Robustness, Information-Processing Constraints, and the Current Account in Small Open Economies∗

Yulei Luo†  
University of Hong Kong

Jun Nie‡  
Federal Reserve Bank of Kansas City

Eric R. Young§  
University of Virginia

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Abstract

We examine the effects of two types of informational frictions, robustness (RB) and finite information-processing capacity (called rational inattention or RI) on the current account, in an otherwise standard intertemporal current account (ICA) model. We show that the interaction of RB and RI improves the model’s predictions for the joint dynamics of the current account and income: (i) the contemporaneous correlation between the current account and income, and (ii) the volatility and persistence of the current account in small open emerging and developed economies. In addition, we show that the two informational frictions better explain consumption dynamics in small open economies: the impulse response of consumption to income shocks and the relative volatility of consumption growth to income growth.

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†School of Economics and Finance, University of Hong Kong, Hong Kong. Email address: yluo@econ.hku.hk.
‡Economic Research Department, Federal Reserve Bank of Kansas City. E-mail: jun.nie@kc.frb.org.
§Department of Economics, University of Virginia, Charlottesville, VA 22904. E-mail: ey2d@virginia.edu.
1 Introduction

Current account models following the intertemporal approach feature a prominent role for the behavior of aggregate consumption. For given total income, consumption is the main determinant of national saving, and the balance of national saving in excess of investment is the major component of the current account. This important role for consumption has naturally led researchers to study current account dynamics using consumption models.\(^1\) For example, the standard intertemporal current account (ICA) model is based on the standard linear-quadratic permanent income hypothesis (LQ-PIH) model proposed by Hall (1978) under the assumption of rational expectations (RE). Within the PIH framework, agents can borrow in the international capital market and optimal consumption is determined by permanent income rather than current income; consequently, permanent income also matters for the current account. For example, consumption only partly adjusts to temporary adverse income shocks, which makes the current account tend to be in deficit. In contrast, consumption fully adjusts to permanent income shocks, with little impact on the current account. We adopt Hall’s LQ-PIH setting in this paper because the main purpose of this paper is to examine the implications of two types of informational frictions, robustness and rational inattention, for the dynamics of the current account, and it is much more difficult to study these informational frictions in non-LQ frameworks.\(^2\)

However, many empirical studies show that the RE-ICA models are often rejected in the post-war data.\(^3\) It is not surprising that the standard RE-ICA models are rejected because the underlying standard permanent income models have encountered their own well-known empirical difficulties, particularly the well-known ‘excess sensitivity’ and ‘excess smoothness’ puzzles. Specifically, the main problems with the standard RE-ICA models are as follows. First, the models cannot generate low contemporaneous correlations between the current account and net income. (Net income is defined as output less than investment and government spending.)\(^4\) If net income is a persistent trend-stationary AR(1) process,\(^5\) The model predicts that the current

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\(^{1}\)See Obstfeld and Rogoff (1995) for a survey.

\(^{2}\)See Hansen and Sargent (2007a) and Sims (2003, 2006) for detailed discussions on the difficulties in solving the non-LQ models with these information imperfections. The primary alternative model is based on Mendoza (1991), a small open economy version of a real business cycle model. That model would be significantly less tractable than the one we use, because it involves multiple state variables.


\(^{4}\)Note that here we follow Aguiar and Gopinath (2007) and Uribe (Chapter 1, 2009) and use the detrended data to compute the reported empirical second moments.

\(^{5}\)It is well known that given the length and structure of the data on real GDP, it is difficult to distinguish persistent trend-stationary AR(1), unit root, and difference-stationary (DS) processes for real GDP. (See Chapter 4 of Deaton 1992 for a detailed discussion on this issue.) We focus on the AR(1) case in this paper; the results for the DS case are available from the authors upon request. In Section 3.2, we will discuss the unit root case. In this case the empirical second moments of the current account and net income are not finite. The RE model
account is a linear function of net income and they are thus perfectly correlated, whereas in the data they are only weakly correlated (Aguiar and Gopinath 2007, Uribe 2009). Second, they cannot generate low persistence of the current account. The standard RE models predict that the current account and net income have the same degree of persistence, whereas in the data the persistence of the current account is much lower than that of net income. Third, the models cannot generate observed volatility of the current account (Bergin and Sheffrin 2000; Gruber 2004). Fourth, they cannot generate volatile consumption growth and the observed hump-shaped impulse responses of consumption to income (Aguiar and Gopinath 2007). Finally, the assumption of certainty equivalence in these models ignore some important channels through which income shocks affect the current account. As shown in Ghosh and Ostry (1997) in post-war quarterly data for the US, Japan, and the UK, the current account is positively correlated with the amount of precautionary savings generated by uncertainty about future net income.

It is, therefore, natural to turn to new alternatives to the standard RE-ICA model and ask what implications they have for the joint dynamics of consumption, the current account, and income. In this paper, we examine how the two types of information imperfections, a preference for robustness (RB) and information-processing constraints (rational inattention, RI), can improve the model’s predictions on these important dimensions we discussed above. Hansen and Sargent (1995, 2007a) first introduce robustness (a concern for model misspecification) into economic models. In robust control problems, agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, they choose their decisions as if the subjective distribution over shocks was chosen by a malevolent nature in order to minimize their expected utility (that is, the solution to a robust decision-maker’s problem is the equilibrium of a max-min game between the decision-maker and nature). Robustness models produce precautionary savings but remain within the class of LQ-Gaussian models, which leads to analytical simplicity. A second class of models that produces precautionary savings but remains within the class of LQ-Gaussian models is the risk-sensitive model.
We show that incorporating robustness can improve the model by along the following three dimensions in all small open countries: generating lower contemporaneous correlation between the current account and net income, lower persistence of the current account, and higher relative volatility of consumption growth to income growth. However, RB by itself fails to generate volatile current accounts and the hump-shaped impulse responses of consumption to income shocks.

Sims (2003) first introduced rational inattention (RI) into economics and argued that it is a plausible method for introducing sluggishness, randomness, and delay into economic models. In his formulation agents have finite Shannon channel capacity, limiting their ability to process signals about the true state of the world. As a result, an impulse to the economy induces only gradual responses by individuals, as their limited capacity requires many periods to discover just how much the state has moved; one key change relative to the RE case is that consumption has a hump-shaped impulse response to changes in income. Using the results in Luo (2008), it is straightforward to show that RI by itself still leads to counterfactual strongly-procyclical current accounts and cannot generate precautionary savings in the LQG setting. However, the combination of RB and RI produces a model that captures many of the facts that are seen as anomalous through the lens of an RE model, while producing consumption dynamics that are consistent with the data.

We briefly list the results of the RB-RI model. First, we can produce a low correlation between the current account and net income, and in fact can even produce negative correlations for some parameter settings; the key requirement to get low correlations is that the agent have a strong fear of model misspecification. Second, we can produce low persistence in the current account, a consequence of the slow movements in consumption that RI produces. Third, if information-processing is sufficiently restricted, current account volatility can match that observed in the data for emerging markets, although not for developed economies. Fourth, the model produces a hump-shaped consumption response to income, a consequence of RI, and can produce highly volatile consumption growth in emerging economies. Fifth, the precautionary savings effect generated by RB is consistent with the positive correlation between income volatility and average current accounts. We detail in the main body of the paper the intuition for all of these results.

8 See Hansen and Sargent (2007a) and Luo and Young (2010) for detailed comparisons of the two models. In our ICA model, it seems more plausible to have different degrees of robustness (θ) across countries than to assume different degrees of risk sensitivity (i.e., enhanced risk aversion) across countries to explain the observed different joint behavior of consumption and current accounts in emerging and developed economies. Backus, Routledge, and Zin (2004) also discuss this issue.


10 Habit formation also worsens the model’s predictions on the current account dynamics; consumption adjusts slowly with respect to income shocks under habit formation, as shown in Gruber (2004), generating procyclical current accounts. Luo (2008) compares the consumption predictions of habit formation and RI.
The remainder of the paper is organized as follows. Section 2 presents key facts of small open economy business cycles. Section 3 reviews the standard RE-ICA model and discusses the puzzling implications of the model. Section 4 presents the RB ICA model and discusses some results regarding the joint dynamics of consumption, the current account, and income. Section 5 solves the RB-RI ICA model and presents the implications for the same variables. Section 6 concludes.

2 Facts

In this section we document key aspects of small open economy business cycles. We follow Aguiar and Gopinath (2007) by dividing these small economies into two groups, labeled emerging economies and developed economies.\textsuperscript{11} We use annual data from World Development Indicators. Net income \((y)\) is constructed as real GDP\(-i-g\), where \(i\) is Gross Fixed Capital Formation and \(g\) is General Government Final Consumption Expenditure. Consumption \((c)\) is defined as Household Final Consumption Expenditure, \(ca\) refers to the Current Account, and holdings of bonds \((b)\) corresponds to Net Foreign Assets.

Tables 2 and 3 report key statistics of emerging economies, and Tables 4 and 5 report the same statistics for developed countries. To provide a comparison for the reader, we report the average values of these moments of both emerging countries and developed countries in Table 1; we report both the results using a linear filter and the Hodrick-Prescott (HP) filter (with a smoothing parameter of 100) in the same table. For the variable growth (with a symbol \(\Delta\)) the unfiltered series are used. The numbers in the parentheses are the GMM-corrected standard errors of the statistics across countries.\textsuperscript{12} Since our permanent income model is stationary, we need to remove the low frequency component from the data. Thus in this paper we focus primarily on the linear filter when we calibrate the parameters and compare models with data.

\textbf{[Insert Tables 1-5 Here]}\textsuperscript{13}

We briefly list the facts we focus on. First, the correlation between the current account and net income is positive but small (and insignificant when detrended with the HP filter). Second, the relative volatility of the current account to net income is smaller in emerging countries than in developed economies, although the difference is not statistically significant when the series are detrended with the HP filter. Third, the persistence of the current account is smaller than that of

\textsuperscript{11}Israel and the Slovak Republic are not in our list because some variables from these two countries are missing from our data set.

\textsuperscript{12}The standard errors are computed under the assumption of independence across the countries. The standard error of \(\sigma(y)/\mu(y)\) in the tables refers to the standard error of \(\sigma(y)\) as the ratio of \(\mu(y)\). \(\mu(y)\) is the average level of net income.

\textsuperscript{13}Insert Tables 1-5 Here
net income, and less persistent in emerging economies. And fourth, the volatility of consumption growth relative to income growth is larger in emerging economies than in developed economies.

3 A Stylized Intertemporal Model of the Current Account

In this section we present a standard RE version of the ICA model and will discuss how to incorporate RB and RI into this stylized model in the next sections. Following common practice in the literature, we assume that the model economy is populated by a continuum of identical infinitely-lived consumers, and the only asset that is traded internationally is a risk-free bond.

3.1 Model Setup

The RE ICA model, the small-open economy version of Hall’s permanent income model, can be formulated as

$$\max_{\{c_t\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

subject to the flow budget constraint

$$b_{t+1} = R b_t + y_t - c_t,$$

where \(u(c_t) = -\frac{1}{2} (c_t - \bar{c})^2\) is the utility function, \(\bar{c}\) is the bliss point, \(c_t\) is consumption, \(R\) is the exogenous and constant gross world interest rate, \(b_t\) is the amount of the risk-free foreign bond held at the beginning of period \(t\), and \(y_t\) is net income in period \(t\) and is defined as output less than investment and government spending. Let \(\beta R = 1\); then this specification implies that optimal consumption is determined by permanent income:

$$c_t = (R - 1) s_t$$

where

$$s_t = b_t + \frac{1}{R} \sum_{j=0}^{\infty} R^{-j} E_t [y_{t+j}]$$

is the expected present value of lifetime resources, consisting of financial wealth (the risk-free foreign bond) plus human wealth. As shown in Luo (2008) and Luo and Young (2010), in order to facilitate the introduction of robustness and rational inattention we reduce the above multivariate model with a general income process to a univariate model with iid innovations to permanent income \(s_t\) that can be solved in closed-form. Specifically, if \(s_t\) is defined as a new state variable, we can reformulate the above PIH model as

$$v(s_0) = \max_{\{c_t, s_t+1\}_{t=0}^{\infty}} \left\{ E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right] \right\}$$
subject to
\[ s_{t+1} = Rs_t - c_t + \zeta_{t+1}, \]  
where the time \((t + 1)\) innovation to permanent income can be written as
\[ \zeta_{t+1} = \frac{1}{R} \sum_{j=t+1}^{\infty} \left( \frac{1}{R} \right)^{j-(t+1)} (E_{t+1} - E_t)[y_j]; \]  
\(v(s_0)\) is the consumer’s value function under RE.\(^{13}\) Under the RE hypothesis, this model with quadratic utility leads to the well-known random walk result of Hall (1978),
\[ \Delta c_t = \frac{R - 1}{R} (E_t - E_{t-1}) \left[ \sum_{j=0}^{\infty} \left( \frac{1}{R} \right)^j y_{t+j} \right] = (R - 1) \zeta_t, \]
which relates the innovations in consumption to income shocks.\(^ {14}\) In this case, the change in consumption depends neither on the past history of labor income nor on anticipated changes in labor income. In addition, the model specification also implies the certainty equivalence property holds, and thus uncertainty has no impact on optimal consumption.

Substituting (2) and (3) into the current account identity,
\[ ca_t = b_{t+1} - b_t = (R - 1) b_t + y_t - c_t, \]
gives
\[ ca_t = - \sum_{j=t+1}^{\infty} \left( \frac{1}{R} \right)^{j-t} E_t[\Delta y_j], \]
which means that the current account equals minus the present discounted value of future expected net income changes. This expression also reflects the fact that consumers smooth income shocks by borrowing or lending in international financial markets. If income is expected to decline in the future, then the current account rises immediately as current consumption determined by permanent income is less than current income; the opposite occurs if income is expected to rise in the future.

### 3.2 Model Predictions for Consumption and the Current Account

We close the model by specifying the stochastic process for net output. Specifically, we assume that the deviation of net output from its mean follows an AR(1) process
\[ y_{t+1} - \overline{y} = \rho(y_t - \overline{y}) + \varepsilon_{t+1}, \]
\(^{13}\)In the next section, we will introduce robustness directly into this ‘reduced’ permanent income model. We may imagine that consumers form the reduced model after many years’ experience.
\(^{14}\)Note that under RE the expression of the change in individual consumption is the same as that of the change in aggregate consumption.
where \( \rho \in (0, 1] \) is the persistence coefficient of output and \( \varepsilon_{t+1} \) is an iid normal shock with mean 0 and variance \( \omega^2 \). In this case, (7) implies that \( \zeta_{t+1} = \frac{1}{1-\rho} \varepsilon_{t+1} \) and \( s_t = b_t + \frac{1}{1-\rho} y_t \). In the RE version of the ICA model, substituting (3) into the current account identity, (9), gives

\[
ca_t = \frac{1 - \rho}{R - \rho} y_t,
\]

which means that given \( \rho \) and \( R \), the current account inherits the properties of the stochastic process for net output (in particular, the persistence of net output). (12) also clearly shows that the value of \( \rho \) affects how output determines the behavior of the current account. Here we discuss two possibilities for the exogenous process of net output.

**Case 1 (0 < \rho < 1).**

When \( \rho < 1 \), the shock is temporary and consumers adjust their optimal plans by only consuming the annuity value of the increase in total income. In this case, the current account works as a shock absorber, and consumers borrow to finance negative income shocks and save in response to positive shocks. In other words, the current account in this case is *procyclical*:

\[
\frac{\partial ca_t}{\partial \varepsilon_t} > 0,
\]

which means that the current account improves during expansions and deteriorates during recessions. The solid line in Figure 1 illustrates the impulse response of the current account to the income shock when \( R = 1.04 \) and \( \rho = 0.7 \). (We set \( R \) to be 1.04 throughout the paper; we treat it as a compromise of different asset returns in the economy.) Equation (12) also means that the contemporaneous correlation between the current account and income, \( \text{corr}(ca_t, y_t) \), is 1. This model prediction contradicts the empirical evidence: in small open economies the correlation between the current account and net output is positive but close to 0. As reported in Panel A (HP filter) of Table 1, \( \text{corr}(ca_t, y_t) = 0.04 \text{ (s.e. 0.04)} \) in emerging countries and 0.06 (s.e. 0.05) in developed economies. Similarly, in Panel B (linear filter) of Table 1, \( \text{corr}(ca_t, y_t) = 0.13 \text{ (s.e. 0.05)} \) in emerging countries and 0.17 (s.e. 0.05) in developed economies. In other words, the model predicts too high a correlation between the current account and net output.

Equation (12) clearly shows that the volatility of the current account is less than that of income:

\[
\mu = \frac{\text{sd}(ca_t)}{\text{sd}(y_t)} = \frac{1 - \rho}{R - \rho} < 1,
\]

where sd denotes standard deviation. Note that \( \frac{\partial \mu}{\partial \rho} < 0 \). Using the estimated \( \rho \) reported in Panel A (HP filter) of Table 1 and assume that \( R = 1.04 \), the RE model predicts that \( \mu = 0.926 \) in emerging countries and \( \mu = 0.933 \) in developed countries. However, in the data (using HP filter) reported in Table 1, \( \mu = 1.53 \text{ (s.e. 0.09)} \) in emerging countries, whereas \( \mu = 1.60 \text{ (s.e. 0.08)} \) in developed countries.\(^{16}\) In other words, given the estimated income processes, the model cannot

\(^{15}\)See Table 1 for our estimates of the output process.

\(^{16}\)Given the estimated \( \rho \) using the linear filter reported in Panel B of Table 1, the RE model predicts that
correctly predict the magnitude of the relative volatility of the current account to net output emerging and developed economies.\textsuperscript{17}

Equation (12) also implies that the persistence of the current account is the same as that of net output. However, in the data the current account is significantly less persistent than net output, and is less persistent in emerging economies than in developed economies. As shown in Panel B (linear filter) of Table 1, $\rho(y_t, y_{t-1}) = 0.8$ (s.e. 0.02) and 0.79 (s.e. 0.02) in emerging and developed countries, respectively, while the corresponding $\rho(ca_t, ca_{t-1}) = 0.53$ (s.e. 0.04) and 0.71 (s.e. 0.02).\textsuperscript{18}

Furthermore, given the AR(1) income specification, the change in aggregate consumption is

$$\Delta c_t = \frac{R - 1}{R - \rho} \varepsilon_t,$$

which means that consumption growth is white noise and the impulse response of consumption to the income shock is flat with an immediate upward jump in the initial period that persists indefinitely. (See the solid line in Figure 2.) However, as well documented in the consumption literature (Reis 2006), the impulse response of aggregate consumption to aggregate income takes a hump-shaped form, which means that aggregate consumption growth reacts to income shocks gradually.

It is worth noting that given the simple one-shock structure of our model, it is impossible to find the corresponding empirical impulse responses of consumption and current accounts to the income shock using the VAR. Specifically, as shown in Fernandez-Villaverde, Rubio-Ramirez, and Sargent (2005) (henceforth FRS), to obtain impulse response functions to VAR innovations that can potentially match impulse response functions to the shocks in an economic model, some conditions need to be satisfied to avoid the invertibility problem. The key reason that we cannot identify the VAR innovations in our model is that the number of economic shocks in our models is not equal to the number of observables from the econometrician’s perspective: consumption $c$, the current account $ca$, net income $y$, asset holdings $b$, and permanent income $s$.\textsuperscript{19} That is, there does not exist an identification of VAR shocks that makes the impulse response associated with a VAR match the one associated with our model.\textsuperscript{20} FRS (2005) proposed a possible way to make the permanent income model overcome the invertibility problem by introducing multiple income components/shocks. However, in our reduced-form permanent income model

\begin{itemize}
  \item $\Lambda = 0.83$ in emerging countries and $\Lambda = 0.84$ in developed countries. However, in the data reported in Table 1, $\Lambda = 0.8$ (s.e. 0.06) in emerging countries and $\Lambda = 1.35$ (s.e. 0.06) in developed countries.
  \item As shown in Panel A of Table 1, using HP filter shows the same pattern.
  \item Our RE and RB models have only one shock: the fundamental shock to permanent income $\zeta$; and our RB-RI model has only two shocks: $\zeta$ and the endogenous noise $\xi$.
  \item As shown in FRS (2005), even in the model in which the number of economic shocks equals the number of observables (and therefore the number of shocks in the VAR), the history of economic shocks can span a larger space than the history of the observable, making it impossible to match up their impulse response functions.
\end{itemize}
what ultimately matters is the innovation to permanent income $\zeta$, which is a linear combination of all separate income shocks. Furthermore, Kim and Roubini (2008) find that the current account reacts positively to a government budget deficit shock within a one-year time frame (i.e., reacts negatively to a positive shock to net income), which is inconsistent with the predictions of standard theoretical current account models.

The relative volatility of consumption growth and income growth can be written as

$$\mu_c = \frac{\text{sd}[\Delta c_t]}{\text{sd}[\Delta y_t]} = \frac{R - 1}{R - \rho} \sqrt{\frac{1 + \rho}{2}},$$

(14)

where we use the facts that $\zeta_t = \frac{\varepsilon_t}{R - \rho}$, $\Delta c_t = (R - 1) \zeta_t$, and $\Delta y_t = \varepsilon_t + (\rho - 1) \frac{\zeta_{t-1}}{1 - \rho L}$, where $L$ is the lag operator. This expression is strictly increasing in $\rho$, implying that consumption growth should be relatively more volatile in emerging economies (which is consistent with the data). However, given the values of $\rho$ from Table 1, the volatility of consumption growth is much too low relative to net output. For example, if $R = 1.04$, the RE model predicts that the relative volatility of consumption growth to income growth in emerging and developed economies would be 0.28 and 0.24, respectively. In contrast, in the data, the corresponding $\mu$ values are 1.35 and 0.98, respectively.\(^{21}\)

Case 2 ($\rho = 1$).

When $\rho = 1$, net output follows a unit root process and the current account becomes constant because consumers allocate all of the increase in net income to current consumption. Intuitively, when the income shocks are permanent, the best response is to adjust consumption plan permanently. (Note that when $\rho = 1$ the empirical second moments of the current account and net income are not finite.) This principle is called “finance temporary shocks, adjust to permanent shocks” in the literature. As a result, var $[ca_t] = 0$, which strongly contradicts the evidence that the current account is highly volatile in all small open economies.

In sum, comparing with the stylized facts reported in Table 1, it is clear that the stylized RE-ICA model with AR(1) income processes cannot account for the following key business cycle features in small open countries:

1. The contemporaneous correlation between the current account and net output is close to 0 in small open economies, and is slightly smaller in emerging markets.

2. The excess relative volatility of the current account to net output in emerging and developed economies.

3. The persistence of the current account is smaller than that of net output, and it is smaller in emerging economies than in developed economies.

\(^{21}\)Here we use the linear filter to obtain these results; using the HP filter leads to similar results.
4. The hump-shaped impulse responses of consumption to income shocks.

5. The relative volatility of consumption growth to income growth is larger in emerging economies than in developed economies.

Finally, in the standard ICA model the current account is independent of the uncertainty in output $\omega^2$; that is, the amount of precautionary savings does not affect the current account surplus. The reason is that the LQ setup satisfies the certainty equivalence property, ruling out any response of saving to uncertainty. However, as shown in Ghosh and Ostry (1997), in the post-war quarterly data for the US, Japan, and the UK, the greater the uncertainty in income, the greater will be the incentive for precautionary saving and, ceteris paribus, the larger the current account surplus.\textsuperscript{22}

4 Intertemporal Models of Current Account with Robustness

In this section, we introduce a concern for model uncertainty (robustness, RB) into the stylized intertemporal current account model (ICA) proposed in Section 3, and explore how this information imperfection affects the dynamics of consumption and the current account in the presence of income shocks.

4.1 Optimal Consumption and the Current Account under Robustness

Robust control emerged in the engineering literature in the 1970s and was introduced into economics and further developed by Hansen, Sargent, and others. A simple version of robust optimal control considers the question of how to make decisions when the agent does not know the probability model that generates the data. In the ICA model present in Section 3, an agent with a preference for robustness considers a range of models surrounding the given approximating model, (6), and makes decisions that maximize expected utility given the worst possible model. Following Hansen and Sargent (2007a), a simple robustness version of the ICA model proposed in Section 3 can be written as

\[
v (s_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (\bar{c} - c_t)^2 + \beta \left[ \varrho \nu_t^2 + \mathbb{E}_t [v (s_{t+1})] \right] \right\}
\]

subject to the distorted transition equation (i.e., the worst-case model):

\[
s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega \nu_t,
\]

\textsuperscript{22}Recent work examines the importance of precautionary savings for current account dynamics, including Sandri (2008), Mendoza, Quadrini, and Ríos-Rull (2009), and Carroll and Jeanne (2009); such models are not analytically tractable (with the exception of Carroll and Jeanne 2009) and the analysis is therefore somewhat less transparent.
where $\nu_t$ distorts the mean of the innovation and $\vartheta > 0$ controls how bad the error can be. As shown in Hansen, Sargent, and Tallarini (1999) and Hansen and Sargent (2007a), this class of models can produce precautionary behavior while maintaining tractability within the LQ-Gaussian framework.

When output follows an AR(1) process, (11), solving this robust control problem and using the current account identity yields the following proposition:

**Proposition 1** Under RB, the consumption function is

$$c_t = \frac{R - 1}{1 - \Sigma} s_t - \frac{\Sigma \varphi}{1 - \Sigma}, \tag{17}$$

the mean of the worst-case shock is

$$\omega \zeta \nu_t = \frac{(R - 1) \Sigma}{1 - \Sigma} s_t - \frac{\Sigma \varphi}{1 - \Sigma}, \tag{18}$$

the current account is

$$ca_t = \frac{1 - \rho}{R - \rho} y_t + \Gamma s_t + \frac{\Sigma \varphi}{1 - \Sigma}, \tag{19}$$

and $s_t \left( = b_t + \frac{1}{R - \rho} y_t \right)$ is governed by

$$s_{t+1} = \rho_s s_t + \zeta_{t+1}, \tag{20}$$

where $\zeta_{t+1} = \varepsilon_{t+1} / (R - \rho)$, $\Sigma = R \omega \zeta^2 / (2 \vartheta) \in (0, 1)$ measures the degree of the preference of robustness, $\Gamma = -\Sigma (R - 1) / (1 - \Sigma) < 0$, and $\rho_s = \frac{1 - R \Sigma}{1 - \Sigma} \in (0, 1)$.

**Proof.** See Appendix 7.1.

The consumption function under RB, (17), shows that the preference for robustness, $\vartheta$, affects the precautionary savings increment, $-\Sigma \varphi / 1 - \Sigma$. The smaller the value of $\vartheta$ the larger the precautionary saving increment. Note that $\Sigma < 1$ comes from the requirement of the second-order condition of the optimization problem. To see this, the second-order condition for a minimization by nature can be rearranged into

$$\vartheta > \frac{1}{2} R^2 \omega \zeta^2. \tag{21}$$

Using the definition of $\Sigma = R \omega \zeta^2 / (2 \vartheta)$, we obtain $1 > R \Sigma$. Since $R > 1$, we must have $\Sigma < 1$. The consumption function also implies that the stronger the preference for robustness, the more consumption responds initially to changes in permanent income; that is, under RB consumption
is more sensitive to unanticipated income shocks. This response is referred to as “making hay while the sun shines” in van der Ploeg (1993).

For the special case $\rho = 1$,

$$ca_t = \Gamma s_t + \frac{\Sigma \sigma}{1 - \Sigma},$$  \hspace{1cm} (22)

which clearly shows that the current account is countercyclical even if output follows a random walk.

### 4.1.1 Impulse Responses of the Current Account

When $\rho \in (0, 1)$, the effect of a change in net output on the current account is determined by the first two terms in (19), and the current account includes a unit root. Specifically,

$$\frac{\partial ca_t}{\partial \epsilon_t} = \frac{\Gamma + 1 - \rho}{R - \rho},$$  \hspace{1cm} (23)

which means that the current account will be procyclical if the preference for robustness is not sufficiently strong:

$$\Sigma < \Sigma_1 = \frac{1 - \rho}{R - \rho} \left(1 - \frac{R - 1}{R - \rho}\right).$$  \hspace{1cm} (24)

The dotted and dash-dotted lines in Figure 1 illustrate the impulse responses of the current account to the income shock when $\Sigma = 0.5$ and 0.95, respectively. For the special case that $\rho = 1$, introducing robustness generates countercyclical behavior of the current account as $\Sigma > 0$.\textsuperscript{24} The pure empirical VAR studies in Kim and Roubini (2008) find that the current account reacts positively to a government budget deficit shock within a one-year time frame (i.e., react negatively to a positive shock to net income), which is consistent with the predictions of our RB model.\textsuperscript{25}

### 4.1.2 Volatility of the Current Account

We now examine how RB affects the relative volatility of the current account to net income. Using (19), the relative volatility of the current account to net income can be written as

$$\mu = \frac{sd(ca_t)}{sd(y_t)} = \sqrt{\left\{ (1 - \rho^2) \left[ \frac{1 - \rho}{1 + \rho} + \frac{\Gamma^2}{1 - \rho^2_s} \right] + \frac{2(1 - \rho)\Gamma}{1 - \rho_s} \right\} / (R - \rho)^2 < 1, \hspace{1cm} (25)$$

\textsuperscript{24}While the current account is not countercyclical with respect to net income, it is countercyclical with respect to GDP in many countries. Standard models attribute this countercyclical behavior to investment flows (Backus, Kehoe, and Kydland 1994). Our model offers an alternative interpretation.

\textsuperscript{25}As we have discussed in Section 3.2, there does not exist an identification of VAR shocks that makes the impulse response associated with a VAR match the one associated with our simple model with one fundamental shock. Consequently, it prevents us from reporting the empirical counterpart impulse responses of consumption and current accounts.
where we use the facts that
\[
\text{var}(ca_t) = \left[ \frac{1 - \rho}{1 + \rho} + \frac{\Gamma^2}{1 - \rho_s} + \frac{2 (1 - \rho) \Gamma}{1 - \rho \rho_s} \right] \frac{\omega^2}{(R - \rho)^2}, \tag{26}
\]
\[
\text{var}(y_t) = \frac{\omega^2}{1 - \rho}, \quad \text{var}(s_t) = \frac{\omega^2}{(R - \rho)^2(1 - \rho_s^2)}, \quad \text{and} \quad \text{cov}(y_t, s_t) = \frac{\omega^2}{(R - \rho)(1 - \rho \rho_s)}.
\]

Given \( R \) and \( \rho \), (25) shows that \( \mu \) is affected by the amount of robustness (\( \Sigma \)). Note that \( \mu \) is not a monotonic function of \( \Sigma \), as \( \frac{\Gamma^2}{1 - \rho_s} \) in (25) is increasing with \( \Sigma \) and \( \frac{2(1 - \rho) \Gamma}{1 - \rho \rho_s} \) in (25) is decreasing with \( \Sigma \). Given the complexity of this expression, we cannot obtain an explicit result about how RB affects \( \mu \). Figure 3 illustrates how RB affects the relative volatility for different values of \( \rho \). It is clear that \( \mu \) is decreasing with \( \Sigma \) when \( \Sigma \) is relatively small and is increasing with \( \Sigma \) when \( \Sigma \) is large. The reason is that when \( \Sigma \) is large, the second term (the volatility term about \( s_t \)) in the bracket of (26) dominates the third term (the negative covariance term about \( s_t \) and \( y_t \)) there. (Note that \( \Gamma < 0 \)). RB thus has a potential to make the model fit the data better along this dimension when \( \Sigma \) in small open economies is large enough and is larger in emerging economies than in developed economies. Note that we have shown in Section 3.2 that the stylized model cannot generate sufficiently-volatile current accounts, and the relative volatility of the current account to income is smaller in emerging economies than in developed economies.

### 4.1.3 Persistence of the Current Account

The persistence of the current account is measured by its first autocorrelation. Using (19), the first autocorrelation of the current account, \( \rho(ca_t, ca_{t-1}) \), can be written as
\[
\rho(ca_t, ca_{t+1}) = \frac{\text{cov}(ca_t, ca_{t+1})}{\sqrt{\text{var}(ca_t)} \sqrt{\text{var}(ca_{t+1})}} = \frac{\rho (1 - \rho)}{1 + \rho} + \frac{\rho_s \Gamma^2}{1 - \rho_s} + \frac{(\rho + \rho_s) (1 - \rho) \Gamma}{1 - \rho \rho_s} \left/ \left[ \frac{1 - \rho}{1 + \rho} + \frac{\Gamma^2}{1 - \rho_s} + \frac{2 (1 - \rho) \Gamma}{1 - \rho \rho_s} \right] \right., \tag{27}
\]
which converges to \( \rho \) (the persistence of net income) as \( \Sigma \) goes to 0. Given the complexity of this expression, we cannot obtain an explicit result about how RB affects \( \rho(ca_t, ca_{t+1}) \). Figure 4 illustrates how RB affects the persistence of the current account for different values of \( \rho \). It is clear that \( \rho(ca_t, ca_{t+1}) \) is decreasing with \( \Sigma \). RB thus has a potential to make the model fit the data better along this dimension. In addition, introducing RB can also explain that \( \rho(ca_t, ca_{t+1}) \) is smaller in emerging countries than in developed countries if \( \Sigma \) is larger in emerging countries.

Note that the standard RE-ICA model predicts that the current account and income have the same degree of persistence, which contradicts the evidence that the current account is significantly less persistent than income in small open economies and the persistence of net income is larger in emerging counties than in developed countries.
4.1.4 Correlation between the Current Account and Income

An alternative description of the comovement of the current account and income is the contemporaneous correlation between the current account and income, \( \text{corr}(ca_t, y_t) \). Under RB, the correlation can be written as:

\[
\text{corr}(ca_t, y_t) = \left( \frac{\Gamma}{1 - \rho \rho_s} + \frac{1}{1 + \rho} \right) \sqrt{\frac{1}{(1 + \rho)^2} + \frac{\Gamma^2}{(1 - \rho^2)(1 - \rho_s^2)} + \frac{2\Gamma}{(1 + \rho)(1 - \rho \rho_s)}}.
\] (28)

which reduces to 1 when \( \Sigma \) converges to 0. Figure 5 illustrates that how RB affects the correlation between the current account and net income for different values of \( \rho \). It is clear that \( \text{corr}(ca_t, y_t) \) is decreasing with \( \Sigma \). (Note that in the figure we restrict the values of \( \Sigma \) to be less than 0.83 such that \( \text{corr}(ca_t, y_t) \) is positive as generated in the data.) RB thus has the potential to align the model and the data more closely along this dimension. In addition, introducing RB can also account for the fact that \( \text{corr}(ca_t, y_t) \) is smaller in emerging countries than in developed countries, provided \( \Sigma \) is larger in emerging countries.

4.1.5 Implications of Macroeconomic Uncertainty for the Current Account under RB

Finally, the last term in (19) determines the effect of precautionary savings on the current account. It is clear that with the preference for robustness, the greater the uncertainty in net income, the greater the amount of precautionary saving, and the larger the current account surplus, as

\[
\frac{\partial ca_t}{\partial \omega^2} > 0.
\] (29)

This result is consistent with the empirical evidence that the current account and volatility are positively correlated (Ghosh and Ostry 1997). Note that the precautionary savings induced by a concern about robustness differs from the usual precautionary savings motive that emerges when labor income uncertainty interacts with the convexity of the marginal utility of consumption. This type of precautionary savings emerges because consumers want to save more as protection against model misspecification and thus occurs even in models with quadratic utility.

Having examined the implications of RB for the relative volatility and persistence of the current account, and the correlation between the current account and income, it is clear that RB has a potential to improve the model’s predictions on the joint dynamics of the current account and net income. A requirement for matching these facts is that the fear of misspecification is stronger in emerging economies. This requirement is obviously subject to empirical testing, so we will use the detection error approach of Hansen and Sargent (2007a) to calibrate \( \Sigma \) for developed and emerging economies in a subsequent section.
4.1.6 Implication for Consumption Volatility

Although introducing robustness has a potential to improve the model’s predictions on the dynamics of the current account and precautionary savings, it worsens the model’s prediction for the joint dynamics of consumption and income. Given (17) and (20), the change in aggregate consumption can be written as

\[
ct_{t+1} = \rho_s ct_t - \frac{(1 - R) \Sigma \tau}{1 - \Sigma} + \frac{R - 1}{(1 - \Sigma) (R - \rho)} \varepsilon_{t+1},
\]

(30)

where \(\rho_s = \frac{1 - R \Sigma}{1 - \Sigma}\) and we use the fact that \(\zeta_{t+1} = \varepsilon_{t+1}/(R - \rho)\). Therefore, aggregate consumption under RB follows an AR(1) process, which contradicts the evidence that in the data consumption reacts to income gradually and with delay. In other words, RB does not produce any propagation in consumption after an income shock. As emphasized in Sims (1998, 2003), VAR studies show that most cross-variable relationships among macroeconomic time series are smooth and delayed. Figure 2 illustrates the response of aggregate consumption growth to an aggregate income shock \(\varepsilon_{t+1}\); comparing the solid line (RE) with the dash-dotted line, it is clear that RB raises the sensitivity of consumption growth to unanticipated changes in aggregate income.

Furthermore, the relative volatility of consumption growth to income growth, \(\mu\), can be written as

\[
\mu_c = \frac{\text{sd} [\Delta c_t]}{\text{sd} [\Delta y_t]} = \frac{R - 1}{(1 - \Sigma) (R - \rho)} \sqrt{1 + \rho} - \frac{1}{1 + \rho_s},
\]

(31)

where we also use the fact that \(\Delta y_t = \varepsilon_t + (\rho - 1) \varepsilon_{t-1} / (1 - \rho)\). It is clear from (31) that RB increases the relative volatility via two channels: first, it strengthens the marginal propensity to consume out of permanent income \(\frac{R - 1}{1 - \Sigma}\); and second, it increases consumption volatility by reducing the persistence of permanent income measured by \(\rho_s\): \(\frac{\partial \rho_s}{\partial \Sigma} < 0\). Furthermore, if \(\Sigma\) is larger in emerging economies, the RB-ICA model will predict that the relative volatility of consumption to income is greater in emerging economies than in developed economies.

4.2 Calibrating the RB Parameter

In this section we use the procedure outlined Hansen, Sargent, and Tallarini (1999) and Hansen and Sargent (2007a) to calibrate the RB parameter (\(\vartheta\) and \(\Sigma\)). Specifically, we calibrate \(\vartheta\) by using the notion of a model detection error probability that is based on a statistical theory of model selection (the approach will be precisely defined below). We can then infer what values of the RB parameter \(\vartheta\) imply reasonable fears of model misspecification for empirically-plausible

\[\text{26}\]

We use the relative volatility of consumption growth to income growth instead of that of consumption to income to compare the implications of RE and RB models, as consumption follows a random walk under RE and the volatility of consumption is not well defined in this model.
approximating models. The model detection error probability is a measure of how far the distorted model can deviate from the approximating model without being discarded; low values for this probability mean that agents are unwilling to discard very many models (as they want errors to be rare), implying that the cloud of models surrounding the approximating model is large.

4.2.1 The Definition of the Model Detection Error Probability

Let model $A$ denote the approximating model and model $B$ be the distorted model. Define $p_A$ as

$$p_A = \text{Prob} \left( \log \left( \frac{L_A}{L_B} \right) < 0 \bigg| A \right), \quad (32)$$

where $\log \left( \frac{L_A}{L_B} \right)$ is the log-likelihood ratio. When model $A$ generates the data, $p_A$ measures the probability that a likelihood ratio test selects model $B$. In this case, we call $p_A$ the probability of the model detection error. Similarly, when model $B$ generates the data, we can define $p_B$ as

$$p_B = \text{Prob} \left( \log \left( \frac{L_A}{L_B} \right) > 0 \bigg| B \right). \quad (33)$$

Following Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007b), the detection error probability, $p$, is defined as the average of $p_A$ and $p_B$:

$$p(\vartheta) = \frac{1}{2} (p_A + p_B), \quad (34)$$

where $\vartheta$ is the robustness parameter used to generate model $B$. Given this definition, we can see that $1 - p$ measures the probability that econometricians can distinguish the approximating model from the distorted model. Now we show how to compute the model detection error probability in the Robustness model.

4.2.2 Calibrating the RB Parameter in the ICA Model

Under RB, assuming that the approximating model generates the data, the state, $s_t$, evolves according to the transition law

$$s_{t+1} = R s_t - c_t + \zeta_{t+1},$$

$$= \frac{1 - R \Sigma}{1 - \Sigma} s_t + \frac{\Sigma}{1 - \Sigma} \varsigma + \zeta_{t+1}. \quad (35)$$

In contrast, assuming that the distorted model generates the data, $s_t$ evolves according to

$$s_{t+1} = R s_t - c_t + \zeta_{t+1} + \omega \nu_t,$$

$$= s_t + \zeta_{t+1}. \quad (36)$$

In order to compute $p_A$ and $p_B$, we use the following procedure:
1. Simulate \( \{s_t\}_{t=0}^T \) using (35) and (36) a large number of times. The number of periods used in the simulation, \( T \), is set to be the actual length of the data for each individual country.

2. Count the number of times that \( \log \left( \frac{L_A}{L_B} \right) < 0 \) \( A \) and \( \log \left( \frac{L_A}{L_B} \right) > 0 \) \( B \) are each satisfied.

3. Determine \( p_A \) and \( p_B \) as the fractions of realizations for which \( \log \left( \frac{L_A}{L_B} \right) < 0 \) \( A \) and \( \log \left( \frac{L_A}{L_B} \right) > 0 \) \( B \), respectively.

In practice, given \( \Sigma \), to simulate the \( \{s_t\}_{t=0}^T \) we need to know a) the volatility of \( \zeta_t \) in (35) and (36), and b) the value of \( \vartheta \). For a), we can compute it from \( \text{sd} (\zeta) = \sqrt{1 - \rho^2} \text{sd} (y) \) where \( \text{sd} (y) \) is the standard deviation of net income. For b), we use the local coefficient of relative risk aversion \( \gamma = -\frac{u''(c)}{u'(c)} = \frac{\bar{c}}{\bar{c} - c} \) to recover the value of \( \vartheta \): \( \vartheta = \left( 1 + \frac{1}{\gamma} \right) E [c] \) where \( E [c] \) is mean consumption. We choose \( \gamma = 2 \).

### 4.3 Calibration Results and Main Findings

After simulating the models and obtaining the detection error probability that circumscribes a neighborhood of models against which consumers want to assure robustness, we can find the values of \( \vartheta \) and \( \Sigma \) associated with that probability. Having shown how the RB parameter is related to the model detection error probability, in this section we report the calibrated values of the RB parameters by setting the model detection error probability to different targeted values. As a benchmark, we choose the RB parameter to match the model detection error probability of \( p = 0.1 \). That is, the probability that the agent can distinguish the approximating model from the distorted model is 0.9.

Tables 7 and 8 report the calibrated values of RB parameter, \( \Sigma \equiv R\vartheta^2/(2\vartheta) \), as well as the associated model detection error, \( p \), the autocorrelation coefficient of GDP, \( \rho \), and the ratios of the standard deviation of real income and permanent income to the mean of real income (undetrended), \( \sigma(y)/\mu(y) \) and \( \sigma(\zeta)/\mu(y) \), respectively.\(^{27}\) For simplicity here we only report the results using the linear filter; using the HP filter generates similar patterns from the model. We use \( \sigma(y)/\mu(y) \) to measure the relative volatility of fundamental uncertainty. Table 6 reports the averages over the emerging countries and that for the developed countries, respectively, and shows that on average:

1. Emerging countries face more volatile income processes than do developed countries. That is, macroeconomic uncertainty is higher in emerging countries.

2. After setting the detection error probability \( p(\vartheta, \Sigma) \) to be the same in the two economies, the recovered \( \Sigma \) is larger in emerging countries.

\(^{27}\)All tables in this paper are generated using the estimated parameters in the exogenous income processes. See Table 1 for the estimated parameters.
Therefore, the preference for robustness in emerging countries is stronger than in developed countries. The intuition is simple: agents in the emerging economy are more concerned about model misspecification because they face larger macroeconomic uncertainty and instability than those in developed countries. It is worth noting that a larger $\Sigma$ does not necessarily imply a smaller value of $\vartheta$ since $\omega_\zeta$ (i.e., $\sigma(\zeta)$) can be different. As we have shown in Section 4.1, RB influences the countercyclical behavior of the current account and the relative volatility of consumption to income in the model through the interaction of $\vartheta$ and $\omega_\zeta$ in $\Sigma$ instead of $\vartheta$.

We first consider a comparison between the standard RE model and the RB model. In Tables 9-10, $p$ is set to 0.1 such that $\Sigma = 0.524$ in emerging countries and 0.205 in developed countries. In this case the first three columns of the tables clearly show that RB can improve the model’s predictions along the following three dimensions: the contemporaneous correlation between the current account and net income, the persistence of the current account, and the relative volatility of consumption growth to income growth, but worsens the model prediction on the relative volatility of the current account to net income. Specifically, for emerging countries, given the calibrated $\Sigma$s RB reduces the correlation between the current account and net income from 1 to 0.62; reduces the first-order autocorrelation from 0.8 to 0.74; increases the relative volatility of consumption growth to income growth from 0.28 to 0.9; and reduces the relative volatility of the current account to income from 0.71 to 0.49. The intuition that RB reduces the volatility of the current account is that RB increases the response of consumption to income shock, and thus reduces the response of the current account.

In Tables 11-12, we reduce the detection error probability to 0.01 and find that in this case RB can improve the model’s predictions along all the four dimensions including the relative volatility of the current account to net income. When the RB parameter is large enough, the second term in the bracket of (26) dominates the third term, and thus the volatility of the current account increases. However, $p = 0.01$ is an extremely low value and means that agents rarely make mistakes and thus can distinguish the models quite well.\(^{28}\) As shown in Tables 11-12, even for this extremely low detection error probability, the RB model still cannot generate the observed volatility of the current account. In the next section, we will show that introducing another informational friction, rational inattention, helps resolve this anomaly.

28 Alternatively, low $p$ means that we impose weak limits on the evil nature who distorts the model.
5 RB-RI Model

5.1 Optimal Consumption and the Current Account under RB and RI

5.1.1 Information-Processing Constraints

Under RI, consumers in the economy face both the usual flow budget constraint and information-processing constraint due to finite Shannon capacity first introduced by Sims (2003). As argued by Sims (2003, 2006), individuals with finite channel capacity cannot observe the state variables perfectly; consequently, they react to exogenous shocks incompletely and gradually. They need to choose the posterior distribution of the true state after observing the corresponding signal. This choice is in addition to the usual consumption choice that agents make in their utility maximization problem.\(^{29}\)

Following Sims (2003), the consumer’s information-processing constraint can be characterized by the following inequality:

\[
\mathcal{H}(s_{t+1}|I_t) - \mathcal{H}(s_{t+1}|I_{t+1}) \leq \kappa, \tag{37}
\]

where \(\kappa\) is the consumer’s channel capacity, \(\mathcal{H}(s_{t+1}|I_t)\) denotes the entropy of the state prior to observing the new signal at \(t+1\), and \(\mathcal{H}(s_{t+1}|I_{t+1})\) is the entropy after observing the new signal.\(^{30}\) The concept of entropy is from information theory, and it characterizes the uncertainty in a random variable. The right-hand side of (37), being the reduction in entropy, measures the amount of information in the new signal received at \(t+1\). Hence, as a whole, (37) means that the reduction in the uncertainty about the state variable gained from observing a new signal is bounded from above by \(\kappa\). Since the ex post distribution of \(s_t\) is a normal distribution, \(\mathcal{N}(\hat{s}_t, \sigma^2_t)\), (37) can be reduced to

\[
\log |\psi^2_t| - \log |\sigma^2_{t+1}| \leq 2\kappa \tag{38}
\]

where \(\hat{s}_t\) is the conditional mean of the true state, and \(\sigma^2_{t+1} = \text{var}[s_{t+1}|I_{t+1}]\) and \(\psi^2_t = \text{var}[s_t|I_t]\) are the posterior variance and prior variance of the state variable, respectively. To obtain (38), we use the fact that the entropy of a Gaussian random variable is equal to half of its logarithm variance plus a constant term.

It is straightforward to show that in the univariate case (38) has a unique steady state \(\sigma^2\).\(^{31}\) In that steady state the consumer behaves as if observing a noisy measurement which

\(^{29}\)More generally, agents choose the joint distribution of consumption and current permanent income subject to restrictions about the transition from prior (the distribution before the current signal) to posterior (the distribution after the current signal). The budget constraint implies a link between the distribution of consumption and the distribution of next period permanent income.

\(^{30}\)We regard \(\kappa\) as a technological parameter. If the base for logarithms is 2, the unit used to measure information flow is a ‘bit’, and for the natural logarithm \(e\) the unit is a ‘nat’. 1 nat is equal to \(\log_2 e \approx 1.433\) bits.

\(^{31}\)Convergence requires that \(\kappa > \log (R) \approx R - 1\); see Luo and Young (2010) for a discussion.
is $s_{t+1} = s_t + \xi_{t+1}$, where $\xi_{t+1}$ is the endogenous noise and its variance $\alpha^2 = \text{var}[\xi_{t+1}|I_t]$ is determined by the usual updating formula of the variance of a Gaussian distribution based on a linear observation:

$$\sigma^2_{t+1} = \psi^2_t - \psi^2_t (\psi^2_t + \alpha^2_t)^{-1} \psi^2_t.$$  

(39)

Note that in the steady state $\sigma^2 = \psi^2 - \psi^2 (\psi^2 + \alpha^2)^{-1} \psi^2$, which can be solved as $\alpha^2 = \left[\left(\sigma^2\right)^{-1} - \left(\psi^2\right)^{-1}\right]^{-1}$. Note that (39) implies that in the steady state $\sigma^2 = \left(\frac{1}{R-\rho}\right)^2 \frac{\omega^2}{\exp(2\kappa) - R^2}$ and $\alpha^2 = \text{var}[\xi_{t+1}] = \frac{\omega^2/(R-\rho)^2 + R^2\sigma^2}{\omega^2/(R-\rho)^2 + (R^2-1)\sigma^2}$.

5.1.2 Incorporating RI into the RB Model

We now incorporate RI into the RB model and examine how the combination of the two types of information imperfections affect the joint dynamics of consumption, the current account, and income. A key assumption in the RB-RI model is that we assume that the consumer not only has doubts about the process for the shock to permanent income $\xi_{t+1}$, but also distrusts his regular Kalman filter hitting the endogenous noise ($\xi_{t+1}$) and updating the estimated state. As a result, our agents have an additional dimension along which they desire robustness.

Specifically, the regular RI-induced Kalman filter equation updating the estimated state ($\hat{s}_t$),

$$\hat{s}_{t+1} = (1 - \theta) (R\hat{s}_t - c_t) + \theta (s_{t+1} + \xi_{t+1}),$$  

(40)

where $\hat{s}_t = E[s_t|I_t]$ is the conditional mean of $s_t$, $\xi_{t+1}$ is the iid endogenous noise with $\alpha^2 = \text{var}[\xi_{t+1}] = \frac{\omega^2/(R-\rho)^2 + R^2\sigma^2}{\omega^2/(R-\rho)^2 + (R^2-1)\sigma^2}$, $\theta = \sigma^2/\alpha^2 = 1 - 1/\exp(2\kappa) \in [0,1]$ is the constant optimal weight on any new observation, and $s_0 \sim N(\hat{s}_0, \sigma^2)$ is fixed. Combining (40) with the budget constraint, $s_{t+1} = Rs_t - c_t + \zeta_{t+1}$, yields the following equation governing the dynamics of the perceived state $\hat{s}_t$ that matters in agents’ decision problems:

$$\hat{s}_{t+1} = R\hat{s}_t - c_t + \eta_{t+1},$$  

(41)

where

$$\eta_{t+1} = \theta R (s_t - \hat{s}_t) + \theta (\xi_{t+1} + \zeta_{t+1})$$  

(42)

is the innovation to the mean of the distribution of perceived permanent income,

$$s_t - \hat{s}_t = \frac{(1 - \theta) \zeta_t}{1 - (1 - \theta)R \cdot L} - \frac{\theta \xi_t}{1 - (1 - \theta)R \cdot L}.$$  

(43)

The RB-RI model proposed in this paper encompasses the hidden state model discussed in Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007b); the main difference is that none of the states in the RB-RI model are perfectly observable (or controllable).

$\theta$ measures how much new information is transmitted each period or, equivalently, how much uncertainty is removed upon the receipt of a new signal.
and \( E_t [\eta_{t+1}] = 0 \). To introduce robustness into the RI model, we assume that the agent thinks that (41) is the approximating model for the true model that governs the data but that he cannot specify. Following Hansen and Sargent (2007a), we surround (41) with a set of alternative models to represent his preference for robustness:

\[
\hat{s}_{t+1} = R\hat{s}_t - c_t + \omega_t \nu_t + \eta_{t+1}.
\]  

(44)

Under RI the innovation \( \eta_{t+1} \) that the agent distrusts is composed of two MA(\( \infty \)) processes and includes the entire history of the exogenous income shock and the endogenous noise, \( \{\zeta_{t+1}, \zeta_t, \cdots, \zeta_0; \xi_{t+1}, \xi_t, \cdots, \xi_0\} \).

The optimizing problem for this RB-RI model is formulated as

\[
\hat{v}(\hat{s}_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (c_t - \bar{c})^2 + \beta E_t \left[ \vartheta \nu_t^2 + \hat{v}(\hat{s}_{t+1}) \right] \right\},
\]

subject to (44)-(43). (45) is a standard dynamic programming problem. The following proposition summarizes the solution to the RB-RI model.

**Proposition 2** Given \( \vartheta \) and \( \theta \), the consumption function under RB and RI is

\[
c_t = \frac{R - 1}{1 - \Sigma} \hat{s}_t - \frac{\Sigma \bar{c}}{1 - \Sigma},
\]

(46)

the mean of the worst-case shock is

\[
\omega_t \nu_t = \frac{(R - 1) \Sigma}{1 - \Sigma} \hat{s}_t - \frac{\Sigma \bar{c}}{1 - \Sigma},
\]

(47)

and \( \hat{s}_t \) is governed by

\[
\hat{s}_{t+1} = \rho_s \hat{s}_t + \eta_{t+1}.
\]

(48)

where \( \rho_s = \frac{1 - R \Sigma}{1 - \Sigma} \in (0, 1) \),

\[
\Sigma = R \omega_\eta^2 / (2 \vartheta) > 0,
\]

(49)

\[
\omega_\eta^2 = \text{var} [\eta_{t+1}] = \frac{\theta}{1 - (1 - \vartheta) R^2 \omega_\xi^2}.
\]

(50)

It is clear from (46)-(50) that RB and RI affect the consumption function via two channels in the model: (1) the marginal propensity to consume (MPC) out of the perceived state \( \left( \frac{R - 1}{1 - \Sigma} \right) \) and (2) the dynamics of the perceived state \((\hat{s}_t)\). Given \( \hat{s}_t \), stronger degrees of RI and RB increase the value of \( \Sigma \), which increases the MPC.

Before proceeding, we want to draw a distinction between the model proposed above and similar ones used in Luo and Young (2010) and Luo, Nie, and Young (2010). In those other papers, agents were assumed to trust the Kalman filter they use to process information, meaning
that decisions were only robust to misspecification of the income process. In that model Σ was
independent of θ, and for the questions at hand here the resulting values were too small. By
adding the additional concern for robustness developed here, we are able to strengthen the
effects of robustness on decisions. In addition, our setup here is arguably more consistent with
the underlying primitive structure of ambiguity that gives rise to robust decision-making (Gilboa
and Schmeidler 1989).

5.1.3 The Joint Dynamics of Consumption, the Current Account, and Net Income
under RB-RI

Furthermore, in the RB-RI model individual dynamics are not identical to aggregate dynamics.
Combining (46) with (44) yields the change in individual consumption in the RI-RB economy:

$$
\Delta c_t = \frac{(1 - R) \Sigma}{1 - \Sigma} (c_{t-1} - \overline{c}) + \frac{R - 1}{1 - \Sigma} \left( \frac{\theta \zeta_t}{1 - (1 - \theta) R \cdot L} + \theta \left( \xi_t - \frac{\theta R \xi_{t-1}}{1 - (1 - \theta) R \cdot L} \right) \right),
$$

where $L$ is the lag operator and we assume that $(1 - \theta) R < 1$.\footnote{This assumption requires $\kappa > \frac{1}{\theta} \log (R) \approx \frac{R - 1}{\theta R}$, which is weaker than the condition needed for convergence of the filter.} This expression shows that
consumption growth is a weighted average of all past permanent income and noise shocks. Since
this expression permits exact aggregation, we can obtain the change in aggregate consumption as

$$
\Delta c_t = \frac{(1 - R) \Sigma}{1 - \Sigma} (c_{t-1} - \overline{c}) + \frac{R - 1}{1 - \Sigma} \left( \frac{\theta \zeta_t}{1 - (1 - \theta) R \cdot L} + \theta \left( \overline{\xi}_t E^i [\xi_t] - \frac{\theta R \overline{\xi}_{t-1}}{1 - (1 - \theta) R \cdot L} \right) \right),
$$

(51)

where $i$ denotes a particular individual, $E^i [\cdot]$ is the population average, and $\overline{\xi}_t = E^i [\xi_t]$ is the common noise.\footnote{For simplicity, here we use the same notation $c$ for aggregate consumption.} This expression shows that even if every consumer only faces the common shock $\zeta$, the RI economy still has heterogeneity since each consumer faces the idiosyncratic noise induced by finite channel capacity. As argued in Sims (2003), although the randomness in an individual’s response to aggregate shocks will be idiosyncratic because it arises from the individual’s own information-processing constraint, there is likely a significant common component. The intuition is that people’s needs for coding macroeconomic information efficiently are similar, so they rely on common sources of coded information. Therefore, the common term of the idiosyncratic error, $\overline{\xi}_t$, lies between 0 and the part of the idiosyncratic error, $\xi_t$, caused by the common shock to permanent income, $\zeta_t$. Formally, assume that $\xi_t$ consists of two independent noises: $\xi_t = \overline{\xi}_t + \xi^i_t$, where $\overline{\xi}_t = E^i [\xi_t]$ and $\xi^i_t$ are the common and idiosyncratic components of the error generated by $\zeta_t$, respectively. A single parameter,

$$
\lambda = \frac{\text{var} [\overline{\xi}_t]}{\text{var} [\xi_t]} \in [0, 1],
$$

[22]
can be used to measure the common source of coded information on the aggregate component (or the relative importance of $\xi_t$ vs. $\xi_t$). Figure 2 also shows how RI can help generate the smooth and hump-shaped impulse response of consumption to the income shock, which, as argued in Sims (1998, 2003), fits the VAR evidence better.

In a recent paper, Angeletos and La'O (2009) show how dispersed information about the underlying aggregate productivity shock contributes to significant noise in the business cycle and helps explain cyclical variations in observed Solow residuals and labor wedges in the RBC setting. In contrast, Lorenzoni (2009) examines how demand shocks, defined as noisy news about future aggregate productivity, contribute to business cycles fluctuations in a new Keynesian model. In the next section, after calibrating the RB parameter we also show that the common noise due to finite capacity can simultaneously increase the relative volatility of consumption growth and income growth and reduce the contemporaneous correlation between the current account and income, which makes the RB-RI model fit the data better.

Substituting (46) into the current account identity, the current account in the RB-RI model economy can be written as

$$ca_t = \frac{1 - \rho}{R - \rho} y_t - \frac{\Sigma(R - 1)}{1 - \Sigma} s_t + \frac{R - 1}{1 - \Sigma} (s_t - \hat{s}_t) + \frac{\Sigma}{1 - \Sigma} c,$$

where

$$s_t - \hat{s}_t = \frac{(1 - \theta) \xi_t}{1 - (1 - \theta) R \cdot L} - \frac{\theta E^i [\xi_t]}{1 - (1 - \theta) R \cdot L},$$

is the error in estimating $s_t$. It is clear that when $\theta = 1$, (52) reduces to (19) in Section 4.1.

The expression for the current account, (52), clearly shows that in the RB-RI model the current account is determined by four factors:

1. The income process, $-\frac{\rho}{R - \rho} \Delta y_t$. Holding other factors constant, the current account deteriorates in response to a positive shock to income.

2. The overreaction in consumption due to the preference for robustness, $-\frac{\Sigma(R - 1)}{1 - \Sigma} s_t$. This expression means that the stronger the preference for robustness, the more countercyclical the current account is. Under robustness, consumption is more sensitive to the unanticipated income shock, and thus the increase in consumption is larger than that of income itself; consequently, the current account deteriorates.

3. The forecast error term due to rational inattention, $\frac{R - 1}{1 - \Sigma} (s_t - \hat{s}_t)$. Consumers with finite capacity cannot observe the state perfectly, and thus adjust optimal consumption gradually and with delay. For a positive income shock, a gradual adjustment in consumption improves the current account.

It is worth noting that the special case that $\lambda = 1$ can be viewed as a representative-agent model in which we do not need to discuss the aggregation issue.
4. The precautionary savings term, $\frac{\Sigma}{1 - \Sigma}$. The precautionary saving premium due to the fear of model misspecification induces a bias toward current account surplus.

Figure 1 also plots the impulse response of the current account to the income shock when $\Sigma = 0.95$ and $\theta = 80\%$. It clearly shows that the current account also responds to the income shock smoothly and gradually, which can better fit the VAR evidence that most cross-variable relationship among macroeconomic time series are smooth and delayed. Using (52) it is straightforward to show that the current account is procyclical if the following inequality is satisfied:

$$\Sigma < 1 - \theta \frac{R - 1}{R - \rho}. \quad (54)$$

5.1.4 Volatility of the Current Account

Under RB-RI, using (52), the relative volatility of the current account and net income can be written as

$$\mu = \frac{\text{sd} (ca_t)}{\text{sd} (y_t)} = \frac{\sqrt{1 - \rho^2}}{R - \rho} \sqrt{\frac{1 - \rho^2}{1 + \rho} + \frac{R - 1}{1 - \Sigma} \left[ \frac{(1 - \theta)^2}{1 - \rho^2} + \frac{\theta \lambda^2}{1 - \rho^2 (1/\theta - R^2)} \right] + \frac{2(1 - \rho) \Gamma}{1 - \rho^2} + \frac{\left( \frac{R - 1}{1 - \Sigma} \right)^2}{1 - \rho^2} + \frac{2(1 - \rho)(1 - \theta)}{1 - \rho^2} + \frac{\left( \frac{R - 1}{1 - \Sigma} \right)^2 \Gamma(1 - \theta)}{1 - \rho^2 \theta}. \quad (55)$$

Given the complexity of this expression, we cannot obtain an explicit result about how the interactions of RI and RB affect the relative volatility. As in the RB case, we thus use a figure to illustrate how RB and RI affect the relative volatility. Figure 6 illustrates the effects of RI on the relative volatility when $R \omega^2_{z_k} / (2 \vartheta) = 0.5$ and $\rho = 0.8$. Note that in the RB-RI case, $\Sigma = R \omega^2_{y_\eta} / (2 \vartheta) = \frac{\theta}{1 - (1 - \theta) R^2} R \omega^2_{z_k} / (2 \vartheta)$ as $\omega^2_{y_\eta} = \frac{\theta}{1 - (1 - \theta) R^2} \omega^2_{z_k}$. It is clear from the figure that given the aggregation factor ($\lambda$), the relative volatility is decreasing with the degree of attention ($\theta$); given $\theta$, the relative volatility is increasing with $\lambda$. The intuition for the first result is that holding the aggregation factor fixed (i.e., given the impact of the common noise), reducing $\theta$ increases the smoothness of aggregate consumption, and thus increases the volatility of the current account. The intuition for the second result is that holding $\theta$ fixed, increasing $\lambda$ strengthens the importance of the common noise, which leads to more volatile consumption and current accounts. Therefore, RI measured by $\theta$ and $\lambda$ has the potential to make the model fit the data better along this dimension. In the next section, we will examine how RI and RB improve the model’s quantitative predictions.
5.1.5 Persistence of the Current Account

Under RB-RI, using (52), the first-order autocorrelation of the current account can be written as:

\[
\rho (ca_t, ca_{t+1}) = \frac{\rho(1-\rho)}{1+\rho} + \frac{\rho^2}{1-\rho^2} + \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\rho_\theta(1-\theta)^2}{1-\rho_\theta^2} + \frac{(\rho+\rho_\omega)(1-\rho)(1-\theta)}{1-\rho_\omega} + \left(\frac{R-1}{1-\Sigma}\right) \frac{(\rho_\theta+\rho)(1-\rho)(1-\theta)}{1-\rho_\omega} + \left(\frac{R-1}{1-\Sigma}\right) \frac{(\rho_\theta+\rho_\omega)(1-\theta)}{1-\rho_\omega} \\
\]

Using this explicit expression, Figure 7 illustrates the effects of RI on \( \rho (ca_t, ca_{t+1}) \) when \( R\omega^2/(2\theta) = 0.5 \) and \( \rho = 0.8 \). It clearly shows that given \( \theta \), the persistence of the current account is decreasing with \( \lambda \). In contrast, the effects of \( \theta \) on the persistence depends on the values of the aggregation factor (\( \lambda \)). When \( \lambda \) is large, (e.g., \( \lambda = 1 \)) the persistence is decreasing with the degree of RI; when \( \lambda \) is small, (e.g., \( \lambda = 0.1 \)) the persistence is increasing with the degree of RI. The intuition behind these results is as follows. Given the degree of attention (\( \theta \)), \( \lambda \) has no impact on the covariance between \( ca_t \) and \( ca_{t+1} \) but increases the variance of the current account, which in turn reduces \( \rho (ca_t, ca_{t+1}) \). It is obvious that RI and RB have the most significant impact on \( \rho (ca_t, ca_{t+1}) \) in the representative agent case (\( \lambda = 1 \)) because the impact of the noise due to RI on the variances of \( ca_t \) that appear in the denominator of (56) is largest in this case. In the next section using the calibrated model we show that the aggregate noise can quantitatively improve the model’s predictions for the first-order autocorrelation of the current account.

5.1.6 Correlation between the Current Account and Income

Similarly, under RB-RI, the correlation between the current account and net income can be written as:

\[
\text{corr} (ca_t, y_t) = \frac{(1-\rho)(R-\rho)}{1-\rho^2} + \frac{(R-\rho)\Gamma}{1-\rho_\omega} + \left(\frac{R-1}{1-\Sigma}\right) \frac{(1-\theta)(R-\rho)}{1-\rho_\omega} \\
\sqrt{\frac{(R-\rho)^2}{1-\rho^2} + \frac{1-\rho}{1-\rho^2} + \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{(1-\theta)^2}{1-\rho_\theta^2} + \frac{\theta\lambda^2}{1-\rho_\theta^2 (1/(1-\theta)-R^2)}} + \frac{(R-1)}{1-\Sigma} \frac{(1-\theta)(1-\theta)}{1-\rho_\omega} + \left(\frac{R-1}{1-\Sigma}\right) \frac{2\Gamma(1-\theta)}{1-\rho_\omega} \\
\]

Using this expression, Figure 8 illustrates the effects of RI on the correlation when \( R\omega^2/(2\theta) = 0.5 \) and \( \rho = 0.8 \). The figure also shows that given \( \theta \), the correlation is increasing with \( \lambda \). In contrast, the effects of \( \theta \) on the correlation are complicated and depend on the value of \( \lambda \). Specifically, when \( \lambda \) is large (\( \lambda = 1 \)), the persistence is decreasing with the degree of RI; when \( \lambda \) is small (\( \lambda = 0.1 \)) the correlation could be increasing with the degree of RI. The intuition behind
these results is similar as that for $\rho(ca_t, ca_{t+1})$: given $\theta$, $\lambda$ has no impact on the covariance between $ca_t$ and $y_t$ but increases the volatility of the current account, which in turn reduces $\text{corr}(ca_t, y_t)$.

5.1.7 Implication for Consumption Volatility

Using (51), the relative volatility of aggregate consumption growth relative to income growth can be written as

$$\mu_c = \frac{\text{sd} [\Delta c_t]}{\text{sd} [\Delta y_t]} = \frac{\theta (R - 1)}{(1 - \Sigma)(R - \rho)} \sqrt{\left(1 + \rho\right) \left(\sum_{j=0}^{\infty} \Gamma_j^2 + \frac{\lambda^2 (1 - \theta)}{\theta (1 - (1 - \theta) R^2)} \sum_{j=0}^{\infty} (\Gamma_j - R \Gamma_{j-1})^2\right)},$$

(58)

where we use the fact that $\omega_\xi^2 = \text{var} [\xi_t] = \frac{1 - \theta}{\theta (1 - (1 - \theta) R^2)} \omega_\varepsilon^2$, $\rho_1 = \rho_s = \frac{1 - R \Sigma}{1 - R} \in (0, 1)$, $\rho_2 = (1 - \theta) R \in (0, 1)$, and

$$\Gamma_j = \sum_{k=0}^{j} \left(\rho_1^{j-k} \rho_2^k\right) - \sum_{k=0}^{j-1} \left(\rho_1^{j-1-k} \rho_2^k\right), \text{ for } j \geq 1,$$

and $\Gamma_0 = 1$. Figure 9 illustrates how the combination of $\theta$ and $\lambda$ affects the relative volatility of consumption growth to income growth when $R \omega_\xi^2 / (2 \vartheta) = 0.5$, $\rho = 0.8$, and $R = 1.04$. It is clear that given $\theta$, the relative volatility $\mu_c$ is increasing with $\lambda$. The effect of $\theta$ on $\mu_c$ is not monotonic, and depends on the values of $\lambda$. Specifically, when $\lambda$ is large ($\lambda = 1$), the relative volatility is decreasing with the degree of attention ($\theta$); when $\lambda$ is small ($\lambda = 0.1$), the relative volatility is decreasing with $\theta$ first and then increasing with $\theta$. The intuition behind this result is as follows. Given $\lambda$ is small, when $\theta$ is low, the presence of the common noise, $\lambda \xi_t$, dominates the smoothness of consumption caused by the gradual responses to fundamental shocks; in contrast, when $\theta$ is large, the gradual response effect dominates the common noise effect, which reduces the relative volatility.

5.2 Comparing the Implications of Different Models

To illustrate the quantitative implications of the RB-RI model on the stochastic properties of the joint dynamics of consumption, the current account, and net income, we fix the RB parameter at the same levels we obtain in Section 4.3 and vary the two RI parameters, $\lambda$ and $\theta$.\textsuperscript{37} As in Section 4.3, we first set the detection error probability, $p$, to be a plausible value, $\vartheta$. The reason why we use the calibrated RB parameter values and vary the two RI parameters is that we want to distinguish the different effects of RB and RI on the model’s dynamics. If we use the DEP to calibrate RB in the RB-RI model, it is difficult to separate the different effects of RB and RI within the model. We recalibrated the value of $\vartheta$ using the DEP in the RB-RI model and found that it does not change our main conclusions about the effects of RB and RI on the joint dynamics of consumption, the current account, and income. The calibration procedure and results are available from the authors by request.

\textsuperscript{37}
10%. Tables 9-10 compare the model performance under different assumptions (RE, RB, and RB-RI) on matching four important dimensions of the data we documented in Section 2: (1) the contemporaneous correlation between the current account and net income, (2) the volatility of the current account, (3) persistence of the current account, and (4) the relative volatility of consumption growth to income growth. The tables clearly show that RI could help further improve the RB model’s predictions along all these four dimensions. Specifically, for emerging countries, in the representative agent case \((\lambda = 1)\), when \(\theta = 50\%\) (i.e., 50% of the uncertainty regarding permanent income can be removed upon receiving a new signal), the interaction of RB and RI reduces the correlation between the current account and net income from 1 to 0.58; reduces the first-order autocorrelation from 0.8 to 0.36; increases the relative volatility of the current account to income from 0.71 to 0.79, and increases the relative volatility of consumption growth to income growth from 0.28 to 1.36, bringing all of them closer to the data.

We make three comments about this result. First, we have seen that in this case \((\lambda = 1\) and \(\theta = 50\%\)) the interaction of RB and RI make the model fit the data quite well along dimensions (3) and (4), while also quantitatively improving the model’s predictions along dimensions (1) and (2). Second, this improvement does not preclude the model from matching the first two dimensions as well (i.e., the contemporaneous correlation between the current account and net income and the volatility of the current account). For example, holding \(\lambda\) equal to 1 and further reducing \(\theta\) can generate a smaller contemporaneous correlation between the current account and net income which is closer to the data. And holding \(\theta = 50\%\) and reducing \(\lambda\) to 0.1 can make the relative volatility of the current account to net income very close to the data. Third, and mostly importantly, all these quantitative results are consistent with the theoretical results we obtained in Section 5.1.

To check how robust these results are, we set the detection error probability to be 0.01 and report the results in Tables 11-12. From these tables, it is clear that in this case RI can improve the model’s predictions on the correlation of the current account and the first-order autocorrelation of the current account. For example, for emerging countries, in the representative agent case \((\lambda = 1)\), when \(\theta = 95\%\) (the agent can process almost all available information about the state), the combination of RB and RI reduces the correlation between the current account and income to 0.09 and reduces the first-order autocorrelation of the current account to 0.52. It is worth noting that given the high calibrated \(\Sigma\) when \(p = 0.01\), the model generates very volatile processes of consumption growth (the relative volatility of consumption growth to income growth increases to 2.09 in this case).

[Insert Tables 9-12 Here]
6 Conclusion

We have examined how introducing two types of information imperfections, robustness and rational inattention, into an otherwise standard intertemporal current account model changes the dynamic effects of income shocks on the joint dynamics of consumption and the current account. We have shown that a model with agents who have both a preference for robustness and limited information processing capacity has the potential to better account for the data along a number of dimensions.

The model proposed in this paper can also be used to address the international diversification and consumption correlations puzzles (Backus, Kehoe, and Kydland, 1992; Baxter and Crucini, 1993; Stockman and Tesar, 1995; Baxter and Jermann, 1997 propose alternative resolutions of these puzzles). In Luo, Nie, and Young (2010) we show that the RB-RI model reduces the correlation of consumption across countries, and can in fact produce consumption correlations lower than income correlations. RB will lower the international consumption correlations by generating heterogenous responses of consumption to income shocks across countries, provided countries differ in terms of their preference for robustness. The effect of RI on consumption correlations depends on the relative importance of the gradual responses to income shocks versus the effect of the common noise due to limited capacity; the first effect increases the correlations, whereas the second reduces them. Whether the model predicts a low correlation turns out to hinge critically on the extent to which noise shocks are correlated across agents.

In addition, in contrast to the intertemporal consumption approach we consider here, the ‘new rule’ approach to the current account assigns the preeminent role to portfolio choice (for conflicting views on the relevance of the new rule, see Kraay and Ventura 2000, 2003 and Tille and van Wincoop 2010). An interesting extension to our study would be to permit portfolio choice and study the dynamics of the current account in the RB-RI model.

Finally, to explicitly explore the mechanisms through which the two informational frictions interact and work, in this paper we have set up the model in a parsimonious way so that we can obtain a closed-form solution. We think that the mechanisms and insights we have explored in this simple framework can be carried over to more general cases. In particular, extending the model to incorporate the global interest rate shock emphasized by Nason and Rogers (2006) and Kano (2009) will be critical for demonstrating conclusively the utility of the RB-RI framework; given that such an extension is nontrivially difficult, we leave it for future work.
Appendix

7.1 Solving the Robust Model

To solve the Bellman equation (15), we conjecture that

\[ v(s_t) = -A s_t^2 - B s_t - C, \]  

(59)

where \( A, B, \) and \( C \) are undetermined coefficients. Substituting this guessed value function into the Bellman equation gives

\[ -A s_t^2 - B s_t - C = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (\nu - c_t)^2 + \beta E_t \left[ \nu + A s_{t+1}^2 - B s_{t+1} - C \right] \right\}. \]

(60)

We can do the min and max operations in any order, so we choose to do the minimization first. The first-order condition for \( \nu_t \) is

\[ 2 \nu_t - 2 A E_t [\omega_\zeta \nu_t + R s_t - c_t] \omega_\zeta - B \omega_\zeta = 0, \]

which means that

\[ \nu_t = \frac{B + 2 A (R s_t - c_t)}{2 (\vartheta - A \omega_\zeta^2)} \omega_\zeta. \]

(61)

Substituting (61) back into (60) gives

\[ -A s_t^2 - B s_t - C = \max_{c_t} \left\{ -\frac{1}{2} (\vartheta - c_t)^2 + \beta E_t \left[ \vartheta \left( \frac{B + 2 A (R s_t - c_t)}{2 (\vartheta - A \omega_\zeta^2)} \omega_\zeta \right)^2 - A s_{t+1}^2 - B s_{t+1} - C \right] \right\}, \]

(62)

where

\[ s_{t+1} = R s_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t. \]

The first-order condition for \( c_t \) is

\[ (\vartheta - c_t) - 2 \beta \vartheta A \omega_\zeta^2 \vartheta - A \omega_\zeta^2 \nu_t + 2 \beta A \left( 1 + \frac{A \omega_\zeta^2}{\vartheta - A \omega_\zeta^2} \right) (R s_t - c_t + \omega_\zeta \nu_t) + \beta B \left( 1 + \frac{A \omega_\zeta^2}{\vartheta - A \omega_\zeta^2} \right) = 0. \]

Using the solution for \( \nu_t \) the solution for consumption is

\[ c_t = \frac{2 A \beta R}{1 - A \omega_\zeta^2 / \vartheta + 2 \beta A} s_t + \frac{\vartheta \left( 1 - A \omega_\zeta^2 / \vartheta \right) + \beta B}{1 - A \omega_\zeta^2 / \vartheta + 2 \beta A}. \]

(63)
Substituting the above expressions into the Bellman equation gives

\[-As_t^2 - Bs_t - C\]

\[= -\frac{1}{2} \left( \frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t + \frac{-2\beta A\vartheta + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} \right)^2 + \beta \vartheta \omega_\zeta^2 \left( \frac{2AR(1 - A\omega_\zeta^2/\vartheta)}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t + B - \frac{2\vartheta \left( 1 - A\omega_\zeta^2/\vartheta \right)}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} \right)^2\]

\[-\beta A \left( \frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A} s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2 \left( 1 - A\omega_\zeta^2/\vartheta + 2\beta A \right)} \right)^2 + \omega_\zeta^2 \right) - \beta C.\]

Given \(\beta R = 1\), collecting and matching terms, the constant coefficients turn out to be

\[A = \frac{R(R - 1)}{2 - R\omega_\zeta^2/\vartheta}, \quad (64)\]

\[B = -\frac{R\vartheta}{1 - R\omega_\zeta^2/\vartheta}, \quad (65)\]

\[C = \frac{R}{2 \left( 1 - R\omega_\zeta^2/2\vartheta \right)} \omega_\zeta^2 + \frac{R}{2 \left( 1 - R\omega_\zeta^2/2\vartheta \right) (R - 1)} \vartheta^2. \quad (66)\]

Substituting (64) and (65) into (63) yields the consumption function (17) in the text. Substituting (17) into the current account identity and using the expression for \(s_t\) yields the expression for the current account (19).

### 7.2 Deriving the Joint Dynamics between the Current Account and Income under RB-RI

#### 7.2.1 Deriving the Volatility of the Current Account under RB-RI

Given (52), \(ca_t = \frac{1 - \varrho \omega_\zeta}{R - \rho y_t - \frac{\bar{\Sigma}(R - 1)}{1 - \Sigma} s_t + \frac{R - 1}{1 - \Sigma} (s_t - \bar{s}_t) + \frac{\bar{\Sigma}}{1 - \Sigma} \vartheta}\), we first derive the variance of the current account as follows:
7.2.2 Deriving the Persistence of the Current Account under RB-RI

Given (52), the first-order autocovariance of the current account can be derived as follows:

\[
\text{var} (\epsilon_t) = \text{var} \left( \frac{1 - \rho}{R - \rho} \frac{\epsilon_t}{1 - \rho} + \frac{\Gamma}{R - \rho} \frac{\epsilon_t}{1 - \rho} + \frac{R - 1}{1 - \Sigma} (\hat{s_t} - \hat{s_t}) \right)
\]

\[
= \text{var} \left( \frac{(1 - \rho) \zeta_t}{1 - \rho \cdot L} + \frac{\Gamma \zeta_t}{1 - \rho \cdot L} + \frac{R - 1}{1 - \Sigma} \left( \frac{(1 - \theta) \xi_t}{1 - \rho \cdot L} - \frac{\theta E^t [\xi_t]}{1 - \rho \cdot L} \right) \right)
\]

\[
= (1 - \rho)^2 \frac{\omega^2_\xi}{1 - \rho^2} + \Gamma^2 \frac{\omega^2_\xi}{1 - \rho^2} + \left( \frac{R - 1}{1 - \Sigma} \right)^2 \text{var} \left( \frac{(1 - \theta) \xi_t}{1 - \rho \cdot L} - \frac{\theta E^t [\xi_t]}{1 - \rho \cdot L} \right)
\]

\[
+ 2 \text{cov} \left( \frac{(1 - \rho) \zeta_t}{1 - \rho \cdot L}, \frac{\Gamma \zeta_t}{1 - \rho \cdot L} \right) + 2 \text{cov} \left( \frac{(1 - \rho) \zeta_t}{1 - \rho \cdot L}, \frac{R - 1}{1 - \Sigma} \left( \frac{(1 - \theta) \xi_t}{1 - \rho \cdot L} - \frac{\theta E^t [\xi_t]}{1 - \rho \cdot L} \right) \right)
\]

\[
= (1 - \rho)^2 \frac{\omega^2_\xi}{1 - \rho^2} + \Gamma^2 \frac{\omega^2_\xi}{1 - \rho^2} + \left( \frac{R - 1}{1 - \Sigma} \right)^2 \left[ \frac{(1 - \theta)^2}{1 - \rho_\theta^2} + \frac{\theta \lambda^2}{1 - \rho_\theta^2} \left( \frac{1}{(1 - \theta) - R^2} \right) \right] \omega^2_\xi
\]

\[
+ 2 \frac{(1 - \rho) \Gamma \omega^2_\xi}{1 - \rho \rho_\theta} + 2 (1 - \rho) (1 - \theta) \left( \frac{R - 1}{1 - \Sigma} \right) \frac{\omega^2_\xi}{1 - \rho \rho_\theta} + 2 \Gamma (1 - \theta) \left( \frac{R - 1}{1 - \Sigma} \right) \frac{\omega^2_\xi}{1 - \rho \rho_\theta}
\]

\[
= \left\{ \frac{1 - \rho}{1 + \rho} + \frac{\Gamma^2}{1 - \rho^2} + \left( \frac{R - 1}{1 - \Sigma} \right)^2 \left[ \frac{(1 - \theta)^2}{1 - \rho_\theta^2} + \frac{\theta \lambda^2}{1 - \rho_\theta^2} \left( \frac{1}{(1 - \theta) - R^2} \right) \right] \right\} \omega^2_\xi.
\]

Using the definition of the relative volatility of the current account and net income, we can obtain (55) in the text.

Given (52), the first-order autocovariance of the current account can be derived as follows:
\[
\text{cov} (ca_t, ca_{t+1}) = \text{cov} \left( \frac{(1-\rho)\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_t}{1-\rho_s \cdot L} + \frac{\Gamma \zeta_{t+1}}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_t}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_t]}{1-\rho^2 - \rho \theta} \right), \frac{(1-\rho)\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_t}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_t}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_t]}{1-\rho^2 - \rho \theta} \right) \right)
\]
\[
= \text{cov} \left( \frac{(1-\rho)\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_t}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_t}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_t]}{1-\rho^2 - \rho \theta} \right), \frac{(1-\rho)\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_{t+1}}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_{t+1}}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_{t+1}]}{1-\rho^2 - \rho \theta} \right) \right)
\]
\[
= \text{cov} \left( (1-\rho)_t \frac{\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_t}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_t}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_t]}{1-\rho^2 - \rho \theta} \right), (1-\rho)_t \frac{\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_{t+1}}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_{t+1}}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_{t+1}]}{1-\rho^2 - \rho \theta} \right) \right)
\]
\[
= \text{cov} \left( (1-\rho)_t \frac{\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_t}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_t}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_t]}{1-\rho^2 - \rho \theta} \right), \frac{(1-\rho)\zeta_t}{1-\rho^2} + \frac{\Gamma \zeta_{t+1}}{1-\rho_s \cdot L} + \frac{R-1}{1-\rho \cdot L} \left( \frac{(1-\theta)\zeta_{t+1}}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_{t+1}]}{1-\rho^2 - \rho \theta} \right) \right)
\]

Using (67) and (68), we can obtain (56) in the text.

**7.2.3 Deriving the Correlation between the Current Account and Net Income under RB-RI**

Given (52), the covariance between the current account and net income is

\[
\text{cov} (ca_t, y_t) = \text{cov} \left( \frac{1-\rho}{R-\rho} y_t + \frac{R-1}{1-\Sigma} (s_t - \bar{s}_t), y_t \right)
\]

\[
= \text{cov} \left( \frac{1-\rho}{R-\rho} y_t, y_t \right) + \text{cov} (y_t, \Gamma s_t) + \text{cov} \left( \frac{R-1}{1-\Sigma} (s_t - \bar{s}_t), y_t \right)
\]

\[
= \frac{1-\rho}{R-\rho} \text{var} (y_t) + \Gamma \text{cov} \left( \frac{(R-\rho)\zeta_t}{1-\rho \cdot L}, \frac{\zeta_t}{1-\rho_s \cdot L} \right) + \frac{R-1}{1-\Sigma} \text{cov} \left( (R-\rho)\zeta_t, \frac{R-1}{1-\rho \cdot L}, \frac{(1-\theta)\zeta_t}{1-\rho^2 - \rho \theta} - \frac{\theta E^{\dagger}[\xi_t]}{1-\rho^2 - \rho \theta} \right)
\]

\[
= \left[ \frac{(1-\rho)(R-\rho)}{1-\rho^2} + \frac{(R-\rho)\Gamma}{1-\rho \cdot L} \frac{R-1}{1-\Sigma} \left( \frac{(1-\theta)(R-\rho)}{1-\rho \cdot L} \right) \right] \omega^2_{\zeta}.
\]
References


Figure 1: Responses of Current Account to Income Shock $\varepsilon$
Figure 2: Responses of Consumption to Income Shock $\varepsilon$
Figure 3: The Relative Volatility of Current Accounts to Net Income under RB
Figure 4: The Persistence of Current Accounts under RB

\[ \rho(c_a, c_{a-1}) = 0.5 \]

\[ \rho = 0.5 \]

\[ \rho = 0.7 \]

\[ \rho = 0.8 \]
Figure 5: The Correlation between the Current Accounts and Net Income under RB
Figure 6: The Relative Volatility of Current Accounts to Net Income under RB and RI
Figure 7: The Persistence of Current Accounts under RB and RI
Figure 8: The Correlation between the Current Accounts and Net Income under RB and RI
Figure 9: The Relative Volatility of Consumption Growth to Income Growth under RB and RI
<table>
<thead>
<tr>
<th></th>
<th>Emerging vs. Developed Countries (Averages)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A: Emerging vs. Developed Countries (HP Filter)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(y)/\mu(y)$</td>
<td>3.19(0.20)</td>
<td>1.83(0.07)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta y)/\mu(y)$</td>
<td>3.82(0.19)</td>
<td>2.07(0.06)</td>
<td></td>
</tr>
<tr>
<td>$\rho(y_t, y_{t-1})$</td>
<td>0.50(0.03)</td>
<td>0.44(0.03)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>1.35(0.08)</td>
<td>0.98(0.04)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>1.53(0.09)</td>
<td>1.60(0.08)</td>
<td></td>
</tr>
<tr>
<td>$\rho(c, y)$</td>
<td>0.33(0.04)</td>
<td>0.46(0.04)</td>
<td></td>
</tr>
<tr>
<td>$\rho(ca_t, ca_{t-1})$</td>
<td>0.30(0.05)</td>
<td>0.41(0.03)</td>
<td></td>
</tr>
<tr>
<td>$\rho(ca, y)$</td>
<td>0.05(0.05)</td>
<td>0.06(0.05)</td>
<td></td>
</tr>
<tr>
<td>$\rho \left( \frac{ca}{y}, y \right)$</td>
<td>0.04(0.04)</td>
<td>0.15(0.04)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B: Emerging vs. Developed Countries (Linear Filter)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(y)/\mu(y)$</td>
<td>9.03(0.43)</td>
<td>4.37(0.18)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta y)/\mu(y)$</td>
<td>3.82(0.19)</td>
<td>2.07(0.06)</td>
<td></td>
</tr>
<tr>
<td>$\rho(y_t, y_{t-1})$</td>
<td>0.80(0.02)</td>
<td>0.79(0.02)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>1.35(0.08)</td>
<td>0.98(0.04)</td>
<td></td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>0.80(0.06)</td>
<td>1.35(0.06)</td>
<td></td>
</tr>
<tr>
<td>$\rho(c, y)$</td>
<td>0.68(0.04)</td>
<td>0.63(0.04)</td>
<td></td>
</tr>
<tr>
<td>$\rho(ca_t, ca_{t-1})$</td>
<td>0.53(0.04)</td>
<td>0.71(0.02)</td>
<td></td>
</tr>
<tr>
<td>$\rho(ca, y)$</td>
<td>0.13(0.05)</td>
<td>0.17(0.05)</td>
<td></td>
</tr>
<tr>
<td>$\rho \left( \frac{ca}{y}, y \right)$</td>
<td>0.03(0.05)</td>
<td>0.16(0.05)</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Summary of Statistics: Emerging Countries

<table>
<thead>
<tr>
<th></th>
<th>Argentina</th>
<th>Brazil</th>
<th>Ecuador</th>
<th>Korea</th>
<th>Malaysia</th>
<th>Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(y)/μ(y)</td>
<td>6.01(1.04)</td>
<td>4.73(0.49)</td>
<td>3.86(0.64)</td>
<td>13.65(1.85)</td>
<td>18.01(2.24)</td>
<td>4.01(0.36)</td>
</tr>
<tr>
<td>σ(Δy)/μ(y)</td>
<td>4.51(0.61)</td>
<td>2.72(0.20)</td>
<td>4.95(0.76)</td>
<td>4.02(0.45)</td>
<td>5.55(1.07)</td>
<td>1.82(0.20)</td>
</tr>
<tr>
<td>ρ(yt, yt−1)</td>
<td>0.71(0.05)</td>
<td>0.83(0.05)</td>
<td>0.01(0.11)</td>
<td>0.95(0.02)</td>
<td>0.95(0.02)</td>
<td>0.90(0.03)</td>
</tr>
<tr>
<td>σ(Δc)/σ(Δy)</td>
<td>1.55(0.05)</td>
<td>1.40(0.14)</td>
<td>0.78(0.25)</td>
<td>1.41(0.35)</td>
<td>0.97(0.14)</td>
<td>2.60(0.56)</td>
</tr>
<tr>
<td>σ(ca)/σ(y)</td>
<td>0.65(0.11)</td>
<td>0.73(0.14)</td>
<td>1.85(0.49)</td>
<td>0.47(0.13)</td>
<td>0.86(0.11)</td>
<td>0.88(0.10)</td>
</tr>
<tr>
<td>ρ(c, y)</td>
<td>0.99(0.00)</td>
<td>0.78(0.09)</td>
<td>−0.28(0.17)</td>
<td>0.38(0.24)</td>
<td>0.74(0.13)</td>
<td>0.90(0.03)</td>
</tr>
<tr>
<td>ρ(ca, ca_{t−1})</td>
<td>0.47(0.14)</td>
<td>0.79(0.06)</td>
<td>−0.18(0.29)</td>
<td>0.40(0.13)</td>
<td>0.80(0.10)</td>
<td>0.60(0.07)</td>
</tr>
<tr>
<td>ρ(y, ca)</td>
<td>−0.70(0.08)</td>
<td>0.21(0.27)</td>
<td>0.40(0.17)</td>
<td>0.08(0.18)</td>
<td>0.89(0.04)</td>
<td>0.29(0.15)</td>
</tr>
<tr>
<td>ρ(\frac{ca}{y}, y)</td>
<td>−0.21(0.16)</td>
<td>0.07(0.18)</td>
<td>0.17(0.15)</td>
<td>−0.01(0.11)</td>
<td>0.28(0.16)</td>
<td>−0.26(0.15)</td>
</tr>
</tbody>
</table>

Table 3: Summary of Statistics: Emerging Countries (Continued)

<table>
<thead>
<tr>
<th></th>
<th>Peru</th>
<th>Philippines</th>
<th>South Africa</th>
<th>Thailand</th>
<th>Turkey</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ(y)/μ(y)</td>
<td>11.89(1.67)</td>
<td>11.95(2.19)</td>
<td>5.24(1.03)</td>
<td>16.00(1.86)</td>
<td>4.05(0.58)</td>
</tr>
<tr>
<td>σ(Δy)/μ(y)</td>
<td>5.06(0.95)</td>
<td>3.54(0.67)</td>
<td>2.39(0.20)</td>
<td>4.78(0.81)</td>
<td>2.62(0.14)</td>
</tr>
<tr>
<td>ρ(yt, yt−1)</td>
<td>0.91(0.04)</td>
<td>0.96(0.02)</td>
<td>0.89(0.05)</td>
<td>0.95(0.02)</td>
<td>0.76(0.08)</td>
</tr>
<tr>
<td>σ(Δc)/σ(Δy)</td>
<td>1.07(0.07)</td>
<td>0.66(0.08)</td>
<td>1.31(0.31)</td>
<td>1.05(0.14)</td>
<td>2.01(0.25)</td>
</tr>
<tr>
<td>σ(ca)/σ(y)</td>
<td>0.45(0.08)</td>
<td>0.37(0.05)</td>
<td>1.05(0.20)</td>
<td>0.69(0.16)</td>
<td>0.79(0.09)</td>
</tr>
<tr>
<td>ρ(c, y)</td>
<td>0.95(0.02)</td>
<td>0.94(0.03)</td>
<td>0.87(0.05)</td>
<td>0.26(0.33)</td>
<td>0.93(0.03)</td>
</tr>
<tr>
<td>ρ(ca, ca_{t−1})</td>
<td>0.57(0.12)</td>
<td>0.54(0.14)</td>
<td>0.72(0.11)</td>
<td>0.64(0.11)</td>
<td>0.45(0.18)</td>
</tr>
<tr>
<td>ρ(ca, y)</td>
<td>0.47(0.23)</td>
<td>0.64(0.16)</td>
<td>−0.58(0.15)</td>
<td>0.58(0.13)</td>
<td>−0.80(0.09)</td>
</tr>
<tr>
<td>ρ(\frac{ca}{y}, y)</td>
<td>0.32(0.13)</td>
<td>0.31(0.20)</td>
<td>−0.12(0.16)</td>
<td>0.27(0.13)</td>
<td>−0.44(0.11)</td>
</tr>
</tbody>
</table>
Table 4: Summary of Statistics: Developed Countries

<table>
<thead>
<tr>
<th></th>
<th>Australia</th>
<th>Austria</th>
<th>Belgium</th>
<th>Canada</th>
<th>Denmark</th>
<th>Finland</th>
<th>Netherlands</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)/\mu(y)$</td>
<td>6.15(0.65)</td>
<td>3.26(0.38)</td>
<td>1.56(0.14)</td>
<td>6.81(0.94)</td>
<td>1.60(0.11)</td>
<td>7.03(1.17)</td>
<td>5.35(0.51)</td>
</tr>
<tr>
<td>$\sigma(\Delta y)/\mu(y)$</td>
<td>2.22(0.35)</td>
<td>1.66(0.13)</td>
<td>1.42(0.14)</td>
<td>2.64(0.35)</td>
<td>1.64(0.20)</td>
<td>2.67(0.29)</td>
<td>2.26(0.20)</td>
</tr>
<tr>
<td>$p(y_t, y_{t-1})$</td>
<td>0.94(0.02)</td>
<td>0.86(0.04)</td>
<td>0.59(0.05)</td>
<td>0.92(0.02)</td>
<td>0.46(0.12)</td>
<td>0.92(0.02)</td>
<td>0.91(0.02)</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.93(0.13)</td>
<td>0.95(0.11)</td>
<td>0.82(0.13)</td>
<td>0.71(0.11)</td>
<td>1.39(0.23)</td>
<td>0.91(0.17)</td>
<td>0.79(0.12)</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>0.47(0.08)</td>
<td>0.97(0.14)</td>
<td>1.99(0.34)</td>
<td>0.55(0.04)</td>
<td>1.79(0.19)</td>
<td>0.86(0.17)</td>
<td>0.74(0.15)</td>
</tr>
<tr>
<td>$p(c, y)$</td>
<td>0.75(0.07)</td>
<td>0.21(0.26)</td>
<td>0.51(0.12)</td>
<td>0.76(0.06)</td>
<td>0.02(0.25)</td>
<td>0.62(0.19)</td>
<td>0.91(0.03)</td>
</tr>
<tr>
<td>$p(ca_t, ca_{t-1})$</td>
<td>0.55(0.08)</td>
<td>0.73(0.10)</td>
<td>0.87(0.03)</td>
<td>0.81(0.08)</td>
<td>0.50(0.15)</td>
<td>0.84(0.06)</td>
<td>0.71(0.06)</td>
</tr>
<tr>
<td>$p(ca, y)$</td>
<td>−0.10(0.20)</td>
<td>0.50(0.19)</td>
<td>−0.40(0.18)</td>
<td>0.81(0.09)</td>
<td>0.00(0.14)</td>
<td>0.44(0.20)</td>
<td>0.21(0.23)</td>
</tr>
<tr>
<td>$p\left(\frac{ca}{y}, y\right)$</td>
<td>0.32(0.17)</td>
<td>0.20(0.20)</td>
<td>0.02(0.14)</td>
<td>0.06(0.14)</td>
<td>0.16(0.12)</td>
<td>0.33(0.18)</td>
<td>0.03(0.15)</td>
</tr>
</tbody>
</table>

Table 5: Summary of Statistics: Developed Countries (Continued)

<table>
<thead>
<tr>
<th></th>
<th>New Zealand</th>
<th>Norway</th>
<th>Portugal</th>
<th>Spain</th>
<th>Sweden</th>
<th>Switerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)/\mu(y)$</td>
<td>5.53(0.69)</td>
<td>3.20(0.19)</td>
<td>2.32(0.32)</td>
<td>3.36(0.28)</td>
<td>8.48(1.31)</td>
<td>2.09(0.29)</td>
</tr>
<tr>
<td>$\sigma(\Delta y)/\mu(y)$</td>
<td>2.44(0.22)</td>
<td>2.67(0.24)</td>
<td>1.88(0.23)</td>
<td>1.13(0.14)</td>
<td>2.44(0.23)</td>
<td>1.80(0.16)</td>
</tr>
<tr>
<td>$p(y_t, y_{t-1})$</td>
<td>0.90(0.03)</td>
<td>0.66(0.08)</td>
<td>0.68(0.07)</td>
<td>0.94(0.02)</td>
<td>0.96(0.01)</td>
<td>0.57(0.10)</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>1.10(0.14)</td>
<td>0.67(0.09)</td>
<td>1.28(0.20)</td>
<td>1.95(0.23)</td>
<td>0.81(0.10)</td>
<td>0.40(0.06)</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>0.91(0.14)</td>
<td>2.54(0.45)</td>
<td>3.02(0.34)</td>
<td>1.49(0.29)</td>
<td>0.65(0.04)</td>
<td>1.53(0.21)</td>
</tr>
<tr>
<td>$p(c, y)$</td>
<td>0.91(0.03)</td>
<td>0.67(0.08)</td>
<td>0.34(0.18)</td>
<td>0.86(0.04)</td>
<td>0.84(0.08)</td>
<td>0.74(0.10)</td>
</tr>
<tr>
<td>$p(ca_t, ca_{t-1})$</td>
<td>0.50(0.11)</td>
<td>0.61(0.09)</td>
<td>0.79(0.06)</td>
<td>0.88(0.04)</td>
<td>0.90(0.03)</td>
<td>0.49(0.15)</td>
</tr>
<tr>
<td>$p(ca, y)$</td>
<td>−0.39(0.16)</td>
<td>0.43(0.17)</td>
<td>0.03(0.18)</td>
<td>−0.60(0.10)</td>
<td>0.92(0.03)</td>
<td>0.32(0.17)</td>
</tr>
<tr>
<td>$p\left(\frac{ca}{y}, y\right)$</td>
<td>−0.07(0.12)</td>
<td>0.27(0.18)</td>
<td>0.06(0.21)</td>
<td>0.01(0.17)</td>
<td>0.34(0.17)</td>
<td>0.31(0.17)</td>
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</tbody>
</table>
Table 6: Emerging vs. Developed Countries (Averages, $p = 0.1$)

<table>
<thead>
<tr>
<th></th>
<th>Emerging Countries</th>
<th>Developed Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma$</td>
<td>0.524</td>
<td>0.205</td>
</tr>
<tr>
<td>$p$</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.802</td>
<td>0.793</td>
</tr>
<tr>
<td>$\frac{\sigma(y)}{\mu(y)}$</td>
<td>0.090</td>
<td>0.044</td>
</tr>
<tr>
<td>$\frac{\sigma(\zeta)}{\mu(y)}$</td>
<td>0.284</td>
<td>0.132</td>
</tr>
</tbody>
</table>

Table 7: Emerging Countries

<table>
<thead>
<tr>
<th></th>
<th>Arg</th>
<th>Bra</th>
<th>Ecu</th>
<th>Kor</th>
<th>Mal</th>
<th>Mex</th>
<th>Per</th>
<th>Phi</th>
<th>Sou</th>
<th>Tha</th>
<th>Tur</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma$</td>
<td>0.749</td>
<td>0.397</td>
<td>0.281</td>
<td>0.470</td>
<td>0.424</td>
<td>0.554</td>
<td>0.684</td>
<td>0.871</td>
<td>0.274</td>
<td>0.631</td>
<td>0.427</td>
</tr>
<tr>
<td>$p$</td>
<td>0.100</td>
<td>0.101</td>
<td>0.100</td>
<td>0.099</td>
<td>0.100</td>
<td>0.099</td>
<td>0.101</td>
<td>0.101</td>
<td>0.101</td>
<td>0.100</td>
<td>0.101</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.705</td>
<td>0.825</td>
<td>0.014</td>
<td>0.952</td>
<td>0.949</td>
<td>0.896</td>
<td>0.907</td>
<td>0.958</td>
<td>0.894</td>
<td>0.953</td>
<td>0.764</td>
</tr>
<tr>
<td>$\frac{\sigma(y)}{\mu(y)}$</td>
<td>0.060</td>
<td>0.047</td>
<td>0.039</td>
<td>0.137</td>
<td>0.180</td>
<td>0.040</td>
<td>0.119</td>
<td>0.119</td>
<td>0.052</td>
<td>0.160</td>
<td>0.040</td>
</tr>
<tr>
<td>$\frac{\sigma(\zeta)}{\mu(y)}$</td>
<td>0.127</td>
<td>0.124</td>
<td>0.038</td>
<td>0.475</td>
<td>0.624</td>
<td>0.124</td>
<td>0.377</td>
<td>0.418</td>
<td>0.161</td>
<td>0.557</td>
<td>0.095</td>
</tr>
</tbody>
</table>
Table 8: Developed Countries

<table>
<thead>
<tr>
<th></th>
<th>Aus</th>
<th>Aut</th>
<th>Bel</th>
<th>Can</th>
<th>Den</th>
<th>Fin</th>
<th>Net</th>
<th>New</th>
<th>Nor</th>
<th>Por</th>
<th>Spa</th>
<th>Swe</th>
<th>Swi</th>
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</thead>
<tbody>
<tr>
<td>Σ</td>
<td>0.385</td>
<td>0.158</td>
<td>0.038</td>
<td>0.307</td>
<td>0.027</td>
<td>0.225</td>
<td>0.163</td>
<td>0.355</td>
<td>0.052</td>
<td>0.303</td>
<td>0.274</td>
<td>0.247</td>
<td>0.125</td>
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<td>p</td>
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<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
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<td>0.101</td>
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<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
<td>0.099</td>
<td>0.100</td>
</tr>
<tr>
<td>ρ</td>
<td>0.935</td>
<td>0.864</td>
<td>0.587</td>
<td>0.924</td>
<td>0.459</td>
<td>0.924</td>
<td>0.912</td>
<td>0.901</td>
<td>0.656</td>
<td>0.678</td>
<td>0.943</td>
<td>0.957</td>
<td>0.570</td>
</tr>
<tr>
<td>σ(y)</td>
<td>0.062</td>
<td>0.033</td>
<td>0.016</td>
<td>0.068</td>
<td>0.016</td>
<td>0.070</td>
<td>0.054</td>
<td>0.055</td>
<td>0.032</td>
<td>0.023</td>
<td>0.034</td>
<td>0.085</td>
<td>0.021</td>
</tr>
<tr>
<td>µ(y)</td>
<td>0.208</td>
<td>0.093</td>
<td>0.028</td>
<td>0.224</td>
<td>0.024</td>
<td>0.232</td>
<td>0.172</td>
<td>0.173</td>
<td>0.063</td>
<td>0.047</td>
<td>0.115</td>
<td>0.296</td>
<td>0.037</td>
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</table>

Table 9: Implications of Different Models (Emerging Countries, p = 0.1)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>RB+RI</th>
<th>RB+RI</th>
<th>RB+RI</th>
<th>RB+RI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(θ = 0.9)</td>
<td>(θ = 0.8)</td>
<td>(θ = 0.7)</td>
<td>(θ = 0.5)</td>
<td>(θ = 0.9)</td>
<td>(θ = 0.8)</td>
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<tr>
<td>(λ = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ(ca, y)</td>
<td>0.13</td>
<td>1.00</td>
<td>0.62</td>
<td>0.57</td>
<td>0.56</td>
<td>0.56</td>
<td>0.58</td>
</tr>
<tr>
<td>ρ(ca_t, ca_t−1)</td>
<td>0.53</td>
<td>0.80</td>
<td>0.74</td>
<td>0.57</td>
<td>0.50</td>
<td>0.45</td>
<td>0.36</td>
</tr>
<tr>
<td>σ(ca)/σ(y)</td>
<td>0.80</td>
<td>0.71</td>
<td>0.49</td>
<td>0.52</td>
<td>0.55</td>
<td>0.59</td>
<td>0.79</td>
</tr>
<tr>
<td>σ(∆c)/σ(∆y)</td>
<td>1.35</td>
<td>0.28</td>
<td>0.90</td>
<td>0.89</td>
<td>0.89</td>
<td>0.91</td>
<td>1.36</td>
</tr>
</tbody>
</table>

(λ = 0.5)

<table>
<thead>
<tr>
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<th>RE</th>
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<th>RB+RI</th>
<th>RB+RI</th>
<th>RB+RI</th>
<th>RB+RI</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>(θ = 0.9)</td>
<td>(θ = 0.8)</td>
<td>(θ = 0.7)</td>
<td>(θ = 0.5)</td>
<td>(θ = 0.9)</td>
<td>(θ = 0.8)</td>
</tr>
<tr>
<td>ρ(ca, y)</td>
<td>0.13</td>
<td>1.00</td>
<td>0.62</td>
<td>0.59</td>
<td>0.58</td>
<td>0.59</td>
<td>0.64</td>
</tr>
<tr>
<td>ρ(ca_t, ca_t−1)</td>
<td>0.53</td>
<td>0.80</td>
<td>0.74</td>
<td>0.63</td>
<td>0.59</td>
<td>0.55</td>
<td>0.46</td>
</tr>
<tr>
<td>σ(ca)/σ(y)</td>
<td>0.80</td>
<td>0.71</td>
<td>0.49</td>
<td>0.50</td>
<td>0.52</td>
<td>0.53</td>
<td>0.64</td>
</tr>
<tr>
<td>σ(∆c)/σ(∆y)</td>
<td>1.35</td>
<td>0.28</td>
<td>0.90</td>
<td>0.85</td>
<td>0.81</td>
<td>0.79</td>
<td>0.99</td>
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(λ = 0.1)

<table>
<thead>
<tr>
<th></th>
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<th>RB</th>
<th>RB+RI</th>
<th>RB+RI</th>
<th>RB+RI</th>
<th>RB+RI</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(θ = 0.9)</td>
<td>(θ = 0.8)</td>
<td>(θ = 0.7)</td>
<td>(θ = 0.5)</td>
<td>(θ = 0.9)</td>
<td>(θ = 0.8)</td>
</tr>
<tr>
<td>ρ(ca, y)</td>
<td>0.13</td>
<td>1.00</td>
<td>0.62</td>
<td>0.61</td>
<td>0.60</td>
<td>0.61</td>
<td>0.67</td>
</tr>
<tr>
<td>ρ(ca_t, ca_t−1)</td>
<td>0.53</td>
<td>0.80</td>
<td>0.74</td>
<td>0.67</td>
<td>0.64</td>
<td>0.62</td>
<td>0.56</td>
</tr>
<tr>
<td>σ(ca)/σ(y)</td>
<td>0.80</td>
<td>0.71</td>
<td>0.49</td>
<td>0.49</td>
<td>0.50</td>
<td>0.51</td>
<td>0.57</td>
</tr>
<tr>
<td>σ(∆c)/σ(∆y)</td>
<td>1.35</td>
<td>0.28</td>
<td>0.90</td>
<td>0.84</td>
<td>0.79</td>
<td>0.75</td>
<td>0.82</td>
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</tbody>
</table>
Table 10: Implications of Different Models (Developed Countries, $p = 0.1$)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>RB+RI ($\theta = 0.9$)</th>
<th>RB+RI ($\theta = 0.6$)</th>
<th>RB+RI ($\theta = 0.3$)</th>
<th>RB+RI ($\theta = 0.1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(ca, y)$</td>
<td>0.17</td>
<td>1.00</td>
<td>0.94</td>
<td>0.94</td>
<td>0.91</td>
<td>0.87</td>
<td>0.83</td>
</tr>
<tr>
<td>$\rho(ca_t, ca_{t-1})$</td>
<td>0.71</td>
<td>0.79</td>
<td>0.78</td>
<td>0.76</td>
<td>0.70</td>
<td>0.64</td>
<td>0.58</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>1.35</td>
<td>0.75</td>
<td>0.64</td>
<td>0.65</td>
<td>0.63</td>
<td>0.79</td>
<td>0.89</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.98</td>
<td>0.24</td>
<td>0.33</td>
<td>0.31</td>
<td>0.26</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td>($\lambda = 0.5$)</td>
<td>($\lambda = 0.5$)</td>
<td>($\lambda = 0.5$)</td>
<td>($\lambda = 0.5$)</td>
</tr>
<tr>
<td>$\rho(ca, y)$</td>
<td>0.17</td>
<td>1.00</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho(ca_t, ca_{t-1})$</td>
<td>0.71</td>
<td>0.79</td>
<td>0.78</td>
<td>0.77</td>
<td>0.73</td>
<td>0.71</td>
<td>0.70</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>1.35</td>
<td>0.75</td>
<td>0.64</td>
<td>0.64</td>
<td>0.68</td>
<td>0.76</td>
<td>0.82</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.98</td>
<td>0.24</td>
<td>0.33</td>
<td>0.30</td>
<td>0.23</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>($\lambda = 0.1$)</td>
<td>($\lambda = 0.1$)</td>
<td>($\lambda = 0.1$)</td>
<td>($\lambda = 0.1$)</td>
</tr>
<tr>
<td>$\rho(ca, y)$</td>
<td>0.17</td>
<td>1.00</td>
<td>0.94</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
<td>0.94</td>
</tr>
<tr>
<td>$\rho(ca_t, ca_{t-1})$</td>
<td>0.71</td>
<td>0.79</td>
<td>0.78</td>
<td>0.77</td>
<td>0.74</td>
<td>0.74</td>
<td>0.76</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>1.35</td>
<td>0.75</td>
<td>0.64</td>
<td>0.64</td>
<td>0.68</td>
<td>0.75</td>
<td>0.79</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.98</td>
<td>0.24</td>
<td>0.33</td>
<td>0.30</td>
<td>0.22</td>
<td>0.16</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Table 11: Implications of Different Models (Emerging Countries, $p = 0.01$)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE (θ = 0.95)</th>
<th>RB (θ = 0.9)</th>
<th>RB+RI (θ = 0.85)</th>
<th>RB+RI (θ = 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(ca,y)$</td>
<td>0.13</td>
<td>0.09</td>
<td>0.13</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>$\rho(ca_t,ca_{t-1})$</td>
<td>0.53</td>
<td>0.52</td>
<td>0.46</td>
<td>0.42</td>
<td>0.37</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>0.80</td>
<td>0.65</td>
<td>0.69</td>
<td>0.74</td>
<td>0.82</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>1.35</td>
<td>2.09</td>
<td>2.22</td>
<td>2.44</td>
<td>2.84</td>
</tr>
</tbody>
</table>

(λ = 0.5)

<table>
<thead>
<tr>
<th></th>
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<th>RE (θ = 0.95)</th>
<th>RB (θ = 0.9)</th>
<th>RB+RI (θ = 0.85)</th>
<th>RB+RI (θ = 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(ca,y)$</td>
<td>0.13</td>
<td>0.07</td>
<td>0.10</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho(ca_t,ca_{t-1})$</td>
<td>0.53</td>
<td>0.58</td>
<td>0.55</td>
<td>0.52</td>
<td>0.48</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>0.80</td>
<td>0.63</td>
<td>0.65</td>
<td>0.67</td>
<td>0.73</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>1.35</td>
<td>2.03</td>
<td>2.08</td>
<td>2.19</td>
<td>2.42</td>
</tr>
</tbody>
</table>

(λ = 0.1)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE (θ = 0.95)</th>
<th>RB (θ = 0.9)</th>
<th>RB+RI (θ = 0.85)</th>
<th>RB+RI (θ = 0.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(ca,y)$</td>
<td>0.13</td>
<td>0.05</td>
<td>0.08</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>$\rho(ca_t,ca_{t-1})$</td>
<td>0.53</td>
<td>0.65</td>
<td>0.64</td>
<td>0.63</td>
<td>0.61</td>
</tr>
<tr>
<td>$\sigma(ca)/\sigma(y)$</td>
<td>0.80</td>
<td>0.62</td>
<td>0.63</td>
<td>0.65</td>
<td>0.69</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>1.35</td>
<td>2.00</td>
<td>2.03</td>
<td>2.10</td>
<td>2.27</td>
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</table>
Table 12: Implications of Different Models (Developed Countries, \( p = 0.01 \))

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>RB+RI ((\theta = 0.9))</th>
<th>RB+RI ((\theta = 0.6))</th>
<th>RB+RI ((\theta = 0.3))</th>
<th>RB+RI ((\theta = 0.2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(ca, y) )</td>
<td>0.17</td>
<td>1.00</td>
<td>0.90</td>
<td>0.85</td>
<td>0.79</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td>( \rho(ca_t, ca_{t-1}) )</td>
<td>0.71</td>
<td>0.79</td>
<td>0.77</td>
<td>0.66</td>
<td>0.56</td>
<td>0.49</td>
<td>0.43</td>
</tr>
<tr>
<td>( \sigma(ca)/\sigma(y) )</td>
<td>1.35</td>
<td>0.75</td>
<td>0.54</td>
<td>0.55</td>
<td>0.62</td>
<td>0.78</td>
<td>1.11</td>
</tr>
<tr>
<td>( \sigma(\Delta c)/\sigma(\Delta y) )</td>
<td>0.98</td>
<td>0.24</td>
<td>0.43</td>
<td>0.41</td>
<td>0.35</td>
<td>0.32</td>
<td>0.49</td>
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</table>

\((\lambda = 1)\)

<table>
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<tr>
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<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>RB+RI ((\theta = 0.9))</th>
<th>RB+RI ((\theta = 0.6))</th>
<th>RB+RI ((\theta = 0.3))</th>
<th>RB+RI ((\theta = 0.2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(ca, y) )</td>
<td>0.17</td>
<td>1.00</td>
<td>0.90</td>
<td>0.87</td>
<td>0.84</td>
<td>0.83</td>
<td>0.84</td>
</tr>
<tr>
<td>( \rho(ca_t, ca_{t-1}) )</td>
<td>0.71</td>
<td>0.79</td>
<td>0.77</td>
<td>0.69</td>
<td>0.62</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>( \sigma(ca)/\sigma(y) )</td>
<td>1.35</td>
<td>0.75</td>
<td>0.54</td>
<td>0.55</td>
<td>0.60</td>
<td>0.70</td>
<td>0.89</td>
</tr>
<tr>
<td>( \sigma(\Delta c)/\sigma(\Delta y) )</td>
<td>0.98</td>
<td>0.24</td>
<td>0.43</td>
<td>0.40</td>
<td>0.31</td>
<td>0.25</td>
<td>0.32</td>
</tr>
</tbody>
</table>

\((\lambda = 0.5)\)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RE</th>
<th>RB</th>
<th>RB+RI ((\theta = 0.9))</th>
<th>RB+RI ((\theta = 0.6))</th>
<th>RB+RI ((\theta = 0.3))</th>
<th>RB+RI ((\theta = 0.2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(ca, y) )</td>
<td>0.17</td>
<td>1.00</td>
<td>0.90</td>
<td>0.89</td>
<td>0.86</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>( \rho(ca_t, ca_{t-1}) )</td>
<td>0.71</td>
<td>0.79</td>
<td>0.77</td>
<td>0.72</td>
<td>0.66</td>
<td>0.70</td>
<td>0.75</td>
</tr>
<tr>
<td>( \sigma(ca)/\sigma(y) )</td>
<td>1.35</td>
<td>0.75</td>
<td>0.54</td>
<td>0.54</td>
<td>0.59</td>
<td>0.67</td>
<td>0.79</td>
</tr>
<tr>
<td>( \sigma(\Delta c)/\sigma(\Delta y) )</td>
<td>0.98</td>
<td>0.24</td>
<td>0.43</td>
<td>0.39</td>
<td>0.29</td>
<td>0.22</td>
<td>0.24</td>
</tr>
</tbody>
</table>

\((\lambda = 0.1)\)