# Long-run Consumption Risk and Asset Allocation under Recursive Utility and Rational Inattention*

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**Abstract**

We study the portfolio decision of a household with limited information-processing capacity in a setting with recursive utility. We find that rational inattention combined with a preference for early resolution of uncertainty leads to a significant drop in the share of portfolios held in risky assets, even when the departure from standard expected utility with rational expectations is small. In addition, we show that the implied equity premium increases because inattentive investors with recursive utility face greater long-run risk and thus require higher compensation in equilibrium.

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# 1. Introduction

The canonical optimal consumption-portfolio choice models implicitly assume that consumers and investors have *unlimited* information-processing capacity and thus can observe the state variable(s) without errors; consequently, they can adjust their optimal plans instantaneously and completely to innovations to equity returns. However, plenty of evidence exists that ordinary people only have *limited* information-processing capacity and face many competing demands for their attention. As a result, agents react to the innovations slowly and incompletely because the channel along which information flows – the Shannon channel – cannot carry an infinite amount of information. In Sims (2003), this type of information-processing limitation is termed "Rational Inattention"(henceforth, RI). In the RI framework, *entropy* is used to measure the uncertainty of a random variable, and the reduction in the entropy is used to measure information flow.[1] For finite Shannon channel capacity, the reduction in entropy is bounded above; as capacity becomes infinitely large, the RI model converges to the standard full-information rational expectations (RE) model.[2]

Luo (2010) applies the RI hypothesis in the intertemporal portfolio choice model with time separable preferences in the vein of Merton (1969) and shows that RI alters the optimal choice of portfolio as well as the joint behavior of aggregate consumption and asset returns. In particular, limited information-processing capacity leads to smaller shares of risky assets. However, to generate the observed share and realistic joint dynamics of aggregate consumption and asset returns, the degree of attention must be as low as 10 percent (the corresponding Shannon capacity is 0.08 bits of information); this number means that only 10 percent of the uncertainty is removed in each period upon receiving a new signal about the aggregate shock to the equity return. Since we cannot estimate the degree of average inattention directly, it may be difficult to justify this low degree of RI. Indirect measurements of capacity uncover significantly higher channel capacity; we discuss them explicitly later in the paper.

[1] Entropy of a random variable $X$ with density $p(X)$ is defined as $E[\log(p(X))]$. Cover and Thomas (1991) is a standard introduction to information theory and the notion of entropy.

[2] There are a number of papers that study decisions within the LQ-RI framework: Sims (2003, 2006), Luo (2008, 2010), Maćkowiak, and Wiederholt (2009), and Luo and Young (2009a, 2009b, 2009c).

The preferences used in Luo (2010) are known to entangle two distinct aspects of preferences. Risk aversion measures the distaste for marginal utility variation across states of the world, while the elasticity of intertemporal substitution measures the distaste for deterministic variation of consumption across time; with expected utility these two attitudes are controlled by a single parameter such that if risk aversion increases the elasticity of intertemporal substitution must fall. The result in Luo (2010) shows that RI interacts with this parameter in a way that raises the apparent risk aversion (lowers the apparent intertemporal substitution elasticity) of the investor; however, it is unclear which aspect of preferences is actually being altered. As a result, interpretation of the results is ambiguous. Here, we develop an RI-Portfolio choice model within the recursive utility (RU) framework and use it to examine the effects of RI and RU on long-run consumption risk and optimal asset allocation. Specifically, we adopt preferences from the class studied by Kreps and Porteus (1978) and Epstein and Zin (1989), where risk aversion and intertemporal substitution are disentangled. These preferences also break indifference to the timing of the resolution of uncertainty, an aspect of preferences that plays an important role in determining the demand for risky assets (see Backus, Routledge, and Zin 2007). Indeed, it turns out that this aspect of preferences is key.

For tractability reasons we are confined to small deviations away from the standard class of preferences. However, we find that even a small deviation from unlimited information-processing capacity will lead to large changes in portfolio allocation if investors prefer early resolution of uncertainty. The intuition for this result lies in the long-term risk that equities pose: with rational inattention, uncertainty about the value of the equity return (and therefore the marginal utility of consumption) is not resolved for (infinitely-many) periods. This postponement of information is distasteful for agents who prefer early resolution of uncertainty, causing them to prefer an asset with an even and certain intertemporal payoff (the risk-free asset); in the standard time-separable expected utility framework, agents must be indifferent to the timing of the resolution of uncertainty, preventing the model in Luo (2010) from producing significant effects without very low channel capacity. Due to the nature of the accumulation of uncertainty, even small deviations from indifference (again, in the direction of preference for early resolution) combined with small deviations from complete information-processing leads to large declines in optimal risky asset shares. Thus, we provide a theory for why agents hold such a small share of risky assets without requiring significant deviations from reasonable preference parameters.

This result complements previous work on asset pricing with rational inattention (in particular Luo and Young 2010). That paper showed that an agent with incomplete information-processing ability will require a higher return to hold a risky asset due to two factors. First, rational inattention introduces higher volatility into consumption given endowment volatility; this extra volatility comes from the presence of estimation errors or noise shocks. Second, rational inattention introduces positive autocorrelation into consumption growth. Since the risk-free component of an asset's price is not affected by RI, the required risk premium rises. Here, we show that this effect may be amplified by a preference for early resolution of uncertainty and can become quite large, even when the deviation from indifference is arbitrarily small. Around the expected utility setting with unitary intertemporal elasticity of substitution and relative risk aversion, what matters for the size of this effect is the relative size of the deviation in IES from 1 as compared to the size of the deviation from relative risk aversion of 1.

We then present the results of adding nontradable labor income into the model, generating a hedging demand for risky equities. We find that our results survive essentially unchanged – rational inattention combined with a preference of early resolution of uncertainty still decreases the share of risky assets in the portfolio for small deviations around standard log preferences. In addition, we find that the importance of the hedging demand for equities is increasing in the degree of rational inattention. As agents become more constrained, they suffer more from uncertainty about consumption (Luo 2008); thus, they are more interested in holding equities if they negatively covary with the labor income shock and less interested if they positively covary. Given that the data support a small correlation between individual wage income and aggregate stock returns (Heaton and Lucas 2000), our results survive this extension intact.

This paper is organized as follows. Section 2 presents and reviews an otherwise standard two-asset portfolio choice model with recursive utility. Section 3 solves an RI version of the RU model and examines the implications of the interactions of RI, the separation of risk aversion and intertemporal substitution, and the discount factor for the optimal portfolio rule. Section 4 discusses the introduction of nontradable labor income. Section 5 concludes and discusses the extension of the results to non-LQ environments. Appendices contain the proofs and derivations that are omitted from the main text.

## 2. A Stylized Portfolio Choice Model with Rational Inattention and Recursive Utility

In this section, we present and discuss a standard portfolio choice model within a recursive utility framework. Following the log-linear approximation method proposed by Campbell (1993), Viceira (2001), and Campbell and Viceira (2002), we then incorporate the RI hypothesis into the standard model and solve it explicitly after considering the long-run consumption risk facing the investors. Another major advantage of the log-linearization approach is that we can approximate the original nonlinear problem by a log-LQ framework when relative risk aversion is close to 1 and thus can justify Gaussian posterior uncertainty under RI. We then discuss the interplay of RI, risk aversion, and intertemporal substitution for portfolio choice.

### 2.1. Specification and Solution of the Standard Recursive Utility Model of Portfolio Choice

Before setting up and solving the portfolio choice model with RI, it is helpful to present the standard portfolio choice model first and then discuss how to introduce RI in this framework. Here we consider a simple intertemporal model of portfolio choice with a continuum of identical investors. Following Epstein and Zin (1989), Giovannini and Weil (1989), and Weil (1990), suppose that investors maximize a recursive utility function $U_t$ by choosing consumption and asset holdings,

$$U_t = \left\{ (1-\beta) C_t^{1-1/\sigma} + \beta \left( E_t \left[ U_{t+1} \right] \right)^{(1-1/\sigma)/(1-\gamma)} \right\}^{\frac{1-\gamma}{1-1/\sigma}}, \tag{2.1}$$

where $C_t$ represents individual's consumption at time $t$, $\beta$ is the discount factor, $\gamma$ is the coefficient of relative risk aversion over wealth gambles (CRRA), and $\sigma$ is the elasticity of intertemporal substitution.[3] Let $\rho = (1-\gamma)/(1-1/\sigma)$; if $\rho > 1$, the household has a preference for early resolution of uncertainty.

We assume that the investment opportunity set is constant and contains only two assets: asset $e$ is risky, with one-period log (continuously compounded) return $r_{e,t+1}$, while the other asset $f$ is riskless with constant log return given by $r_f$. We refer to asset $e$ as the market portfolio of equities, and to asset $f$ as the riskless bond. $r_{e,t+1}$ has expected return $\mu$, $\mu - r_f$ is the equity premium, and $r_{e,t+1}$ has an unexpected component $u_{t+1}$ with $\text{var}[u_{t+1}] = \omega^2$.[4]

The intertemporal budget constraint for the investor is

$$A_{t+1} = R_{p,t+1}(A_t - C_t) \tag{2.2}$$

where $A_{t+1}$ is the individual's financial wealth (the value of financial assets carried over from period $t$ at the beginning of period $t+1$), $A_t - C_t$ is current period savings, and $R_{p,t+1}$ is the one-period gross return on savings given by

$$R_{p,t+1} = \alpha_t (R_{e,t+1} - R_f) + R_f \tag{2.3}$$

where $R_{e,t+1} = \exp(r_{e,t+1})$, $R_f = \exp(r_f)$, and $\alpha_t = \alpha$ is the proportion of savings invested in the risky asset.[5] As in Campbell (1993), we can derive an approximate expression for the log return on wealth:

$$r_{p,t+1} = \alpha(r_{e,t+1} - r_f) + r_f + \frac{1}{2}\alpha(1-\alpha)\omega^2. \tag{2.4}$$

Given the above model specification, it is well known that this simple discrete-time model can not be solved analytically. We therefore follow the log-linearization method

[3]When $\gamma = \sigma^{-1}$, $\rho = 1$ and the recursive utility reduces to the standard time-separable power utility with RRA $\gamma$ and intertemporal elasticity $\gamma^{-1}$. When $\gamma = \sigma = 1$ the objective function is the time-separable log utility function.

[4]Under unlimited information-processing capacity two-fund separation theorems imply that this investment opportunity set is sufficient. All agents would choose the same portfolio of multiple risky assets; differences in preferences would manifest themselves only in terms of the share allocated to this risky portfolio versus the riskless asset. We believe, but have not proven, that this result would go through under rational inattention as well.

[5]Given iid equity returns and a power utility function, $\alpha_t$ will be constant over time.

proposed in Campbell (1993), Viceira (2001), and Campbell and Viceira (2002) to obtain a closed-form solution to an approximation of this problem.[6] Specifically, the original intertemporal budget constraint, (2.2), can be written in log-linear form:

$$\Delta a_{t+1} = \left(1 - \frac{1}{\phi}\right)(c_t - a_t) + \psi + r^p_{t+1}, \tag{2.5}$$

where $c - a = E\left[c_t - a_t\right]$ is the unconditional (log of) consumption's share of financial wealth, $\phi = 1 - \exp(c - a)$, $\psi = \log(\phi) - (1 - 1/\phi)\log(1 - \phi)$, and lowercase letters denote logs. The optimal consumption and portfolio rules are then

$$c_t = b_0 + a_t, \tag{2.6}$$

$$\alpha = \frac{\mu - r_f + 0.5\omega^2}{(\rho/\sigma + 1 - \rho)\,\omega^2}, \tag{2.7}$$

where $b_0 = \log\left(1 - \beta^{\sigma}\left(E_t\left[R^{1-\gamma}_{p,t+1}\right]\right)^{\frac{\sigma-1}{1-\gamma}}\right)$ and $\frac{\rho}{\sigma} + 1 - \rho = \gamma$.[7]

Two aspects of preferences play a role in determining the portfolio share $\alpha$: intertemporal substitution, measured by $\sigma$, and the preference for the timing of the resolution of uncertainty, measured by $\rho$. A household who is highly intolerant of intertemporal variation in consumption will have a high share of risky assets. If $\sigma < 1$, a household who prefers earlier resolution of uncertainty (larger $\rho$) will have a lower share of risky assets. Using the identity this statement is equivalent to noting that larger $\rho$ means larger $\gamma$ for fixed $\sigma$, so that more risk aversion also implies lower share of risky assets. Thus, as noted in Epstein and Zin (1989), risk aversion and intertemporal substitution, while disentangled from each other, are entwined with the preference for the timing of uncertainty resolution.

We choose to focus on the temporal resolution aspect of preferences, rather than risk aversion, for two reasons. First, results in Backus, Routledge, and Zin (2007)

[6]This method proceeds as follows. First, both the flow budget constraint and the consumption Euler equations are log-approximated around the steady state. The Euler equations are log-approximated by a second-order Taylor expansion so that the second-moment are included; these terms are constant and thus the resulting equation is log-linear. Second, the optimal consumption and portfolio choices that satisfy these log-linearized equations are chosen as log-linear functions of the state. Finally, the coefficients of these optimal decision rules are pinned down using the method of undetermined coefficients.

[7]See Appendix in Giovannini and Weil (1989) for detailed deviations.

show a household with infinite risk aversion and infinite intertemporal elasticity actually holds almost entirely risky assets, and the opposite household (risk neutral with zero intertemporal elasticity) holds almost none. The second household prefers early resolution of uncertainty, a preference that cannot be expressed within the expected utility framework, and thus prefers paths of consumption that are smooth, while the first household prefers paths of *utility* that are smooth. Holding equities makes consumption risky, but not future utility, and therefore even a risk-neutral agent will avoid them. Second, it will turn out that rational inattention will have a strong effect when combined with a preference for early resolution of uncertainty, independent of the values of risk aversion and intertemporal elasticity.

## 3. RI-Recursive Utility Model

Given the specification of the above standard recursive utility model, it is straightforward to show that when $\sigma$ goes to 1, (2.1) can be rewritten as

$$V_t = C_t^{1-\beta} \left( E_t \left[ V_{t+1}^{1-\gamma} \right] \right)^{\beta/(1-\gamma)}, \tag{3.1}$$

where $V_t = U_t^{1/(1-\gamma)}$. Taking logs on both sides and dividing by $(1-\beta)$ and $1/(1-\gamma)$ gives

$$J_t = \log \left( C_t^{1-\gamma} \right) + \beta \log \left( E_t \left[ \exp \left( J_{t+1} \right) \right] \right), \tag{3.2}$$

where $J_t = \frac{1-\gamma}{1-\beta} \log V_t$.[8] When $\gamma$ is close to 1 (specifically, we assume that $\gamma$ is slightly above 1), the original nonlinear function $\log \left( C_t^{1-\gamma} \right)$ can be approximated around $\gamma = 1$:

$$\begin{aligned} \log \left( C_t^{1-\gamma} \right) &\simeq \log \left( 1 - (\gamma - 1) \log (C_t) + \frac{1}{2} (\gamma - 1)^2 (\log (C_t))^2 \right) \\ &\simeq -(\gamma - 1) c_t + \frac{1}{2} (\gamma - 1)^2 c_t^2. \end{aligned} \tag{3.3}$$

[8]See Appendix 6.1 for more details.

Substituting this approximation into (3.2) gives the following risk-sensitive linear quadratic (LQ) setup:

$$J_t = -(\gamma - 1) c_t + \frac{1}{2} (\gamma - 1)^2 c_t^2 + \beta \log (E_t [\exp (J_{t+1})]), \tag{3.4}$$

which implies that

$$W_t = c_t - \frac{1}{2} (\gamma - 1) c_t^2 - \frac{\beta}{\gamma - 1} \log (E_t [\exp ((1 - \gamma) W_{t+1})]), \tag{3.5}$$

where $W_t = \frac{1}{1-\gamma} J_t$. This representation is just the standard risk-sensitive linear quadratic control problem. In this expression $\gamma$ plays the role of enhanced risk aversion (or equivalently measures the household's desire to have decision rules that are robust to misspecification of the process for risky returns).[9]

### 3.1. Introducing RI

Following Sims (2003), we introduce rational inattention (RI) into the otherwise standard intertemporal portfolio choice model by assuming consumers/investors face information-processing constraints and have only finite Shannon channel capacity to observe the state of the world. Specifically, we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow. Formally, entropy is defined as the expectation of the negative of the log of the density function, $-E [\log (f (X))]$. For example, the entropy of a discrete distribution with equal weight on two points is simply $E [\log_2 (f (X))] = -0.5 \log_2 (0.5) - 0.5 \log_2 (0.5) = 1$, and the unit of information contained in this distributed is one "bit".[10] In this case, an agent can remove all uncertainty about $X$ if the capacity devoted to monitoring $X$ is $\kappa = 1$ bit.

With finite capacity $\kappa \in (0, \infty)$, the true state $a$ (a continuous variable) cannot be observed without error; thus the information set at time $t + 1$, $\mathcal{I}_{t+1}$, is generated by

[9] Tallarini (2000) contains a discussion of the role of $\gamma$ as risk-sensitivity (enhanced risk aversion) and fear of misspecification. Luo and Young (2009b) contains a discussion of the consumption-savings problem with limited information-processing capacity and risk sensitivity.

[10] For alternative bases for the logarithm, the unit of information differs: with natural log the unit of information is the 'nat' and with base 10 it is a 'hartley.'

the entire history of noisy signals $\left\{a_j^*\right\}_{j=0}^{t+1}$. Following Sims (2003), we assume in this paper that the noisy signal takes the additive form: $a_{t+1}^* = a_{t+1} + \xi_{t+1}$, where $\xi_{t+1}$ is the endogenous noise caused by finite capacity. We further assume that $\xi_{t+1}$ is an iid idiosyncratic shock and is independent of the fundamental shock. Note that the reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer's own internal information-processing constraint. Investors with finite channel capacity will choose a new signal $a_{t+1}^* \in \mathcal{I}_{t+1} = \left\{a_1^*, a_2^*, \cdots, a_{t+1}^*\right\}$ that reduces the uncertainty about the state variable $a_{t+1}$ as much as possible. Formally, this idea can be described by the information constraint

$$\mathcal{H}\left(a_{t+1}|\mathcal{I}_t\right) - \mathcal{H}\left(a_{t+1}|\mathcal{I}_{t+1}\right) = \kappa, \tag{3.6}$$

where $\kappa$ is the investor's information channel capacity, $\mathcal{H}\left(a_{t+1}|\ \mathcal{I}_t\right)$ denotes the entropy of the state *prior to* observing the new signal at $t+1$, and $\mathcal{H}\left(a_{t+1}|\ \mathcal{I}_{t+1}\right)$ is the entropy *after* observing the new signal. $\kappa$ imposes an upper bound on the amount of information – that is, the change in the entropy – that can be transmitted in any given period. We assume that the noise $\xi_{t+1}$ is Gaussian. Finally, following the literature, we suppose that the *ex ante* $a_{t+1}$ is a Gaussian random variable. As shown in Sims (2003), the optimal posterior distribution for $a_{t+1}$ will also be Gaussian.

The dynamic optimization problem of the typical investor with finite channel capacity is

$$\widehat{Q} = \max_{c_t, \mathcal{D}_t} \left\{ c_t - \frac{1}{2}\left(\gamma - 1\right) c_t^2 - \frac{\beta}{\gamma - 1} \log\left(E_t\left[\exp\left(-\left(\gamma - 1\right) Q_{t+1}\right)\right]\right) \right\} \tag{3.7}$$

subject to

$$\Delta a_{t+1} = \left(1 - \frac{1}{\phi}\right)\left(c_t - a_t\right) + \psi + r_{t+1}^p,$$

$$a_{t+1}|\mathcal{I}_{t+1} \sim \mathcal{D}_{t+1}, \tag{3.8}$$

$$a_t|\mathcal{I}_t \sim \mathcal{D}_t, \tag{3.9}$$

given $a_0|\ \mathcal{I}_0 \sim N\left(\widehat{a}_0, \Sigma_0\right)$, and the requirement that the rate of information flow at $t+1$ implicit in the specification of the prior and posterior distributions, $\mathcal{D}_t$ and $\mathcal{D}_{t+1}$, be less than channel capacity, and $\mathcal{I}_t$ is the information available at time $t$.

We assume that all individuals in the model economy have the same channel capacity; hence the average capacity in the economy is equal to individual capacity.[11] In this case the effective state variable is not the traditional state variable (the wealth level $a_t$), but rather the so-called *information state*: the distribution of the state variable $a_t$ conditional on the information set available at time $t$, $\mathcal{I}_t$.

As noted earlier, Gaussian uncertainty is optimal:

$$a_{t+1}|\mathcal{I}_{t+1} \sim N\left(\widehat{a}_{t+1}, \Sigma_{t+1}\right), \tag{3.10}$$

where $\widehat{a}_{t+1} = E\left[a_{t+1}|\, \mathcal{I}_{t+1}\right]$ and $\Sigma_{t+1}$ =var$\left[a_{t+1}|\, \mathcal{I}_{t+1}\right]$ are the conditional mean and variance of $a_{t+1}$, respectively. The information constraints (3.6) can thus be reduced to

$$\frac{1}{2}\left(\log\left(\Psi_t\right) - \log\left(\Sigma_{t+1}\right)\right) = \kappa \tag{3.11}$$

where $\Sigma_{t+1} = \text{var}_{t+1}\left[a_{t+1}\right]$ and $\Psi_t = \text{var}_t\left[a_{t+1}\right]$ are the posterior and prior variance, respectively. Given a finite transmission capacity of $\kappa$ bits per time unit, the optimizing consumer chooses a signal that reduces the conditional variance by $\frac{1}{2}\left(\log\left(\Psi_t\right) - \log\left(\Sigma_{t+1}\right)\right)$.[12] In the univariate state case this information constraint completes the characterization of the optimization problem and everything can be solved analytically.[13]

[11] Assuming that channel capacity follows some distribution in the cross-section complicates the problem when aggregating, but would not change the main findings. Luo and Young (2009c) show that RI with heterogeneous channel capacity can rationalize the infrequent adjustment used in Abel, Eberly, and Panageas (2009) or Huang and Liu (2007). Luo (2010) uses that model to discuss the effect of RI on stock market participation.

[12] Note that given $\Sigma_t$, choosing $\Sigma_{t+1}$ is equivalent to choosing the noise var$\left[\xi_t\right]$, since the usual updating formula for the variance of a Gaussian distribution is

$$\Sigma_{t+1} = \Psi_t - \Psi_t\left(\Psi_t + \text{var}\left[\xi_t\right]\right)^{-1}\Psi_t$$

where $\Psi_t$ is the ex ante variance of the state and is a function of $\Sigma_t$.

[13] With more than one state variable, there is an additional constraint that requires the difference between the prior and the posterior variance-covariance matrices be positive semidefinite; the resulting optimal posterior cannot be characterized analytically, and generally poses significant numerical challenges as well. See Sims (2003) for some examples.

Then intertemporal budget constraint (2.5) then implies that

$$\mathrm{E}_t\left[a_{t+1}\right] = E_t\left[r_{p,t+1}\right] + \psi + \widehat{a}_t, \tag{3.12}$$

$$\mathrm{var}_t\left[a_{t+1}\right] = \mathrm{var}_t\left[r_{p,t+1}\right] + \left(\frac{1}{\phi}\right)^2 \Sigma_t. \tag{3.13}$$

Substituting (3.12) into (3.11) yields

$$\kappa = \frac{1}{2}\left[\log\left(\mathrm{var}_t\left(r_{p,t+1}\right) + \left(\frac{1}{\phi}\right)^2 \Sigma_t\right) - \log\left(\Sigma_{t+1}\right)\right], \tag{3.14}$$

which has a unique steady state $\Sigma = \frac{\mathrm{var}_t[r_{p,t+1}]}{\exp(2\kappa)-(1/\phi)^2}$ with $\mathrm{var}_t\left[r_{p,t+1}\right] = \alpha^2\omega^2$. Note that here $\phi$ is close to $\beta$ as both $\gamma$ and $\sigma$ are close to 1.

The separation principle applies to this problem.[14] Hence, the optimal consumption rule is

$$c_t = b_0 + \widehat{a}_t \tag{3.15}$$

and the perceived state $\widehat{a}_t$ evolves according to a Kalman filter equation

$$\widehat{a}_{t+1} = (1-\theta)\,\widehat{a}_t + \theta a^*_{t+1} + \Omega, \tag{3.16}$$

where $\theta = 1 - 1/\exp(2\kappa)$ is the optimal weight on a new observation, $a^*_{t+1} = a_{t+1} + \xi_{t+1}$ is the observed signal, $\Omega$ is an irrelevant constant term, and $\xi_{t+1}$ is the iid noise with $\mathrm{var}\left[\xi_{t+1}\right] = \Sigma/\theta$.[15]

We make the following assumption.

**Assumption 1.**

$$2\kappa > \log\left(1/\beta\right). \tag{3.17}$$

Equation 3.17 ensures that agents have sufficient information-processing ability to "zero out" the unstable root in the Euler equation. It will also ensure that certain

[14] This principle says that under the LQ assumption optimal control and state estimation can be decoupled. See Whittle (1996) for detailed discussions.

[15] The observed signal about future permanent income $a^*_{t+1}$ is equivalent to making the signal current consumption.

infinite sums converge. Note that using the definition of $\theta$ we can write this restriction as $1-\theta<\beta^2<\beta$; the second inequality arises because $\beta<1$.

Combining (2.5), (3.15), and (3.16) gives the expression for individual consumption growth:

$$\Delta c_{t+1}=\theta\left\{\frac{\alpha u_{t+1}}{1-((1-\theta)/\beta)\cdot L}+\left[\xi_{t+1}-\frac{(\theta/\beta)\,\xi_t}{1-((1-\theta)/\beta)\cdot L}\right]\right\}, \tag{3.18}$$

where $L$ is the lag operator.[16] Note that all the above dynamics for consumption, perceived state, and the change in consumption are not the final solutions because the optimal share invested in stock market $\alpha$ has yet to be determined. To determine the optimal allocation in risky assets, we have to use an intertemporal optimality condition. However, the standard Euler equation is not suitable for determining the optimal asset allocation in the RI economy because consumption adjusts slowly and incompletely, making the relevant intertemporal condition one that equates the marginal utility of consumption today to the covariance between marginal utility and the asset return *arbitrarily far* into the future; that is, it is the "long-run Euler equation" that determines optimal consumption/savings plans. We now turn to deriving this equation.

### 3.2. Long-run Risk and the Demand for the Risky Asset

Bansal and Yaron (2004), Hansen, Heaton, and Li (2006), Parker (2001, 2003) and Parker and Julliard (2005) argue that long-term risk is a better measure of the true risk of the stock market if consumption reacts with delay to changes in wealth; the contemporaneous covariance of consumption and wealth understates the risk of equity.[17] Long-term consumption risk is the appropriate measure for the RI model.

Following Parker (2001, 2003), we define the long-term consumption risk as the covariance of asset returns and consumption growth over the period of the return and many subsequent periods. Because the RI model predicts that consumption reacts to the innovations to asset returns gradually and incompletely, it can rationalize the conclusion

[16] Here we have used the approximation result that $\phi=\beta$.

[17] Bansal and Yaron (2004) also document that consumption and dividend growth rates contain a long-run component. An adverse change in the long-run component will lower asset prices and thus makes holding equity very risky for investors.

in Parker (2001, 2003) that consumption risk is long term instead of contemporaneous. Given the above analytical solution for consumption growth, it is straightforward to calculate the ultimate consumption risk in the RI model. Specifically, when agents behave optimally but only have finite channel capacity, we have the following equality for the risky asset $e$ and the risk free asset $f$:

$$E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\left(R_{e,t+1}-R_f\right)\right]=0, \tag{3.19}$$

where $U_{i,t}$ for any $t$ denotes the derivative of the aggregate function with respect to its $i$th argument evaluated at $\left(C_t, E_t\left[U_{t+1}\right]\right)$.[18] Note that with time additive expected utility, the discount factor $U_{2,t+1+j}$ is constant and equal to $\beta$. (3.19) implies that the expected excess return can be written as

$$E_t\left[R_{e,t+1}-R_f\right]=-\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right]},$$

so that

$$\mu-r_f+\frac{1}{2}\omega^2=\operatorname{cov}_t\left[\frac{\rho}{\sigma}\left(\sum_{j=0}^{S}\Delta c_{t+1+j}\right)+(1-\rho)\left(\sum_{j=0}^{S}r_{p,t+1+j}\right), u_{t+1}\right], \tag{3.21}$$

where we have used $\gamma\simeq 1$, $c_{t+1+S}-c_t=\sum_{j=0}^{S}\Delta c_{t+1+j}$, and $\Delta c_{t+1+j}$ as given by (3.18). Furthermore, since the horizon $S$ over which consumption responds completely to income

[18]This long-term Euler equation can be obtained by combining the standard Euler equation for the excess return

$$E_t\left[U_{1,t+1}\left(R_{e,t+1}-R_f\right)\right]=0$$

with the Euler equation for the riskless asset between $t+1$ and $t+1+S$,

$$U_{1,t+1}=E_{t+1}\left[\left(\beta_{t+1}\cdots\beta_{t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right], \tag{3.20}$$

where $\beta_{t+1+j}=U_{2,t+1+j}$, for $j=0,\cdots,S$. In other words, the equality can be obtained by using $S+1$ period consumption growth to price a multiperiod return formed by investing in equity for one period and then transforming to the risk free asset for the next $S$ periods. See Appendix 6.2 for detailed derivations.

shocks under RI is infinite, the right hand side of (3.21) can be written as

$$\lim_{S\to\infty}\left\{\sum_{j=0}^{S}\operatorname{cov}_t\left[\frac{\rho}{\sigma}\Delta c_{t+1+j}+(1-\rho)\left(\sum_{j=0}^{S}r_{p,t+1+j}\right),u_{t+1}\right]\right\}=\alpha\left[\frac{\rho}{\sigma}\varsigma+1-\rho\right]\omega^2. \tag{3.22}$$

$\varsigma$ is the ultimate consumption risk measuring the accumulated effect of the equity shock to consumption under RI:

$$\varsigma=\theta\sum_{i=0}^{\infty}\left(\frac{1-\theta}{\beta}\right)^i=\frac{\theta}{1-(1-\theta)/\beta}>1 \tag{3.23}$$

when Assumption 1 holds.

### 3.3. Optimal Consumption and Asset Allocation

Combining Equations (3.21), (3.22), with (3.15) gives us optimal consumption and portfolio rules under RI. The following proposition gives a complete characterization of the model's solution for optimal consumption and portfolio choice.

**Proposition 2.** *Suppose that $\gamma$ is close to 1 and Assumption 1 is satisfied. The optimal share invested in the risky asset is*

$$\alpha^*=\left(\frac{\rho}{\sigma}\varsigma+1-\rho\right)^{-1}\frac{\mu-r^f+0.5\omega^2}{\gamma\omega^2}. \tag{3.24}$$

*The consumption function is*

$$c_t^*=\log(1-\beta)+\widehat{a}_t, \tag{3.25}$$

*actual wealth evolves according to*

$$a_{t+1}=\frac{1}{\beta}a_t+\left(1-\frac{1}{\beta}\right)c_t^*+\psi+\left[\alpha^*\left(r_{t+1}^e-r^f\right)+r^f+\frac{1}{2}\alpha^*(1-\alpha^*)\omega^2\right], \tag{3.26}$$

*and estimated wealth $\widehat{a}_t$ is characterized by the following Kalman filtering equation*

$$\widehat{a}_{t+1}=(1-\theta)\widehat{a}_t+\theta\left(a_{t+1}+\xi_{t+1}\right), \tag{3.27}$$

*where* $\psi = \log(\beta) - (1 - 1/\beta)\log(1-\beta)$, $\theta = 1 - \frac{1}{\exp(2\kappa)}$ *is the optimal weight on a new observation,* $\xi_t$ *is an* iid *idiosyncratic noise shock with* $\omega_\xi^2 = \mathrm{var}\left[\xi_{t+1}\right] = \Sigma/\theta$*, and* $\Sigma = \frac{\alpha^{*2}\omega^2}{\exp(2\kappa)-(1/\beta)^2}$ *is the steady state conditional variance. The change in individual consumption is*

$$\Delta c^*_{t+1} = \theta \left\{ \frac{\alpha^* u_{t+1}}{1 - ((1-\theta)/\beta) \cdot L} + \left[ \xi_{t+1} - \frac{(\theta/\beta)\,\xi_t}{1 - ((1-\theta)/\beta) \cdot L} \right] \right\}. \tag{3.28}$$

***Proof.*** *The proof is straightforward.* ■

The proposition clearly shows that optimal consumption and portfolio rules are *interdependent* under RI. Expression (3.24) shows that although the optimal fraction of savings invested in the risky asset is proportional to the risk premium ($\mu - r^f + 0.5\omega^2$), the reciprocal of both the coefficient of relative risk aversion ($\gamma$), and the variance of the unexpected component in the risky asset ($\omega^2$), as predicted by the standard Merton solution, it also depends on the interaction of RI and RU measured by $\frac{\rho}{\sigma}\varsigma + 1 - \rho$. We now examine how the interplay of RI and the preference for the timing of uncertainty resolution affects the long-term consumption risk and the optimal share invested in the risky asset. Denote $\frac{\rho}{\sigma}\varsigma + 1 - \rho$ in (3.24) the long-run consumption risk, and rewrite it as

$$\frac{\rho}{\sigma}\varsigma + 1 - \rho = \gamma + \Gamma, \tag{3.29}$$

where

$$\Gamma = \frac{\gamma - 1}{1 - \sigma}(\varsigma - 1) \tag{3.30}$$

measures how the interaction of recursive utility $\left(\frac{\gamma-1}{1-\sigma}\right)$ and the long-run impact of the equity return on consumption under RI ($\varsigma$) affect the risk facing the inattentive investors. Expression (3.29) clearly shows that both risk aversion ($\gamma$) and $\Gamma$ determine the optimal share invested in the risky asset. Specifically, suppose that investors prefer early resolution of uncertainty: $\gamma > \sigma$; even a small deviation from infinite information-processing capacity due to RI will generate large increases in long-run consumption risk and then reduce the demand for the risky asset.[19] In fact, it is not the scale of the deviation from $\sigma = \gamma = 1$ that matters, but the relative size of the deviations from $\sigma = 1$ and $\gamma = 1$.

[19] That is, $\theta$ is very close to 100% and therefore $\varsigma$ is only slightly greater than 1.

Figures 1 and 2 illustrate how RI affects the long-run consumption risk $\Gamma$ when $\sigma$ is close to 1 (here we set it to be 0.99999). (Following Viceira 2001 and Luo 2010, we set $\beta = 0.91$.) The figures show that the interaction of RI and RU can significantly increase the long-run consumption risk facing the investors. In particular, it is obvious that even if $\theta$ is high (so that investors can process nearly all the information about the equity return), the long-run consumption risk is still non-trivial. For example, when $\gamma = 1.01$ and $\theta = 0.9$ (i.e., 90 percent of the uncertainty about the equity return can be removed upon receiving the new signal), $\Gamma = 11$; if $\theta$ is reduced to 0.8, $\Gamma = 25$. That is, a small difference between risk aversion $\gamma$ and intertemporal substitution $\sigma$ has a significant impact on optimal portfolio rule.

Note that Expression (3.24) can be rewritten as

$$\alpha^* = \frac{\mu - r^f + 0.5\omega^2}{\widetilde{\gamma}\omega^2}, \tag{3.31}$$

where $\widetilde{\gamma} = \gamma\left(\frac{\rho}{\sigma}\varsigma + 1 - \rho\right)$ is the *effective* coefficient of relative risk aversion.[20] When $\theta = 1$, $\varsigma = 1$ and optimal portfolio choice (3.24) under RI reduces to (2.7) in the standard RU case, which we have discussed previously. Similarly, when $\rho = 1$ (3.24) reduces to the optimal solution in the expected utility model discussed in Luo (2010). Later we will show that $\widetilde{\gamma}$ could be significantly greater than the true coefficient of relative risk aversion ($\gamma$). In other words, even if the true $\gamma$ is close to 1 as assumed at the beginning of this section, the effective risk aversion that matters for the optimal asset allocation is $\gamma + \Gamma$, which will be greater than 1 if the capacity is low and $(\gamma - 1)$ is greater than $(1 - \sigma)$ (indeed, it can be a lot larger even for small deviations from $\gamma = \sigma = 1$). Therefore, both the degree of attention ($\theta$) and the discount factor ($\beta$) amount to an increase in the effective coefficient of relative risk aversion. Holding $\beta$ constant, the larger the degree of attention, the less the ultimate consumption risk. As a result, investors with low attention will choose to invest less in the risky asset. For example, with RI, a 1 percent negative shock in investors' financial wealth would affect their consumption more than that predicted by the standard RE model. Therefore, investors with finite capacity are

[20]By effective, we mean that if we observed a household's behavior and interpreted it as coming from an individual with unlimited information-processing ability, $\widetilde{\gamma}$ would be our estimate of the risk aversion coefficient.

less willing to invest in the risky asset.[21]

As argued in Campbell and Viceira (2002), the effective investment horizon of investors can be measured by the discount factor $\beta$. In the standard full-information RE portfolio choice model (such as Merton 1969), the investment horizon measured by $\beta$ is *irrelevant* for investors who have power utility functions, have only financial wealth, and face constant investment opportunities. In contrast, it is clear from (3.23) and (3.24) that the investment horizon measured by $\beta$ does matter for optimal asset allocations under RU and RI because it affects the valuation of long-term consumption risk. Expression (3.24) shows that the higher the value of $\beta$ (the longer the investment horizon), the higher the fraction of financial wealth invested in the risky asset. Figure 3 illustrates how the investment horizon affects the long-run consumption risk $\Gamma$ when $\gamma = 1.01$, $\sigma = 0.99999$, $\theta = 0.8$, and $\beta = 0.91$. The figure shows that the investment horizon can significantly affect the long-run consumption risk facing the investors. For example, when $\beta = 0.91$, $\Gamma = 25$; if $\beta$ is increased to 0.93, $\Gamma = 19$. That is, a small reduction in the discount factor has a significant impact on long-run consumption risk and the optimal portfolio share when combined with RI.

Given RRA ($\gamma$), IES ($\sigma$), and $\beta$, we can calibrate $\theta$ using the share of wealth held in risky assets. Specifically, we start with the annualized US quarterly data in Campbell (2003), and assume that $\omega = 0.16$, $\pi = \mu - r^f = 0.06$, $\beta = 0.91$, $\sigma = 0.99999$, and $\gamma = 1.001$. We then calibrate $\theta$ to match the observed $\alpha = 0.22$ estimated in Section 5.1 of Gabaix and Laibson (1999) to obtain

$$\alpha^* = \left[\gamma + \frac{\gamma - 1}{1 - \sigma}(\varsigma - 1)\right]^{-1} \frac{\pi + 0.5\omega^2}{\gamma\omega^2} = 0.22, \tag{3.32}$$

which means that $\theta = 0.48$.[22] That is, approximately 48 percent of the uncertainty is removed upon receiving a new signal about the equity return. Note that if $\gamma = 1$, the RE version of the model generates a highly unrealistic share invested in the stock market: $\alpha = \frac{\pi + 0.5\omega^2}{\omega^2} = 2.84$. To match the observed fraction in the US economy (0.22),

[21] Luo (2010) shows that with heterogeneous channel capacity the standard RI model would predict some agents would not participate in the equity market at all. It is clear that the same result would obtain with recursive utility.

[22] Gabaix and Laibson (2001) assume that all capital is stock market capital and that capital income accounts for 1/3 of total income.

$\gamma$ must be set to 13.

Equation (3.28) shows that individual consumption under RI reacts not only to fundamental shocks $(u_{t+1})$ but also to the endogenous noise $\left(\xi_{t+1}\right)$ induced by finite capacity. The endogenous noise can be regarded as a type of "consumption shock" or "demand shock". In the intertemporal consumption literature, some transitory consumption shocks are often used to make the model fit the data better. Under RI, the idiosyncratic noise due to RI provides a theory for these transitory consumption movements. Furthermore, Equation (3.28) also makes it clear that consumption growth adjusts slowly and incompletely to the innovations to asset returns but reacts quickly to the idiosyncratic noise.

Using (3.28), we can obtain the stochastic properties of the joint dynamics of consumption and the equity return. The following proposition summarizes the major stochastic properties of consumption and the equity return.

**Proposition 3.** *Given finite capacity $\kappa$ (i.e., $\theta$) and optimal portfolio choice $\alpha^*$, the volatility of consumption growth is*

$$\operatorname{var}\left[\Delta c_t^*\right] = \frac{\theta\beta^2\alpha^{*2}}{\beta^2+\theta-1}\omega^2, \tag{3.33}$$

*the relative volatility of consumption growth to the equity return is*

$$\mu = \frac{\operatorname{sd}\left[\Delta c_t^*\right]}{\operatorname{sd}\left[u_t\right]} = \sqrt{\frac{\theta}{1-(1-\theta)/\beta^2}}\alpha^*, \tag{3.34}$$

*the first-order autocorrelation of consumption growth is*

$$\rho_{\Delta c(1)} = \operatorname{corr}\left[\Delta c_t^*, \Delta c_{t+1}^*\right] = 0, \tag{3.35}$$

*and the contemporaneous correlation between consumption growth and the equity return is*

$$\operatorname{corr}\left[\Delta c_{t+1}^*, u_{t+1}\right] = \sqrt{\theta\left(1-(1-\theta)/\beta^2\right)}. \tag{3.36}$$

**Proof.** See Appendix 6.3. ∎

Expression (3.34) shows that RI affects the relative volatility of consumption growth to the equity return via two channels: (i) $\frac{\theta\beta^2}{\beta^2+\theta-1}$ and (ii) $\alpha^*$. Holding the optimal share invested in the risky asset $\alpha^*$ fixed, RI increases the relative volatility of consumption growth via the first channel because $\partial\left(\frac{\theta\beta^2}{\beta^2+\theta-1}\right)/\partial\theta < 0$. (3.28) indicates that RI has two effects on the volatility of $\Delta c$: the gradual response to a fundamental shock and the presence of the RI-induced noise shocks. The former effect reduces consumption volatility, whereas the latter one increases it; the net effect is that RI increases the volatility of consumption growth holding $\alpha^*$ fixed. Furthermore, as shown above, RI reduces $\alpha^*$ as it increases the long-run consumption risk via the interaction with the RU preference, which tends to reduce the volatility of consumption growth as households switch to safer portfolios. Figure 4 illustrates how RI affects the relative volatility of consumption to the equity return for different values of $\beta$ in the RU model; for the parameters selected RI reduces the volatility of consumption growth in the presence of optimal portfolio choice.

Expression (3.35) means that there is no persistence in consumption growth under RI. The intuition of this result is as follows. Both MA($\infty$) terms in (3.28) affect consumption persistence under RI. Specifically, in the absence of the endogenous noises, the gradual response to the shock to the equity return due to RI leads to positive persistence in consumption growth: $\rho_{\Delta c(1)} = \frac{\theta(1-\theta)}{\beta} > 0$. (See Appendix 6.3.) The presence of the noises generate negative persistence in consumption growth, exactly offsetting the positive effect of the gradual response to the fundamental shock under RI.

Expression (3.36) shows that RI reduces the contemporaneous correlation between consumption growth and the equity return because $\partial\,\mathrm{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right)/\partial\theta > 0$. Figure 5 illustrates the effects of RI on the correlation when $\beta = 0.91$. It clearly shows that the correlation between consumption growth and the equity return is increasing with the degree of attention ($\theta$).

### 3.4. Implications for the Equilibrium Equity Premium

According to the standard consumption-based capital asset pricing theory (CCAPM), the expected excess return on any risky portfolio over the risk-free interest rate is determined by the covariance of the excess return with contemporaneous consumption growth and the coefficient of relative risk aversion. Given the observed low contemporaneous covariance between equity returns and contemporaneous consumption growth, the standard CCAPM theory predicts that equities are not very risky. Consequently, to generate the observed high equity premium (measured by the difference between the average real stock return and the average short-term real interest rate), the coefficient of relative risk aversion must be very high. Given that $\omega = 0.16$, $\pi = \mu - r^f = 0.06$, and $\operatorname{cov}\left[\Delta c^*_{t+1}, u_{t+1}\right] = 6 \times 10^{-4}$ (annualized US quarterly data from Campbell 2003), to generate the observed equity premium we need a risk aversion coefficient of $\gamma = 100$.

In the RI-RU model, because every investor is assumed to have the same $\kappa$, the following pricing equation linking consumption growth and the equity premium holds when $\gamma \simeq 1$:

$$\pi = \alpha^* \left(\frac{\rho}{\sigma}\varsigma + 1 - \rho\right)\omega^2 - 0.5\omega^2. \tag{3.37}$$

Suppose, as in the consumption-based asset pricing literature, that the risk-free asset is an inside bond, so that in equilibrium the net supply of the risk free asset is 0 and then the share of wealth invested in the risky asset ($\alpha^*$) is 1. (3.37) becomes

$$\pi = (\gamma + \Gamma)\,\omega^2, \tag{3.38}$$

where $\Gamma = \frac{\gamma-1}{1-\sigma}(\varsigma - 1)$ and $\gamma \simeq 1$. It is clear from (3.38) that the interaction between RI and RU induces a higher equity premium because risk aversion and intertemporal substitution are disentangled and the accumulated effect of the innovation to the equity on consumption $\varsigma = \frac{\theta}{1-(1-\theta)/\beta} > 1$. The intuition behind this result is that for inattentive investors the uncertainty about consumption changes induced by changes in the equity return takes many periods to be resolved and this postponement is distasteful for these investors who prefer early uncertainty resolution; consequently, they require higher risk compensation in equilibrium.

Figures 1 and 2 can be used again to illustrate how RI affects the equity premium in equilibrium via increasing the long-run consumption risk $\Gamma$ when $\beta = 0.91$ and both $\gamma$ and $\sigma$ are close to 1. Using the same example in the portfolio choice problem, when $\gamma = 1.01$ and $\theta = 0.9$, $\Gamma = 11$, which means that the required equity premium would be increased by 11 times; when $\theta$ is smaller $\Gamma$ is larger, as we showed earlier, so the required return must be larger. That is, a small difference between risk aversion $\gamma$ and intertemporal substitution $\sigma$ can have a significant impact on the equilibrium equity return if agents have limited attention.

### 3.5. Channel Capacity

Our required channel capacity ($\theta = 0.48$ or $\kappa = 0.33$ nats) may seem low; 1 bit of information transmitted is definitely well below the total information-processing ability of human beings.[23] However, it is not implausible for little capacity to be allocated to the portfolio decision because individuals also face many other competing demands on their attention. For an extreme case, a young worker who accumulates balances in his $401\,(k)$ retirement savings account might pay no attention to the behavior of the stock market until he retires. In addition, in our model for simplicity we only consider an aggregate shock from the equity return, while in reality consumers/investors face substantial idiosyncratic shocks (in particular labor income shocks) that we do not model in this paper; Sims (2010) contains a more extensive discussion of low information-processing limits in the context of economic models.

As we noted in the Introduction, there are some existing estimation and calibration results in the literature, albeit of an indirect nature. For example, Adam (2005) found $\theta = 0.4$ based on the response of aggregate output to monetary policy shocks; Luo (2008) found that if $\theta = 0.5$, the otherwise standard permanent income model can generate realistic relative volatility of consumption to labor income; Luo and Young (2009) found that setting $\theta = 0.57$ allows a otherwise standard RBC model to match the post-war US consumption/output volatility. Finally, Melosi (2009) uses a model of firm rational inattention (similar to Maćkowiak and Wiederholt 2009) and estimates it to match the dynamics of output and inflation, obtaining $\theta = 0.66$. Thus, it seems that somewhere between 0.4 and 0.7 is a reasonable range, and our number lies right in the middle of this interval while the one required in Luo (2010) is much lower.

[23]See Landauer (1986) for an estimate.

## 4. Nontradable Labor Income

It is known that some of the anomalous predictions of the portfolio model can be reduced, although not eliminated, by the introduction of nontradable labor income. Following Viceira (2001) and Campbell and Viceira (2002), we assume that labor income $Y_t$ is uninsurable and nontradable in the sense that investors cannot write claims against future labor income; thus, labor income can be viewed as a dividend on the implicit holdings of human wealth. We will only sketch the results here; formal derivations are a straightforward extension of our existing results and are omitted.

We assume that the process for labor income is

$$Y_{t+1} = Y_t \exp\left(\nu_{t+1} + g\right) \tag{4.1}$$

where $g$ is a deterministic growth rate and $\nu_{t+1}$ is an iid normal random variable with mean zero and variance $\omega_\nu^2$. Log labor income therefore follows a random walk with drift; to keep the exposition simpler, we abstract from any transitory income shocks. In order to permit the risky asset to play a hedging role against labor income risk, we suppose that the two shocks are potentially correlated contemporaneously:

$$\text{cov}_t\left(u_{t+1}, \nu_{t+1}\right) = \omega_{u\nu}.$$

If $\omega_{u\nu} = 0$ then labor income can be viewed as purely idiosyncratic. The flow budget constraint then becomes

$$A_{t+1} = R_{t+1}^p\left(A_t + Y_t - C_t\right) \tag{4.2}$$

Log-linearizing 4.2 around the long-run means of the log consumption-income ratio and the log wealth-income ratio, $c - y = E\left[c_t - y_t\right]$ and $a - y = E\left[a_t - y_t\right]$, yields the approximate budget constraint

$$a_{t+1} - y_{t+1} = \rho_0 + \rho_a\left(a_t - y_t\right) + \rho_c\left(c_t - y_t\right) - \Delta y_{t+1} + r_{t+1}^p \tag{4.3}$$

where $\rho$, $\rho_a$, and $\rho_c$ are constants:

$$\begin{aligned}
\rho_a &= \frac{\exp(a-y)}{1+\exp(a-y)-\exp(c-y)} > 0 \\
\rho_c &= \frac{\exp(c-y)}{1+\exp(a-y)-\exp(c-y)} > 0 \\
\rho_0 &= -(1-\rho_a+\rho_c)\log(1-\rho_a+\rho_c) - \rho_a \log(\rho_a) + \rho_c \log(\rho_c) .
\end{aligned}$$

Starting from the standard time-separable rational expectations model ($\gamma = \sigma$ and $\theta = 1$) we obtain the decision rules

$$c_t = y_t + b_0 + b_1 (a_t - y_t) \tag{4.4}$$

$$\alpha^* = \frac{1}{b_1}\left(\frac{\mu - r_f + \frac{1}{2}\omega^2}{\gamma\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\omega_{u\nu}}{\omega^2} \tag{4.5}$$

where

$$\begin{aligned}
b_1 &= \frac{\rho_a - 1}{\rho_c} \in (0,1] \\
b_0 &= \frac{1}{1-\rho_a}\left[\left(\frac{1}{\gamma} - b_1\right) E\left[r^p_{t+1}\right] + \frac{1}{\gamma}\log(\beta) + \frac{\Xi}{2\gamma} - \rho_0 - (1-b_1)\, g\right];
\end{aligned}$$

$b_1$ is the elasticity of consumption with respect to financial wealth, making $1 - b_1$ the elasticity with respect to labor income, and $\Xi$ is an irrelevant constant term. If labor income is tradable, $b_1 = 1$ and the model reduces to the one studied previously.

To introduce rational inattention, we define the state variable

$$s_t = a_t + \lambda y_t \tag{4.6}$$

where

$$\lambda = \frac{1 - \rho_a + \rho_c}{\rho_a - 1}.$$

(As we have noted earlier, multivariate rational inattention models are analytically intractable, so the reduction of the state space to a single variable is critical for our results). Using this new state variable, the log-linearized budget constraint (4.3) can be rewritten as

$$s_{t+1} = \rho + \rho_a s_t - \rho_c c_t - g + \rho_\varepsilon \varepsilon_{t+1} + \lambda \nu_{t+1} + r^p_{t+1}. \tag{4.7}$$

The consumption function then becomes

$$c_t = b_0 + b_1 s_t. \tag{4.8}$$

Applying the separation principle yields

$$c_t = b_0 + b_1 \widehat{s}_t \tag{4.9}$$

and we obtain the law of motion for the conditional mean of permanent income

$$\widehat{s}_{t+1} = (1-\theta)\,\widehat{s}_t + \theta\left(s_{t+1} + \xi_{t+1}\right) + \Upsilon, \tag{4.10}$$

where $\Upsilon$ is an irrelevant constant and all other notation is the same as above. We require the following assumption.

**Assumption 1.**

$$1 - (1-\theta)\,\rho_a > 0. \tag{4.11}$$

Note that $\rho_a$ has replaced $\beta^{-1}$, but otherwise Equation 4.11 is the same as Equation 3.17. The optimal share invested in equity is now given by:

$$\alpha^* = \frac{1}{\varsigma}\left[\frac{1}{b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\varsigma\omega_{u\nu}}{\omega^2}\right] \tag{4.12}$$

where

$$\varsigma = \frac{\theta}{1-(1-\theta)\,\rho_a} > 1.$$

Luo (2010) contains details about the effects of labor income risk and rational inattention on strategic asset allocation in the expected utility setting.

Deviating from expected utility in the same manner as above, we obtain

$$\alpha^* = \frac{1}{\widetilde{\varsigma}}\left[\frac{1}{b_1}\left(\frac{\mu - r^f + 0.5\omega^2}{\omega^2}\right) + \left(1 - \frac{1}{b_1}\right)\frac{\widetilde{\varsigma}\omega_{u\nu}}{\omega^2}\right] \tag{4.13}$$

where

$$\widetilde{\varsigma} = \frac{\rho}{\sigma}\varsigma + 1 - \rho.$$

$\widetilde{\varsigma} > 1$ measures the long-run (accumulated) impacts of financial shocks on consumption. It is clear that our key result – that the presence of rational inattention combined with a preference for early resolution of uncertainty will dramatically reduce the share of risky assets and increase the required equity premium – survives the introduction of labor income risk.

Expression 4.13 contains two components. The first part is the so-called speculative asset demand, driven by the gap between the return to equity and the riskfree rate. Note that without labor income risk, the optimal asset allocation is solely determined by the speculative demand, that is, the allocation is proportional to the expected excess return of the risky asset, and is inversely relative to the variance of the equity return and to the elasticity of consumption to perceived wealth, $b_1$. The second part is the hedging demand, controlled by the correlation between returns and labor income. Given that $\rho_a > 1$ and $\theta \in (0, 1)$, RI affects the optimal allocation in the risky asset via the following two channels:

1. Reducing both the speculative demand and the income-hedging demand by the long-run consumption risk parameter $\widetilde{\varsigma}$.
2. In addition, as shown in the second term in the bracket of (4.13), RI increases the income hedging demand by $\widetilde{\varsigma}$ because $u_t$ and $\nu_t$ are correlated and consumption reacts to the shock to total wealth $\zeta_t = \alpha u_t + \lambda \nu_t$ gradually and indefinitely.

To make these points clear, we rewrite (4.13) as:

$$\alpha^* = \frac{1}{\widetilde{\varsigma} b_1} \left( \frac{\mu - r^f + 0.5\omega^2}{\omega^2} \right) + \left( 1 - \frac{1}{b_1} \right) \frac{\omega_{u\nu}}{\omega^2} \tag{4.14}$$

This expression clearly shows that RI increases the *relative* importance of the income-hedging demand to the speculative demand via the long-run consumption risk $\widetilde{\varsigma}$; under RI, the ratio of the income hedging demand to the speculative demand increases by $\widetilde{\varsigma}$. As inattention increases ($\theta$ declines), the hedging aspect of the demand for risky assets increases in importance, since $\frac{\partial \widetilde{\varsigma}}{\partial \theta} < 0$. To see where this positive relationship derives from, results from Luo (2008) and Luo and Young (2009b) imply that the welfare cost of labor income uncertainty is increasing in the degree of inattention (as $\theta$ falls, the cost rises). If equity returns are positively correlated with labor income, the agent will decrease demand for the asset as an insurance vehicle; similarly, a negative correlation will increase hedging demand. The data suggest this correlation is negative, but so small as to be quantitatively unimportant.[24] In addition, the second term in (4.14) also shows that RI has no impact on the absolute value of the income hedging demand. The reason is simple: under RI the innovation to the equity return has systematic effects on consumption growth affected by both the equity return and labor income.

As in Section 3.4, we can examine the asset pricing implications of the twin assumptions of recursive utility and rational inattention in the presence of nontradable labor income. Given that every investor has the same degree of RI, the following pricing equation linking consumption growth and the equity premium holds when $\gamma \simeq 1$:

$$\pi = \alpha^* \widetilde{\varsigma} b_1 \omega^2 - \widetilde{\varsigma} \left( b_1 - 1 \right) \omega_{u\nu} - 0.5\omega^2. \tag{4.15}$$

Under the same assumptions made above (zero net supply of bonds so that $\alpha^* = 1$), (4.15) becomes

$$\pi = \widetilde{\varsigma} \left[ b_1 \omega^2 + \left( 1 - b_1 \right) \omega_{u\nu} \right] - 0.5\omega^2, \tag{4.16}$$

which clearly shows that the positive correlation between the equity return and labor income, $\omega_{uv} > 0$, increases the equilibrium equity premium. Specifically, the magnitude of the hedging demand, $\left(1 - b_1\right) \omega_{uv}$, is increased by $\varsigma$ in the presence of information-processing constraints. In sum, the interaction between RI and positive correlations between the equity return and labor income will increase the equity premium in equilibrium by

$$\varpi = \widetilde{\varsigma} \left[ 1 + \left( \frac{1}{b_1} - 1 \right) \frac{\rho_{uv} \omega \omega_\nu}{\omega^2} \right]. \tag{4.17}$$

Note that in the case without RI and $\rho_{uv} = 0$, $\pi + 0.5\omega^2 = b_1 \omega^2$.

[24]For example, Heaton and Lucas (2000) find that individual labor income is weakly correlated with equity returns, with support for both positive and negative correlations. Aggregate wages have a correlation of $-0.07$ with equity returns.

## 5. Conclusion

In this paper we have studied the portfolio choice of a household with Kreps-Porteus/Epstein-Zin preferences and limited information-processing capacity (rational inattention). Rational inattention interacts with a preference for early resolution of uncertainty to generate significant decreases in the demand for risky assets; small deviations from indifference over timing and infinite channel capacity are magnified over the infinite future produce empirically-reasonable portfolios with actual risk aversion essentially equal to 1, whether the agent has nontradable labor income or not. This result raises important questions about empirical assessment, such as how to identify risk aversion separately from channel capacity, that we will not pursue here.

We have focused on solutions in which *ex post* uncertainty is Gaussian. Recent results in the rational inattention literature (Matejka and Sims 2010 and Saint-Paul 2010) have noted that there exist *discrete* optimal solutions to the decision problem, even in the LQ-Gaussian case, that may dominate the Gaussian one; the intuition for this result is that information costs can be reduced by dividing the state space into regions and only permitting solutions to differ across these regions instead of inside them. Of particular relevance to this paper is Batchuluun, Luo, and Young (2008), who show that fully-nonlinear portfolio decisions are discrete in a simple two-period economy. The discrete solutions from Batchuluun, Luo, and Young (2008) have the property that agents will place positive measure on only a small number of different portfolio shares. For reasonable degrees of risk aversion and low enough channel capacity, one group of these points involves zero equity holdings, because agents who want to borrow from future income will do so using the risk-free asset and there is always a positive probability that wealth is actually such that borrowing would be optimal. A second group of points involves a small amount of risky assets (and generally this set of points has the most mass), while a third group has a significant amount of risky assets. Extending these results to study portfolios in long-horizon models has the potential to rationalize why few households hold assets that do not appear very risky (in terms of consumption or utility), why those that hold these assets hold so few of them, and why these assets pay such high rates of return. The mechanism identified here will still be present, if somewhat obscured by the numerical solution.

# 6. Appendix

## 6.1. Deriving Risk-sensitive Utility from the Original Recursive Utility

Here, we detail the straightforward steps omitted in the main part of the paper that derive the form of the Bellman equation we use. Tallarini (2000) contains a similar derivation.

$$\log\left(V_t\right) = (1-\beta)\log C_t + \frac{\beta}{1-\gamma}\log\left(E_t\left[V_{t+1}^{1-\gamma}\right]\right) \Longrightarrow$$
$$(1-\gamma)\log\left(V_t\right) = (1-\gamma)(1-\beta)\log C_t + \beta\log\left(E_t\left[\exp\left((1-\gamma)\log\left(V_{t+1}\right)\right)\right]\right) \Longrightarrow$$
$$\frac{1-\gamma}{1-\beta}\log\left(V_t\right) = (1-\gamma)\log C_t + \beta\log\left(E_t\left[\exp\left(\frac{1-\gamma}{1-\beta}\log\left(V_{t+1}\right)\right)\right]\right),$$

which can be rewritten as

$$J_t = (1-\gamma)\log\left(C_t\right) + \beta\log\left(E_t\left[\exp\left(J_{t+1}\right)\right]\right). \tag{6.1}$$

## 6.2. Deriving Long-term Euler Equation within the Recursive Utility Framework

Within the recursive utility framework, when wealth is allocated efficiently across assets, the marginal investment in any asset yields the same expected increase in future utility,

$$E_t\left[\frac{U_{2,t}U_{1,t+1}}{U_{1,t}}\left(R_{e,t+1}-R_f\right)\right] = 0, \tag{6.2}$$

which means that

$$E_t\left[U_{1,t+1}\left(R_{e,t+1}-R_f\right)\right] = 0, \tag{6.3}$$

where $U_{i,t}$ for any $t$ denotes the derivative of the aggregator function with respect to its $i-th$ argument, evaluated at $(C_t, E_t\left[U_{t+1}\right])$.

Using the Euler equation for the risk free asset between $t+1$ and $t+1+S$,

$$\begin{aligned} U_{1,t+1} &= E_{t+1}\left[\left(\beta_{t+1}\cdots\beta_{t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right] \\ &= E_{t+1}\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right], \end{aligned} \tag{6.4}$$

where we denote $\beta_{t+1+j} = U_{2,t+j}$, for $j = 0, \cdots, S$. Substituting (6.4) into (6.3) yields

$$\begin{aligned} & E_t\left[E_{t+1}\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R^f\right)^S U_{1,t+1+S}\right]\left(R^e_{t+1}-R^f\right)\right] \\ &= E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R^f\right)^S U_{1,t+1+S}\left(R^e_{t+1}-R^f\right)\right] = 0. \end{aligned}$$

Hence, the expected excess return can be written as

$$\begin{aligned} E_t\left[R_{e,t+1}-R_f\right] &= -\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{E_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}\right]} \\ &= -\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{E_t\left[U_{1,t+1}\right]} \\ &= -\frac{\operatorname{cov}_t\left[\left(U_{2,t+1}\cdots U_{2,t+S}\right)\left(R_f\right)^S U_{1,t+1+S}, R_{e,t+1}-R_f\right]}{U_{1,t}/\left(U_{2,t}R_f\right)} \\ &= -\operatorname{cov}_t\left[\left(U_{2,t}\cdots U_{2,t+S}\right)\left(R_f\right)^{S+1}\frac{U_{1,t+1+S}}{U_{1,t}}, R_{e,t+1}-R_f\right] \\ &= -\operatorname{cov}_t\left[\left(R_f U_{2,t}\frac{U_{1,t+1}}{U_{1,t}}\right)\cdots\left(R_f U_{2,t+S}\frac{U_{1,t+1+S}}{U_{1,t+S}}\right), R_{e,t+1}-R_f\right] \\ &\simeq \operatorname{cov}_t\left[\left(\frac{\theta}{\rho}\Delta c_{t+1}+(1-\theta)\, r_{p,t+1}\right)+\cdots+\left(\frac{\theta}{\rho}\Delta c_{t+1+S}+(1-\theta)\, r_{p,t+1+S}\right), u_{t+1}\right] \\ &= \operatorname{cov}_t\left[\frac{\theta}{\rho}\left(\sum_{j=0}^{S}\Delta c_{t+1+j}\right)+(1-\theta)\left(\sum_{j=0}^{S} r_{p,t+1+S}\right), u_{t+1}\right]. \end{aligned}$$

### 6.3. Deriving the Stochastic Properties of Consumption Dynamics

Taking unconditional variance on both sides of (3.28) yields

$$\begin{aligned}
\operatorname{var}[\Delta c_t^*] &= \theta^2 \left\{ \frac{\alpha^{*2}\omega^2}{1-(1-\theta)^2/\beta^2} + \left[1 + \frac{\theta^2/\beta^2}{1-(1-\theta)^2/\beta^2}\right]\omega_\xi^2 \right\} \\
&= \theta^2 \left[ \frac{1}{1-(1-\theta)^2/\beta^2} + \frac{1+(2\theta-1)/\beta^2}{1-(1-\theta)^2/\beta^2} \frac{1}{\left(1/(1-\theta)-1/\beta^2\right)\theta} \right] \alpha^{*2}\omega^2 \\
&= \theta^2 \left\{ \frac{1}{1-(1-\theta)^2/\beta^2} + \left[ \frac{1}{\left(1-(1-\theta)/\beta^2\right)\theta} - \frac{1}{1-(1-\theta)^2/\beta^2} \right] \right\} \alpha^{*2}\omega^2 \\
&= \frac{\theta\beta^2}{\beta^2+\theta-1}\alpha^{*2}\omega^2.
\end{aligned}$$

Using (3.28), we can compute the first-order autocovariance of consumption growth:

$$\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = \operatorname{cov}\left( \begin{array}{c} \theta\left\{ \frac{\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L} + \left[\xi_t - \frac{(\theta/\beta)\xi_{t-1}}{1-((1-\theta)/\beta)\cdot L}\right] \right\}, \\ \theta\left\{ \frac{\alpha^* u_{t+1}}{1-((1-\theta)/\beta)\cdot L} + \left[\xi_{t+1} - \frac{(\theta/\beta)\xi_{t}}{1-((1-\theta)/\beta)\cdot L}\right] \right\} \end{array} \right)$$

$$= \operatorname{cov}\left( \theta\left\{ \frac{\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L} + \left[\xi_t - \frac{(\theta/\beta)\,\xi_{t-1}}{1-((1-\theta)/\beta)\cdot L}\right] \right\}, \theta\left\{ \frac{((1-\theta)/\beta)\,\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L} - \frac{(\theta/\beta)\,\xi_t}{1-((1-\theta)/\beta}\right.\right.$$

$$= \frac{1-\theta}{\beta}\operatorname{cov}\left( \frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L}, \frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L} \right)$$

$$+ \operatorname{cov}\left( \theta\left[\xi_t - \frac{(\theta/\beta)\,\xi_{t-1}}{1-((1-\theta)/\beta)\cdot L}\right], -\frac{\theta\,(\theta/\beta)\,\xi_t}{1-((1-\theta)/\beta)\cdot L} \right)$$

$$= \frac{1-\theta}{\beta}\operatorname{var}\left( \frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L} \right) + \operatorname{cov}\left( \theta\left[\xi_t - \frac{(\theta/\beta)\,\xi_{t-1}}{1-((1-\theta)/\beta)\cdot L}\right], -\frac{\theta\,(\theta/\beta)\,\xi_t}{1-((1-\theta)/\beta)\cdot L} \right)$$

$$= \frac{1-\theta}{\beta}\operatorname{var}\left( \frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L} \right) + \operatorname{cov}\left(\theta\xi_t, -\theta\,(\theta/\beta)\,\xi_t\right) + \operatorname{cov}\left( -\frac{(\theta^2/\beta)\xi_{t-1}}{1-((1-\theta)/\beta)\cdot L}, -\frac{(\theta^2\,(1-\theta)/\beta}{1-((1-\theta)/}\right.$$

$$= \frac{1-\theta}{\beta}\frac{(\theta\alpha^*)^2\,\omega^2}{1-(1-\theta)^2/\beta^2} - \frac{\theta^3}{\beta}\frac{\alpha^{*2}\omega^2}{\left(1/(1-\theta)-1/\beta^2\right)\theta} + \frac{\theta^4\,(1-\theta)}{\beta^3}\frac{1}{1-(1-\theta)^2/\beta^2}\frac{\alpha^{*2}\omega^2}{\left(1/(1-\theta)-1/\beta^2\right)\theta}$$

$$= \left[ \frac{1-\theta}{\beta}\frac{\theta^2}{1-(1-\theta)^2/\beta^2} - \frac{\theta^2}{\beta}\frac{1}{1/(1-\theta)-1/\beta^2} + \frac{\theta^3\,(1-\theta)}{\beta^3}\frac{1}{1-(1-\theta)^2/\beta^2}\frac{1}{1/(1-\theta)-1/\beta^2} \right]\alpha^{*2}\omega^2.$$

$$= 0.$$

We thus have

$$\operatorname{corr}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = \frac{\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right)}{\sqrt{\operatorname{var}\left(\Delta c_t^*\right)}\sqrt{\operatorname{var}\left(\Delta c_{t+1}^*\right)}} = 0.$$

Note that in the absence of the endogenous noise shocks, we have

$$\begin{aligned}
\operatorname{corr}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) &= \frac{1-\theta}{\beta} \frac{\left(\theta\alpha^*\right)^2 \omega^2}{1-(1-\theta)^2/\beta^2} \left(\frac{\theta\beta^2}{\beta^2+\theta-1}\alpha^{*2}\omega^2\right)^{-1} \\
&= \frac{1-\theta}{\beta} \frac{\theta^2}{1-(1-\theta)^2/\beta^2} \left(\frac{\theta}{1-(1-\theta)^2/\beta^2}\right)^{-1} = \frac{\theta(1-\theta)}{\beta} > 0
\end{aligned}$$

because

$$\begin{aligned}
&\operatorname{cov}\left(\Delta c_t^*, \Delta c_{t+1}^*\right) = \operatorname{cov}\left(\frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L}, \frac{\theta((1-\theta)/\beta)\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L}\right) \\
&= \frac{1-\theta}{\beta} \operatorname{cov}\left(\frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L}, \frac{\theta\alpha^* u_t}{1-((1-\theta)/\beta)\cdot L}\right) \\
&= \frac{1-\theta}{\beta} \frac{\left(\theta\alpha^*\right)^2\omega^2}{1-(1-\theta)^2/\beta^2}.
\end{aligned}$$

Finally, using (3.28), it is straightforward to show that

$$\begin{aligned}
\operatorname{corr}\left(\Delta c_{t+1}^*, u_{t+1}\right) &= \frac{\operatorname{cov}\left(\Delta c_{t+1}^*, u_{t+1}\right)}{\operatorname{sd}\left(\Delta c_{t+1}^*\right)\operatorname{sd}\left(u_{t+1}\right)} \\
&= \frac{\theta\alpha^*\omega^2}{\sqrt{\frac{\theta\beta^2}{\beta^2+\theta-1}}\alpha^*\omega^2} \\
&= \sqrt{\theta\left(1-(1-\theta)/\beta^2\right)},
\end{aligned}$$

where we use the result that

$$\operatorname{cov}\left(\Delta c_{t+1}^*, u_{t+1}\right) = \operatorname{cov}\left(\theta\alpha^* u_{t+1}, u_{t+1}\right) = \theta\alpha^*\omega^2$$

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Figure 1: The Effects of RI and RU on Long-run Consumption Risk

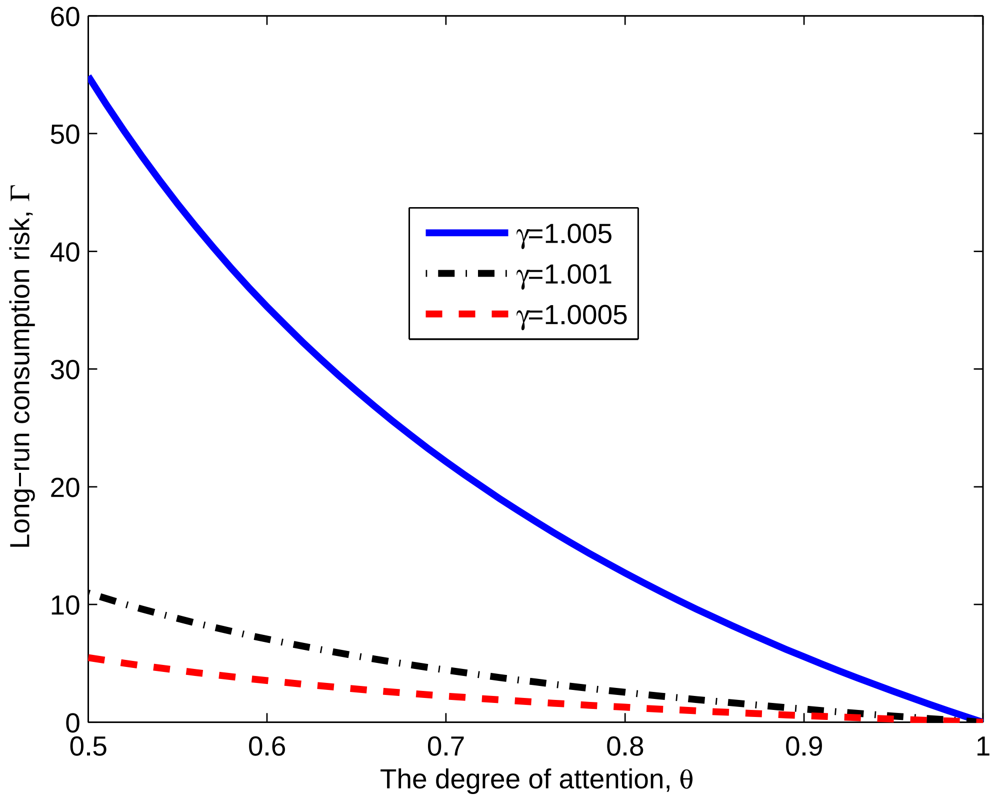

Figure 2: The Effects of RI and RU on Long-run Consumption Risk

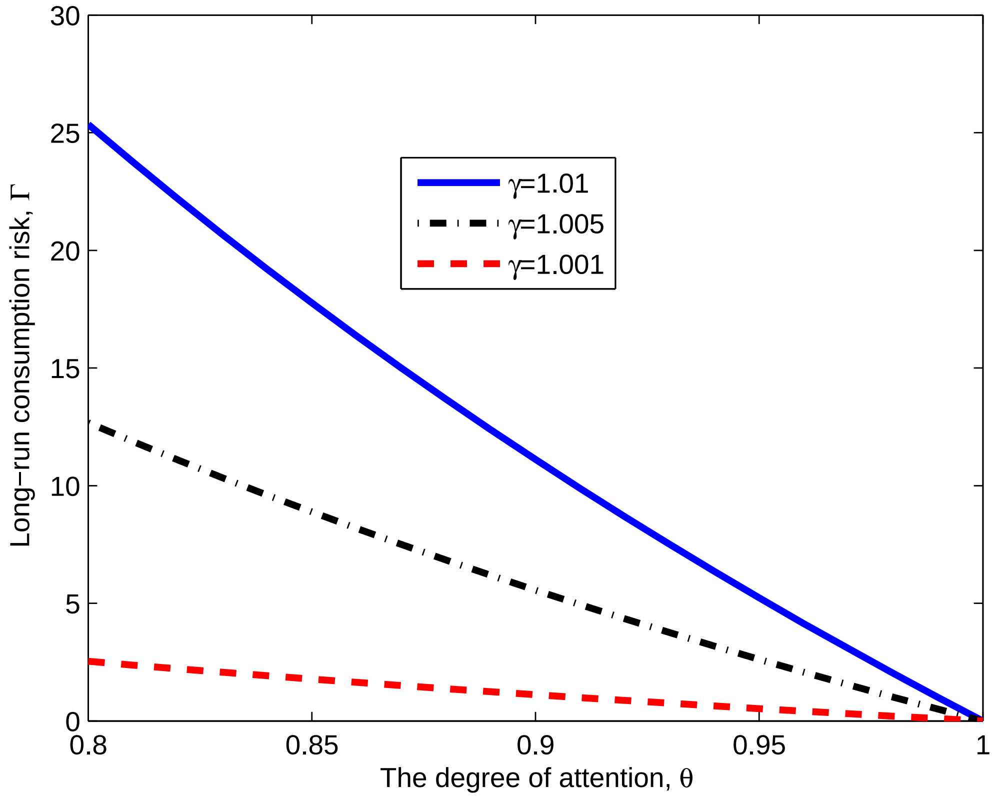

Figure 3: The Effects of the Investment Horizon on Long-run Consumption Risk

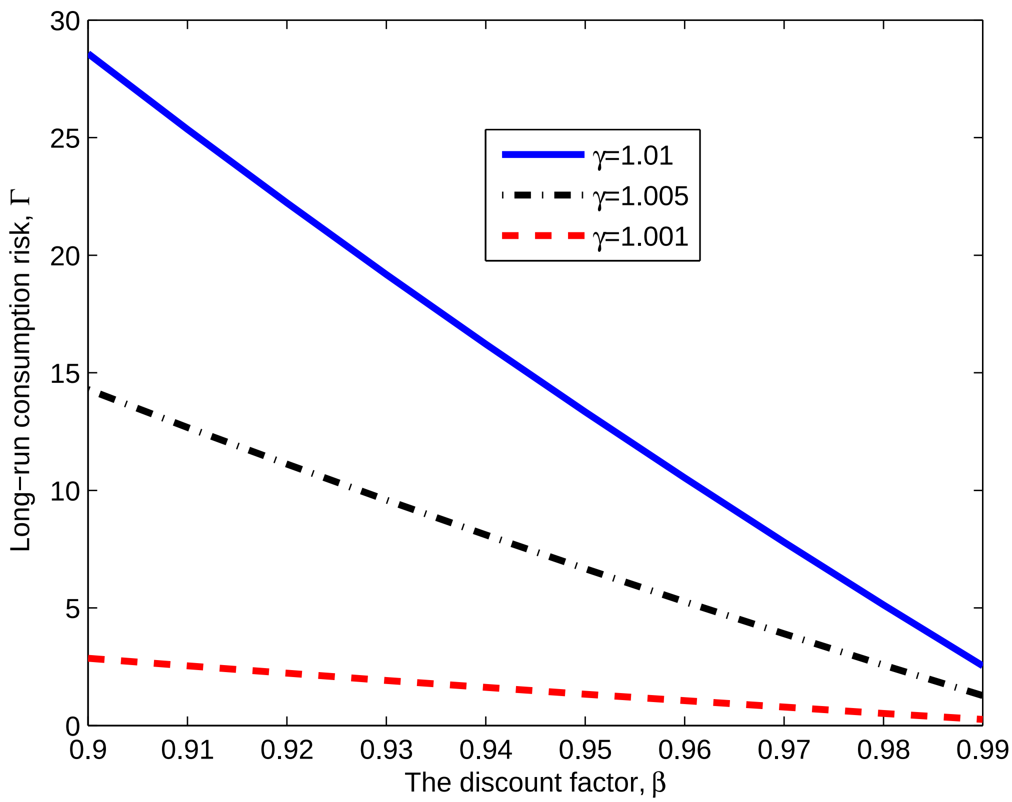

Figure 4: The Effects of RI on Consumption Volatility

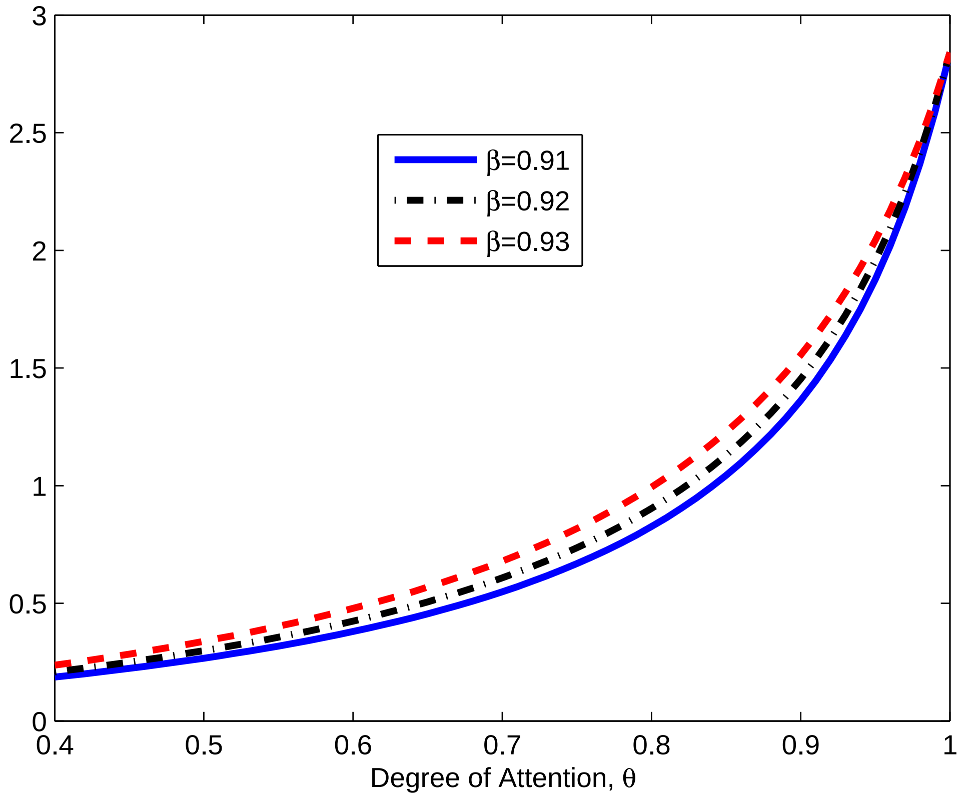

Figure 5: The Effects of RI on Consumption Correlation