

Crowding Out or Crowding In? UI and Private Insurance*

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Abstract

This paper explores the welfare consequences of unemployment insurance when private insurance markets are subject to enforcement frictions that limit debt accumulation, endogenously determining individual borrowing limits as a function of the length of unemployment. Reducing the replacement rate in the model initially increases borrowing, as the value of default declines significantly. As the replacement rate continues to fall, borrowing eventually is restricted, with zero borrowing sustainable at a zero replacement rate. In the neighborhood of the current US policy UI crowds out private insurance. The paper attributes only a small fraction of the changes caused by UI to endogenous debt limits.

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1. Introduction

In a recent paper Young (2004) shows that unemployment insurance has a strong negative effect on welfare and aggregate activity in a standard Aiyagari (1994) model extended to permit elastic search effort. When debt limits are set to the natural debt limit, negative UI becomes optimal; in order to align incentives to search, it is efficient to tax the unemployed and rebate proceeds to the employed.¹ That example illustrates an important characteristic: the debt limits depend on the policy settings through general equilibrium effects. This paper extends the model from Young (2004) to incorporate endogenous debt limits that prevent default, as in Kehoe and Levine (1993). These limits are less generous than the natural debt limit and dependent on individual behavior through the value of the outside option. The purpose of this paper is to assess the welfare implications of unemployment insurance in this environment.

Permitting changes in policy to affect private risk sharing possibilities is a recent development in dynamic general equilibrium modelling of inequality; two prominent examples are Krueger and Perri (2006) and Attanasio and Ríos-Rull (2000).² The first paper is closely related to the issues discussed here. Krueger and Perri (2006) show that progressive tax/lump-sum transfer systems can have important consequences for the provision of private risk sharing: public insurance tends to crowd-out private insurance opportunities in an endowment economy with a full set of contingent claims. As the authors note, their model abstracts away from a large number of important details, but it still provides a unique result that points the way toward a more complete understanding of risk sharing – government policy matters not just because it provides insurance but also because it may affect the functioning of private markets. The second paper is interested in exploring the possibility that aggregate insurance policies may actually reduce the total amount of insurance, again by reducing the supply of private insurance.

In this model unemployment insurance has general equilibrium consequences. First, increasing the generosity of the UI system tends to reduce aggregate labor input through the reduction in search. Second, it also tends to reduce the aggregate capital input, since precautionary savings falls. Third, the larger UI transfer requires a higher distortionary payroll tax rate, further reducing search effort (this effect is typically not quantitatively important). All of these effects are present in

¹As shown in Figure 10 of Young (2004), negative replacement rates combined with natural debt limits create existence of equilibrium problems; the supply curve is no longer monotone.

²Golosov and Tsyvinski (2007) approaches the problem from the other direction: they study how the opportunities for private insurance affect the optimal settings for public insurance.

Young (2004). The additional effects present here involve the endogenous debt limits. The effect of UI on the outside option of the household is generated by two opposing forces. With lower after-tax wages the variance of labor income is smaller; this change tends to raise the value of autarky and inhibit private risk-sharing. But higher returns make the opportunity cost of default larger (as households are restricted to non-interest-bearing assets), leading to lower autarky values and more private risk-sharing. It is therefore a quantitative question whether UI crowds in or crowds out private insurance; the finding here is that crowding out occurs in the region where the equilibrium interest rate lies, leading to more restrictive borrowing constraints and less private insurance. For values of the interest rate that are not close to the equilibrium one UI can actually crowd in private insurance; thus, a general equilibrium perspective is important, as partial equilibrium approaches would generate misleading inference.

I then provide a decomposition of the effects of UI into three components – the income-smoothing effect, the general equilibrium price effect, and the insurance effect. The first effect is the change generated by the reduction in individual income variance, the second effect is the additional feedback produced by the changes in general equilibrium prices and taxes, and the third is the effect of changes in the risk-sharing arrangements in the private sector. The third effect is quite weak, contributing almost no change to two measures of consumption smoothing – the unconditional standard deviation of consumption and the average change in consumption at the onset of an unemployment spell. The small impact of the endogenous borrowing constraints can be traced to a nonmonotone response to the replacement rate – decreasing the replacement rate from 50 percent initially increases debt limits but eventually decreases them. For zero replacement rates borrowing is not sustainable, in fact, and the maximal amount of borrowing is never large. Thus, most of the aggregate consequences of changing UI can be traced to the first two effects, both of which are present in Young (2004).

That result does not imply that endogenous debt constraints are irrelevant for the study of unemployment insurance, however. If debt constraints are set to rule out default and punishment is exclusion from borrowing and saving in the credit market (only non-interest-bearing storage is permitted), the optimal replacement rate for the first two periods is around 25 percent, about half the value in the US. In contrast, if the punishment from default is weakened to only exclude borrowing, debt effectively vanishes from the economy (all borrowing limits become very close to 0) and the optimal replacement rate is 0, as in Young (2004). That is, endogenous changes in debt limits do matter for optimal policy settings, even though they have only minor consequences for

aggregates. These results point to a clear need to explicitly model the consequences of equilibrium default – as in the recent literature on unsecured credit markets – when assessing the value of unemployment insurance. The conclusion contains a more complete discussion of this point.

2. Model

The model considered here introduces endogenous debt limits in the spirit of Kehoe and Levine (1993). Unemployment insurance affects credit market conditions because it alters the value of the outside option, but it does so in complicated ways because human capital is inalienable and wages are determined endogenously. Households solve the dynamic program

$$v(k, e) = \max_{k'_1 \geq k^b(0), k'_{0e} \geq k^b(\min\{e+1, \bar{e}\}), a \geq 0} \left\{ \begin{array}{l} [1 - \exp(-\gamma_e a)] [\log(c_1) + \beta v(k'_1, 0)] + \\ \exp(-\gamma_e a) [\log(c_{0e}) + \beta v(k'_{0e}, \min\{e+1, \bar{e}\})] - a^x \end{array} \right\} \quad (2.1)$$

subject to the budget constraints

$$c_1 + k'_1 \leq (1 + r - \delta)k + (1 - \tau)w \quad (2.2)$$

$$c_{0e} + k'_{0e} \leq (1 + r - \delta)k + \theta_e w. \quad (2.3)$$

τ is the payroll tax rate and θ_e is the replacement rate for an individual whose current spell of unemployment has lasted e periods and \bar{e} is the maximum relevant unemployment length (here set to 2).

The debt limits are set to the lowest value that would not induce default at the beginning of next period; since there is no information revealed at the beginning of next period default occurs with probability zero or one. Following Kehoe and Levine (1993), I assume that the punishment from default is the expulsion from asset markets forever; however, I do permit the banished households to store consumption goods. The autarky problem is

$$v^{Aut}(m, e) = \max_{m'_1 \geq 0, m'_{0e} \geq 0, a \geq 0} \left\{ \begin{array}{l} [1 - \exp(-\gamma_e a)] [\log(c_1^{Aut}) + \beta v^{Aut}(m'_1, 0)] + \\ \exp(-\gamma_e a) [\log(c_{0e}^{Aut}) + \beta v^{Aut}(m'_{0e}, \min\{e+1, \bar{e}\})] - a^x \end{array} \right\} \quad (2.4)$$

subject to

$$c_1 + m'_1 \leq m + (1 - \tau)w \quad (2.5)$$

$$c_{0e} + m'_{0e} \leq m + \theta_e w. \quad (2.6)$$

The debt limits are then

$$v(k_1^b, 0) = v^{Aut}(0, 0) \quad (2.7)$$

$$v(k_{0e}^b, \min\{e + 1, \bar{e}\}) = v^{Aut}(0, \min\{e + 1, \bar{e}\}). \quad (2.8)$$

One important factor is that the autarky household has two options to smooth expected consumption – buffer stocks of assets and variations in search effort.

The asset market setup is similar to [Ábráham and Carceles-Poveda \(2007\)](#), where holdings of uncontingent assets are limited by the same enforcement constraints considered here. One important difference is that in [Ábráham and Carceles-Poveda \(2007\)](#) savings in the asset is chosen *before* the realization of the relevant uncertainty, while here it is chosen *after*. The asset that is absent due to moral hazard considerations is an insurance contract exchanged at the beginning of the current period, before the realization of the employment lottery, and the debt constraints then limit the ability of the household to respond *ex post*.

The competitive production sector rents capital and labor and produces output. The stand-in firm solves the profit maximization problem

$$\max_{K, 1-u} \left\{ K^\alpha (1-u)^{1-\alpha} - rK - w(1-u) \right\} \quad (2.9)$$

which leads to the usual factor pricing conditions:

$$r = \alpha K^{\alpha-1} (1-u)^{1-\alpha} \quad (2.10)$$

$$w = (1-\alpha) K^\alpha (1-u)^{-\alpha}. \quad (2.11)$$

The government budget constraint is

$$\sum_e \theta_e w u_e = \tau w (1-u) \quad (2.12)$$

where u_e is the measure of unemployed in state e . The UI agency is not permitted to use government debt as a means of providing revenue and for simplicity I ignore any wasteful government spending.³

Market equilibrium requires

$$K = \sum_e \int k \Gamma(k, e) \quad (2.13)$$

$$1 - u = \sum_e \int \exp(-\gamma_e a) \Gamma(k, e) \quad (2.14)$$

$$\begin{aligned} \sum_e \int_k ([1 - \exp(-\gamma_e a)] (c_1 + k'_1) + \exp(-\gamma_e a) (c_{0e} + k'_{0e})) \Gamma(dk, e) \\ = K^\alpha (1 - u)^{1-\alpha} + (1 - \delta) K. \end{aligned} \quad (2.15)$$

$\Gamma(dk, e)$ is the distribution of households over individual values of wealth and employment status.

3. Results

The benchmark unemployment insurance system used to calibrate the model involves the sequence of replacement rates $\{\theta_e\} = \{0.5, 0.5, 0.17, 0.17, \dots\}$; after 2 quarters, the benefit should be interpreted as a combination of low income support programs and other government transfers. Under this system, I choose $(\beta, \delta, \gamma_0, \gamma_1, \gamma_2)$ to match the same calibration targets as in Young (2004) – wealth/GDP ratio of 11.5, an investment/GDP ratio of 0.25, aggregate unemployment rate of 7.4 percent, an fraction of the unemployed who are in their first quarter of 69.8 percent, and a fraction who are in their second quarter of 15.5 percent. $\chi = 2$ is arbitrary; the model's implications are robust to changes in this parameter, provided the other parameters are recalibrated. Computation of the stationary equilibrium of the model is detailed in an appendix. Alternative calibrations – such as one in which the wealth/GDP ratio is reduced by excluding housing and consumer durables from the definition of wealth or matching a different unemployment rate – do not change the main conclusions.

I then compute new stationary equilibria for various different combinations of $\{\theta_1 = \theta_2\}$. Holding fixed general equilibrium values for (r, w, τ) , changes in the replacement ratio alter the amount of risk sharing that is privately sustainable; as θ decreases the variance of individual income rises, making autarky relatively less attractive and therefore increasing the amount of debt that will be

³Since the welfare distortions of taxation are (roughly) proportional to the square of the tax, this assumption may not be inconsequential. Since the effect of distortionary taxation changes in the model is relatively minor, I doubt the quantitative results would be substantially different.

willingly repaid. At the same time, however, lower θ reduces the amount of debt that can be feasibly repaid with probability one, since there is always a positive probability of being unemployed; as a result, debt limits can contract or expand depending on the strength of the two mechanisms. When $\theta = 0$, though, the result is clear – no borrowing. The general equilibrium consequences work in complicated ways. More generous UI tends to reduce aggregate search effort and therefore aggregate labor input, but it also tends to reduce precautionary savings and therefore aggregate capital – as a result, interest rates and wages may either rise or decline. Furthermore, more generous UI requires higher distortionary tax rates, an effect which is reinforced by the decline in aggregate activity, which reduce after-tax wages. As a result, it is unclear whether UI would crowd out or crowd in private insurance, since the direction of the change in the value of autarky is ambiguous.

3.1. Aggregate Effects

Given the benchmark UI system, Table 1 presents the details of the calibrated stationary equilibrium with endogenous debt constraints; note that the ability to borrow is a decreasing function of e , meaning that those agents who are most in need of debt to smooth their consumption are the ones who face the most severe restrictions. Furthermore, very little borrowing can be sustained here; the debt limit for currently employed agents is only about one quarter of pre-tax labor income. Autarky is simply not very bad if you can save, even if there is an opportunity cost in terms of return, particularly since the household can also vary their search effort. The column labelled ' Δ_c ' is the average percent drop in consumption for agents who transition from employment to unemployment (that is, $e = 0$ to $e' = 1$). This number is commonly used a measure of the consumption-smoothing effects of UI; in the data, Gruber (1997) estimates a 7 percent drop in food consumption upon entering an unemployment spell, likely an understatement due to the relative temporal inflexibility of food consumption. With no borrowing the drop in consumption is 9 percent, a very close fit to the data; at the benchmark calibrated equilibrium the drop is only 1 percent, suggesting that the model is overstating the extent to which consumption is smoothed over states despite having what appear to be excessively-tight debt constraints.⁴ The column ' Σ_c ' is the

⁴This discrepancy is a consequence of the assumption that all wealth is liquid. In reality, households likely face significant transactions costs when accessing some components of wealth, such as housing and consumer durables, and so are compelled to permit consumption to drop further than they otherwise would. Young (2004) shows that the quantitative results do not depend on whether the model is calibrated to exclude illiquid assets, and the same robustness holds here.

standard deviation of consumption in the cross-section; both Δ_c and Σ_c will be used to measure consumption-smoothing.

Table 2 presents the results of cutting the replacement rate to 25 percent for the first two periods of unemployment, leaving the long-term replacement rates unchanged. Since the long-term income support is not typically considered part of the UI system, and frequently is attached to means-testing for income or wealth, this experiment seems the most reasonable as an illustration of the relative strength of the various effects. Figure 1 presents an illustration of the asset market equilibrium as it changes. The asset supply curve 'A^s' shifts to the right by a small amount, leading to a decline in returns and a rise in total capital. Since wages and returns are related by the function

$$w(r) = (1 - \alpha) \left(\frac{r}{\alpha} \right)^{\frac{\alpha}{\alpha-1}}$$

and $\alpha < 1$, wages fall when returns rise. After-tax wages rise even more because the payroll tax rate falls. Figure 1 also clearly shows that private risk-sharing is altered by the presence of public insurance but not perhaps in an intuitive way; the divergence between the 50 percent and 25 percent supply curves is most pronounced at negative levels of asset holdings, but the 50 percent economy seems to permit more borrowing. That belief is misleading, however, as the following argument will show.

Figure 2 presents the debt limits as a function of r for $e = 0$; note that the debt limit is not a monotone function of the interest rate.⁵ When r is very low, w is very high, meaning that the variance of income in autarky is quite large. High debt limits are therefore sustainable, as autarky is relatively quite unpleasant, even though the return gap is relatively small.⁶ As r rises, the variance of income falls and the return gap rises; for low enough r the first effect dominates, leading to smaller debt limits. When r gets large enough, however, the return gap is dominant and more borrowing can be sustained; when r is large, autarky provides very low relative returns, and low returns make consumption smoothing more difficult. It is clear here that a decrease in θ reduces borrowing for small r but increases it for larger values; since the equilibrium r is in the region where the solid curve is above the dashed curves, cutting the replacement rate will lead to

⁵The debt limit functions for $e > 0$ are identical in shape, simply shifted up a small amount (so that $e = 1$ households can borrow a little less than $e = 0$ households and $e = 2$ households a little less than $e = 1$). For readability they have been omitted.

⁶Having high wages lead to relatively low autarky utilities may seem counterintuitive. What matters for the determination of the debt limit is how much worse autarky is; when wages are high income variance is also high and therefore autarky is quite bad.

more private insurance being available.

Measured in terms of welfare, cutting the replacement rate from $\theta = 0.5$ to $\theta = 0.25$ for the first two periods leads to a welfare increase equivalent to 2.1 percent of lifetime consumption. As a reference point, eliminating the UI system in Young (2004) is optimal and leads to a welfare increase equivalent to about 1 percent of lifetime consumption. Here, completely eliminating UI (in every period) leads to no borrowing and results in a welfare loss of 1.6 percent, rather than a gain.⁷ The difference between the two results is that debt limits are generous enough in the model with UI to make consumption smoothing relatively easy. With no UI payments, any negative asset holdings would imply negative consumption in the event that search was unsuccessful (and this probability is always positive) and therefore lead to expected utility equal to $-\infty$; such a situation cannot therefore arise. Thus, eliminating UI implies a significant adverse change in the conditions in the credit market, generating the welfare loss; when debt limits are always zero, this effect is not present and therefore eliminating UI is a net welfare gain. Note that this welfare gain comes despite only mild effects on aggregates.

3.2. Decomposition

A change in the unemployment insurance system creates three effects that induce changes in household behavior. The first effect is obtained by holding the debt limits and the vector of general equilibrium objects (r, w, τ) fixed; I call this the income-smoothing effect, because it is the change in behavior induced by the change in the variance of individual income.⁸ The second effect is due to the change in prices caused by general equilibrium forces; to compute this effect I keep the debt limits fixed and solve for the new (and different) equilibrium price vector. Finally, changing the UI system alters the value of the repayment relative to default and thus alters the extent to which private markets can provide insurance; this last effect I term the insurance effect.⁹

The income-smoothing effect (Effect I) dominates the change in the unemployment rate. UI reduces the incentive to search by reducing the relative value of employment to unemployment.

⁷In Figure 2, setting $\theta = b = 0$ would result in a flat borrowing limit of zero.

⁸Technically the variance of income is controlled by the household through variations in search effort. Since utility is concave, the variance of income does increase when the gap between the after-tax wage and the replacement rate rises.

⁹When isolating the effects of each part, I do not permit agents who would find autarky attractive to default. This assumption is important because otherwise pricing would reflect estimated default risk and returns would change. The interaction between default risk and unemployment insurance is studied in Athreya and Simpson (2006).

The first-order necessary and sufficient condition for optimal effort shows this clearly:

$$\chi a^{\chi-1} = \beta \gamma_e \exp(-\gamma_e a) (v(k'_1, 0) - v(k'_0, \min\{e+1, \bar{e}\})) . \quad (3.1)$$

The gap between the two outcomes of the employment search is negatively related to the value of unemployment insurance, holding general equilibrium effects and debt constraints constant; from there it is a straightforward calculation to show that search effort would decrease, leading to higher unemployment. UI also reduces the amount of capital in the economy through the income-smoothing effect, as precautionary savings motives decline. Since the equilibrium return to savings is smaller than the time rate of preference in this economy,

$$r - \delta < \frac{1}{\beta} - 1,$$

in the absence of idiosyncratic risk savings would disappear. Any policy change which reduces risk therefore also reduces capital accumulation; in the current model this change is large because the economy operates on the very elastic portion of the 'A^s' curve (see Figure 1). Since the reduction in the replacement rate raises the variance of individual income, capital rises.

The new equilibrium has a lower r and τ and a higher w . When I add these changes (labelled Effect II) to Effect I, they reinforce the drop in the unemployment rate but almost wipe out the capital accumulation change. With w rising and τ falling, the after-tax wage $(1 - \tau)w$ rises, increasing the income gap between the employed and unemployed states. With a larger gap more search effort is undertaken, leading to a lower equilibrium unemployment rate. At the same time, the decline in r , although small, leads to a large drop in total capital for reasons noted above – the economy is on the very elastic portion of the asset supply curve. The additional drop in the unemployment rate is relatively modest.

The effect of the debt limits (Effect III) adjusting is relatively small along both dimensions. With a less generous replacement rate, borrowing limits become more generous (that is, public insurance is a substitute for private insurance markets). Quantitatively, the debt limits move from those listed in Table 1 to $(-0.476, -0.394, -0.379)$, more than twice their previous values; it seems that public insurance does significantly crowd out the private insurance market. Since the default option becomes less attractive due to the rise in the variance of individual income, fewer individuals have an incentive to default, leading to higher debt positions being sustainable. As a result, precautionary motives for saving are weakened, leading to less capital accumulation; as

noted already, this change is modest relative to the other effects – it is not quantitatively important because few agents are near the borrowing limits. Effect III also increases search effort and thus decreases unemployment, despite reducing wages; this change is small as well.

3.3. Consumption Smoothing

I now turn to the issue of whether consumption smoothing is better when public insurance is large and private insurance is small or vice versa. I use two measures of consumption insurance – the coefficient of variation of consumption in the invariant distribution Σ_c and the innovation to consumption at the onset of an unemployment spell Δ_c . Since the invariant distribution in this model is also the time series distribution for any individual – see Aiyagari and Alvarez (2001) – Σ_c measures the consumption risk faced by a household over their infinite life; Σ_c is therefore an unconditional measure of consumption smoothing. In contrast, Δ_c is a measure of conditional smoothing – for those individuals who are unlucky in search, is consumption smoother? In Young (2004) UI increased Σ_c while reducing Δ_c ; Δ_c was smaller because UI reduces the variance of individual income but Σ_c was larger because the invariant distribution had more wealth-poor agents who were located on the steeply-sloped region of their consumption functions. Figure 3 plots the consumption functions for $\theta = 0.5$. Effect I shifts the consumption functions ‘Employed’ and ‘Unemployed, $e = 0$ ’ further apart, raising Δ_c , while also pushing more agents to higher values of k , reducing Σ_c .

Table 2 shows that conditional consumption smoothing is inhibited by the cut in unemployment benefits, but the effect is not large – the average drop in consumption is only 0.4 percent larger when $\theta = 0.25$. Because agents have relatively large debt limits when $\theta = 0.25$, they are able to effectively insulate their consumption against shocks which are not very persistent; since the average duration of unemployment in the model is only 1.5 quarters, buffer stocks of assets are effective insurance.¹⁰ On the other hand, decreasing θ leads to more unconditional consumption smoothing; as in Young (2004), the presence of more agents on the steeply-sloped portion of the consumption function leads to a rise in the volatility of consumption. Effect I leads to a decline in consumption smoothing according to both measures; the more volatile income stream makes it

¹⁰This value is larger than the data; calibration to shorter periods is difficult because the discount factor becomes very close to 1, generating excessively-long runtimes and numerical instability. Long runtimes are particularly problematic here, given that the household problem must be solved several times to compute the debt limits and the equilibrium r must be computed several times to solve the calibration equations. The extreme sensitivity of the model to r means that solving the system of market clearing conditions and calibration targets simultaneously is not computationally feasible.

more difficult to smooth consumption over a lifetime while it also makes it more difficult to preserve consumption conditional on bad luck. Effect II has a lower r , implying that assets are less effective as insurance, leading to a decrease in consumption smoothing along both dimensions; however, the higher w tends to move agents to higher levels of k , increasing the amount of consumption smoothing in the cross-section.

3.4. Alternative Punishments

Since the key difference between this paper and Young (2004) is the introduction of endogenous debt constraints, it seems critical to explore how the arbitrary assumption regarding the consequences of default changes the results. I consider alternative punishment in this subsection. Because the model does not permit default on the equilibrium path, it is not appropriate to calibrate the punishments to US bankruptcy code. Instead, I simply consider how variations in the rate of return on saving after default affect the outcomes.

The first punishment I consider assumes that in autarky a household can save at the equilibrium rate of return but cannot borrow, yielding the dynamic program

$$v^{Aut}(k, e) = \max_{k'_1 \geq 0, k'_{0e} \geq 0, a \geq 0} \left\{ \begin{array}{l} [1 - \exp(-\gamma_e a)] [\log(c_1^{Aut}) + \beta v^{Aut}(k'_1, 0)] + \\ \exp(-\gamma_e a) [\log(c_{0e}^{Aut}) + \beta v^{Aut}(k'_{0e}, \min\{e + 1, \bar{e}\})] - a^x \end{array} \right\} \quad (3.2)$$

subject to

$$c_1^{Aut} + k'_1 \leq (1 + r - \delta)k + (1 - \tau)w \quad (3.3)$$

$$c_{0e}^{Aut} + k'_{0e} \leq (1 + r - \delta)k + \theta_e w. \quad (3.4)$$

The punishment for default is now very small, as the loss of the right to borrow is the only penalty; as a result, borrowing essentially disappears from this economy for any level of the replacement rate, making it very similar to the one in Young (2004). Indeed, the welfare implications of unemployment insurance are essentially identical to those in that paper – eliminating UI completely is the optimal policy. The return gap is therefore seen to be an important mechanism for sustaining borrowing in this model. If savings produces a positive interest rate, repeated bad shocks cannot drive the agent to zero assets; instead, the marginal propensity to consume out of wealth converges to a constant strictly less than one, as shown in Carroll (2004), and the agent consumes out of asset income in the limit. An even weaker version of this punishment adopts the strategy from Krueger

and Uhlig (2006) – the household can borrow and save at the equilibrium interest rate because the outside option is driven by competitive lenders; under this assumption, borrowing completely disappears and the model reduces precisely to Young (2004).

More generally, I can assume that savings after autarky is taxed at rate $1 - \zeta < 1$, so that the agent solves (3.2) but faces the constraints

$$c_1^{Aut} + k'_1 \leq (1 + \zeta(r - \delta))k + (1 - \tau)w \quad (3.5)$$

$$c_{0e}^{Aut} + k'_{0e} \leq (1 + \zeta(r - \delta))k + \theta_e w; \quad (3.6)$$

because this taxation occurs off the equilibrium path I do not need to incorporate it into the government budget constraint. I set $\zeta = 0.5$ and recompute optimal UI settings; the benchmark economy is equivalent to $\zeta = 0$ and the punishment above is equivalent to $\zeta = 1$. I find here that eliminating UI dominates again, even though the absence of UI eliminates borrowing; thus, it is critical that the return gap is sufficiently large or UI is not desirable. An alternative interpretation is that punishment for default must be sufficiently large, as weak institutions make the moral hazard costs of UI too large to be worth it. The model in Young (2004) can then be viewed as the case where institutional punishment is absent, implying no borrowing can be sustained.

The standard punishment in the consumption-smoothing literature is complete banishment from the credit market forever, forcing the agent to live on labor income alone (see Krueger and Perri 2001); that is, the tax on savings is $\zeta = \frac{1}{r - \delta}$. This punishment implies that agents get $v^{Aut} = -\infty$ when they default if unemployment insurance is eliminated, since they cannot guarantee employment in the current period. In endowment economies it is always better to punish agents more severely off the equilibrium path, as more severe punishments implement greater risk sharing.¹¹

4. Conclusion

This paper has barely scratched the surface of the important question posed at the beginning – how does the provision of public insurance affect the extent to which markets for private insurance can operate? In this model, private markets for insurance are limited by the inability of the market to enforce repayment of debt; as shown in Alvarez and Jermann (2001), this friction limits only the amount of assets that can be traded, not their type. I rule out state-contingent claims by

¹¹This argument is used in Krueger and Perri (2006) to account for the failure of consumption volatility to rise with income volatility in the US.

assumption, although the effect of this assumption is limited by permitting agents to save after the realization of their idiosyncratic uncertainty; thus, consumption does not absorb the entire impact of an adverse shock to employment.¹² Cole and Kocherlakota (2001) present a model of hidden savings which can motivate this assumption of no state-contingent claims (Hopenhayn and Werning 2008 obtains a similar result by assuming outside options are private information). Ongoing work is exploring a model in which private UI is provided through a dynamic intermediary that faces a moral hazard problem because it cannot observe search effort.¹³ Another direction to proceed would be to permit the endogenous determination of the punishment for default, as developed by Koepl (2007) and Popov (2008); the desire to provide goods to the enforcement mechanism would clearly not be independent of the government-provided insurance.

One paper closely related to this one is Athreya and Simpson (2006), which studies the intersection of private insurance through bankruptcy and public insurance through UI. Bankruptcy converts uncontingent savings returns into contingent ones by permitting the household to default, but the instrument is very blunt: only two returns are possible.¹⁴ The main conclusion is that generous bankruptcy provisions and generous unemployment insurance should not coincide, because the incentives are reinforcing – both policies tend to reduce search effort and reduce saving, leading to more default and higher unemployment. Since the borrowing limits appear to play a role in determining whether UI is desirable or not, careful modelling of the factors which determine credit market limits seems important. The presence of the government in the risk-free market for credit – government debt, capital income taxation, and various loan guarantee programs – could also play an important role.

In this paper wages are homogeneous. In contrast, there is considerable heterogeneity in wages evident in US data, even conditioned on observables; quantitative models in the literature only account for a tiny fraction of wage dispersion.¹⁵ As a result, the literature clearly understates the provision of insurance obtained through the labor market, a feature which may alter the extent

¹²The inefficiency comes from households being forced to *ex ante* adjust effort to deal with risk; *ex post* their marginal rate of substitution between consumption and effort is suboptimal.

¹³This work is distinct from the large literature on dynamic optimal unemployment insurance – such as Wang and Williamson (1996, 2001) – since it takes the policy setting as given. Instead, the dynamic contracting problem is part of a competitive equilibrium, similar to Smith and Wang (2006). Chiu and Karni (1998) provides a theoretical explanation for the absence of private unemployment insurance. Solving this model is challenging because the insurance market faces severe adverse selection problems.

¹⁴In the model considered here bankruptcy would be exercised with probability 0 or 1 because all relevant uncertainty is realized before savings occurs. One could permit the agent to default partially while in bankruptcy, although current US law limits the extent to which debt restructuring can be fine-tuned in this manner. Exemption levels for non-debt instruments also play a role by introducing more state contingency into the return.

¹⁵See Krusell, Mukoyama, and Şahin (2007). Costain (1997) is an early paper that involves no wage heterogeneity.

to which government insurance can play an important role. Unions and other labor market institutional arrangements would also seem to be important for measuring the amount of insurance that the labor market can provide. Comparative studies of unemployment insurance between countries – such as Pallage and Zimmermann (2005) – would seem to be misspecified without these features. In addition, the presence of informal credit arrangements – particularly risk sharing through extended families – may alter the ability of the government to provide insurance (Kaplan 2008 studies the extent to which family arrangements, in particular cohabitation of adult children, is a source of insurance).

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Table 1

Properties of Calibrated Equilibrium

r	w	τ	u	K	k_0^b	k_1^b	k_2^b	Σ_c	Δ_c
0.03504	2.3728	0.036	0.072	11.552	-0.269	-0.206	-0.192	0.126	-0.010

Table 2Switching to $\theta = 0.25$

Effect	Δu	ΔK	Σ_c	Δ_c
I	-15.3%	28.2%	0.150	-0.012
I+II	-18.1%	1.6%	0.119	-0.014
Full	-18.9%	1.5%	0.119	-0.014

Figure 1

Equilibrium in Asset Market

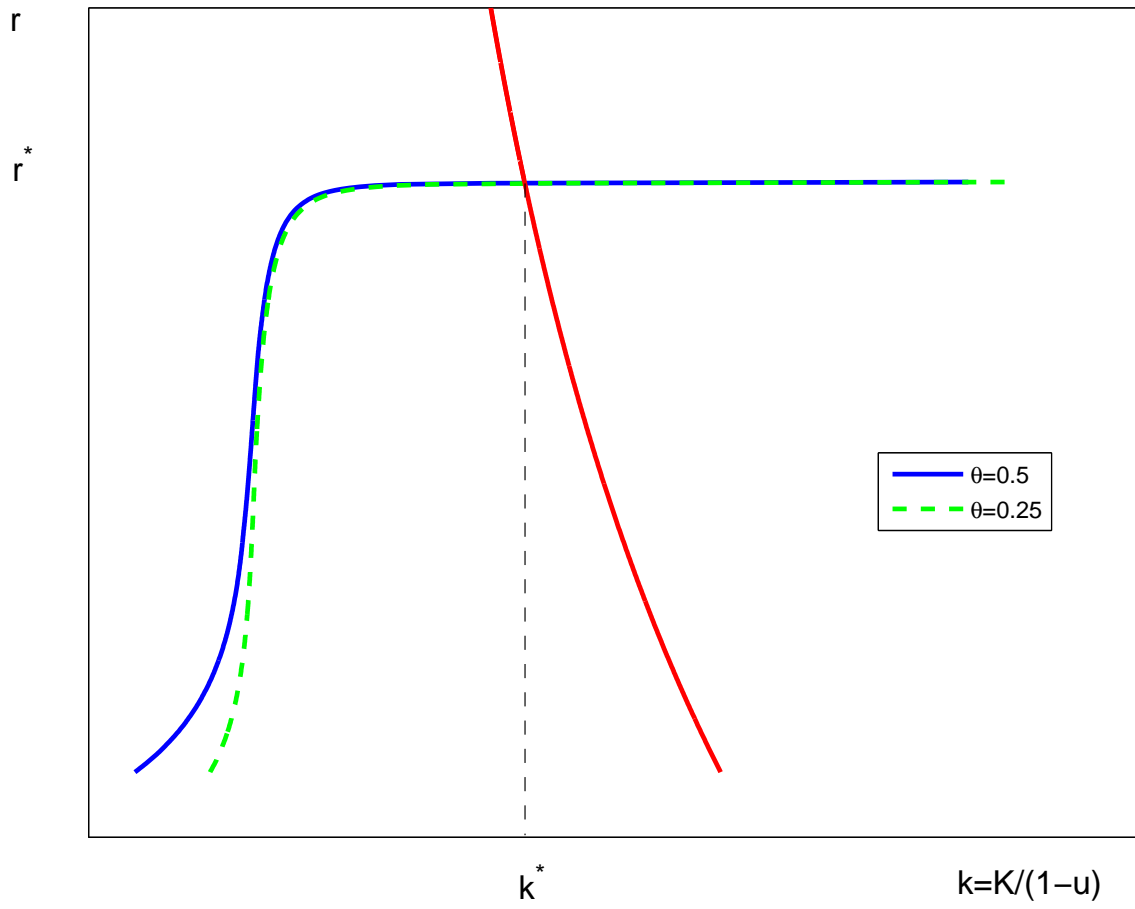


Figure 2

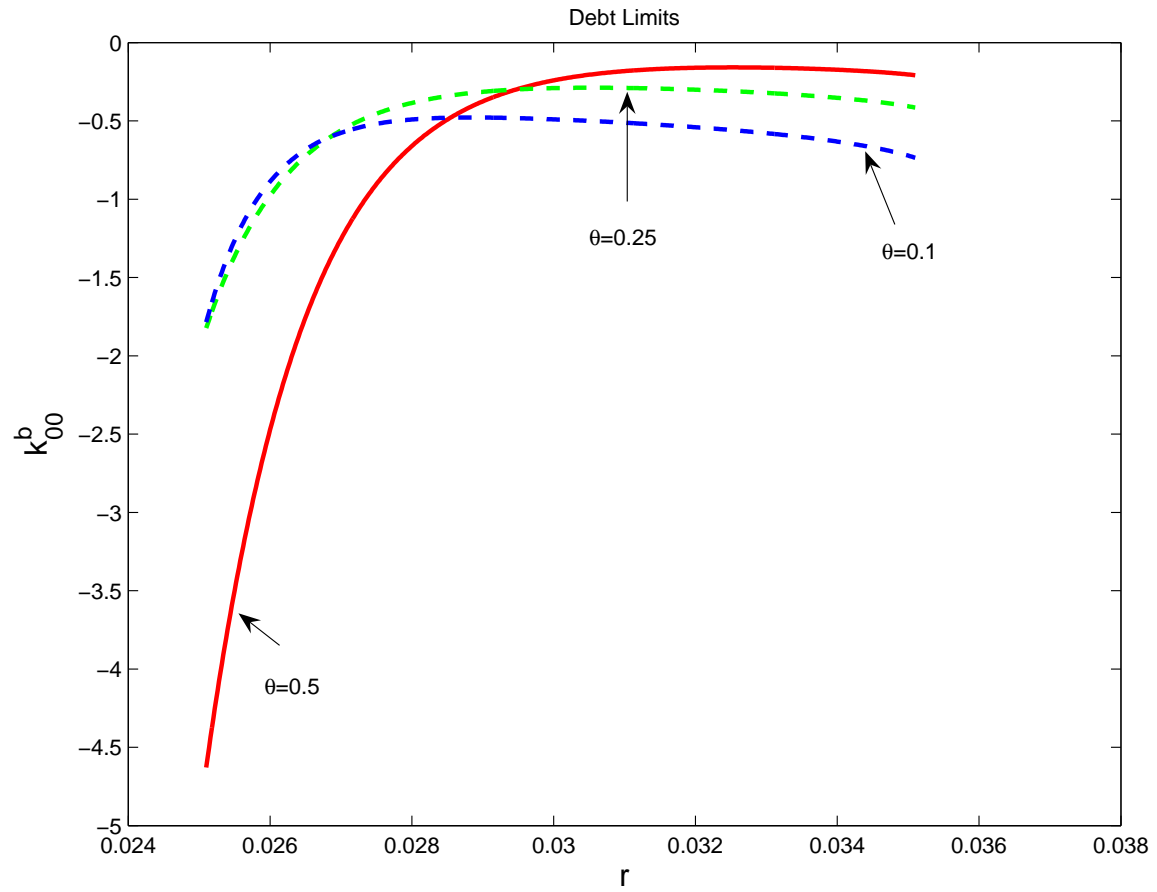
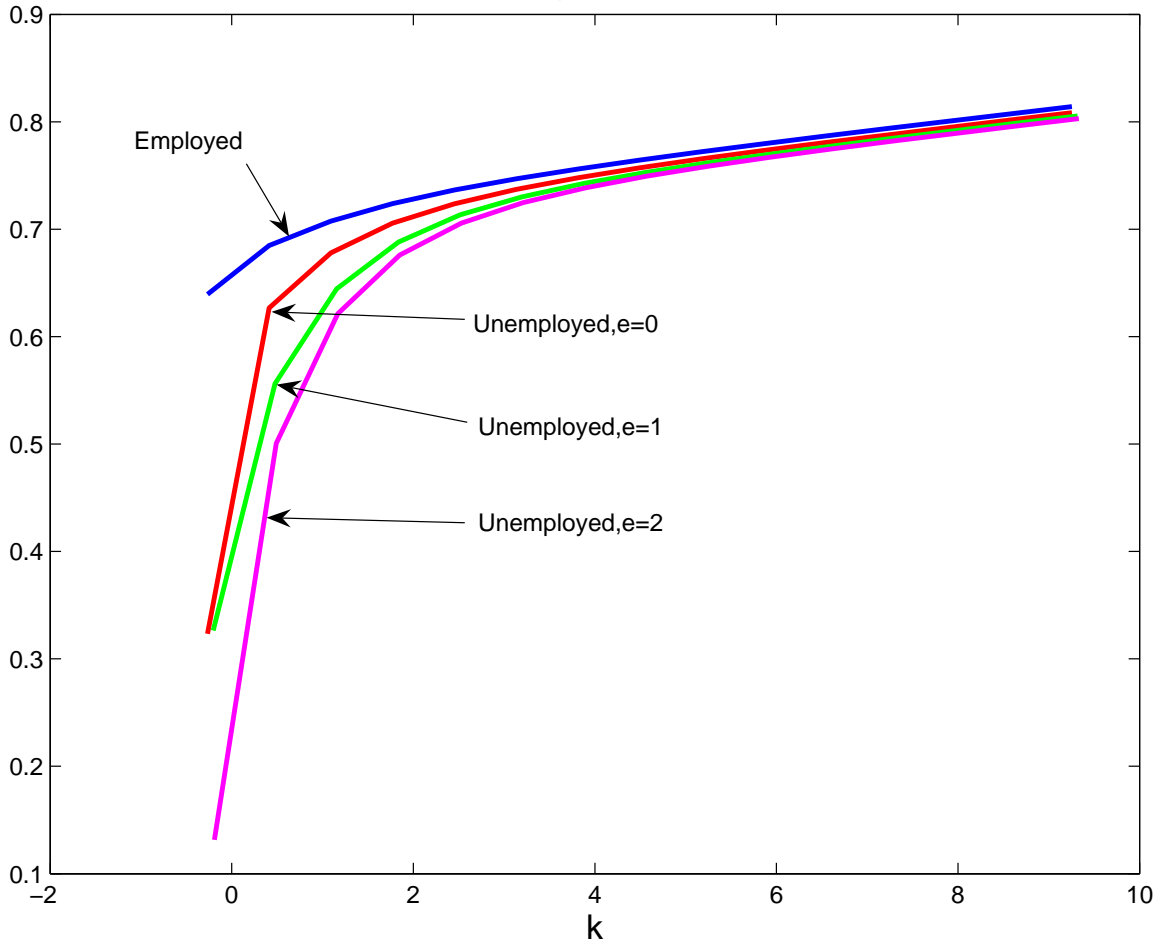


Figure 3

Consumption Functions



5. Computational Appendix (Not for Publication)

This appendix details the computation of the equilibrium. The equilibrium for this economy is a stationary distribution over the states of the world, denoted $\Gamma(k, e)$, which satisfies the conditions given in Definition 1. Computing this object involves obtaining 3 value functions, $v(k, 0)$, $v(k, 1)$, and $v(k, 2)$, in the baseline case, and endogenous debt limits $k^b(e)$ for each e . These functions are computed as follows; the autarky problem is solved in an analogous fashion.

The first step is to guess a debt limit for each value of e . I then choose a grid in the k direction whose lower bound is the debt limit; thus, the grid is not rectangular. As the value function has much more curvature near the borrowing constraint, the grid is not chosen in a uniform fashion; rather, the number of grid points is much larger for values close to the borrowing constraint. For the current model, I chose 150 grid points with 40 lying between 0 and 1 (above the debt limit) and the majority of the rest between 1 and 25 (above the debt limit). I chose an upper bound for the grid of $k = 1250$. Households are free to choose levels of capital above this point – they simply choose not to do so. For each point on the grid $K \times \{0, 1, 2\}$ I solve the consumer's problem in the following fashion. My results are not sensitive to increases in the number of grid points.

I first partition the Bellman equation as follows. Let $\varepsilon = 1$; that is, the consumer has been offered an employment opportunity this period. Next period's wealth must then solve the problem

$$\tilde{v}_1(k) = \max_{k'_1} \left\{ \frac{1}{1-\sigma} [(r+1-\delta)k + (1-\tau)wh - k'_1]^{1-\sigma} + \beta v(k'_1, 0) \right\} \quad (\text{B1})$$

subject to the borrowing constraint. Note that employed agents' savings decisions do not depend on their past employment status. Now let $\varepsilon = 0$; no employment opportunity has been offered. Next period's wealth must now solve the problem

$$\tilde{v}_0(k, e) = \max_{k'_0} \left\{ \frac{1}{1-\sigma} [(r+1-\delta)k + B_e wh - k'_0]^{1-\sigma} + \beta v(k'_0, \min\{e+1, j\}) \right\} \quad (\text{B2})$$

subject to the borrowing constraint. Finally, I solve for the optimal search effort by solving

$$v(k, e) = \max_a \{ [1 - \exp(-\gamma_e a)] \tilde{v}_1(k) + \exp(-\gamma_e a) \tilde{v}_0(k, e) - a^\chi \} \quad (\text{B3})$$

subject to $a \geq 0$. Partitioning is useful because solving multivariate constrained optimization problems is much more difficult than solving one nonlinear equation. This three-part procedure

defines a recursive algorithm which can be iterated upon, from an initial guess, to obtain the value function as the fixed point of the Bellman operator. All conditions needed to ensure the existence of this object are satisfied as well as the conditions that the value function be continuous, differentiable, and concave. To increase speed, I implement a form of Howard’s improvement algorithm, in which computed policy functions are used repeatedly to update value functions without reoptimizing at each step. This approach can be shown to be a contraction operator as well, with a modulus of contraction strictly smaller than β .¹⁶

I solve the above problems with a two-part method. For values near the borrowing constraint – where it might bind – I use a bisection method to find the optimal level of next period’s capital: I zero the first-order condition with the added constraint that k' cannot fall below k_b . For values of k that are not likely to lead to binding borrowing constraints, I use a fast Newton-Raphson routine to maximize the function.¹⁷ Both of these methods require that the derivative be continuous and the Newton-Raphson routine requires a continuous second derivative as well. For these reasons I use cubic spline interpolation for evaluating the value function at points off the grid. This method has advantages over completely grid-bound methods; namely, it does not introduce discreteness and is much faster as it requires fewer grid points for a given level of accuracy. It also has a speed advantage over methods based on approximating the value function by polynomials – solving for the coefficients in the cubic spline involves nothing more than inverting a tridiagonal matrix, rather than some minimization routine. The solution for a is located using a Newton-Raphson routine after checking that the condition $\tilde{v}_1 > \tilde{v}_0$ holds; if that condition does not hold, then $a = 0$.

The autarky problem is solved in an identical fashion with the obvious differences in the return on assets. The only other difference is that the grid for the autarky problem has a lower limit of 0, consistent with the borrowing constraint imposed upon default.

The next step is to update the guess at the debt limit. I use the solution from the equilibrium and autarky problems to evaluate the constraint

$$v\left(k^b(e'), \min\{e+1, \bar{e}\}\right) = v^{Aut}(0, \min\{e+1, \bar{e}\}); \quad (\text{B4})$$

I use a nonlinear equation solver to obtain the new implied debt limit for each e . Since I use

¹⁶The program contains a variable `hfix` that rescales aggregate output; it has no effect on the results but is important because it interacts with the grid and determines the range over which the value function has significant curvature. It is set to 0.3271 which puts output close to 1.

¹⁷It is in general not safe to use Newton-Raphson close to the constraint because extrapolation of cubic splines is not recommended and the routine is likely to attempt negative values when the solution is close to zero.

cubic splines to interpolate the value function, I must ensure that the implied debt limit is strictly above the initial guess, as extrapolation with cubic splines is generally not advisable. To ensure that this is the case, I initialize the search with the natural debt limit (that is, the one implied by nonnegative consumption) for each e . I then use a fixed point iteration to update my guess for the debt limit and resolve the consumer problem until this iterative procedure converges. It must converge if it is monotone because it is bounded – the debt limit cannot be positive – and it was monotone for all examples.

Having solved the consumer’s problem, I am now left with computing prices. I begin by guessing values for r and τ ; w can be obtained from the firm’s first-order conditions. Next I solve the consumer’s problem as detailed above. Then, having obtained the value function, I resolve it on a much finer uniform grid; this grid has 5000 evenly spaced points in the k direction. A uniform grid is required here, so the number of grid points must be substantially increased. Using this new grid, I iterate on the consumers’ decision rules until a stationary distribution emerges. This iteration proceeds as follows.

Take a point (k, e) in the state space. Locate $k'_1(k)$ and $k'_0(k, e)$ for this point using a search routine – an efficient one can be found in Press *et.al* (1993). Also compute $\Pr(\varepsilon = 1|e)$. Then allocate the mass located at (k, e) to the two grid points bracketing each of the four new asset holdings as follows. Let ω_1 be the weight attached to the lower grid point, denoted k_{1L} , for an agent who obtains a job. Then, I have

$$\begin{aligned}\Gamma^{n+1}(k_{1L}, 0) &= \Pr(\varepsilon = 1|e) * \omega_1 * \Gamma^n(k, e) \\ \Gamma^{n+1}(k_{1H}, 0) &= \Pr(\varepsilon = 1|e) * (1 - \omega_1) * \Gamma^n(k, e).\end{aligned}$$

where Γ^n is the distribution obtained in the n th iteration of this procedure and

$$\omega_1 = 1 - \frac{k'_1 - k_{1L}}{k_{1H} - k_{1L}}.$$

Similarly, I have

$$\begin{aligned}\Gamma^{n+1}(k_{0eL}, e^*) &= [1 - \Pr(\varepsilon = 1|e)] * \omega_{0e} * \Gamma^n(k, e) \\ \Gamma^{n+1}(k_{0eH}, e^*) &= [1 - \Pr(\varepsilon = 1|e)] * (1 - \omega_{0e}) * \Gamma^n(k, e).\end{aligned}$$

I terminate the loop when the changes in the distribution are small. This method of computing

the cross-sectional distribution of wealth is better than the simulation methods typically used in the literature because it does not involve sampling error and is continuous in γ_e . A uniform grid is required; otherwise, there will be a discrete upward jump at any point where the distance between grid points increases.

Once the stationary distribution is obtained, I can use the firm's first-order condition to obtain an implied value for r and the government budget constraint to find an implied value for τ . I then iterate on these values until they converge. In practice, I use a nested version of Brent's method to ensure a well-behaved convergence – the problem is extremely sensitive to small changes in r and it is convenient to have a solution method which brackets the zero for r . It can be proven that r must satisfy

$$r \leq \frac{1}{\beta} - 1 + \delta \tag{B5}$$

which provides an upper bound on r . A lower bound is 0, although this is obviously not an efficient choice. The algorithm guesses a value for τ , then computes the equilibrium value for r . For this value of r the government budget surplus is computed. τ is then iterated on, solving at each step for the equilibrium interest rate, until the budget is balanced. For the calibration procedure, I use Brent's method to find equilibrium r for given values of $\{\tau, \gamma_0, \gamma_1, \gamma_2\}$ then update that vector using a Newton-Raphson multidimensional routine. Since Laffer curve considerations are important here I make sure to select the lowest tax rate consistent with a particular steady state – there will also exist a high tax rate steady state where aggregate activity is smaller. By starting my search at a low value for τ these 'bad' equilibria are not encountered.