exp (g(S)) B'(S)}{1 + r(S)}, \]  
(6.10)

where \( E_p \) is the distorted expectations operator

\[ E_p \left[ f(S') \right] = \frac{E_\pi \left[ \exp (s V(S')) f(S') \right]}{E_\pi \left[ \exp (s V(S')) \right]} \]

and \( S = (B, z, g, r) \). I solve these equations using an iterative approach, as in Coleman (1989). I fix a grid for \( B \), with 300 points concentrated over a small interval at the lower end of the interval and 50 additional points more dispersed at the upper end.; for stability it turns out to be important to include points where \( B_i > 0 \) even though they lie outside the ergodic set of debt levels. To solve the model with the asset constraint, I simply add the pricing equation

\[ Q(S) \left( \lambda(S) - \varpi \max \{ \mu(S), 0 \}^2 \right) = \beta \exp \left( -g(S) \rho \right) E_p \left[ \lambda(S' | S) \right] (Q(S' | S)) + K(S' | S) \left| S \right| \]

to the above system and modify the constraint equation to

\[ \max \{ -\mu(S), 0 \}^2 = \frac{B'(S)}{1 + r(S)} + \varphi \left( w(S) (H_T(S) + H_N(S)) + D(S) \right) + \varpi Q(S). \]
I guess functions
\[
\lambda^0 (B', z, g, r) = E_p \left[ \lambda (B', z', g', r') | z, g, r \right]
\]
and
\[
\mathcal{V}^0 (B', z, g, r) = \log \left( E_{\pi} \left[ \exp \left( \mathcal{V} (B', z', g', r') \right) | z, g, r \right] \right).
\]

Using a hybrid Powell method (see Judd 1998), I solve the system of equations
\[
u_1 (S) = \lambda (S)
\]
\[
u_2 (S) = (1 + \tau_N) p_N (S) \lambda (S)
\]
\[
u_3 (S) = w (S) \left( \lambda (S) + \varphi \max \left\{ \mu (S), 0 \right\} \right)^2
\]
\[
\frac{(1 + \tau_B) \lambda (S) - \max \left\{ \mu (S), 0 \right\}^2}{1 + r (S)} = \beta \exp \left( - g (S) \rho \right) \lambda^0 (B', z (S), g (S), r (S))
\]
\[
\max \left\{ -\mu (S), 0 \right\}^2 = \frac{\exp (g (S)) B' (S)}{1 + r (S)} + \varphi \left( w (S) (H_T (S) + H_N (S)) + D (S) \right)
\]
\[
w (S) = \alpha \exp (z (S)) H_T (S)^{\alpha - 1}
\]
\[
w (S) = \theta p_N (S) A H_N (S)^{\theta - 1}
\]
\[
D (S) = (1 - \alpha) \exp (z (S)) H_T (S)^\alpha + (1 - \theta) p_N (S) A H_N (S)^{1 - \theta}
\]
\[
C_T (S) = B (S) + \exp (z (S)) H_T (S)^\alpha - \frac{\exp (g (S)) B' (S)}{1 + r (S)}
\]
\[
C_N (S) = A H_N (S)^\theta
\]
\[
T (S) = \tau_N p_N (S) C_N (S) + \tau_B \frac{\exp (g (S)) B' (S)}{1 + r (S)}
\]
\[
V (S) = u (S) + \beta \exp (g (S) (1 - \rho)) \mathcal{V}^0 (B', z (S), g (S), r (S))
\]

for the functions \((C_T, C_N, H_T, H_N, B', \lambda, \mu, T, V, w, d, p_N) (S)\), and update my guesses; values for the functions \(\lambda^0 (B', z, g, r)\) and \(\mathcal{V}^0 (B', z, g, r)\) at points \(B'\) not on the grid are computed using cubic splines. This process is repeated until (if?) it converges; as noted already, existence and uniqueness have not been established for this model, but in no case did I detect any multiplicity. The true Lagrange multiplier can be recovered using the equation
\[
\mu^* (S) = \max \left\{ \mu (S), 0 \right\}^2.
\]

I then construct the stationary distribution and the ergodic set using the nonstochastic method
from Young (2010), which converges more rapidly than simulation-based methods and deals better with rare events (like the interest rate shocks); this process uses an evenly-spaced grid of 1000 points for $B$.

The optimal policy problem is solved using standard dynamic programming methods, assuming

$$W^0(B', z, g, r) \equiv \log \left( E_\pi \left[ \exp \left( \sigma W(B', z', g', r') \right) \mid z, g, r \right] \right) \quad (6.25)$$

is evaluated using cubic splines whenever necessary. The constrained maximization is conducted using a feasible sequential quadratic programming method (see Zhou, Tits, and Lawrence 1997) with the equilibrium conditions imposed as equality constraints for given functions $\lambda^0(B', z, g, r)$ and $V^0(B', z, g, r)$. For numerical stability, I also impose a constraint that the argument of $u$ be positive. All three guesses are updated and the process is iterated to convergence. Convergence does not always obtain here, particularly for cases where $\varsigma$ and $\sigma$ are large in value or far apart; at least part of the reason is that when agents face a lot of model uncertainty, their precautionary savings motive pushes them into positive $B$ regions where the worst-case interest rate shock changes identity. When I recalibrate $\beta$ to match average debt levels in the $\varsigma = \sigma = 0$ case, I can explore higher values for these parameters without convergence becoming a problem.

A similar dynamic programming approach is used to calculate the welfare function $V^*(B, z, g, r, \chi)$ (I define a grid for $\chi$ and treat it as a state variable with a trivial transition function $\chi' = \chi$) and then Brent’s method is used to solve the nonlinear equation

$$V^*(B, z, g, r, \chi(B, z, g, r)) = V(B, z, g, r), \quad (6.26)$$

using cubic splines again to interpolate in the $\chi$ direction.

The worst-case stochastic process is governed by the probabilities

$$p(z', r', g' \mid B, z, r, g) = \frac{\pi(z', r', g' \mid z, r, g) \exp (\varsigma V(B'(B, z, r, g), z', r', g'))}{\sum \pi(z', r', g' \mid z, r, g) \exp (\varsigma V(B'(B, z, r, g), z', r', g'))}; \quad (6.27)$$

as noted previously by Bidder (2011), the worst-case process is not Markovian in the usual sense because the probabilities depend on $B$. I construct the ergodic distribution of the worst-case scenario using the non-stochastic approach from Young (2010); because the worst-case distribution will generally involve correlation between the shocks, I construct a joint process for $(z, r, g)$ involving
a single state $s$:

$$p(s'|B,s) = \frac{\pi(z(s'), r(s'), g(s') | z(s), r(s), g(s)) \exp (\varsigma V(B', z(s), r(s), g(s)), z(s'), r(s'), g(s'))}{\sum \pi(z(s'), r(s'), g(s') | z(s), r(s), g(s)) \exp (\varsigma V(B', z(s), r(s), g(s)), z(s'), r(s'), g(s'))}$$

(6.28)

where $(z(s), r(s), g(s))$ is the unique trio of fundamental shocks associated with a particular $s$.

### 6.3. Social Planning Allocation

There are two ways to formulate a social planning problem for the benchmark economy. The first is used in Bianchi and Mendoza (2011) and will be denoted the conditionally-efficient problem:

$$V^P(B, z, g, r) = \max \left\{ u(C_T, C_N, H_T + H_N) + \beta \exp (g(1 - \rho)) E[V^P(B', z', g', r') | z, g, r] \right\}$$

subject to the resource constraints, the borrowing constraint, and the function $p_N(B, z, g, r)$ from the competitive equilibrium; that is, conditional on the state of the world being the same as in the CE, the price of nontradables must also be the same. In contrast, Benigno et al. (2011b) use the constrained-efficient problem

$$V^P(B, z, g, r) = \max \left\{ u(C_T, C_N, H_T + H_N) + \beta \exp (g(1 - \rho)) E[V^P(B', z', g', r') | z, g, r] \right\}$$

subject to the resource constraints, the borrowing constraint, and the equation

$$p_N = \frac{u_1}{u_2}$$

The two formulations are not equivalent in general, but are in an endowment economy (see Benigno et al. 2011b).

### References


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Figure 1: Current Account Dynamics in Mexico

Trade Balance and Current Account in Mexico, Quarterly

- Percentage of GDP
- Trade Balance/GDP
- Current Account/GDP


Tequila Crisis
Figure 2: Consumption and Output Growth in Mexico
Figure 3: Optimal Policy, RE Model

Dynamics of a Sudden Stop

$B(t)$

$B(t+1)$

$A$

$B$

$C$

Low $z$

High $z$

$45^0$
Figure 4: Endogenous Variables, RE Model
Figure 5: Endogenous Variables, RE Model
Figure 6: Nontradable Price, RE Model

![Graph showing the price of nontraded goods as a function of B(t). The graph includes lines representing different equilibrium scenarios: Competitive Equilibrium, OP Both Instruments, OP $\tau_B$ Only, and OP $\tau_N$ Only.](image-url)
Figure 7: Optimal Policy, RE Model

Dynamics of a Sudden Stop, OP

- Low z, CE
- High z, CE
- Low z, OP
- High z, OP
Figure 8: Nontradable Price, RE Model

Ergodic Set, RE Model

- Competitive Equilibrium
- OP Both Instruments
- OP $\tau_B$ Only
- OP $\tau_N$ Only

Density

$B(t)$

-1.55 to -1.3

44
Figure 9: Welfare Gains, RE Model

Welfare Gains from OP, % of $C_T$

$\chi(B(t), z(t), g(t), r(t))$

$\tau_B$ and $\tau_N$

$\tau_B$

$\tau_N$

$\tau_B$ only

$B(t)$
Figure 10: Optimal Policy, RE Model

Optimal Taxes, One vs. Two Instrument Cases

\( \tau_B, \text{Both} \)
\( \tau_B, \text{One} \)
\( \tau_N, \text{Both} \)
\( \tau_N, \text{One} \)

\( B(t) \) vs. \( (\tau_B, \tau_N)(t) \)
Figure 11: Optimal Policy, RE Model

Optimal Policy, One Step Equilibrium

\[ \tau_B(t) \]

\[ B(t) \]

- Converged
- One Step
Figure 12: Optimal Policy, RB Model

Optimal Taxes When Government Wants Robustness

\( \tau_B(t) \) vs. \( B(t) \) for different values of \( \sigma \):
- \( \sigma = 0 \) (solid blue line)
- \( \sigma = -1 \) (dashed red line)
- \( \sigma = -5 \) (dotted green line)
Figure 13: Optimal Policy, RB Model

![Ergodic Sets Graph]

- $B(t)$
- Density
- Ergodic Sets
- $\text{CE, RE}$
- $\text{OP, RE}$
- $\text{OP, } \sigma = -1$
- $\text{OP, } \sigma = -5$
Figure 14: Optimal Policy, RB Model

Stationary Distribution, $\sigma = -5$

- Density
- $B(t)$

Lines:
- Blue: OP
- Red Dash: OP Worst-Case
Figure 15: Optimal Policy, RB Model

\[ \chi(B(t), z(t), g(t), r(t)) \]

Welfare Loss, \( \sigma = -5 \)

B(t)

\[ \begin{array}{c|c|c|c}
\hline
\text{B(t)} & -1.6 & -1.4 & -1.2 & -1 & -0.8 & -0.6 & -0.4 & -0.2 & 0 \\
\hline
\text{Welfare Loss, } \sigma = -5 & \end{array} \]
Figure 16: Optimal Policy, RB Model

Optimal Taxes When Both Want Robustness

\[ \tau_B(t) \]

\( \varsigma = \sigma = 0 \)
\( \varsigma = \sigma = -1 \)
\( \varsigma = \sigma = -5 \)
Figure 17: Optimal Policy, RB Model

Ergodic Sets

$\zeta = \sigma = 0$

Density

$\zeta = \sigma = -1$

$\zeta = \sigma = -5$

CE

OP Two Instruments
Figure 18: Optimal Policy, RB Model

Optimal Taxes, Recalibrated $\beta$ and $\zeta = \sigma = -15$
Figure 19: Optimal Policy, RB Model

Stationary Distribution, $\zeta = \sigma = -15$

Density

B(t)
Figure 20: Optimal Policy, RE Model

Optimal Policy, Two Instruments and Distortionary Taxation

$(\tau_B, \tau_N, \tau_L)(t)$
Figure 21: Optimal Policy, RE Model

Optimal Policy, One Instrument and Distortionary Taxation

The graph shows the optimal policy with different tax instruments and distortionary taxation. The x-axis represents $B(t)$, and the y-axis represents $(\tau_B, \tau_N)(t)$. The graph includes lines for different tax instruments with and without distortionary taxation.
Figure 22: Optimal Policy, RE Model

Optimal Taxes, Asset Constraint

$B(t)$

$\tau_B(t), \tau_N(t)$

Optimal Taxes, Asset Constraint

$\tau_B(t)$

$\tau_N(t)$

$B(t)$