

Optimal Policy for Macro-Financial Stability*

Gianluca Benigno
London School of Economics

Huigang Chen
JD Power

Christopher Otrok
University of Virginia

Alessandro Rebucci
Inter-American Development Bank

Eric R. Young
University of Virginia

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Abstract We study optimal policy in an small open economy in which a foreign borrowing constraint binds only occasionally and a financial crisis is an endogenous event. In this environment, the scope for policy arises because of a pecuniary externality stemming from the presence of a key relative price in the borrowing constraint. We study how optimal policy should be set. We first examine the merit of alternative policy tools. In the endowment case, we compare exchange rate policy and controls on capital flows and find that exchange rate policy dominates capital controls since it replicates the unconstrained first best allocation while capital controls achieve only the constrained efficient one. In this special endowment case, optimal policy (in the Ramsey sense) is time consistent and replicates the social planner allocation. We then examine the case of a production economy and compute the time-consistent optimal policy for alternative instruments confirming that, even in this more realistic case, exchange rate policy dominates capital controls but with the important difference that, when used individually, neither of these policies can replicate the constrained efficient outcome. For this reason, we examine the optimal use of the two policy instrument. A methodological contribution of the paper is the development of computational algorithms to solve optimal policy problems in environments with constraints that bind only occasionally.

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1 Introduction

The global financial crisis and ensuing great recession of 2007-2009 have ignited a debate on the role of policy for the stability of the financial system and hence the economy as a whole (i.e., macro-financial stability). In advanced economies, this debate is revolving around the role of monetary and regulatory policies in causing the global crisis and how the conduct of monetary policy and supervision of financial intermediaries should be altered in the future. In emerging markets, financial imperfections have long been recognized as an important source of business cycle fluctuations and crises informing the debate on the relative merit of alternative macroeconomic policy regimes. But the resurgence of very strong inflows of foreign capital from advanced economies after the global crisis has refocused the ongoing discussion on policies that are desirable from a macro-financial stability perspective.

The key question in this broad debate in advanced and emerging economies alike is whether or how policy should act before a financial crisis strikes.¹ In this paper we address this general question in a Ramsey optimal policy framework and find that it is desirable to intervene in a precautionary manner before a crisis strikes; how to intervene, however, depends on the characteristics of the economy and the instruments that the policy maker has available.

We address the general question above from the perspective of the economy as a whole (i.e., the financial stability of the whole economy as opposed to specific sectors, or individual intermediaries). A growing, related literature addresses the same issue with specific reference to the financial sector or the regulation of individual financial intermediaries with a system-wide stability objective in mind. That is we ask how much of a precautionary component should there be in the optimal stabilization policy of the economy as a whole in normal times? At what point before a possible financial crisis should the government intervene with its instruments (e.g., monetary, fiscal, quasi fiscal, regulatory, etc.)? Should the government wait until the crisis strikes, or should it intervene as the probability of the crisis rises? In addressing these questions, we solve for optimal policy both in and away from the crisis period.

These questions are meaningful only in a model in which there are both crisis and non-crisis states and in which a crisis event is an endogenous outcome. We model financial crises as a situation in which a financial friction (e.g., an international borrowing constraint in our model) becomes binding. The constraint in our model binds endogenously, depending

¹This is different than asking what is the optimal policy responses in models in which the economy is in a financial crisis (or a sudden stop of financial flows). On the latter, see for instance Braggion, Christiano, Roldos (2007), Caballero and Panageas (2007), Christiano, Gust, and Roldos (2004), Cúrdia (2007), Caballero and Krishnamurthy (2005), or Hevia (2008).

on agents' choices as well as the state of the economy. When the constraint does not bind the model economy exhibits normal business cycle fluctuations. The presence of the borrowing constraint, though, leaves the economy vulnerable to the possibility that a small negative shock pushes it into the binding region, for certain levels of indebtedness. When this happens, the economy enters a crisis state and suffers the economic dislocation typically associated with a financial crisis episode.

Our endogenous borrowing credit constraint is embedded in a standard two-sector (tradable and non-tradable good) small open economy (e.g., Obstfeld and Rogoff, 1996, Chapter 4) in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002, 2010). The asset menu is restricted to a one period risk-free bond paying off the exogenously given foreign interest rate. In addition, we assume that access to foreign financing is constrained to a fraction of households' total income. Foreign borrowing is denominated in units of the tradable good but it is leveraged on income generated at different relative prices (i.e. the relative price of non-tradeable good), a specification of the borrowing constraint that captures "liability dollarization" a key feature of emerging market capital structure (e.g., Krugman 2002).²

Within this framework, a scope for policy arises because of a price externality (or a pecuniary or credit externality) stemming from the presence of a key market price in the occasionally binding financial friction (see also Benigno *et al.* (2010, 2011), Bianchi (2011), Bianchi and Mendoza (2010), Caballero and Lorenzoni (2008), Chang, Cespedes, and Velasco (2011), Jeanne and Korinek (2010), Korinek (2008), or Lorenzoni (2008)) who analyze the same kind of externality). Individual agents take prices as given and do not internalize the effect of their individual decisions on a key market price that enters the specification of the financial friction—see Arnott, Greenwald, and Stiglitz (1994) for a discussion. Because of this externality, it has been shown that in models like the one we analyze the competitive equilibrium is constrained inefficient. This inefficiency is typically measured and quantified by comparing the competitive equilibrium (CE) of the economy with the amount that a social planner would choose in an economy subject to the same occasionally binding credit constraint (SP).

The policy analysis is then conducted by choosing a policy instrument and determining the policy rule that implements the constrained efficient allocation under commitment (e.g., Bianchi 2011). The social planner approach provides a useful normative benchmark but is not necessarily useful to design actual policies to move the economy toward it. One important limitation of this approach is that the social planner is assumed to be able to commit in advance to a full set of state contingent policies. In this set up, however, there

²The latest wave of crises in emerging Europe is striking evidence of the importance of such feature.

is no interaction between the policy maker and the private sector, thus omitting to consider time-consistency considerations, a fundamental aspect of the typical macroeconomic stabilization problems—e.g., Kydland and Prescott (1977). In contrast with the existing literature, an important contribution of our paper is to allow for this interaction in the analysis of the key policy question in the ongoing policy debate. Specifically, in this paper we develop optimal policies to address the same pecuniary externality analyzed in the literature under discretion. These policies are the optimal responses of a government that is unable to commit to policies in advance and must decide period by period how to set policy. Our optimal policy approach is more closely related to the problem faced by actual governments that are unable to commit in advance to a set of policies.³

To our knowledge, there are no contributions in the literature on the analysis of optimal policy in an environment in which a borrowing constraint both binds occasionally and is endogenous to the decisions in the model. The most closely related work to ours are Adams and Billi (2006a,2006b), who study optimal monetary policy in a closed economy, new Keynesian model in which there is zero lower bound on interest rates. Their zero-bound constraint is fixed and does not evolve endogenously. Bordo and Jeanne (2002) and Devereaux and Poon (2004), Jeanne and Korinek (2011) investigate precautionary components of optimal monetary policy responses to asset prices and sudden stops, respectively, but not in the context of fully-specified DSGE models. We thus address key economic and computational issues related to the design of optimal policy with occasionally binding financial frictions.

To solve for optimal policy in this model we develop a global solution method. That is, we solve for a policy rule across both states of the world, when the constraint binds and when it does not. Such an approach enforces that the rule away from the crisis periods is designed with full knowledge of what the rule will be when the economy enters the sudden stop. This statement holds for both the policymaker and the agents in the economy. This solution method, while computationally costly, is critical for understanding the interaction between precautionary behavior on the part of the private sector with precautionary behavior on the part of the policy maker. The technical challenge in solving such a model is that the constraint binds only occasionally and changes location in the state space of the model depending on the state of the economy.

We employ two tax instruments in our optimal policy exercise. The first is a distortionary tax on non-traded consumption, which can be interpreted as exchange rate policy.

³As Chari and Kehoe (2010) note, discretion is a more realistic assumption. While a social planner will be able to reach an equilibrium with higher welfare than the optimal policy, gains from the optimal policies we solve for are more in line with the actual constraints faced by governments, particularly those in emerging markets.

The second is a tax on debt and can be interpreted as a control on capital flows (or a capital control for brevity). In both cases, tax policy is financed through lump-sum transfers so that the government budget constraint is balanced in each period. Ideally one would like to analyze optimal policy for macro-financial stability alongside a more traditional stabilization objective (i.e., monetary, fiscal, external). However, computational limitations restrict the analysis to the sole macro-financial stability objective (see below on this). This means that, if we were to remove the friction that is the source of financial instability in the economy, optimal policy would be "no action" in all periods and states of the world.

The main results of the analysis are as follows. In an endowment version of our economy (i.e., Bianchi, 2011), if the policy maker has only one instrument, we find analytically that (i) a tax or subsidy on consumption (i.e., exchange rate policy) can achieve the first best allocation, while a tax or subsidy on foreign borrowing (i.e., a capital control) can only achieve a second best allocation; (ii) Ramsey optimal policy under discretion is the same as under commitment and achieves the same allocation selected by a social planner that is not constrained by the behavior of the private sector. In a production version of our economy (i.e., Benigno *et al.* 2011), we find numerically that, (iii) if the Ramsey planner has only one instrument with discretion, the optimal intervention is a prudential tax regardless of the instrument used. However (iv), if the Ramsey planner has both a consumption tax and a tax on debt as instruments (e.g., Jeanne and Korinek, 2011), the optimal intervention is a tax on debt and a subsidy on consumption during financial crises and no intervention in normal times. Finally, we also find numerically that (v) exchange rate policy dominates capital controls if the Ramsey planner has only one instrument with discretion, and neither achieves the second best constrained efficient allocation selected by a social planner in our production economy. The results reported have a number of general policy implications for macro-financial stability.

The rest of the paper is organized as follows. Section 2 presents analytic results from an endowment economy. Section 3 describes the fully specified DSGE model we use. Section 4 discusses the notion of equilibria that we compute and describe the solution methods. Section 5 calibrates the model and evaluate its performance against the data. Section 5 reports the numerical analysis of the production economy we consider, characterizing the optimal policy and discussing its working. Section 7 concludes.

2 Optimal Policies in a Two-Goods Endowment Economy

Before turning to our general model, we first discuss the optimal policy problem for its simpler version, an endowment economy, which can be solved analytically with one alternative policy instrument at the time. There are two reasons to do so. First, we want to understand the relationship between the social planner problem and optimal policies with and without commitment. Second, we use this analysis to gain some intuition for the working of the instruments we choose in the more general optimal policy problem that we solve numerically.

2.1 Tax on Non-Tradable Consumption (Exchange Rate Policy)

Consider the two-goods endowment economy studied by Korinek (2009) and Bianchi (2010). In the first case, we study the taxation on non-tradable consumption (this policy tool can be interpreted in terms of managing the exchange rate).

2.1.1 Competitive Equilibrium

The competitive equilibrium of this economy is characterized as follows. There is a continuum of households $j \in [0, 1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1-\rho} (C_{j,t})^{1-\rho} \right\}, \quad (1)$$

with C_j denoting the individual consumption basket. The consumption basket, C_t , is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \quad (2)$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of the two goods in the utility function.

We normalize the price of traded goods to 1. The relative price of the nontraded good is denoted P^N . The aggregate price index is then given by

$$P_t = \left[\omega + (1-\omega) (P_t^N)^{1-\kappa} \right]^{\frac{1}{1-\kappa}},$$

where we note that there is a one-to-one link between the aggregate price index P and the relative price P^N . Households maximize utility subject to their budget constraint, which is expressed in units of tradeable consumption. The constraint each household faces is

$$C_t^T + P_t^N(1 + \tau_t^N)C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + i) B_t, \quad (3)$$

where $\tau_t > 0$ (< 0) is a tax (or a subsidy) on non-tradeable consumption. Each household has stochastic endowment streams in tradeable and non-tradeable outputs, $\{Y_t^T\}$ and $\{Y_t^N\}$. For simplicity, we assume that both $\{Y_t^T\}$ and $\{Y_t^N\}$ are Markov processes with finite, strictly positive support. Therefore the current state of the economy can be completely characterized by the triplet (B_t, Y_t^T, Y_t^N) .

International financial markets are incomplete and access to them is also imperfect. The asset menu includes only a one-period bond denominated in units of tradeable consumption. The amount that each individual can borrow internationally is limited by a fraction of his current total income

$$B_{t+1} \geq -\frac{1 - \phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (4)$$

We also assume that there is a lower bound of natural debt limit, \underline{B} , such that $B_t \geq \underline{B}$, for all t .⁴ In order to assure that the constraint (4) plays a non-trivial role, we assume that no policy, given current state $\{B_t = \underline{B}, Y_t^T, Y_t^N\}$, permits the competitive equilibrium allocation to violate the borrowing constraint. The Lagrangian of this problem is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{1-\rho} C_{j,t}^{1-\rho} + \lambda_t \left(B_{t+1} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right) \\ \mu_t (Y_t^T + P_t^N Y_t^N - B_{t+1} + (1 + i) B_t - C_t^T - P_t^N (1 + \tau_t^N) C_t^N) \end{array} \right]$$

Households maximize (1) subject to (3) and (4) by choosing C_t^N, C_t^T, B_{t+1} . The first order conditions of this problem are

$$C_T : u'(C_t) C_{C^T} = \mu_t, \quad (5)$$

$$C_N : u'(C_t) C_{C^N} = \mu_t P_t^N (1 + \tau_t^N), \quad (6)$$

$$B_{t+1} : \mu_t = \lambda_t + \beta (1 + i) E_t [\mu_{t+1}] \quad (7)$$

The competitive equilibrium allocation is defined these conditions and the goods market

⁴Note here that, by imposing the lower limit on borrowing \underline{B} , the competitive equilibrium allocation without government tax instrument and international borrowing constraint (4) (first best unconstrained allocation) has a steady state distribution with finite support, given our assumptions. In addition, tradeable and non-tradeable consumption in steady state have a lower bound that is strictly greater than 0, and thus the non-tradeable price has finite support with a strictly positive lower bound.

equilibrium conditions. We can then combine (5) and (6) to obtain the intratemporal allocation of consumption:

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N (1 + \tau_t^N). \quad (8)$$

2.1.2 Social Planner Equilibrium

We now consider the social planner problem. We focus on a problem in which the planner faces the same constraint faced by private agents. We note here that it is possible to show that, in the one sector (endowment or production) economy with the same international borrowing constraints, the first welfare theorem would hold (see also Kehoe and Levine (1993)). In contrast, in the two-goods economy we consider, private decisions fail to internalize their effect on equilibrium prices, and, in turn, these prices affect the financial constraints.

To define the social planner problem we need to specify the way prices are determined in that allocation. There are two alternatives. One possibility is to consider what Kehoe and Levine (1993) refer to as "conditionally-efficient problem" in which market prices are determined by the competitive equilibrium *function* — in our case $P_t^N = f^{CE}(B_t, Y_t^N, Y_t^T)$. The other alternative is to consider what Kehoe and Levine would refer to as "general constrained-efficient" problem where market prices are determined by the competitive equilibrium *rule* — in our case, that would be equation (35).

We follow the second characterization of the social planner problem so that the planner maximizes (1) subject to the resource constraints, the international borrowing constraint from an aggregate perspective, and the pricing rules as in (35). Combining the household budget constraint with the government budget constraint ($T_t = \tau C_t^N$) and the equilibrium condition in the nontradables good market, we obtain the current account equation of our small open economy:

$$C_t^T = Y_t^T - B_{t+1} + (1 + i) B_t. \quad (9)$$

The nontradable goods market equilibrium condition implies that

$$C_t^N = Y_t^N.$$

The international borrowing constraint can be expressed, from the perspective of the planner as

$$B_{t+1} \geq -\frac{1 - \phi}{\phi} [Y_t^T + P_t^N Y_t^N], \quad (10)$$

where the relative price is determined by the competitive rule (35). This condition allows us to rewrite (4) as

$$B_{t+1} \geq -\frac{1-\phi}{\phi} \left[Y_t^T + \left(\frac{(1-\omega)(C_t^T)}{\omega Y_t^N} \right)^{\frac{1}{\kappa}} (1+\tau_t^N) Y_t^N \right].$$

The Lagrangian of the planner problem becomes

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\rho} (C_{j,t})^{1-\rho} + \mu_{1,t} (Y_t^T - B_{t+1} + (1+i) B_t - C_t^T) + \mu_{2,t} (Y_t^N - C_t^N) + \lambda_t \left(B_{t+1} + \frac{1-\phi}{\phi} \left[Y_t^T + \left(\frac{(1-\omega)(C_t^T)}{\omega Y_t^N} \right)^{\frac{1}{\kappa}} (1+\tau_t^N) Y_t^N \right] \right) \right],$$

in which the planner chooses the optimal path of C_t^T, C_t^N, B_{t+1} . The first order conditions for the planner problem are given by

$$C_T : u'(C_t) C_{C^T} = \mu_{1,t} - \lambda_t \Sigma_t, \quad (11)$$

$$C_N : u'(C_t) C_{C^N} = \mu_{2,t}, \quad (12)$$

$$B_{t+1} : \mu_{1,t} = \lambda_t + \beta (1+i) E_t [\mu_{1,t+1}]. \quad (13)$$

where

$$\Sigma_t \equiv \frac{1-\phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left(\frac{(1-\omega)(C_t^T)}{\omega} \right)^{\frac{1}{\kappa}-1} (1+\tau_t^N) (Y_t^N)^{\frac{\kappa-1}{\kappa}}.$$

Note here that Σ_t contains the instrument $(1+\tau_t^N)$.

We now discuss the extent to which it is possible to use $(1+\tau_t^N)$ in the competitive equilibrium allocation to replicate the social planner equilibrium. The planner Euler equation can be rewritten as

$$u'(C_t) C_{C^T} + \lambda_t \Sigma_t = \lambda_t + \beta (1+i) E_t [u'(C_{t+1}) C_{C^T} + \lambda_{t+1} \Sigma_{t+1}]$$

with

$$\lambda_t \left[B_{t+1} + \frac{1-\phi}{\phi} \left[Y_t^T + \left(\frac{(1-\omega)(C_t^T)}{\omega Y_t^N} \right)^{\frac{1}{\kappa}} (1+\tau_t^N) Y_t^N \right] \right] = 0.$$

For the competitive equilibrium we have

$$u'(C_t)C_{C^T} = \lambda_t + \beta(1+i)E_t[u'(C_{t+1})C_{C^T}]$$

with

$$\lambda_t \left[B_{t+1} + \frac{1-\phi}{\phi} [Y^T + P_t^N Y^N] \right] = 0$$

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N (1 + \tau_t^N).$$

Note here that our policy instrument (non-tradable consumption tax) does not affect the marginal utility of consumption in the competitive endowment case, so that at first it seems that the only role of policy would be to change the relative prices with no implications for real quantities. In contrast, in the social planner case, the non-tradable tax does affect the way the planner internalize the externality through its effect on the relative prices. Despite this apparent difference, the two-allocation can coincide: tax policy needs to be designed in such a way that the constraint is never binding in both equilibria, i.e. $\lambda_t = 0$ for all t .

Proposition 1. *In an economy defined by (1), (3) and (4) with tax on non-tradable consumption τ^N as the government's policy instrument, there exists policy for τ^N under which the social planner allocation is implemented competitively. Such a tax policy is also a time-consistent optimal policy without commitment. In addition, it achieves the first best unconstrained allocation.*

Proof. For given stochastic processes of $\{Y_t^N, Y_t^T\}$ and a given state B_t , let \hat{B}_{t+1} is the policy function for next-period debt in the economy defined by (1) and (3) but without a credit constraint (4). Define \hat{P}_t^N to be the minimum price such that the credit constraint would be met if it existed,

$$\hat{P}_t^N = \max \left\{ 0, -\frac{\hat{B}_{t+1} + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} Y_t^N} \right\}.$$

For an economy with a credit constraint, one can just set τ^N is such that $\hat{P}_t^N(1 + \tau^N) \leq P_t^N$ so that the credit constraint does not bind. In other words, let $\hat{\tau}^N = P_t^N / \hat{P}_t^N - 1$. Then any $\tau^N \in (-1, \hat{\tau}^N]$ is the tax rate which eliminates the credit constraint. Under such a tax policy, $\lambda_t = 0$ for all t and the competitive equilibrium coincides with the social planner's solution to achieve the first best unconstrained allocation.

In addition, since the competitive equilibrium from the above tax policy achieves the social planner optimum, and by the fact that Ramsey policy can at best achieve

the social planner optimum, we reach the conclusion that the tax policy is also a Ramsey solution. Such policy is completely determined by the current state $\{B_t, Y_t^T, Y_t^N\}$ and therefore it is time-consistent. ■

2.2 Tax on Debt (Capital Controls)

Consider now the same model in which we impose a tax on capital flows. The competitive equilibrium is going to be modified in the following way. Households maximize utility subject to their budget constraint, which is expressed in units of tradeable consumption. The constraint each household faces is

$$C_t^T + P_t^N C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau^B) + (1 + i) B_t, \quad (14)$$

where $\tau_t^B > 0 (< 0)$ is a subsidy (or a tax) on bond purchases and T_t denotes lump sum transfer/tax. As before, the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geq -\frac{1 - \phi}{\phi} [Y_t^T + P_t^N Y_t^N]$$

with $\underline{B} \leq B_t$ for all t . We can write the Lagrangian of this problem as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{l} \frac{1}{1-\rho} C_{j,t}^{1-\rho} + \lambda_t \left(B_{t+1} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right) \\ \mu_t (Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau^B) + (1 + i) B_t - C_t^T - P_t^N C_t^N) \end{array} \right]$$

Households maximize (1) subject to (14) and (4) by choosing C_t^N, C_t^T and B_{t+1} . The first order conditions of this problem are

$$C_T : (C_{j,t})^{-\rho} \omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}} = \mu_t, \quad (15)$$

$$C_N : (C_{j,t})^{-\rho} (1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}} = \mu_t P_t^N, \quad (16)$$

$$B_{t+1} : \mu_t(1 + \tau^B) = \lambda_t + \beta(1 + i) E_t [\mu_{t+1}]. \quad (17)$$

To define the competitive equilibrium allocation we add the goods market equilibrium conditions. From the previous conditions, we can then combine (5) and (6) (without τ^N this time) to obtain the intratemporal allocation of consumption and (5)

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N.$$

2.2.1 Social Planner Equilibrium

We now consider the social planning problem. In this case the planner maximizes (1) subject to the resource constraints, the international borrowing constraint from an aggregate perspective, the government budget constraint ($T_t = \tau B_{t+1}$), and the pricing rule for the competitive equilibrium allocation. Combining the household budget constraint and the government budget constraint with the equilibrium condition in the nontradables good market, we obtain the current account equation of our small open economy:

$$C_t^T = Y_t^T - B_{t+1} + (1 + i) B_t. \quad (18)$$

The nontradable goods market equilibrium condition implies that

$$C^N = Y^N.$$

The international borrowing constraint can be expressed, from the perspective of the planner, as

$$B_{t+1} \geq -\frac{1 - \phi}{\phi} [Y^T + P_t^N Y^N]. \quad (19)$$

where the relative price is determined by the competitive rule (35). This condition allows us to rewrite (4) as

$$B_{t+1} \geq -\frac{1 - \phi}{\phi} \left[Y^T + \left(\frac{(1 - \omega) (C_t^T)}{\omega Y^N} \right)^{\frac{1}{\kappa}} Y^N \right].$$

The Lagrangian of the planner problem becomes

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{aligned} & \frac{1}{1-\rho} (C_{j,t})^{1-\rho} + \mu_{1,t} (Y_t^T - B_{t+1} + (1+i) B_t - C_t^T) + \\ & + \mu_{2,t} (Y_t^N - C_t^N) + \lambda_t \left(B_{t+1} + \frac{1-\phi}{\phi} \left[Y^T + \left(\frac{(1-\omega)(C_t^T)}{\omega Y^N} \right)^{\frac{1}{\kappa}} Y^N \right] \right) \end{aligned} \right]$$

and the planner chooses C_t^T, C_t^N, B_{t+1} . The first order conditions for the planner problem are

$$C_T : (C_{j,t})^{-\rho} \omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}} = \mu_{1,t} - \lambda_t \Sigma_t, \quad (20)$$

$$C_N : (C_{j,t})^{-\rho} (1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}} = \mu_{2,t}, \quad (21)$$

$$B_{t+1} : \mu_{1,t} = \lambda_t + \beta (1 + i) E_t [\mu_{1,t+1}] \quad (22)$$

with

$$\Sigma_t \equiv \frac{1 - \phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1 - \phi}{\phi} \frac{1}{\kappa} \frac{(1 - \omega)}{\omega} \left(\frac{(1 - \omega) (C_t^T)}{\omega} \right)^{\frac{1}{\kappa} - 1} (Y_t^N)^{\frac{\kappa - 1}{\kappa}}.$$

Note here an important difference from the social planner problem with non-tradable consumption tax: in (47) the planner cannot affect the marginal utility of consumption through the tax rate.

We now discuss the extent to which it is possible to use $(1 + \tau_t^B)$ in the competitive equilibrium allocation to replicate the social planner equilibrium. We can rewrite the planner's Euler equation (22) as

$$u'(C_t) C_{C_t^T} + \lambda_t \Sigma_t = \lambda_t + \beta(1 + i) E_t [u'(C_{t+1}) C_{C_{t+1}^T} + \lambda_{t+1} \Sigma_{t+1}]. \quad (23)$$

The Euler equation for the competitive equilibrium (15) is

$$(1 + \tau_t^B) u'(C_t) C_{C_t^T} = \lambda_t + \beta(1 + i) E_t [u'(C_{t+1}) C_{C_{t+1}^T}]. \quad (24)$$

We then have the following proposition.

Proposition 2. *In an economy defined by (1), (3) and (4) with tax on capital flow τ^B as the government policy instrument, there exists policy for τ^B under which the competitive equilibrium implements the social planner allocation. Such tax policy is a time-consistent optimal policy without commitment. However it does not achieve the first best unconstrained allocation.*

Proof. Since the resource constraints and the credit constraint are all identical in the competitive equilibrium and the social planner problem, we are only concerned with the intertemporal Euler equations (23) and (24). In order for the competitive equilibrium to replicate the solution of the social planner problem, the government must set the capital flow tax τ_t^B in the following way:

$$\tau_t^B = \left(u'(C_t^{SP}) C_{C_t^{SP}} \right)^{-1} \left(\lambda_t^{SP} \Sigma_t^{SP} - \beta(1 + i) E_t [\lambda_{t+1}^{SP} \Sigma_{t+1}^{SP}] \right),$$

where the superscript *SP* denotes the values coming from the social planner problem. By this prescription, the competitive equilibrium will match the social planner problem since the Euler equations become identical. Notice here in the case when $\lambda_t = 0$ and $E_t[\lambda_{t+1} \Sigma_{t+1}] > 0$, which means the credit constraint is not currently binding but in the next period it can be binding with a positive probability, τ^B is negative. So the tax policy exhibits precautionary behavior by taxing capital flows.

By the same reasoning from Proposition 1 we conclude that this tax policy is a time consistent optimal policy. Since the policy also comes from the solution of social planner problem, which can be represented by value iteration, such policy is also a no-commitment optimum.

To see the social planner problem does not achieve the first best unconstrained allocation, notice that the FOCs of the social planner problem (20), (21), and (22) would be identical to the FOCs of competitive equilibrium without the international borrowing constraint (4) if the first best unconstrained allocation were achieved ($\lambda_t \equiv 0$). Therefore since $\beta(1+i) < 1$, B_t would eventually converge to the lower limit \underline{B} where the credit constraint (4) would be violated by assumption. ■

2.3 Tax on Tradable Consumption

As exchange rate policy could be mimicked by taxing either the consumption of tradables or non-tradables, we also consider the case in which there is a tax on tradable goods. Households face the following budget constraint

$$(1 + \tau_t^T)C_t^T + P_t^T C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + i)B_t.$$

The first order conditions for the competitive equilibrium become

$$C_T : (C_{j,t})^{-\rho} \omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}} = \mu_t(1 + \tau_t^T),$$

$$C_N : (C_{j,t})^{-\rho} (1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} C_t^{\frac{1}{\kappa}} = \mu_t P_t^N,$$

$$B_{t+1} : \mu_t = \lambda_t + \beta(1+i) E_t [\mu_{t+1}].$$

Given the goods market equilibrium conditions, we can combine (5) and (6) to obtain the intratemporal allocation of consumption:

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = \frac{P_t^N}{(1 + \tau_t^T)}$$

The Euler equation in the competitive equilibrium then becomes

$$\frac{u'(C_t)C_{C_t^T}}{1 + \tau_t^T} = \lambda_t + \beta(1+i)E_t \left[\frac{u'(C_{t+1})C_{C_{t+1}^T}}{1 + \tau_{t+1}^T} \right].$$

In the social planner problem, we have the resource constraints

$$\begin{aligned} C_t^T &= Y_t^T - B_{t+1} + (1+i)B_t, \\ C_t^N &= Y_t^N, \end{aligned}$$

the international borrowing constraint

$$B_{t+1} \geq -\frac{1-\phi}{\phi} \left[Y_t^T + (1+\tau_t^T) \left(\frac{(1-\omega)C_t^T}{\omega C_t^N} \right)^{\frac{1}{\kappa}} Y_t^N \right], \quad (25)$$

and the lower limit on the international borrowing $\underline{B} \leq B_t$ as the previous cases. The Euler equation of social planner problem can thus be expressed as

$$u'(C_t)C_{C^T} + \lambda_t \Sigma_t = \lambda_t + \beta(1+i) E_t [[u'(C_{t+1})C_{C^T} + \lambda_{t+1} \Sigma_{t+1}]]$$

where

$$\Sigma_t \equiv \frac{1-\phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left(\frac{(1-\omega)(C_t^T)}{\omega} \right)^{\frac{1}{\kappa}-1} (1+\tau_t^T) (Y_t^N)^{\frac{\kappa-1}{\kappa}}.$$

As in the case of non-tradable consumption taxation, but unlike the case of debt taxation, the tax rate enter the expression for Σ_t . Therefore, as in the case of the tax on non-tradable consumption, we have the following proposition.

Proposition 3. *In an economy defined by (1), (3) and (4) with tax on tradable consumption τ^T as the government policy instrument, there exists policy for τ^T under which competitive equilibrium implements social planner solution. Such tax policy is also a time-consistent optimal policy without commitment. In addition, it achieves the first best unconstrained allocation.*

Proof. Let the optimal non-tradable consumption tax be τ_t^N . It is easy to see that in the social planner problem, if we set $\frac{1}{1+\tau_t^T} = 1 + \tau_t^N$, we achieve the first best unconstrained allocation and $\lambda_t^{SP} \equiv 0$. Here the superscript *SP* denotes the social planner solution.

We will show there exists a constant tax policy that replicates the social planner optimum. Such policy is naturally time-consistent and non-commitment. By comparing Euler equations in both social planner problem and competitive equilibrium, and using $\lambda_t^{SP} \equiv 0$,

it is sufficient to find τ_t^T so that

$$\frac{1}{1 + \tau_t^T} = \frac{E_t \left[\frac{w'(C_{t+1}^{SP}) C_{t+1}^{SP}}{1 + \tau_{t+1}^T} \right]}{E_t [w'(C_{t+1}^{SP}) C_{t+1}^{SP}]}, \quad (26)$$

and the international borrowing constraint (25) is satisfied, in order for the competitive equilibrium to achieve social planner optimum.

First we note that a constant tax policy will satisfy (26). Secondly, by our observation of the first best unconstrained allocation, non-tradable price has strictly positive lower limit. Therefore there exists $\underline{\tau}^T$ such that the borrowing constraint (25) is always satisfied for any $\tau^T \geq \underline{\tau}^T$. So any constant tax policy of the form $\tau_t^T \equiv \tau^T \geq \underline{\tau}^T$ is an optimal policy such that the competitive equilibrium replicates the social planner optimal. ■

Whether exchange rate policy is enacted by taxing the tradable or the non-tradable sector does not affect the main results of this part of the analysis. In the rest of the paper we shall focus only on tax on debt and tax on non-tradable consumption.

3 A Two-Good, Two-Sector Production Economy

In this section we present the two-good, two-sector production economy in which we will study optimal policy numerically. This is the same simple two-good (tradable and non-tradable) small open economy discussed above, in which financial markets are not only incomplete but also imperfect like in Mendoza (2010), and in which production occurs in both sectors as in the model used by Benigno *et al.* (2011).

3.1 Households

There is a continuum of households $j \in [0, 1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{1}{1 - \rho} \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{1-\rho} \right\}, \quad (27)$$

with C_j denoting the individual consumption basket and H_j the individual supply of labor for the tradeable and non-tradeable sectors ($H_j = H_j^T + H_j^N$). The assumption of perfect substitutability between labor services in the two sectors insures that there is a unique labor market. For simplicity we omit the j subscript for the remainder of this section,

but it is understood that all choices are made at the individual level. The elasticity of labor supply is δ , while ρ is the coefficient of relative risk aversion. In (27), the preference specification follows from Greenwood, Hercowitz and Huffman (1988) (hereafter, GHH). In the context of a one-good economy this specification eliminates the wealth effect from the labor supply choice. Here it is important to emphasize that in a multi-good economy, the sectoral allocation of consumption will affect the labor supply decision through relative prices.

As before, the consumption basket, C_t , is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[\omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \quad (28)$$

The parameter κ is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while ω is the relative weight of tradable goods in the consumption basket. We normalize the price of traded goods to 1. The relative price of the nontradable good is denoted P^N . The aggregate price index is then given by

$$P_t = \left[\omega + (1-\omega) (P_t^N)^{1-\kappa} \right]^{\frac{1}{1-\kappa}},$$

where we note that there is a one to one link between the aggregate price index P and the relative price P^N .

Households maximize utility subject to their budget constraint, which is expressed in units of tradeable consumption. The constraint each household faces is:⁵

$$C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i) B_t, \quad (29)$$

where W_t is the wage in units of tradable goods, B_{t+1} denotes the net foreign asset position at the end of period t with gross real return $1+i$. Households receive profits, π_t , from owning the representative firm. Their labor income is given by $W_t H_t$.

International financial markets are incomplete and access to them is also imperfect. The asset menu includes only a one-period bond denominated in units of tradable consumption. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [\pi_t + W_t H_t]. \quad (30)$$

⁵Note here that, as we want to compute optimal policy for alternative instruments, and also their combined use, the government is not explicitly introduced in the agent's budget constraint to keep the notation manageable.

This constraint captures a balance sheet effect (e.g., Krugman (1999) and Aghion, Bacchetta and Banerjee (2004)) since foreign borrowing is denominated in units of tradables while the income that can be pledged as collateral is generated also in the non-tradable sector. The value of the collateral is endogenous in this model as it depends on the current realization of profits and wage income. We don't derive explicitly the credit constraint as the outcome of an optimal contract between lenders and borrowers. However, we can interpret this constraint as the outcome of an interaction between lenders and borrowers in which the lenders is not willing to permit borrowing beyond a certain limit.⁶ This limit depends on the parameter ϕ that measures the tightness of the borrowing constraint and it depends on current income that could be used as a proxy of future income.⁷

Households maximize (27) subject to (29) and (30) by choosing C_t^N, C_t^T, B_{t+1} , and H_t . The first order conditions of this problem are

$$C_T : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} \omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t, \quad (31)$$

$$C_N : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} (1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t P_t^N, \quad (32)$$

$$B_{t+1} : \mu_t = \lambda_t + \beta (1 + i) E_t [\mu_{t+1}], \quad (33)$$

and

$$H_t : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} (H_{j,t}^{\delta-1}) = \mu_t W_t + \frac{1 - \phi}{\phi} W_t \lambda_t. \quad (34)$$

μ_t is the multiplier on the period budget constraint and λ_t is the multiplier on the international borrowing constraint. When the credit constraint is binding ($\lambda_t > 0$), the Euler equation (33) incorporates an effect that can be interpreted as arising from a country-specific risk premium on external financing. In this framework, even if the constraint is not binding at time t , there is an intertemporal effect coming from the possibility that the constraint might be binding in the future. This effect is embedded in the term $E_t [\mu_{t+1}]$,

⁶As emphasized by Arellano and Mendoza (2003), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, that would be the outcome of the interaction between a risk-averse borrower and a risk-neutral lender in a contracting framework as in Eaton and Gersovitz (1981). It is also consistent with anecdotal evidence on lending criteria and guidelines used in mortgage and consumer financing.

⁷As we discuss in Benigno *et al.* (2009), a constraint expressed in terms of future income which could be the outcome of the interaction between lenders and borrowers in a limited commitment environment would introduce further computational difficulties that we need to avoid for tractability. If future consumption choices affect current borrowing decisions the planning problem would not be time-consistent in the usual set of state variables.

which implies that current consumption of tradeable goods would be lower compared to an economy in which access to foreign borrowing is unconstrained.

From the previous conditions, we can combine (31) and (32) to obtain the intratemporal allocation of consumption and (31) with (34) to obtain the labor supply schedule, respectively:

$$P_t^N = \frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \quad (35)$$

$$(H_{j,t}^{\delta-1}) = \left(\frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}} W_t \left(1 + \frac{1 - \phi \lambda_t}{\phi \mu_t} \right). \quad (36)$$

Note here that

$$\left(\frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}} = (\omega)^{\frac{1}{\kappa-1}} \left(1 + \left(\frac{1 - \omega}{\omega} \right) (P_t^N)^{1-\kappa} \right)^{\frac{1}{\kappa-1}}.$$

If we were in a one good economy model, there would be no effect coming from the marginal utility of consumption on the labor supply choice because of the GHH preference specification. In a two-sector model, however, a decrease in P_N increases $\left(\frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}}$, and the labor supply curve becomes steeper as P_N falls.⁸ Note also that, when the constraint is binding ($\lambda_t > 0$), the marginal utility of supplying one more unit of labor is higher, and this helps to relax the constraint: when $\lambda_t > 0$, the labor supply becomes steeper and agents substitute leisure with labor to increase the value of their collateral for given wages and prices. Given that P_N falls when the constraint is binding, these two effects imply an increase in labor supply for given wages in the constrained region.

Importantly, the labor supply is also affected by the possibility that the constraint may be binding in the future. If in period t the constraint is not binding but it may bind in period $t + 1$, we have

$$\left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{-\rho} (H_{j,t}^{\delta-1}) = \mu_t W_t$$

and

$$\mu_t = \beta (1 + i) E_t [\lambda_{t+1} + \beta (1 + i) E_t [\mu_{t+2}]],$$

so that the marginal benefit of supplying one more unit of labor today is higher, the higher is the probability that the constraint will be binding in the future. This effect will induce agents to supply more labor for any given wage, and again the labor supply curve will be steeper relative to the case in which there is no credit constraint. For given wages then, this effect tend to increase the level of non-tradable production and consumption and

⁸In what follows, we refer to the labor supply curve in a diagram in which labor is on the vertical axis and the wage rate on the horizontal one.

affects tradable consumption depending on the substitutability between tradable and non-tradable goods. When goods are complements, an increase in nontradable consumption is associated with an increase in tradable consumption that reduces the amount agents save in the competitive equilibrium. The opposite would occur if goods were substitute.

3.2 Firms

Firms produce tradable and nontradable goods with a variable labor input and decreasing return to scale technologies

$$\begin{aligned} Y_t^N &= A_t^N H_t^{1-\alpha^N}, \\ Y_t^T &= A_t^T H_t^{1-\alpha^T}, \end{aligned}$$

where A^N and A^T are the productivity levels that are assumed to be random variables in the non-tradable and tradable sectors, respectively. The firm's problem is static and current-period profits (π_t) are

$$\pi_t = A_t^T (H_t^T)^{1-\alpha^T} + P_t^N A_t^N (H_t^N)^{1-\alpha^N} - W_t H_t.$$

The first order conditions for labor demand in the two sectors are

$$W_t = (1 - \alpha^N) P_t^N A_t^N (H_t^N)^{-\alpha^N}, \quad (37)$$

$$W_t = (1 - \alpha^T) A_t^T (H_t^T)^{-\alpha^T}, \quad (38)$$

so that the value of the marginal product of labor equals the wage in units of tradable goods (W_t). By taking the ratio of (37) over (38) we obtain

$$P_t^N = \frac{(1 - \alpha^T) A_t^T (H_t^T)^{-\alpha^T}}{(1 - \alpha^N) A_t^N (H_t^N)^{-\alpha^N}}, \quad (39)$$

from which we note that the relative price of non-tradable goods determines the allocation of labor between the two sectors. For given productivity levels, a fall in P_t^N drives down the marginal product of non-tradable and induces a shift of labor toward the tradable sector.

3.3 Aggregation and equilibrium

3.3.1 Labor Market Equilibrium in a Two-Sector Production Economy

The distinguishing and novel feature of our two-sector production economy is the implication of sector labor allocation for precautionary saving behavior. To analyze our mechanism, we characterize the labor market equilibrium and the sector labor allocation in terms of three equilibrium conditions. We can express the labor supply schedule as

$$(H_t^{\delta-1}) = \left(1 + \left(\frac{1-\omega}{\omega}\right) (P_t^N)^{1-\kappa}\right)^{\frac{1}{\kappa-1}} W_t \left(1 + \frac{1-\phi}{\phi} \frac{\lambda_t}{\mu_t}\right),$$

where W_t is determined by (38), and note that the wage rate falls when tradable labor input increases:

$$(H_t^{\delta-1}) = \left(1 + \left(\frac{1-\omega}{\omega}\right) (P_t^N)^{1-\kappa}\right)^{\frac{1}{\kappa-1}} (1-\alpha^T) A_t^T (H_t^T)^{-\alpha^T} \left(1 + \frac{1-\phi}{\phi} \frac{\lambda_t}{\mu_t}\right). \quad (40)$$

We then combine (39) with (35) to obtain the sector allocation of labor:

$$P_t^N = \frac{(1-\alpha^T) A_t^T (H_t^T)^{-\alpha^T}}{(1-\alpha^N) A_t^N (H_t^N)^{-\alpha^N}} \quad (41)$$

$$P_t^N = \frac{(1-\omega)^{\frac{1}{\kappa}} \left(A_t^N (H_t^N)^{1-\alpha^N}\right)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \quad (42)$$

with $H = H^T + H^N$. The system of equations (40)-(42) determines H_t , P_t^N , H_t^N for given consumption of tradables C_t^T , productivity levels in the two sector (i.e. A_t^N and A_t^T), and the possibility that the constraint is binding, λ_t .⁹ When the constraint is not binding (i.e., $\lambda_t = 0$), (40), (41) and (42) determine the labor market equilibrium along with the relative prices, while changes in equilibrium C_t^T capture the effect of the possibility that the constraint might be binding in the future.¹⁰

The general equilibrium interaction of labor market equilibrium, relative price of non-tradable goods, and precautionary saving is complex in our two-sector production economy. This interaction can generate, in equilibrium, stronger precautionary saving than a one sector production or endowment economy.

⁹In the appendix we determine the sign of the response to total labor supply, the demand of non-tradable and tradable labor and the relative price of non-tradable for a given change in C^T .

¹⁰As we explained above, when $\lambda_t = 0$ agents will save more compared to the unconstrained economy as they take into account the possibility that the constraint might bind in the future.

As in the two-sector endowment economy, lower tradable consumption for precautionary saving reason leads to a decline in the relative price of non-tradable. For given wages, the decline in the relative price of non-tradable will induce changes in labor supply and production decisions that eventually have implications for the saving behavior. While total labor supply always increases, because of the income effect generated by the relative price change, the associated sector reallocation of labor implies a decline in non-tradable labor that, in equilibrium, tends to increase the relative price of non-tradable goods. If goods are complements, as we assume in the model calibration, the ensuing decline in non-tradable consumption might induce agents to save even more compared to the endowment economy, and hence amplify the precautionary saving effect coming from the possibility of a binding borrowing constraint in the future.

The magnification of the precautionary saving effect of a possibly binding borrowing constraint is a property of a two-sector production economy and does not depend on the way the borrowing constraint is specified. In a one-sector production economy with elastic labor supply and standard preferences, the first order condition for labor supply would be equal to $(H_t^{\delta-1}) = U_C(C_t)W_t$ and the labor supply schedule would be affected by consumption choices.¹¹

The mechanism induced by the two-sector production structure is also robust to the way the collateral constraint is specified. If we add land to the model and express the collateral constraint in terms of the land price (as in Jeanne and Korinek (2009) or Bianchi and Mendoza (2010)) the labor supply and intrasectoral reallocation effects would still operate. This mechanism would also survive in the context in which there is a working capital constraint like in Bianchi and Mendoza (2010): as long as the constraint is not binding, the labor market equilibrium conditions would be identical to the one proposed here ((40), (41) and (42) (with $\lambda_t = 0$)).

3.3.2 Goods Market Equilibrium Conditions

To determine the good market equilibrium, combine the household budget constraint and the firm's profits with the equilibrium condition in the nontradable good market to obtain the current account equation of our small open economy:

$$C_t^T = A_t^T H_t^{1-\alpha^T} - B_{t+1} + (1+i) B_t. \quad (43)$$

¹¹With GHH preferences, the same condition would become $(H_t^{\delta-1}) = W_t$ and labor supply would be independent of the consumption choices.

The nontradable good market equilibrium condition implies that

$$C_t^N = Y_t^N = A_t^N (H_t^N)^{1-\alpha^N}. \quad (44)$$

Finally, using the definitions of firm profits and wages, the credit constraint implies that the amount that the country, as a whole, can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N], \quad (45)$$

so that (43) and (45) determines the evolution of the foreign borrowing.

3.4 Social Planner Problem

To understand the working of the pecuniary externality we focus on, we now focus on the social planner problem. The planner maximizes (27) subject to the resource constraints (43) and (44), the international borrowing constraint from an aggregate perspective (45), and the pricing rule of the competitive equilibrium allocation. By constraining the social planner problem to the pricing *rule* of the competitive equilibrium allocation we follow Kehoe and Levine (2003) in the characterization of the constrained efficient outcome. Another possibility would be to use the concept of conditional efficiency in which the planner problem is constrained by the competitive equilibrium pricing *function* in which P_t^N would be a function of state variables as in the competitive equilibrium allocation (i.e. $P_t^N = f(B_t, A_t^N, A_t^T)$). Here in the constrained efficient case we note that the relative price is determined by the competitive rule (35), so that we can rewrite (45) as

$$B_{t+1} \geq -\frac{1-\phi}{\phi} \left[A_t^T (H_t^T)^{1-\alpha^T} + \frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \left(A_t^N (H_t^N)^{1-\alpha^N} \right)^{1-\frac{1}{\kappa}} \right]. \quad (46)$$

In particular, the planner chooses the optimal path of $C_t^T, C_t^N, B_{t+1}, H_t^T$ and H_t^N , and the first order conditions for its problem are

$$C_T : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} \left(\frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}} = \mu_{1,t} +$$

$$-\frac{\lambda_t}{\kappa} \frac{1-\phi}{\phi} \frac{(1-\omega)}{\omega} \left(\frac{(1-\omega)(C_t^T)}{\omega} \right)^{\frac{1-\kappa}{\kappa}} \left(A_t^N (H_t^N)^{1-\alpha^N} \right)^{\frac{\kappa-1}{\kappa}}, \quad (47)$$

$$C_N : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{-\rho} (1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_{2,t}, \quad (48)$$

$$B_{t+1} : \mu_{1,t} = \lambda_t + \beta (1 + i) E_t [\mu_{1,t+1}], \quad (49)$$

and

$$H_t^T : \left(C_t - \frac{H_t^\delta}{\delta} \right)^{-\rho} (H_t^{\delta-1}) = (1 - \alpha^T) \mu_{1,t} A_t^T H_t^{-\alpha^T} + \frac{1 - \phi}{\phi} \lambda_t (1 - \alpha^T) \mu_{1,t} A_t^T H_t^{-\alpha^T}. \quad (50)$$

$$H_t^N : \left(C_t - \frac{H_t^\delta}{\delta} \right)^{-\rho} (H_t^{\delta-1}) = (1 - \alpha^N) \mu_{2,t} A_t (H_t^N)^{-\alpha^N} \quad (51)$$

$$+ \frac{1 - \phi}{\phi} \lambda_t \frac{(1 - \omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \frac{\kappa - 1}{\kappa} (1 - \alpha^N) (A_t^N)^{\frac{\kappa-1}{\kappa}} (H_t^N)^{(1-\alpha^N)\frac{\kappa-1}{\kappa}-1}.$$

where $\mu_{1,t}$ is the Lagrange multiplier on (43), $\mu_{2,t}$ is the Lagrange multiplier on (44) and λ_t is the multiplier on (46).

There are two main differences between the competitive equilibrium first order conditions and those of the planner's problem introduced by the presence of the occasionally binding borrowing constraint. First, equation (47) shows that, in choosing tradable consumption, the planner takes into account the effects that a change in tradable consumption has on the value of the collateral (see also Korinek, 2010 and Bianchi, 2009). This effect is what is usually referred as a "pecuniary externality" in the related literature and it occurs when the constraint is binding (i.e. $\lambda_t > 0$). As we noted above, however, even if the constraint is not binding today, the possibility that it might bind in the future can affect the marginal value of tradable consumption today (i.e. the marginal value of saving). The Euler equation from the planner perspective becomes

$$\mu_{1,t} = \beta (1 + i) E_t [\lambda_{t+1} + \beta (1 + i) E_t [\mu_{1,t+2}]],$$

where $E_t [\mu_{1,t+2}]$ is given by (47) and takes into account the future effect of the pecuniary externality. This crucially implies that, at the same allocation, the marginal social value of saving (the marginal value in the SP allocation), through this effect, will be higher than the private value (in the CE allocation). Thus, the decentralized equilibrium might display overborrowing. This effect of the price externality is common in economies in which the collateral constraint is expressed in terms of a relative price (see Benigno *et al.* (2010)).

A different effect would arise in an economy in which the price externality is modeled

through the presence of an asset price in the credit constraint (e.g., when the value of an asset serves as a collateral rather than income). Because of the forward-looking nature of asset prices, the planner takes also into account the effect of its consumption choices on asset prices through their effects on the stochastic discount factor. This effect might induce a higher increase in tradable consumption in the social planner allocation and go in the opposite direction of the price externality one.

In the production economy that we study, the presence of the occasionally binding borrowing constraint generates an additional mechanism. To see this effect, we can rewrite the first order conditions for the labor allocation in the tradable sector as

$$H_t^T : \left(C_t - \frac{H_t^\delta}{\delta} \right)^{-\rho} (H_t^{\delta-1}) = (1 - \alpha^T) \mu_{1,t} A_t^T H_t^{-\alpha^T} \left(1 + \frac{1 - \phi}{\phi} \frac{\lambda_t}{\mu_{1,t}} \right),$$

and rewrite the nontradable labor supply equation by using (48) and the equilibrium condition in the nontradable good market as

$$H_t^N : \left(C_t - \frac{H_t^\delta}{\delta} \right)^{-\rho} (H_t^{\delta-1}) = (1 - \alpha^N) \mu_{2,t} A_t^N (H_t^N)^{-\alpha^N} \left(1 + \frac{1 - \phi}{\phi} \frac{\lambda_t}{\mu_{2,t}} \frac{(1 - \omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \frac{\kappa - 1}{\kappa} (A_t^N)^{-\frac{1}{\kappa}} (H_t^N)^{-\frac{1}{\kappa}(1 - \alpha^N)} \right).$$

These expressions show that, when the constraint is binding, the social marginal utility of supplying one extra unit of tradable labor is always positive, while the social marginal value of supplying one extra unit of nontradable labor depends on the degree of substitutability between tradable and nontradable goods. When goods are substitutes and the borrowing constraint is binding, the planner always supplies one more unit of non-tradable labor for given marginal product of labor, as that helps in relaxing the constraint. However, when goods are complements, the planner decreases the amount of nontradable labor supplied at the margin.

Note here that there is an effect on labor supply also when the constraint is not binding ($\lambda_t = 0$). To see this, note that the labor market equilibrium is determined by the following three equations. The first is

$$H_t^T : (H_t^{\delta-1}) = (1 - \alpha^T) \left(\frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}} A_t^T (H_t^T)^{-\alpha^T}. \quad (52)$$

We can then rewrite the non tradable labor supply equation by using (48) and the equilib-

rium condition in the non-tradable good market to obtain:

$$H_t^N : (H_t^{\delta-1}) = (1 - \alpha^N) \left(\frac{(1 - \omega)C}{C^N} \right)^{\frac{1}{\kappa}} A_t^N (H_t^N)^{-\alpha^N}. \quad (53)$$

where total labor supply is defined as

$$H = H^T + H^N. \quad (54)$$

The system of equations given by (52), (53) and (54) determines total labor supply and the sectoral allocation of labor for given C^T , A_t^T and A_t^N .

There are two effects in our production economy coming from the possibility that the constraint might bind in the future. The first one is on total labor supply, while the second is on the substitution between tradable and nontradable labor (intratemporal labor reallocation effect). Both effects are induced by the fact that, in the social planner allocation, current marginal utility of tradable consumption is higher compared to the competitive equilibrium allocation. Higher current marginal utility of tradable consumption increases the marginal utility of supplying one unit of labor today. As a result, in the social planner allocation, labor supply is higher compared to the CE even when the constraint is not binding. This effect alone can cause underborrowing in equilibrium.

The second effect depends on the intrasectoral labor allocation. Higher current marginal utility of tradable consumption (i.e. $\mu_{1,t}$) in the SP implies that, for given total labor supply, the planner will shift resources towards the tradable sector. This shift will reduce the production and the consumption of non-tradable goods. When goods are complement this reduction in the consumption of non-tradable consumption will also imply a reduction in tradable consumption, and hence increasing the amount agents save in the SP allocation relative to the CE allocation. The shift of labor towards tradable production then will tend to strengthen overborrowing in the competitive allocation compared to the social planner one.¹² When goods are substitutes, the decline in nontradable consumption leads to an

¹²It is possible to see the effect on total labor supply by combining (51) and (50) when the constraint is not binding to get

$$2 \left(C_t - \frac{H_t^\delta}{\delta} \right)^{-\rho} (H_t^{\delta-1}) = (1 - \alpha^T) \mu_{1,t} A_t^T H_t^{-\alpha^T} \left(1 + \frac{(1 - \alpha^N) A_t^N (H_t^N)^{-\alpha^N} \mu_{2,t}}{(1 - \alpha^T) A_t^T H_t^{-\alpha^T} \mu_{1,t}} \right)$$

and note that when the constraint is not binding

$$\frac{\mu_{2,t}}{\mu_{1,t}} = \left(\frac{(1 - \alpha^N) A_t^N (H_t^N)^{-\alpha^N}}{(1 - \alpha^T) A_t^T H_t^{-\alpha^T}} \right)^{-1}$$

increase in tradable consumption and as such to a decrease in the amount agents save in the SP allocation compared to the CE allocation. Under substitutability sectoral allocation of labor might induce underborrowing in the competitive equilibrium allocation. Note finally that, in equilibrium, sectoral reallocation will have a feedback effect on total labor supply by affecting wages in units of tradable goods.

In contrast to what we discussed for the competitive equilibrium, the specification of the borrowing constraint has implications for the characterization of the social planner allocation. While the production/labor supply choice are independent from the way the constraint is specified (equations (52), (53) and (54) will remain the same), the intertemporal consumption pattern is affected by the way the planner manipulates the stochastic discount factor when the borrowing constraint is specified in terms of asset prices.¹³ Consider the following experiment in which the planner decreases future consumption while increasing current consumption: by doing so, the planner increases the pricing kernel and inflate asset prices. When the incentive of the planner to manipulate the intertemporal consumption pattern dominates, marginal utility of tradable consumption today is lower than in the competitive equilibrium the possibility of underborrowing arises.

In the papers by Bianchi and Mendoza (2010) and Korinek and Jeanne (2010) this effect is not present despite the fact that they consider economies in which the borrowing constraint depend on a key asset price. Bianchi and Mendoza (2010) do not have this effect because to solve for the social planner problem they use the concept of conditional efficiency (i.e. they assume that the asset price is determined by the asset price function that links current asset price to the exogenous and endogenous state variables). By construction then the planner cannot influence the intertemporal path of consumption.¹⁴

4 Solution Methods

In this section we define the equilibria we consider and describe the global solution methods that we use to compute them. We will present results for three different equilibria in

so that

$$\left(C_t - \frac{H_t^\delta}{\delta}\right)^{-\rho} (H_t^{\delta-1}) = (1 - \alpha^T) \mu_{1,t} A_t^T H_t^{-\alpha^T}.$$

¹³The following reasoning is based on characterizing the constrained efficient social planner problem as in Kehoe and Levine (1993) so that the equilibrium condition that determines asset prices in the competitive allocation is taken as a constraint of the social planner problem.

¹⁴Using the concept of conditional efficiency has implications also for the behavior of the economy in the binding region. When the amount of borrowing is constrained, conditional efficiency eliminates the possibility that the planner can manipulate asset prices, forcing the social planner allocation to be closer to the competitive one.

Section 4. The first is the solution to the competitive equilibrium of the model. This is the benchmark where there is no intervention on the part of a government. The second is the social planners equilibrium. The third is the solution to the model with Ramsey optimal policy. We will present optimal policies for different types of taxes. The solution algorithm is the same in each case so here we explain only the solution for the case in which the Ramsey planner has two instruments as an example. The solution of the cases in which either of the alternative tax instruments are used individually are based on the same algorithm.¹⁵ The competitive and social planner solution algorithms are those developed by Benigno *et al.* (2010, 2011) so here we simply summarize them. The algorithm for the solution of the optimal policy problem is a novel contribution of this paper, so we provide extensive detail.

4.1 Competitive Equilibrium and Social Planner Solutions

The competitive equilibrium problem is defined by the first order conditions for the model in Section 2. Following Benigno *et al.* (2010, 2011) the algorithm for the solution of the competitive equilibrium is derived from Baxter (1990) and Coleman (1989), and involves iterating on the functional equations that characterize a recursive competitive equilibrium in the states (B, A^T) . The key step is to transform the complementary slackness conditions on the borrowing constraint into a set of nonlinear equations that can be solved using standard solvers (in particular, a modified Powell's method). The key steps are to replace the Lagrange multiplier, λ_t , with the expression $\max\{\lambda_t^*, 0\}$ ² and to replace the complementary slackness conditions

$$\begin{aligned} \lambda_t &\geq 0 \\ B_{t+1} + \frac{1-\varphi}{\varphi} \left(A_t^T (H_t^T)^{1-\alpha_T} + P_t^N A (H_t^N)^{1-\alpha_N} \right) &\geq 0 \\ \lambda_t \left(B_{t+1} + \frac{1-\varphi}{\varphi} \left(A_t^T (H_t^T)^{1-\alpha_T} + P_t^N A (H_t^N)^{1-\alpha_N} \right) \right) &= 0 \end{aligned}$$

with the single nonlinear equation

$$\max\{-\lambda_t^*, 0\}^2 = B_{t+1} + \frac{1-\varphi}{\varphi} \left(A_t^T (H_t^T)^{1-\alpha_T} + P_t^N A^N (H_t^N)^{1-\alpha_N} \right).$$

¹⁵The code to solve the model is written in Fortran95 and available upon request. The optimal policy problem is particularly sensitive to parameters and initial conditions, and there is no guarantee made that the programs work for parameter settings not considered explicitly here.

We then guess a function $\eta_{t+1} = G_\eta(B_{t+1}, A_{t+1}^T)$ and solve for $\{\lambda_t^*, \eta_t, B_{t+1}, C_t^T, C_t^N, H_t^T, H_t^N, P_t^N\}$ at each value for (B_t, A_t^T) . This solution is used to update the G_η function to convergence. Note that if the constraint binds, $\lambda_t^* > 0$ so that $\max\{-\lambda_t^*, 0\}^2 = 0$.¹⁶

Given the solution for the equilibrium decision rules, we can compute the equilibrium value of lifetime utility by solving the functional equation

$$V(B_t, A_t^T) = \frac{1}{1-\rho} \left(\left(\omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}} - \frac{1}{\delta} (H_t^T + H_t^N)^\delta \right)^{1-\rho} + \beta E [V(B_{t+1}, A_{t+1}^T) | A_t^T];$$

this equation defines a contraction mapping and thus has a unique solution.

As in Benigno *et al.* (2010, 2011) to solve for the social planning equilibrium we set up a standard dynamic programming problem

$$V^{SP}(B_t, A_t^T) = \max_{C^T, C^N, H^T, H^N, B'} \frac{1}{1-\rho} \left(\left(\omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}} - \frac{1}{\delta} (H_t^T + H_t^N)^\delta \right)^{1-\rho} + \beta E [V^{SP}(B_{t+1}, A_{t+1}^T) | A_t^T]$$

subject to the resource constraints, the borrowing constraint, and the marginal condition that determines P^N :

$$\begin{aligned} C_t^T &= (1+r) B_t + A_t^T (H_t^T)^{1-\alpha_T} - B_{t+1} \\ C_t^N &= A_t^N (H_t^N)^{1-\alpha_N} \\ B_{t+1} &\geq -\frac{1-\varphi}{\varphi} \left(A_t^T (H_t^T)^{1-\alpha_T} + P_t^N A_t^N (H_t^N)^{1-\alpha_N} \right) \\ P_t^N &= \left(\frac{1-\omega}{\omega} \right)^{\frac{1}{\kappa}} \left(\frac{C_t^N}{C_t^T} \right)^{-\frac{1}{\kappa}}. \end{aligned}$$

We approximate the function V^{SP} using cubic splines, and solve the maximization using feasible sequential quadratic programming. This functional equation also defines a contraction mapping, so the planning solution is unique.

4.2 Optimal Policy Solution

The optimal policy solution assumes no commitment on the part of the policymaker. As an example we describe the solution for optimal debt and non-tradable consumption taxation.

¹⁶Note also that $\lambda_t = \max\{\lambda_t^*, 0\}^2 \geq 0$, $\max\{-\lambda_t^*, 0\}^2 \geq 0$, and $\max\{\lambda_t^*, 0\}^2 \max\{-\lambda_t^*, 0\}^2 = 0$ so the complementary slackness conditions are satisfied. See Garcia and Zangwill (1981) for details.

Optimal policies for other tax instruments are computed using the same algorithm. Optimal policy for one instrument, say debt taxation, simply sets τ_N to zero in the following equations. An optimal policy is a state contingent tax plan that maximizes the agents welfare. Welfare here is defined by the household's value function evaluated at the equilibrium state. The optimal policy solution is the solution to the functional equation subject to the constraints in the model (i.e. borrowing, production, budget etc.). The optimization problem is given by the following functional equation and constraints:

$$V(B, \epsilon) = \max_{\tau_B, \tau_N} \left\{ \frac{1}{1 - \rho} \left((\omega C_T^\kappa + (1 - \omega) C_N^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T + H_N)^\delta \right)^{1 - \rho} + \beta E [V(B', \epsilon') | \epsilon] \right\}$$

subject to

$$C_T(B, \epsilon) = (1 + R)B + \epsilon H_T(B, \epsilon)^{1 - \alpha_T} - B'(B, \epsilon) \quad (55)$$

$$C_N(B, \epsilon) = A H_N(B, \epsilon)^{1 - \alpha_N} \quad (56)$$

$$(1 + \tau_B) \mu(B, \epsilon) = \beta (1 + R) E [\mu(B', \epsilon'), \epsilon'] + \max \{ \lambda(B, \epsilon), 0 \}^2 \quad (57)$$

$$\mu(B, \epsilon) = \left((\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T(B, \epsilon) + H_N(B, \epsilon))^\delta \right)^{-\rho} \times \quad (58)$$

$$(\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa} - 1} \omega C_T(B, \epsilon)^{\kappa - 1}$$

$$p_N(B, \epsilon) = \frac{1 - \omega}{\omega} \left(\frac{C_N(B, \epsilon)}{C_T(B, \epsilon)} \right)^{\kappa - 1} \frac{1}{1 + \tau_N(B, \epsilon)} \quad (59)$$

$$\left((\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T(B, \epsilon) + H_N(B, \epsilon))^\delta \right)^{-\rho} (H_T(B, \epsilon) + H_N(B, \epsilon))^{\delta - 1} \quad (60)$$

$$= p_N(B, \epsilon) (1 - \alpha_N) A H_N(B, \epsilon)^{-\alpha_N} \left(\mu(B, \epsilon) + \frac{1 - \varphi}{\varphi} \max \{ \lambda(B, \epsilon), 0 \}^2 \right)$$

$$p_N(B, \epsilon) (1 - \alpha_N) A H_N(B, \epsilon)^{-\alpha_N} = \epsilon (1 - \alpha_T) H_T(B, \epsilon)^{-\alpha_T} \quad (61)$$

$$\max \{ -\lambda(B, \epsilon), 0 \}^2 = B'(B, \epsilon) + \frac{1 - \varphi}{\varphi} (\epsilon H_T(B, \epsilon)^{1 - \alpha_T} + p_N(B, \epsilon) A H_N(B, \epsilon)^{1 - \alpha_N}) \quad (62)$$

$$\tau_B(B, \epsilon) B'(B, \epsilon) + \tau_N(B, \epsilon) p_N(B, \epsilon) C_N(B, \epsilon) + T(B, \epsilon) = 0. \quad (63)$$

The constraints imposed are the complete set of equilibrium conditions of the model. Note that this setup defines a game, and not a single-agent optimization problem, because $\mu(B', \epsilon')$ appears in the constraints (it depends on decisions that are made by a future

government). This dependence is clear if we iterate (3) forward:

$$(1 + \tau_{B,t}) \mu_t = \sum_{j=0}^{\infty} \beta^j (1 + i)^j E [\max \{\lambda_{t+j}, 0\}^2 | \epsilon_t];$$

provided that μ_t remains bounded (so that the transversality condition is respected and the limit term disappears from the above expression) and $\beta(1 + i) < 1$ (which by itself does not guarantee μ_t is bounded because $C - H^\delta/\delta$ could go to zero), the marginal utility of tradable consumption today is the discounted expected value of future Lagrange multipliers on the borrowing constraint. Since these multipliers depend on choices made in the future, which are assumed to be outside the control of the current government, they define a game between current and future governments. A formal definition of the equilibria now follows.

Definition 1. A *recursive competitive equilibrium*, given the tax functions $(\tau_N, \tau_B)(B, \epsilon)$, is an equilibrium value function $V(B, \epsilon)$ and equilibrium functions $(C_T, C_N, H_T, H_N, B', p_N, \lambda, \mu, T)(B, \epsilon)$ such that

1. $(C_T, C_N, H_T, H_N, B', p_N, \lambda, \mu, T)(B, \epsilon)$ solve equations (1)-(9);
2. $V(B, \epsilon)$ solves

$$V(B, \epsilon) = \frac{1}{1 - \rho} \left((\omega C_T^\kappa + (1 - \omega) C_N^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T + H_N)^\delta \right)^{1 - \rho} + \beta E [V(B', \epsilon') | \epsilon]$$

given $(C_T, C_N, H_T, H_N, B', p_N, \lambda, \mu, T)(B, \epsilon)$.

Definition 2. A *Markov-perfect policy equilibrium* requires that $(\tau_N, \tau_B)(B, \epsilon)$ satisfies

$$(\tau_N, \tau_B)(B, \epsilon) \in \operatorname{argmax}_{\tau_B} \left\{ \frac{1}{1 - \rho} \left((\omega C_T^\kappa + (1 - \omega) C_N^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T + H_N)^\delta \right)^{1 - \rho} + \beta E [V(B', \epsilon') | \epsilon] \right\}$$

given $(C_T, C_N, H_T, H_N, B', p_N, \lambda, \mu, T, V)(B, \epsilon)$.

Note that this definition does not deal with the value function out of equilibrium, $v(b, B, \epsilon)$, nor any policy functions out of equilibrium. It is therefore implicit in this definition the requirement that $b = B$. This definition also ignores the issue that all equilibrium functions must depend on τ_B and τ_N , because the Markov-perfect equilibrium replaces τ_B and τ_N with functions of the state (this is the "compact" definition from Krusell (2002) or Klein, Krusell, and Ríos-Rull (2009)).

The computational algorithm used is backward iteration. Smooth equilibria of the sort considered by Klein, Krusell, and Ríos-Rull (2009) would not exist here, because the policy functions are not differentiable at the point where the constraint binds exactly (that is, where $\lambda(B, \epsilon) = 0$). As is common in dynamic games, uniqueness may be an issue; the computational method locates the infinite-horizon equilibrium (if it exists) that is the limit of finite-horizon equilibria. Since the operator defined below is not a contraction mapping, nor even guaranteed to be monotone, there are no known conditions under which it converges. No evidence of multiple equilibria have been found at any stage in the iteration, however.

The algorithm consists of three steps. First, a guess for both the value function and next period's multiplier are needed. Second, we solve the optimization problem to find government taxes and corresponding resource allocations induced by the competitive equilibrium given those taxes. Third, we update the value function given the results of the optimization step. The last two steps are iterated until convergence. We now state the algorithm explicitly.

1. Guess $V^0(B, \epsilon)$ and $\mu^0(B, \epsilon)$;
2. Solve the constrained maximization problem for $(\tau_N, \tau_B, C_T, C_N, B', \lambda, \mu, p_N, H_T, H_N, T)$ as functions of (B, ϵ) :

$$(\tau_N, \tau_B)(B, \epsilon) \in \operatorname{argmax}_{\tau_B, \tau_N} \left\{ \frac{1}{1 - \rho} \left((\omega C_T^\kappa + (1 - \omega) C_N^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T + H_N)^\delta \right)^{1 - \rho} + \beta E [V^0(B', \epsilon') | \epsilon] \right\}$$

subject to

$$C_T(B, \epsilon) = (1 + R)B + \epsilon H_T(B, \epsilon)^{1-\alpha_T} - B'(B, \epsilon) \quad (64)$$

$$C_N(B, \epsilon) = AH_N(B, \epsilon)^{1-\alpha_N} \quad (65)$$

$$(1 + \tau_B) \mu^1(B, \epsilon) = \beta(1 + R) E[\mu^0(B'(B, \epsilon), \epsilon') | \epsilon] + \max\{\lambda(B, \epsilon), 0\}^2 \quad (66)$$

$$\mu^1(B, \epsilon) = \left((\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T(B, \epsilon) + H_N(B, \epsilon))^\delta \right)^{-\rho} \times \quad (67)$$

$$\begin{aligned} & (\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa}-1} \omega C_T(B, \epsilon)^{\kappa-1} \\ p_N(B, \epsilon) &= \frac{1 - \omega}{\omega} \left(\frac{C_N(B, \epsilon)}{C_T(B, \epsilon)} \right)^{\kappa-1} \frac{1}{1 + \tau_N(B, \epsilon)} \end{aligned} \quad (68)$$

$$\left((\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T(B, \epsilon) + H_N(B, \epsilon))^\delta \right)^{-\rho} (H_T(B, \epsilon) + H_N(B, \epsilon))^{\delta-1} \quad (69)$$

$$= p_N(B, \epsilon) (1 - \alpha_N) AH_N(B, \epsilon)^{-\alpha_N} \left(\mu^1(B, \epsilon) + \frac{1 - \varphi}{\varphi} \max\{\lambda(B, \epsilon), 0\}^2 \right)$$

$$p_N(B, \epsilon) (1 - \alpha_N) AH_N(B, \epsilon)^{-\alpha_N} = \epsilon (1 - \alpha_T) H_T(B, \epsilon)^{-\alpha_T} \quad (70)$$

$$\max\{-\lambda(B, \epsilon), 0\}^2 = B'(B, \epsilon) + \frac{1 - \varphi}{\varphi} (\epsilon H_T(B, \epsilon)^{1-\alpha_T} + p_N(B, \epsilon) AH_N(B, \epsilon)^{1-\alpha_N}) \quad (71)$$

$$\tau_B(B, \epsilon) B'(B, \epsilon) + \tau_N(B, \epsilon) p_N(B, \epsilon) C_N(B, \epsilon) + T(B, \epsilon) = 0. \quad (72)$$

Note here that, the program also imposes nonnegativity constraints on those variables that require them (consumption, labor supply, prices). The constraint that the argument of the utility function is positive is imposed explicitly to avoid convergence issues:

$$(\omega C_T^\kappa + (1 - \omega) C_N^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T + H_N)^\delta > 0.$$

There is a potential complication: since the constraint set is not generally convex, the value function may not be concave (or even continuous). As a result, the program is solved using a number of methods (a Nelder-Mead search method, a feasible sequential quadratic programming approach FQSP, and a global search method called GLOBAL) designed to ensure we achieve a global maximum; depending on the instruments used, one or more of these approaches may fail. The failure of concavity also implies that there may exist multiple solutions and/or the decisions rules may not be continuous. The feasible sequential quadratic programming approach has proven to be the fastest and converges in all cases considered to date.

3. Update the guesses to

$$V^1(B, \epsilon) = \frac{1}{1 - \rho} \left((\omega C_T(B, \epsilon)^\kappa + (1 - \omega) C_N(B, \epsilon)^\kappa)^{\frac{1}{\kappa}} - \delta^{-1} (H_T(B, \epsilon) + H_N(B, \epsilon))^\delta \right)^{1 - \rho} + \beta E [V^0(B'(B, \epsilon), \epsilon') | \epsilon]$$

and $\mu^1(B, \epsilon)$ and repeat to convergence.

4.3 Welfare

In Section 4 we will rank equilibria under various policies by calculating the welfare gain of that policy. As it is standard, welfare is quantified as the percent of consumption that the representative agent will pay at every date and state to move from one equilibrium to the other. Here this amount is constructed by calculating the welfare gain at each state. The state specific gains are then aggregated using the unconditional probability of being in each state. Technical details of this calculation can be found in Benigno *et al.* (2010, 2011).

5 Parameter Values and Model Evaluation

The model is calibrated at quarterly frequency on Mexican data. There are several reasons to focus on Mexico. First Mexico is a representative emerging market economy whose experience is particularly relevant for the main issue addressed in the paper. Mexico in fact experienced three major episodes of international capital flows reversals since 1980 that are unambiguously regarded as typical examples of sudden stops: the first one leading to the 1982 debt crisis; the second one, the well known "Tequila crisis" in 1994-1995; and the third one in 2008-09 during the global financial crisis that led Mexico to seek (or accept) IMF financial assistance. Second, Mexico is a well functioning and relatively large market-based economy in which production in both the tradable and non-tradable sectors of the economy goes well beyond the extraction of natural resources such as oil or other commodities. Finally, there is a substantial body of previous quantitative work on Mexico, starting from Mendoza (1991), which greatly facilitates the choice of the parameter values of the model. In particular, we choose model parameters following the work of Mendoza (2002, 2010) and Kehoe and Ruhl (2008) to the extent possible, and use available data where necessary to complement or update this previous work. In the rest of this section we discuss the parameter values chosen and the model's ability to fit the data for a typical emerging market economy like Mexico.

5.1 Parameter Values

The specific set of parameter values that we use in our baseline calibration are reported in Table 1. The inverse of the elasticity of intertemporal substitution is set to a standard value of $\rho = 2$, like in Mendoza (2002, 2010). We set then the world interest rate to $i = 0.01587$, which yields an annual real rate of interest of about 6.5 percent like in Mendoza (2002): a value that is between the 5 percent of Kehoe and Ruhl (2008) and the 8.6 percent of Mendoza (2010).

The elasticity of intratemporal substitution in consumption between tradables and non-tradables is an important parameter in the analysis as we discussed in the previous section. There is significant consensus in the literature on its value. We follow Ostry and Reinhart (1992), who estimates a value of $\kappa = 0.760$ for developing countries. This is a conservative assumption compared to the value of 0.5 used by Kehoe and Ruhl (2008) and closer to the one assumed for an advanced, more closed economy like the United States.

Estimates of the wage elasticity of labor supply in Mexico are uncertain at best (Mendoza, 2002 and 2010). We set the value of $\delta = 1.75$, close to the value of 1.84 adopted by Mendoza (2010).¹⁷

The labor share of income, $(1 - \alpha^T)$ and $(1 - \alpha^N)$ is set to 0.66 in both tradable and nontradable sectors: a standard value, close to that used by Mendoza (2002), and consistent with empirical evidence on the aggregate share of labor income in GDP in the household survey of Garcia-Verdu (2005).

The shock to tradable total factor productivity specified as

$$\log(A_t^T) = \rho_A \log(A_{t-1}^T) + \varepsilon_t,$$

where ε_t is an iid $N(0, \sigma_A^2)$ innovation. The parameters of this process are set to $\rho_A = 0.537$ and $\sigma_A = 0.0134$ which are the first autocorrelation and the standard deviation of aggregate total factor productivity reported by Mendoza (2010). Both the average value of A^T and the constant A^N are set to one.

The remaining three model parameters — the share of tradable consumption in the consumption basket (ω), the credit constraint parameter (ϕ), and the discount factor (β) — are set by iterating on a routine that minimizes the sum of squared differences between the moments in the ergodic distribution of the competitive equilibrium of the model and three data targets. The data targets are a C^N/C^T ratio of 1.643, a 35 percent debt-to-GDP

¹⁷The value of this parameter is 2 in Benigno *et al.* (2011). The higher elasticity used here allows us to replicate the higher volatility of consumption growth relative to income growth common in emerging market economies.

ratio, and an unconditional probability of a sudden stop equal to 2 percent per quarter. This C^N/C^T ratio is the value implied by the following ratios estimated by Mendoza (2002): $Y^T/Y^N = 0.648$, $C^T/Y^T = 0.665$, and $C^N/Y^N = 0.708$ as in Mendoza (2002).¹⁸ The debt-to-GDP target is Mexico’s average net foreign asset to annual GDP ratio, from 1970 to 2008, in the updated version of the Lane and Milesi-Ferretti (2006) data set.

The target for the unconditional probability of a sudden stop is more difficult to pin down. Despite a significant body of empirical work on identifying sudden stops in emerging markets to describe the macroeconomic dynamics around these events, there is no consensus in the literature on how to define sudden stops empirically, and hence no accepted measure of the unconditional probability of these events. By focusing on Mexico, we can pin down this target simply and unambiguously, measuring it as the relative frequency, on a quarterly basis, of Mexico’s sudden stops years over the period 1975-2010. We assume that, as generally accepted, 1982, 1995, and 2009 were sudden stop years for Mexico. The resulting 2 percent is very close to the 1.9 percent implied by the empirical analysis of Jeanne and Ranciere (2010) over the period 1975-2003, who use an “absolute” definition of sudden stops as current account reversals larger than 5 percent of GDP. Our number is also similar to the 2.2 percent value implied by Calvo, Izquierdo, and Mejia (2008) for the period 1990-2004, based on a “relative” definition of sudden stops as current account reversals larger than two standard deviations. The 2 percent value, however, is at the low end of the range of values estimated in these studies by pooling data for the whole sample of emerging markets considered.

In order to contrast Mexico data with model outcomes during sudden stop episodes, consistent with both the model and the empirical literature above, we define a sudden stop in the model as an event in which: (a) $\lambda_t > 0$ (i.e. the international borrowing constraint is binding) and (b) $(B_{t+1} - B_t) > 2\sigma(B_{t+1} - B_t)$ (i.e. the current account or changes in the net foreign asset position in a given period exceed two times its standard deviation). The first criterion is a purely model based definition sudden stop. The second criterion allows us to consider only model events in which there are large current account reversals, in line with the aforementioned empirical literature.^{19 20}

¹⁸Ratios computed with updated data are essentially the same. As we evaluate the model’s ability to replicate the 1995 Tequila crisis we use the exact values reported by Mendoza (2002).

¹⁹The definition of sudden stop typically used in the empirical literature focuses on large capital flows reversals because some smaller ones may be due to terms of trade changes or other factors Jeanne and Ranciere (forthcoming), for instance, excludes commodity importers and oil producers, while Calvo *et al.* (DATE) add other criteria to the second one we use above.

²⁰Note that national accounts data typically have a trend, and hence the empirical literature focuses changes in the current account, or the first difference of the capital flows. As our model has no trend growth and the data are in percent deviation from HP filter, we focus on the current account rather than its change. We obtain similar results when we define the sudden stop with respect to changes in the current

With the targets above we obtain $\omega = 0.3526$, $\beta = 0.9717$, and $\phi = 0.415$. The implied value of ω is slightly higher than in Mendoza (2002) and slightly lower than targeted by Kehoe and Ruhl (2008). The implied annual value of β is yield an annual discount factor of 0.8915, only slightly lower than in Kehoe and Ruhl (2008).²¹ The implied value of ϕ is lower than in Mendoza (2002), who however calibrates it to the deterministic steady state of the model, and there are no standard benchmarks for this model parameter in the literature.

5.2 Model Evaluation

The class of models we study is potentially capable of describing well both the cycle and the crisis periods of an emerging market economy like Mexico (Mendoza 2010). However, in our implementation of the model, we shut down a number of shocks used in other work and focus on the mechanisms driving our policy results. With our one shock we clearly cannot match all the moments of the data that this class of models is capable of replicating. Nonetheless, it is useful to see how well our one shock model does do in describing both the business cycle and the dynamics around a typical sudden stop event, as the first exercise is standard and helps to understand the findings in the second one.

To conduct this comparison we use the variable as defined in Table 2. All data variables are reported in percent deviations from the HP-filter trend (over the 1993Q1-2007Q4 period) except the current account, which is reported as a share of GDP. All model variables are reported in percent deviation from ergodic mean except the current account that is reported, as in the data, as a share of GDP. To calculate model moments we simulate the model for 1,000,000 time periods, and retain the final 10,000 simulation periods to calculate moments and identify sudden stop events.

Table 3 reports data and simulated second moments. Despite its simplicity, the model describes the data reasonably well except for the behavior of the tradable GDP that is counterfactual because of the behavior of labor supply when the constraint is binding in our model economy. As we can see, once we normalize all standard deviations relative to GDP in units of tradable goods (as in Bianchi, 2010), the model roughly matches the ranking of the data volatilities consistent with the results in Mendoza (2002), despite the fact that the model has only one shock. In particular, the model generates consumption volatility that is almost as high as GDP volatility and a current account that is less volatile than aggregate GDP or its components. The model however produces higher relative price

account.

²¹This value is not comparable to the one assumed by Mendoza (2002) as he uses an endogenous discount factor specification. In our model, the presence of the borrowing constraint removes the necessity to introduce any device to induce a stationary ergodic distribution of foreign borrowing.

volatility and too low tradable GDP volatility relative to the data (i.e. relative to GDP volatility).²² Like in the data, all model variables are similarly persistent, but less than in the data (especially for the relative price on nontradable goods and tradable GDP). All correlations with GDP except the relative price one are also all roughly consistent with the data. The correlation between CA and GDP is positive contrary to what we observe in the data. This is because, as calibrated to Mexican data, the constraint does not alter consumption smoothing enough in the ergodic distribution of our model to generate such negative correlation.²³ Note in addition that, the correlation between CA and net income (defined as GDP minus investment and government expenditure, and hence closer to our model definition) may be either slightly positive or zero in the average emerging market economy (Luo, Nie, and Young 2010). Indeed, as it is well known (Backus, Kehoe, and Kydland 1994), a model with investment would generate a negative correlation.

Similar strengths and weaknesses emerge by comparing the macroeconomic dynamics around a typical sudden stop event. For this comparison, we focus on the 1995 Tequila crisis, the same episode studied by Kehoe and Ruhl (2008) and Mendoza (2010). Specifically, Figure 1 compares the model and the Mexican data for key variables four quarters before and after 1995Q1, where the model variables are average across the identified sudden stop episodes, four periods before and four periods after our sudden stop definition is initially met.²⁴

As we can see from Figure 1, the model qualitatively reproduces the large declines in expenditure on consumption and output (both expressed in units of tradable goods), and the relative price of tradable during the 1995 Tequila crisis in Mexico. However in the model this relative price decline is less persistent than in the data. Similarly, qualitatively, non tradable output and expenditure on non-tradable consumption measured in units of tradables are described relatively well by the model. The same lack of persistence characterizes all model variables that generally recover much faster than in the data. We note also that consumption expenditure falls much more than output in our model economy since, in the model, tradable output increases in sudden stop. Consistent with the data, tradable GDP also starts to fall sharply before the sudden stops, but it increases during the sudden stop period, counterfactually. As a result, tradable consumption falls much less

²²Note that, using data up to 2007, as we do, the absolute value of consumption volatility in the data is much lower than reported by Mendoza (2002), and hence much closer to GDP volatility.

²³For instance, Bianchi (2010) calibrates to Argentine data and obtains a negative correlation by using the combination of a high shock variance and a low discount factor.

²⁴As it is evident in the capital flow data (not reported), while capital flows into Mexico started to revert in the fourth quarter of 1994, they were initially accommodated by a very large decrease in official reserves that eventually lead to collapse of the fixed exchange rate regime in December 1994. As a result, the current account started to revert only in 1995Q1.

than nontradable consumption, while in the data the opposite occurs.

Quantitatively, however, the model produces a sudden stop dynamics of amplitude roughly one-order of magnitude smaller than in the data. This occurs for two reasons. First, as we noted above, the model is too simple to provide an accurate quantitative account of the data: in particular we limit ourselves to only one shock in tradable productivity while other shocks (for example foreign interest rate shocks) might have contributed in amplifying the dynamic of the economy during sudden stop. Second, and more importantly, the model counterfactually predicts an increase in total employment at the sudden stop, driven by a sharp increase in labor supply and fall in the real wage (not reported).

As Kehoe and Ruhl (2008) discuss there are three ways to generate a falling employment in the model: a friction in the labor mobility across sectors, variable capital utilization, and a working capital constraint, but none produces satisfactory account of labor market dynamics during the Tequila crisis in their model. In addition, in our model they pose additional complications. Imperfect labor mobility and variable capital utilization introduce an additional state variable. But, as we noted earlier, the comparison between the competitive and the social planner allocation that is the focus of the paper constrains the number of endogenous state variable that can feature in our model. A working capital constraint could produce falling output, but would complicate the specification of the borrowing constraint. In addition a working capital constraint would generate output falling at the sudden stop, but would not alter the underlying mechanism at work in the region in which we examine inefficient borrowing (i.e. during tranquil times) so that our discussion on the role of macro-prudential policies would be robust to this change. For these reasons, at first pass, we prefer to keep the model simple.

6 Optimal Policies in A Two-Goods, Two-Sectors Production Economy

In this section we study optimal policies numerically. We first consider two optimal policy problems in which the Ramsey planner has one instrument only, either exchange rate policy (i.e., a tax on non-tradable consumption, τ_t^N) or direct controls on capital flows (a tax on debt, τ_t^B) as in Section 2.²⁵ As neither of these two instruments alone achieves constrained efficiency, we also consider the case in which the planner uses both instruments at the same time.

²⁵Results for optimal policy with a tax on tradable consumption are not reported to conserve space but are available from the authors on request. They are consistent with the analysis in Section 2, and confirm that, in our set up, exchange rate policy could be analyzed considering either of these two tax instruments.

6.1 Optimal Policies with One Instrument

Figure 2 reports the decision rules for each of the policy instrument considered, τ_t^N or τ_t^B (where a positive value is a tax and a negative value is a subsidy). Figure 3 reports the decision rules for key endogenous variables under these two alternative optimal policies (OPs), as well as in the competitive equilibrium (CE) and the social planner allocation of the economy (SP) for comparison. Figure 4 reports the ergodic distribution of debt in these four different cases. Tables 4, 5 and 6 report the following: the mean and the standard deviation of debt, the real exchange rate (the relative price of non-tradable goods in the model), consumption, and the tax rates; the unconditional probability of a financial crisis; and the welfare gain compared to the CE equilibrium for all the alternative policy regimes, respectively.

If the Ramsey planner has only one instrument, the optimal intervention is a tax before the constraint binds and a subsidy while the constraint binds strictly, regardless of the instrument used. Thus, the optimal Ramsey policy (OP) is non linear and has a precautionary component regardless of the instrument used. Both type of interventions can be interpreted as "leaning against the wind" kind of policies in normal times, either against the real exchange rate or capital flows, and "bail outs" in crisis times, either in the domestic good market or the international capital market.²⁶ But, unlike the endowment case analyzed in Section 2, in the case of a production economy, neither exchange rate policy nor controls on capital flows can achieve constrained efficiency (i.e., the SP allocation in Figure 3).

Notwithstanding these similarities, there are important differences in the effects and the working of the two optimal policies. Relative to the CE, exchange rate policy allows for a more appreciated but less volatile exchange rate than the control on capital flows (Figure 3 and Table 4), more and more volatile borrowing (Figure 4 and Table 5), and much higher welfare (both overall and during periods of financial crisis as illustrated by Table 6). Welfare is higher because the average level of consumption is higher with exchange rate policy. This increase occurs despite the fact that, in the OP regime with exchange rate policy as an instrument, the probability of financial crises is much higher than in the OP regime with controls on capital flows (albeit smaller than in the CE). Note also that the welfare gains from OP with capital controls are not only one order of magnitude smaller than those with exchange rate policy, but also arise primarily in normal times, while with exchange rate policy they arise mostly in crisis times.

²⁶Note that, both type of interventions in crisis times can potentially be financed by the government intervention (in opposite direction) in normal times.

From a quantitative perspective, the exchange rate intervention rule computed requires a tax on non-tradable consumption of about 5 percent per quarter on average, inducing only a small exchange rate appreciation relative to the CE equilibrium and similar volatility (Table 4). In contrast, the capital control rule computed requires a much smaller tax on debt of about 0.2 percent per quarter on average, but produces a more volatile exchange rate at the same average level of the CE. Given that the average size of the interventions in crisis states are smaller or equal to the size of the intervention in normal times (not reported), and that the economy spends only a very small fraction of the time in crisis states, the cost of intervening in crisis states can be easily financed by the tax proceeds in normal times under both policy regimes.

6.2 Optimal Policy with Two Instruments

As neither of the two policies considered can replicate the constrained efficient outcome, we also examine the optimal mix of the two policy instruments. Figure 5 reports the decision rules for each of the policy instrument considered, τ_t^N and τ_t^B , when used one at a time or together. Figure 3 reports the decision rules for key endogenous variables in all previous cases as well as in the new case in which OP is implemented using both exchange rate policy and controls on capital flows. Table 4, 5 and 6 report the relevant statistics also for this additional case.

If the Ramsey planner has both a consumption tax and a tax on debt as instruments, the optimal intervention is no intervention in normal times and a tax on debt combined with a subsidy on non-tradable consumption during financial crises (Figure 5). Thus, in states of the world in which the constraint is not binding the optimal policy is “no action”. This result means that there is no precautionary motive in the optimal policy related to the presence of the borrowing constraint if there is enough instruments to intervene more effectively when a crisis strikes. Note that, as we illustrated in Section 3, the pecuniary externality that justifies policy intervention, is active when the constraint binds or it is expected to bind in the future. Therefore, if OP is more effective in intervening during a crisis, there is no need to intervene before it strikes because the cost of the tax distortions is larger than the benefit from using them in normal times. Note also that the average size of the intervention, for both instruments, is much larger in the OP with two instruments than the case in which there is only one instrument. One intuition for this result is that as each of this instrument push the economy in a different direction, when both are used, both can be used more aggressively without facing a trade off in their effectiveness on different margins. With two instruments, it is conceivable that the proceeds from taxing debt might

finance part of the subsidy to finance the subsidy.

With two instruments, welfare is much higher than with exchange rate policy or capital controls alone, but yet it does not achieve the level associated with constrained efficiency by the SP allocation (Table 6).²⁷ Debt is also higher (but less volatile) than the case in which there is only exchange rate policy (Table 5). The real exchange rate is more appreciated and even less volatile than the exchange rate policy case.

7 Conclusions

In this paper we study optimal policy for macro-financial stability in an small open economy in which a foreign borrowing constraint binds only occasionally and a financial crisis is an endogenous event. In this environment, scope for policy before and during financial crises arises because of a pecuniary externality stemming from the presence of a key relative price in the borrowing constraint. In an endowment version of our economy (i.e., Bianchi 2011), if the policy maker has only one instrument, we find analytically that (i) a tax or subsidy on consumption (i.e., exchange rate policy) can achieve the first best allocation, while a tax or subsidy on foreign borrowing (i.e., a capital control) can only achieve a second best allocation; (ii) Ramsey optimal policy under discretion is the same as under commitment and achieves the same allocation selected by a social planner that is not constrained by the behavior of the private sector. In a production version of our economy (i.e., Benigno *et al.* 2011), we find numerically that, (iii) if the Ramsey planner has only one instrument, the optimal intervention is a prudential tax regardless of the instrument used. However (iv), if the Ramsey planner has both a consumption tax and a tax on debt as instruments, the optimal intervention is a tax on debt and a subsidy on consumption during financial crises and no intervention in normal times. Finally, we also find numerically that (v) exchange rate policy dominates controls on capital flows if the Ramsey planner has only one instrument, but neither achieves the second best constrained efficient allocation selected by a social planner.

Overall, these results suggest that, if there is only one instrument available to deal with multiple distortions, intervening on the exchange rate may be a superior way to enact optimal policy taking macro-financial stability considerations into account. Note that both policy rules entails a "leaning against the wind" component in normal times and a "bailout" component in crisis times (an exchange rate defense in the case of exchange rate policy and

²⁷There are three distorted margins in this economy and only two instruments. Adding a tax on labor income, which enters the third margin, has little effect and still does not achieve constrained efficiency; the commitment issue thus appears to be critical.

a subsidy element to foreign borrowing with capital controls). The leaning against the wind components in normal times may potentially finance the bailout components in crisis times.

An important methodological contribution of the paper is the development of computational algorithms to solve optimal policy problems in environments with constraints that bind on occasion.

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Appendix A: Labor Market Equilibrium in CE allocation

By taking a total differential of the system of equations (40), (41) and (42) we get that

$$\text{sign} \left(\frac{dH_t}{dC_t^T} \right) = \text{sign} \left(\frac{\alpha^T}{\alpha^N} \frac{1 - \alpha^N}{1 - \alpha^T} - \frac{Y_t^T}{C_t^T} \right).$$

so that, among other things, the response of total hours worked to a change in precautionary savings depends on labor intensities in the two sector and on whether the country is producing more tradable output than what it consumes during the current period. Moreover it is possible to show that

$$\text{sign} \left(\frac{dH^N}{dC^T} \right) = \text{sign} \left((\delta - 1) \frac{h^T}{\alpha^T} + 1 - \varepsilon_{pn} \right) > 0,$$

where $h^T = \frac{H^T}{H}$ and $h^N = \frac{H^N}{H}$ with

$$\varepsilon_{pn} = \frac{\frac{1-\omega}{\omega}(P^N)^{1-k}}{1 + \frac{1-\omega}{\omega}(P^N)^{1-k}} < 1$$

so that unambiguously $dH^N/dC^T > 0$. The response of H^T to a change in precautionary savings can then be found using

$$\frac{dH^T}{H^T} ((\delta - 1)h^T + \alpha^T (1 - \varepsilon_{pn})) = -\frac{dH^N}{H^N} ((\delta - 1)h^N + \varepsilon_{pn}\alpha^N),$$

which implies that H^T and H^N always move in opposite directions after a change in precautionary savings and so that $dH^T/dC^T < 0$. Finally, $dH^N/dC^T > 0$, $dH^T/dC^T < 0$ implies that $dP^N/dC^T > 0$.

Table 1. Model Parameters

Structural parameters	Values
Elasticity of substitution between tradable and non-tradable goods	$\kappa = 0.760$
Intertemporal substitution and risk aversion	$\rho = 2$
Labor supply elasticity	$\delta = 1.75$
Credit constraint parameter	$\phi = 0.415$
Labor share in production	$1 - \alpha^T = 1 - \alpha^N = 0.66$
Relative weight of tradable and non-tradable goods	$\omega = 0.3526$
Discount factor	$\beta = 0.9717$
Exogenous variables	Values
World real interest rate	$i = 0.01587$
Steady state productivity level	$A^N = A^T = 1$
Productivity process	
Persistence	$\rho_{\varepsilon^T} = 0.5370$
Volatility	$\sigma_{\varepsilon^T} = 0.0134$
Average values in the ergodic distribution	
NFA to Annual GDP	$B/Y = -0.35$
Qaurterly GDP	$Y = 0.856$
Qaurterly Tradable GDP	$Y^T = 0.334$
Qaurterly Non-Tradable GDP	$Y^N = 0.522$

Table 2. Variable Definitions

Variables	Model	Data 1/
GDP	$Y=YT+YN$	National Accounts, production accounts, GDP, 2003 prices.
Non-Tradable GDP	$YN=PN*HN^{0.66}$	National Accounts, production accounts, GDP, tertiary sectors, 2003 prices.
Tradable GDP	$YT=EXP(ESPILON)*HT^{0.66}$	National Accounts, production accounts, GDP, secondary sectors, 2003 prices.
Relative Price of Non-Tradable	$PN=((1-\omega)/\omega)(CN/CT)^{(\kappa-1)}$	Consumer price of services relative to consumer price of merchandise, indexes, base 2002Q2.
Consumption Expenditure	$C=CT+PN*CN$	National Accounts, expenditure accounts, Private Consumption, 1993 prices.
Non-Tradable Consumption	$CN=YN$	National Accounts, expenditure accounts, Services plus nationally produced non-durable goods, 1993 prices.
Tradable Consumption	$CT=(1+i)*B(t)+YT-B(t+1)$	National Accounts, expenditure accounts, Imported goods plus nationally produced durable goods, 1993 prices.
Current Account	$CA(t)=(B(t+1)-B(t))/Y$	Balance of payment statistics, Current account balance to GDP.

1/ Data sources:

National accounts are from INEGI, Banco de Información Económica (BIE), <http://dgcnesyp.inegi.org.mx/bdiesi/bdie.html>.

Consumer price indexes are from Banco de Mexico (Consulta; series SP68277 and SP56335, respectively), <http://www.banxico.org.mx/sitioingles/polmoneinflacion/estadisticas/cpi/cpi.htm>.

Current account and GDP in US dollar are from the IDB Latin Macro Watch (LMW), <http://www.iadb.org/Research/LatinMacroWatch/lmw.cfm>.

Table 3. Model Evaluation: Second Moments of the Data and the Competitive Equilibrium (CE)

	St. Dev.	St. Dev. Relative to GDP		First Autocorrelation		Correlation with GDP	
	Data	Data	CE	Data	CE	Data	CE
GDP	2.4%	1.0	1.0	0.8	0.5	1.00	1.00
Non-Tradable GDP	2.2%	0.9	0.7	0.8	0.5	0.97	0.97
Tradable GDP	3.4%	1.4	0.4	0.8	0.3	0.96	0.91
Consumption Expenditure	2.6%	1.1	0.9	0.8	0.5	0.91	0.98
Relative Price of Non-Tradable	2.5%	1.0	2.9	0.9	0.5	0.26	0.85
Current Account (In percent of GDP)	2.1%	0.9	0.5	0.8	0.5	-0.61	0.98

Table 4. Ergodic Mean and Volatility of Selected Variables

(In units of tradable consumption unless noted; standard deviation in parenthesis)

	CE	SP	OP(τ_N)	OP(τ_B)	OP(τ_N, τ_B)
B_{t+1}	-1.19 (0.0038)	-1.33 (0.0117)	-1.23 (0.0078)	-1.19 (0.0020)	-1.21 (0.0041)
P_n	1.16 (0.0219)	1.39 (0.0238)	1.18 (0.0086)	1.16 (0.0196)	1.20 (0.0064)
C	0.840 (0.004)	0.939 (0.009)	0.866 (0.007)	0.841 (0.002)	0.856 (0.003)
τ_N	<i>na</i>	<i>na</i>	-5.3% ()	<i>na</i>	<i>na</i>
τ_B	<i>na</i>	<i>na</i>	<i>na</i>	+0.2% ()	<i>na</i>
τ_B, τ_N	<i>na</i>	<i>na</i>	<i>na</i>	<i>na</i>	-9.0%, +4.1% ()

Table 5. Quarterly crisis probabilities

(In percent, unconditional)

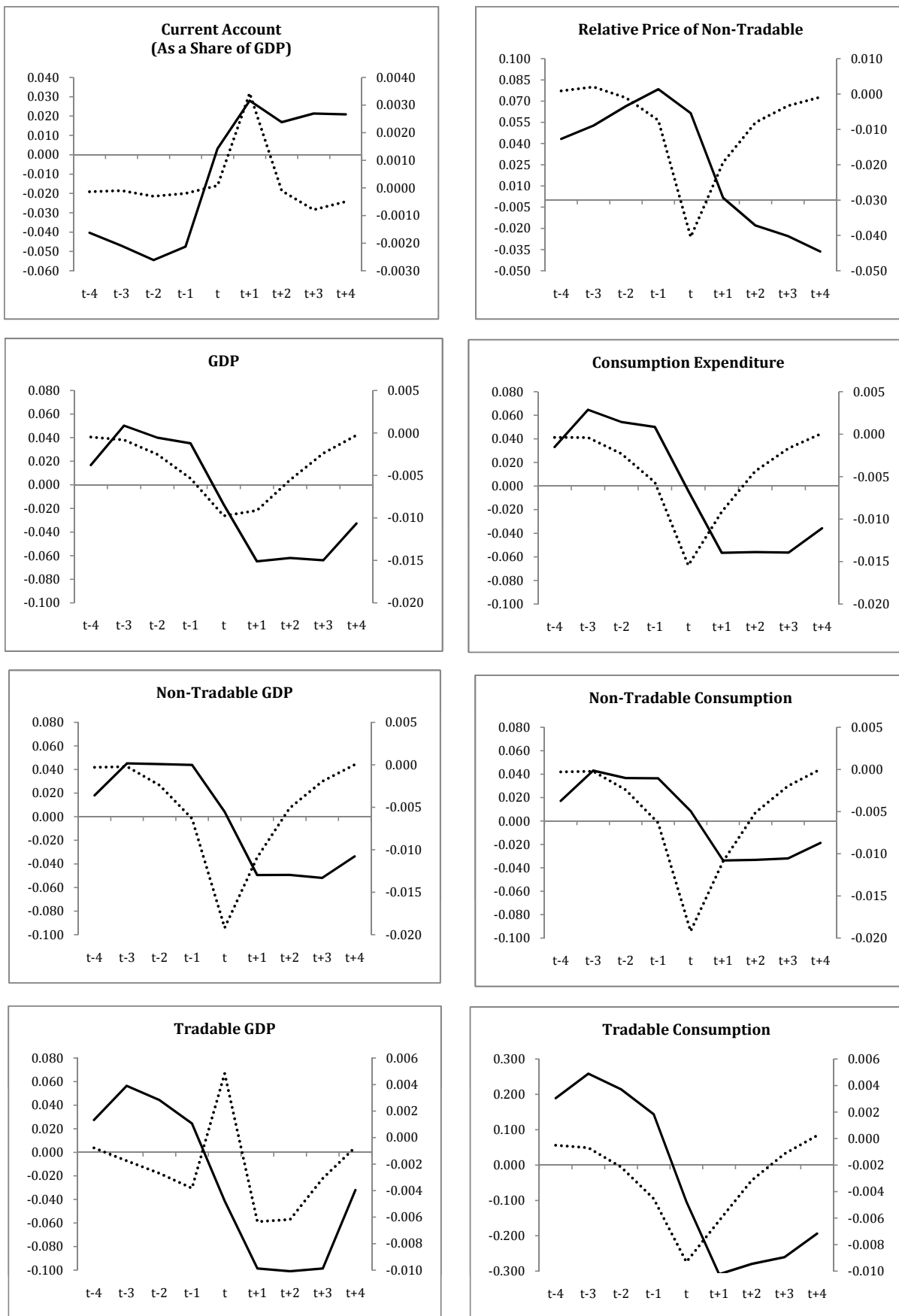
CE	SP	OP(τ_N)	OP(τ_N)	OP(τ_N, B)
2.00	2.19	1.72	0.31	1.41

Table 6. Welfare gain of moving from the CE

(In percent of permanent consumption)

	Overall	At the sudden stop
CE	<i>na</i>	<i>na</i>
SP	0.930%	1.05%
OP(τ_N)	0.080%	0.10%
OP(τ_N)	0.007%	0.01%
OP(τ_N, B)	0.180%	0.22%

Figure 1. Model Evaluation: Sudden Stop Dynamics in the Data and the Competitive Equilibrium 1/



1/ Data (In percent deviation from HP trend unless otherwise noted): Solid line, left axis.
 Competitive Equilibrium (In percent deviation from ergodic mean unless otherwise noted): Dotted line, right axis.
 See Table 2 for variable definitions and data sources.

Figure 1: Optimal Policy with One Instrument

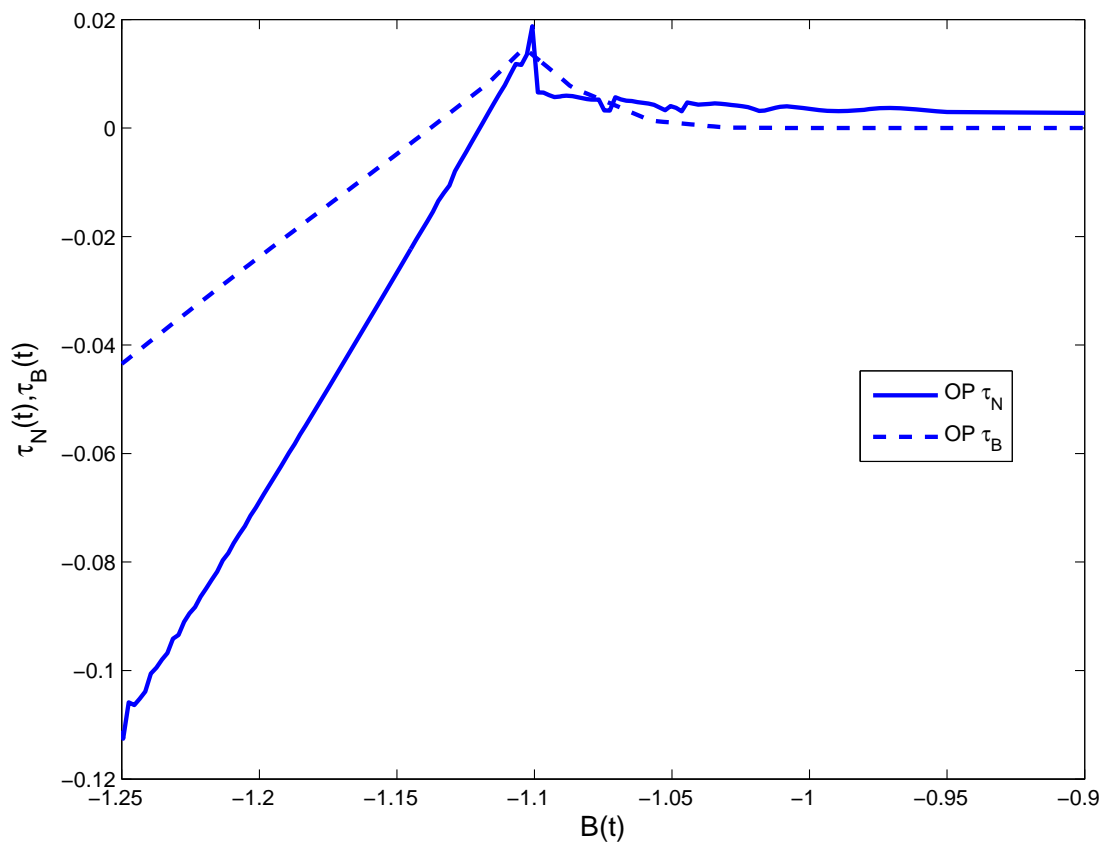


Figure 2: Decision Rules for Key Variables
 (Benchmark Allocations and Optimal Policy with One Instrument)

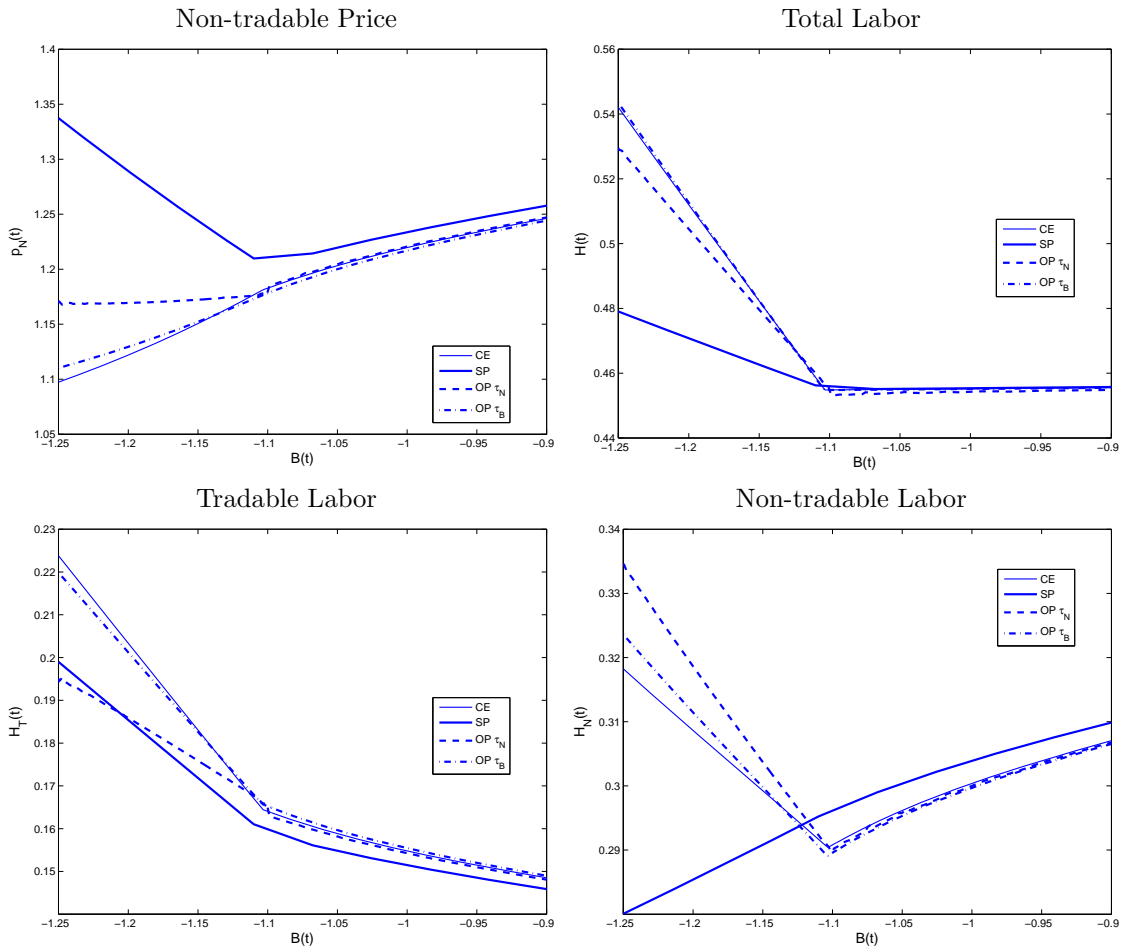


Figure 3: Ergodic Distribution of Debt
(Benchmark Allocations and Optimal Policy with One Instrument)

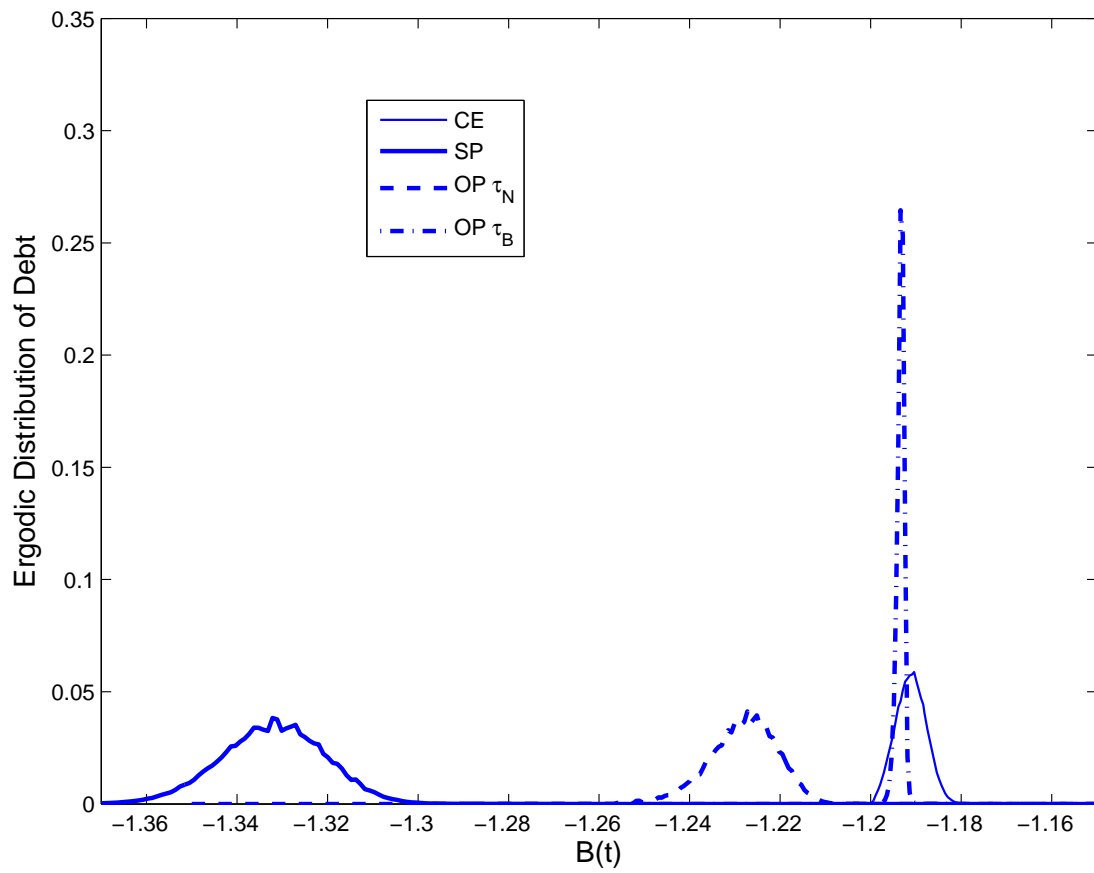


Figure 4: Optimal Policy with Two Instruments

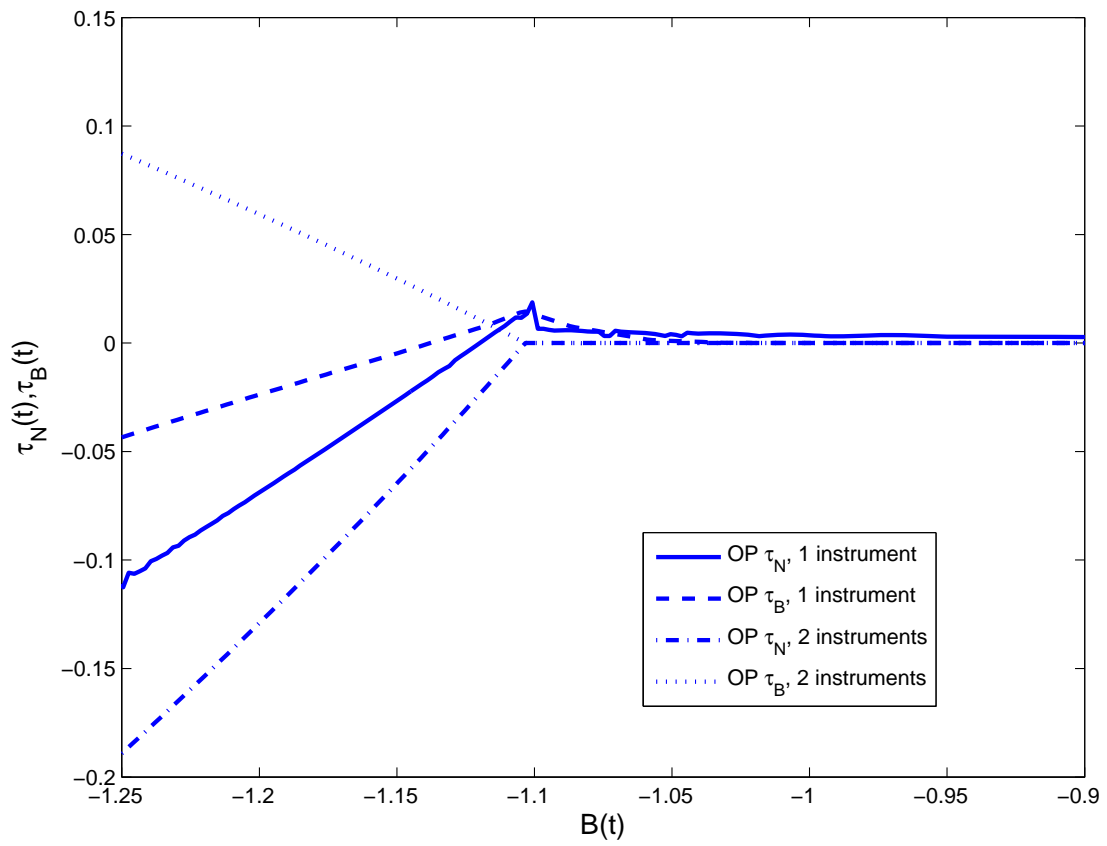


Figure 5: Decision Rules for Key Variables
 (Benchmark Allocations and Optimal Policy with Two Instruments)

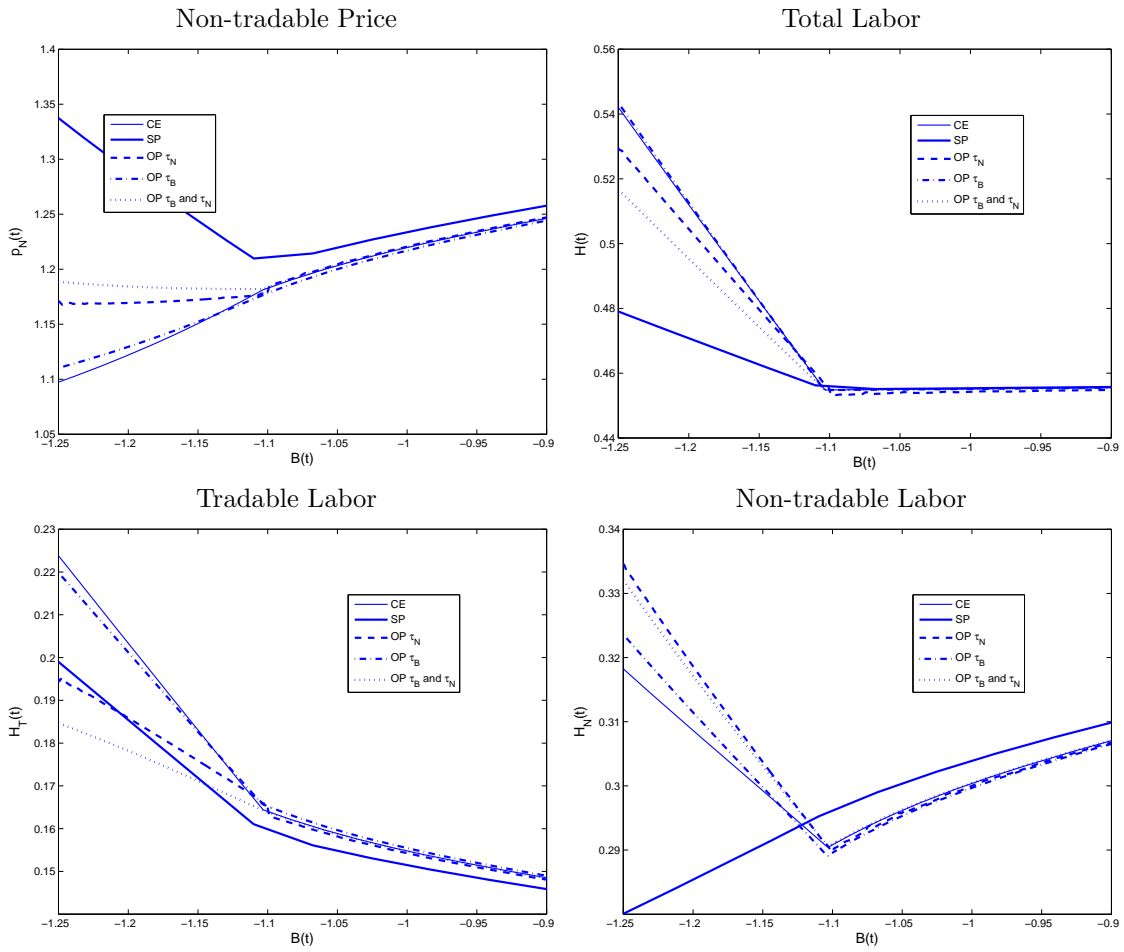


Figure 6: Ergodic Distribution of Debt
(Benchmark Allocations and Optimal Policy with Two Instruments)

