Monetary and Macro-Prudential Policies: An Integrated Analysis*

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Abstract This paper studies the interaction between monetary and macro-prudential policies in a simple model with both nominal and financial frictions. The nominal friction gives rise to conventional monetary policy objectives emphasized in the New Keynesian literature. The financial friction, in the form of an occasionally binding collateral constraint, gives rise to a financial stability objective. We study how rules developed for the nominal rigidity perform in a model that also has the financial friction. We then study how two alternative macro-prudential regimes perform. The first is a macroprudential adjusted monetary policy. The second is a two-part rule—a standard Taylor rule and a tax rule on the amount that the economy borrows. There are three main findings. First, in the economy with a nominal rigidity and a financial friction, a relatively accommodative monetary policy may be welfare improving, suggesting a role for positive inflation. By the same token, we find that there may be a trade off between macroeconomic and financial stability with a relatively aggressive monetary policy. Second, macro-prudential policy is most effective in our model (from a welfare ranking point of view) when it is designed in terms of macro-prudential augmented interest rate rules rather than through an independent tax rule on debt. Third, independent macro-prudential tax policies rules can be welfare reducing when monetary policy is accommodative and welfare increasing when it is aggressive toward inflation as in this case it helps to address the possible trade off between macroeconomic and financial stability. An important caveat to these results is that they depend on both the model specification and the parameter values adopted. They are therefore illustrative of the complex interactions at play rather than definitive.

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1 Introduction

The recent financial crisis has raised fundamental questions on the role and objectives of monetary policy. For instance, Taylor (2009) argued that excessively lax monetary policy before the crisis contributed to its occurrence and severity. A large literature is emerging that responds to this idea by designing monetary policy rules that curtail growth in credit or asset prices.\(^1\) In contrast, others believe that the crisis was the result of regulatory failures, and financial stability should be pursued by macroprudential policy, not monetary policy. For example, Svensson (2010) argues that monetary policy should continue to focus squarely on macroeconomic objectives (i.e., price and output stability).

The contribution of this paper is to study the interaction between monetary and macro-prudential policies in a framework in which there is a scope for macroeconomic and financial stability. In doing so, the model developed in this paper represents a departure from most of the existing literature that has focused on one objective at a time (notable exceptions are Cesa-Bianchi and Rebuffi, 2011, Fornaro, 2011 and Unsal, 2011). In particular, in our model a financial stability objective arises since financial crises are endogenous events captured, from a model perspective, by the situation in which the credit constraint becomes binding. The advantage of our approach is that it allows us to study the implications of “conventional” monetary policy design for financial stability (broadly defined by the frequency and the severity of crises) and to examine the extent to which monetary policy can be used in a precautionary manner to guard against the occurrence of such events.

This paper builds upon two distinct strands of literatures. The first is an extensive literature on the design of monetary policy rules to achieve macroeconomic stability in the face of nominal frictions (e.g., Woodford, 2003). This New-Keynesian literature has proposed a policy framework (inflation targeting) that performs well at stabilizing output and inflation fluctuations using interest rate rules in the presence of nominal rigidities. The second is a literature that has emerged since the great recession and focuses on designing stabilization policies before and after a financial crisis in environments with credit constraints that bind only occasionally (Benigno at al 2009; Bianchi, 2011, Bianchi and Mendoza 2010, Jeanne and Korinek 2011, Korinek 2011). This neo-Fisherian literature works in environments where the non-crisis policy is a seemingly trivial no-action because there are no other frictions in the models. While this approach focuses on the issue of financial stability, it leaves open the question of how financial stability objectives interacts with macroeconomic

\(^1\)See below for a partial list of contributions. In addition there is widespread work on such rules at central banks and IFIs, the spirit of which is captured by the call of the Basel Committee on the Global Financial System (2010) for the development of macroprudential rules that target financial stability.
stability traditionally defined.

Once we build our model we ask a series of questions about the design of both monetary and macroprudential policies. First, what are the consequences of following a monetary policy rule designed to address the nominal friction in an economy with an occasionally binding financial frictions? Second, what are the consequences of adding a macroprudential component to a conventional Taylor rule? Specifically, does this component improve welfare by contributing to macro-financial stability? Third, how well does a two part rule—one to address the nominal friction and one to address the financial constraint—in delivering both macroeconomic and financial stability? A common feature of all these questions is the role that a monetary policy instrument can play as part of a macroprudential policy toolkit.

We address these questions from the perspective of a small open economy that borrows from the rest of the world in foreign currency at a given interest rate. The world lasts for three periods and our small open economy is a two-sector production economy of tradeable and non-tradeable goods. We allow for nominal price rigidities in the tradeable sector while prices in the non-tradeable sector are perfectly flexible. Fluctuations in the model are driven by a technology shock to the production of tradable goods. The key feature of the model is an international borrowing or collateral constraint that depends on the price of a domestically traded asset and affects the intertemporal choices of domestic household like in Jeanne and Korinek (2010).

Monetary policy in this framework has real effects through multiple channels of transmission: via the nominal rigidity, the price of the asset, or the exchange rate; and each of them can have an impact on the tightness of the borrowing constraint, which binds endogenously in this framework. In this context, we consider conventional monetary policy in terms of a Taylor-type rule and an augmented monetary policy rule that targets also the amount that agents borrow (macro-prudential augmented monetary policy rule). Independent macro-prudential policy is similarly modelled as a tax rule on domestic agents' borrowing, based on the principle that taxing the amount that agents borrow limits the possibility that a crisis might occur or ameliorates its severity. As the model has no closed form solution, we conduct a numerical analysis of its equilibrium under alternative policy regimes aimed at understanding the interaction between the policy design and the behavior of the economy.

The numerical analysis that we report highlights the complex interactions involved in designing macro-prudential policies. The general policy message is that macro-prudential

\[2\text{While this may be true in some special cases, it is not necessarily true in general or an optimal policy to conduct even in the context of a simple neo-Fisherian environment (see Benigno et al, 2010 and 2011 for more details on this).}\]
policies are not necessarily welfare improving and the interaction with traditional monetary policy is crucial. Note here first that in our framework both a macro-prudential monetary policy rule and an independent macro-prudential tax rule affect the relative return of domestic versus foreign currency bonds. In the case of a tax rule on debt, borrowing in foreign currency is made relatively more expensive, while in the case of an augmented monetary policy rule domestic interest rates are relatively higher and, as such, intertemporal consumption choices will be also directly distorted. Yet, as we shall see, the two specifications yield very different outcomes from a welfare perspective.

Second, here also note that the specification of the borrowing constraint is crucial for understanding the financial stability implications of monetary policy. In fact the amount that agents borrow depends not only on the price of the domestically traded asset but also on the behavior of the nominal exchange rate since borrowing occurs in foreign currency units. Monetary policy (through domestic nominal interest rate) can influence the borrowing limit of agents by affecting the behavior of asset prices and the nominal exchange rate. While higher nominal interest rates tend to depress asset prices and tighten the agents’ borrowing limit, they also generate a relatively more appreciated nominal exchange rate that loosens agents’ borrowing limit. The relative strength of these two opposing effects determines the extent to which traditional monetary policy can entail a prudential component. When that is the case, i.e. when traditional monetary policy embeds its own prudential component, an additional policy tool for specific prudential objectives might be harmful as its costs can outweigh the benefits. In other cases, as we shall see, an additional instrument is beneficial as it helps address possible trade offs between monetary and financial stability.

More specifically, we report three main findings. First, when we compare relatively more or less aggressive Taylor rules toward inflation, in the economy with sticky price and the collateral constraint, we find that an accommodative monetary policy dominates an aggressive one from a welfare ranking point of view. This is contrary to the typical welfare ranking that arises when we consider the same policies in an economy with only sticky prices. This result suggests that in economies with financial frictions, positive inflation might be optimal from a welfare point of view (e.g., Greenwald, Michael and Joseph Stiglitz, 1993).\(^3\) This also shows that traditional monetary policy may face a trade off between macroeconomic and financial stability depending on its design rather the nature of the shock that buffets the economy (e.g., Woodford, 2011).

Second, we find that the scope for macro-prudential policies depends crucially on the design of the traditional component of monetary policy. When monetary policy is aggressive

\(^3\)Along this line of argument, Koening (2011) argues that inflation targeting may contribute to financial instability by concentrating risk in a world with nominal debt.
(the parameter on the Taylor rule is higher) macro-prudential policy is always welfare improving, regardless of how it is implemented (in terms of augmenting the interest rate rule with a macro-prudential argument or by adding an independent tax rule on debt). However, when monetary policy is accommodative, macro-prudential policy is welfare improving only if implemented through an augmented interest rate rule while it is welfare decreasing if conducted through an independent tax rule. This is because, with both the nominal rigidity and the financial friction, the separate distortion introduced by the tax rule on debt helps only when there is a trade off between macroeconomic and financial stability induced by the traditional component of monetary policy.

Third, conducting macro-prudential policies through an interest rate rule augmented with a prudential argument dominates alternative policy regimes from a welfare ranking point of view under our parametrization. The main feature of this regime is to produce, all else equal, relatively higher interest rates when borrowing is higher in normal times, and no interference with the interest rates when the occasional crisis occurs. An aggressive traditional monetary policy instead, would react strongly to inflation in both normal and crisis times. This suggests that the non-linear feature of the augmented interest rate rule is key to understand its good welfare properties.

An important caveat to these results is that they depend on both the model specification and the parameter values adopted. They are therefore illustrative of the complex interactions at play rather than definitive. A robust and common theme across the whole set of results we report, however, is that welfare enhancing policies work by supporting the borrowing (and hence consumption) capacity of the economy and hence by relaxing the borrowing constraint of our production economy. This is consistent with the main result of Benigno et al (2011) who have shown that the policy maker’s ability to affect the economy in the crisis state is what determines the scope for its policy action outside the crisis state.

As we noted above, there is an emerging and growing literature that studies augmented interest rate rules with macro-prudential arguments or two-part rules like the one we study in this paper. The basic premise of this literature is that by smoothing cycles in financial variables it may be possible to bring about greater macroeconomic stability. For example, Quint and Rabanal (2011) find that there are reductions in macroeconomic volatility from targeting financial variables, but optimizing the interest rate response to inflation and output is quantitatively more important in reducing macroeconomic volatility. On the other hand, Lambertini, Mendicino and Punzi (2011), find that an interest rate rule augmented with credit growth or house price growth is welfare improving, and that a two-part rules (one

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4Braggion, Christiano and Roldos (2007) examine how optimal monetary policy is designed in an environment in which the credit constraint becomes binding unexpectedly and remains binding forever.
for financial stability, one for macroeconomic stability) dominate the one instrument rule in the presence of news shocks in the model. The models in these exercises typically have many shocks and frictions and are linearized around a deterministic steady state. Hence they can focus only on the regular cyclical fluctuations of the economy or on the large deviations from the steady state caused by large shocks. In these environments, therefore, the notion of designing monetary and macro-prudential policies for financial stability is ambiguous. In contrast, in this paper, we build a smaller model in which the constraint binds only occasionally and there are both crisis and non-crisis states that interact and realize endogenously. We then study how monetary and macro-prudential policies should be designed and interact in such environment.

The rest of the paper is organized as follows. In section 2 we set up the model. In Section 3 we report and discuss equilibrium allocations under alternative frictions and policy rules. In section 4 we conclude. An appendix reports key equilibrium conditions of the model.

2 Model

We study a two-country world composed of a small open economy and the rest of the world. For simplicity, we assume that the world economy lasts for three periods (periods 0, 1, and 2). The specification of preferences and parameters is such that there is a one-way interaction between the two economies: the rest of the world affects the small open economy, but the latter does not have any effect on the former. The key difference between the two economies is that households in the small open economy face a constraint on the amount that they can borrow from abroad. They also face nominal rigidities in their price-setting behavior.

In this model, a financial crisis is defined as the event in which the borrowing constraint is binding, and this is an endogenous event. This feature of the model resembles the debt-deflation mechanism as in Fisher (1933) since, in a crisis, when the constraint binds, there is feedback loop between asset prices and the tightness of the borrowing constraint, which amplifies the effect of negative shocks and magnifies bust dynamic in credit and asset prices.

2.1 Households

We consider two countries, H (Home) and F (Foreign). The home country is a small open economy that takes prices as given, while the foreign country represents the rest of the

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5Financial stability, therefore, is broadly defined by the frequency and the severity of these events in the model.
world. We will use a * to denote prices and quantities of the foreign country. Note that
the home country issues bonds in the foreign currency (held by foreign agents) and hence
a * variable will appear in the home country’s budget constraints. The world economy is
populated with a continuum of agents of unit mass, where the population in the segment
[0; n) belongs to country H and the population in the segment (n; 1] belongs to country F.

The utility function of a consumer in country H is given by:

\[ U_0 = E_0 \left[ \frac{C_0^{1-\rho}}{1-\rho} + \beta \frac{C_1^{1-\rho}}{1-\rho} + \beta^2 \frac{C_2^{1-\rho}}{1-\rho} + \beta^3 \right], \]

where \( \rho \) is the elasticity of intertemporal substitution and \( \beta \in (0, 1] \) is the subjective
discount factor. The consumption basket, \( C_t \), is a composite good of tradable and non-
tradable goods:

\[ C_t \equiv \left[ \omega \frac{1}{\kappa} \left( C_t^{T} \right)^{\frac{1-\gamma}{\gamma}} + \left( 1 - \omega \right) \frac{1}{\kappa} \left( C_t^{N} \right)^{\frac{1-\gamma}{\gamma}} \right]^{\frac{\kappa}{1-\gamma}}. \]

The parameter \( \kappa > 0 \) is the elasticity of intratemporal substitution between consumption
of tradable and nontradable goods, while \( \omega \) is the relative weight of tradable goods in the
consumption basket. We denote with \( P^T \) the price of tradeable goods and with \( P^N \) the
price of nontradable goods. We further assume that tradeable goods are a composite of
home and foreign produced tradeables (\( C^H \) and \( C^F \), respectively):

\[ C_t^{T} = \left[ v^{\frac{1}{\gamma}} \left( C_t^{H} \right)^{\frac{\gamma-1}{\gamma}} + \left( 1 - v \right)^{\frac{1}{\gamma}} \left( C_t^{F} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \]

where \( \theta > 0 \) is the intratemporal elasticity of substitution. The parameter \( v \) is the relative
weight of home tradable goods in \( C^T \) and is related to the size of the small economy relative
to the rest of the world (\( n \)) and the degree of openness, \( \gamma : (1-v) = (1-n)\gamma \) (see Sutherland,
2004). Foreigners share a similar preference specification as domestic agents with \( v^* = n\gamma : \)

\[ C_t^{T*} = \left[ v^{\frac{1}{\gamma}} \left( C_t^{H*} \right)^{\frac{\gamma-1}{\gamma}} + \left( 1 - v^* \right)^{\frac{1}{\gamma}} \left( C_t^{F*} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}. \]

That is, foreign consumers preferences for home goods depend on the relative size of the
home economy and the degree of openness.

Consumption preferences towards domestic and foreign goods are given by

\[ C^H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z) \frac{1}{\sigma} dz \right]^{\frac{1}{\sigma}}, \quad C^F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z) \frac{1}{\sigma} dz \right]^{\frac{1}{\sigma}}, \]

where \( \sigma > 1 \) is the elasticity of substitution for goods produced within a country. \( C^{H*} \) and
are specified in the same manner.

Accordingly, the consumption-based price-index for the small open economy can be written as

\[ P_t = \left[ \omega (P^T_t)^{1-\kappa} + (1 - \omega) (P^N_t)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}, \]

with

\[ P^T = \left[ v (P^H_t)^{1-\theta} + (1 - v) (P^F_t)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{3} \]

where \( P^H \) is the price sub-index for home-produced goods expressed in the domestic currency and \( P^F \) is the price sub-index for foreign produced goods expressed in the domestic currency:

\[ P^H = \left[ \left( \frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad \text{and} \quad P^F = \left[ \left( \frac{1}{1-\sigma} \right) \int_1^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}. \tag{4} \]

The law of one price holds (for tradeable goods): \( p(h) = S p^*(h) \) and \( p(f) = S p^*(f) \), where \( S \) is the nominal exchange rate (i.e., the price of foreign currency in terms of domestic currency). Our preference specification implies that \( P^H = S P^H^* \) and \( P^F = S P^F^* \), while \( P^T \neq S P^{T^*} \), since

\[ P^{T^*} = \left[ v^*(P^H_t^*)^{1-\theta} + (1 - v^*) (P^F_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{5} \]

We define the real exchange rate as \( RS \equiv S P^* / P \). Note that because of our small open economy assumption (i.e., \( n \to 0 \)) \( P^F^* = P^* \), which implies that \( RS = S P^F^* / P \). Essentially nothing that occurs in the small open economy will affect the rest of the world.

The period budget constraints, expressed in units of domestic currency, for the home country are:

\[ Q_0 A_1 + P_0 C_0 + B_1 + S_0 B_1^* = B_0 (1 + i_{-1}) + S_0 B_0^* (1 + i_{-1}^*) + A_0 (D_0 + Q_0) + W_0 L_0 + F_0 \]

\[ Q_1 A_2 + P_1 C_1 + B_2 + S_1 B_2^* = B_1 (1 + i_0) + S_1 B_1^* (1 + i_0^*) + A_1 (D_1 + Q_1) + W_1 L_1 + F_1 \]

\[ P_2 C_2 = B_2 (1 + i_1) + S_2 B_2^* (1 + i_1^*) + A_2 D_2 + W_2 L_2 + F_2 \]

where we denote with \( A_{t+1} \) the individual asset holding at the end of period \( t \), \( Q_t \) is the price of the asset in units of domestic currency, with \( D_t \) the exogenous dividends from holding the asset at time \( t \), \( W_t \) is the wage rate at time \( t \), \( L_t \) is the amount of total labor supplied at time \( t \), \( F_t \) are firms’ profit and \( i_t \) is the nominal interest rate from holding debt \( B_t \) at time \( t \). We denote with \( B_t \) the amount of domestic-currency denominated bonds (which is
traded only within the small open economy) and with \( B_t^* \) the foreign-currency denominated bond which is traded internationally. In writing the budget constraint we used the fact that \( B_3 = Q_2 = 0 \).

The collateral constraints are expressed as limits on foreign borrowing:

\[
S_0 B_1^* \geq -\psi Q_0 A_1 \\
S_1 B_2^* \geq -\psi Q_1 A_2 \\
S_2 B_3^* \geq 0.
\]

We can rewrite the borrowing constraints in period 0 and 1 as:

\[
B_1^* \geq -\frac{\psi Q_0 A_1}{S_0} \\
B_2^* \geq -\frac{\psi Q_1 A_2}{S_1}.
\]

It is now evident that, for given asset holding \((A_1 \text{ and } A_2)\), asset price and exchange rate appreciation increase the value of the collateral and allow agents to borrow more.

The dependence of the borrowing constraint from both exchange rate and asset price is behind the interplay between monetary policy and financial crises in the model. As we shall describe below, the determination of both prices is affected by the design of monetary policy both when the constraint is binding and when is not.

**Intratemporal Consumption Choices**  
The intratemporal first order conditions determines how the household allocate their consumption expenditure among the different goods:

\[
C^N = \omega \left( \frac{P^N}{P} \right)^{-\kappa} C, \quad C^T = (1 - \omega) \left( \frac{P^T}{P} \right)^{-\kappa} C
\]

with

\[
C^H = v \left( \frac{P^H}{P^T} \right)^{-\theta} C^T, \quad C^F = (1 - v) \left( \frac{P^F}{P^T} \right)^{-\theta} C^T
\]

and

\[
c(h) = \left[ \frac{p(h)}{P^H} \right]^{-\sigma} C^H = v \left[ \frac{p(h)}{P^H} \right]^{-\sigma} \left[ \frac{P^H}{P^T} \right]^{-\theta} C^T
\]

\[
c(f) = \left[ \frac{p(f)}{P^F} \right]^{-\sigma} C^F = (1 - v) \left[ \frac{p(f)}{P^F} \right]^{-\sigma} \left[ \frac{P^F}{P^T} \right]^{-\theta} C^T
\]

There are corresponding conditions for the foreign economy and given our preference
specification, the total demands of the generic good \( h \), produced in Home country, and of the good \( f \), produced in Foreign country, are respectively:

\[
y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} [C^H + C^{H*}]
\]

and

\[
y^d(f) = \left[ \frac{p^*(f)}{P_F} \right]^{-\sigma} [C^F + C^{F*}]
\]

with \((1 - v) = (1 - n)\gamma\) and \(\nu^* = n\gamma\). Because of our characterization of the small open economy as an economy in which \( n \rightarrow 0 \), we can rewrite our demand equations as:

\[
y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left( \frac{P^H}{P^F} \right)^{-\theta} (1 - \omega) \left( \frac{P^T}{P} \right)^{-\kappa} \left[ (1 - \gamma) C + \gamma \left( \frac{P^T}{SP^T} \right)^{\kappa-\theta} \left( \frac{1}{RS} \right)^{-\kappa} C^* \right]
\]

and

\[
y^d(f) = \left[ \frac{p^*(f)}{P_F} \right]^{-\sigma} \left\{ \left[ \frac{P^*_F}{P^*_T} \right]^{-\kappa} (1 - \omega) C^* \right\}.
\]

We note here that the demand of home produced goods is affected by movements in two international relative prices: the real exchange rate \((RS)\) and the real exchange rate at the level of tradeable goods \((\frac{SP^T}{P^T})\). If we assume that \(\theta > \kappa\) (the elasticity of substitution among tradeable goods is higher than the one between tradeable and nontradeable), a depreciation of both real exchange rate measures redirect demand towards home produced goods. Foreign demand on the other hand is not affected by developments in the small open economy and it is determined only by foreign factors.

**Intertemporal Consumption Choices**  The intertemporal first order conditions for consumption are then given by:

\[
C_0^{-\rho} = \lambda_0 P_0
\]

\[
\beta C_1^{-\rho} = \lambda_1 P_1
\]

\[
\beta^2 C_2^{-\rho} = \lambda_2 P_2.
\]

where we have denoted with \(\lambda_t\) the multipliers on the period budget constraints. Using the expression for the Lagrange multiplier from the previous conditions we can write the first order conditions for foreign-currency denominated bond holdings as:

\[
S_0 \frac{C_0^{-\rho}}{P_0} = S_0 \mu_0 + \beta E_t \left[ S_1 \frac{C_1^{-\rho}}{P_1} (1 + i^*) \right]
\]
where $\mu_t$ denotes the Lagrange multiplier on the collateral constraints. From the first order conditions for domestic-currency denominated bond holdings we can retrieve the familiar Euler equations:

\[
\frac{1}{(1 + i_0)} = E_t \left[ \beta \frac{C_{-\rho}^1 P_0}{P_1 C_{-\rho}^0} \right] \tag{6}
\]

\[
\frac{1}{(1 + i_1)} = E_t \left[ \beta \frac{C_{-\rho}^2 P_1}{P_2 C_{-\rho}^1} \right] \tag{7}
\]

Using the expression for the Lagrange multiplier from the previous conditions we can then rewrite the first order conditions for the asset holdings as:

\[
\frac{C_{-\rho}^0}{P_0} Q_0 = \mu_0 \psi Q_0 + E \left[ \frac{C_{-\rho}^1}{P_1} (D_1 + Q_1) \right] \tag{8}
\]

\[
\frac{C_{-\rho}^1}{P_1} Q_1 = \mu_1 \psi Q_1 + E \left[ \frac{C_{-\rho}^2}{P_2} D_2 \right]. \tag{9}
\]

By rearranging these conditions, we have:

\[
Q_t = \frac{\lambda_{t+1} (D_{t+1} + Q_{t+1})}{\lambda_t - \mu_t \psi} \quad t = 0, 1.
\]

All else being equal, this expression shows that when the constraint binds agents have an extra incentive to buy the asset and use it as collateral since the asset price is increasing in $\mu_t$. In fact, the previous equation is almost identical to a standard asset price condition in which the price of an asset is equal to the the expected present discounted value of future dividends. The discount is now given by the term $\frac{\lambda_{t+1}}{\lambda_t - \mu_t \psi}$ and differs from the standard one (i.e. $\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{(1+i_0)}$) only because of the multiplier associated with the credit constraint. This implies that in general (both when the constraint is binding and when is not), the discount factor is going to be higher, other things being equal, since agents take into account the shadow value of relaxing the credit constraint by purchasing an extra unit of the asset whenever the collateral constraint binds or it is expected to bind at a future date $t + i$ (for $i = 0, 1$). Equations (8) and (9) thus highlight the first channel of interaction between monetary policy and the credit constraint: the asset price is given by the present discounted value of dividends and more aggressive policy in normal time reduces the asset price and hence the value of the collateral.
No-arbitrage implies the following modified version of international parity condition:

\[
E_t \left[ \frac{C_1^\rho}{P_1} (1 + i_0) \right] = \left[ \mu_0 + E_t \left[ \frac{C_1^\rho}{P_1} \frac{S_1}{S_0} (1 + i^*) \right] \right] \quad (10)
\]

and

\[
E_t \left[ \frac{C_2^\rho}{P_2} (1 + i_1) \right] = \left[ \mu_1 + E_t \left[ \frac{C_2^\rho}{P_2} \frac{S_2}{S_1} (1 + i^*) \right] \right] \quad (11)
\]

The international parity conditions are now modified to take into account the possibility that the constraint is binding \((\mu_t > 0)\) or might be binding in the future. Equations (10) and (11) determine a second channel of interaction between monetary policy and the borrowing constraint operating via the nominal exchange rate. When the constraint binds, agents reallocate their wealth towards domestic asset and in particular towards domestic currency bonds. This generates an increase in the real return on domestic currency bonds through an expected appreciation of the nominal exchange rate or an increase in the domestic nominal interest rate. This in turn implies that, when the constraint is binding, a relatively more aggressive monetary policy is coupled with a relatively more appreciated currency that tends to relax the constraint. When the constraint is not binding, a similar mechanism operates: for given future exchange rate, a more aggressive monetary policy is accompanied by a more appreciated exchange rate.

To summarize, in normal times, monetary policy affects the borrowing capacity of agents (i.e., the possibility that the constraint might be binding) through two channels. Higher interest rates can increase the borrowing capacity by appreciating the nominal exchange rate while they decrease it by lowering the asset price that serve as a collateral. The relative strength of these two channels determine the extent to which monetary policy can entail an indirect prudential component that contains the amount of foreign currency-denominated borrowing of the small open economy and hence contribute to contain the frequency and the severity of financial crises.

2.2 Firms

Our economy is a two-sector economy that produces tradeables and non-tradeables goods. We assume that only domestic agents hold shares in home firms. Firms in the tradeables sector operate in a monopolistic competitive environment and face a technology that might prevent them from adjusting prices in period 0 and 1. In period 2, prices are fully flexible for all firms. On the other hand, firms in the non-tradeables sector operate under decreasing return to scale in a competitive environment.

In the non-tradeable sector, firms produce according to the following production func-
tion:
\[ Y_t^N = z_t^N \left( L_t^N \right)^\delta \]
where \( z_t^N \) is the sector-specific productivity shock, \( L_t^N \) is the amount of labor employed in the non-tradeables sector and \( \delta < 1 \) is the return to scale parameter. The profit of non-tradeable firms, \( \pi_t^N \), is given by:
\[ \pi_t^N = P_t^N z_t^N \left( L_t^N \right)^\delta - W_t L_t^N. \]

From the maximization problem of non-tradeable firms we obtain the following standard first order condition:
\[ W_t = P_t^N z_t^N \delta \left( L_t^N \right)^{\delta - 1}. \] (12)

In the tradable sector the firms’ production function is linear in labor:
\[ y_t(h) = z_t^T L_t^T(h) \]
with \( z_t^T \) denoting a sector-specific productivity shock. These firms operate in a monopolistic competitive market and face a technology constraint that prevents them from adjusting prices every period. In particular, we assume that only a fraction \((1 - \alpha)\) can change price in period 0 and 1, while prices are fully flexible in period 2. Here we also assume that when firms can reset prices they have observed the relevant uncertainty.

Starting from period 2, we write the individual firm problem as:
\[ \pi_2(h) = p_2(h) y_2(h) - W_2 y_2(h) z_2, \]
where
\[ y_2(h) = \left( \frac{p_2(h)}{P_{H,2}} \right)^{-\sigma} Y_{H,2}, \]
is the total demand faced by the individual firm for the single differentiated good. Period 2’s maximization problem renders that the optimal price is a mark-up over nominal marginal cost:
\[ p_2(h) = \frac{\sigma}{\sigma - 1} \frac{W_2}{z_2^T}. \] (13)
Given that all firms in period 2 face the same marginal cost, the optimal price is the same across firms \( p_2(h) = P_{2}^H \), with
\[ 1 = \frac{\sigma}{\sigma - 1} \frac{W_2}{P_{2}^H z_2^T}. \]

Consider now firm pricing in period 0 and 1. In period 0 only a fraction \((1 - \alpha)\) of
firms can reset prices taking into account that prices might be fixed in period 1. So the maximization problem is given by

$$\max E_0 \left[ \pi^T_0 + \beta \alpha Q_{0,1} \pi^T_1 \right] = \left[ p_0(h) \tilde{y}_0(h) - W_0 \frac{\tilde{y}_0(h)}{z_0^T} \right]$$

$$+ \beta \alpha Q_{0,1} \left[ z_1^T \tilde{p}_0(h) \tilde{y}_1(h) - W_1 \frac{\tilde{y}_1(h)}{z_1^T} \right],$$

where

$$\tilde{y}_0(h) = \left( \frac{\tilde{p}_0(h)}{P_{H,0}} \right)^{-\sigma} Y_{H,0}, \quad (14)$$

$$\tilde{y}_1(h) = \left( \frac{\tilde{p}_0(h)}{P_{H,1}} \right)^{-\sigma} Y_{H,1} \quad (15)$$

are the total demands that the individual firm face in period 0 and 1, conditional on the choice of price in period 0, while $Q_{0,1}$ is the nominal stochastic discount factor between period 0 and 1. The first order condition for the individual firm’s maximization problem gives:

$$\tilde{p}_0(h) = \frac{\sigma}{\sigma - 1} \frac{E_0 \left( \frac{W_0 \tilde{y}_0(h)}{z_0^T} + \beta \alpha Q_{0,1} \frac{W_1 \tilde{y}_1(h)}{z_1^T} \right)}{E_0 (\tilde{y}_0(h) + \beta \alpha Q_{0,1} \tilde{y}_1(h))}$$

By using (14), we can rewrite the above condition as:

$$\tilde{p}_0(h) = \frac{\sigma}{\sigma - 1} \frac{E_0 \left( \frac{W_0 \tilde{y}_0(h)}{z_0^T} + \beta \alpha Q_{0,1} \frac{W_1 \tilde{y}_1(h)}{z_1^T} \Pi_{1}^{H} \right)^{1+\sigma} Y_{H,1}}{E_0 [Y_{H,0} + \beta \alpha Q_{0,1} (\Pi_{1}^{H})^{\sigma} Y_{H,1}]} \quad (16)$$

with $\Pi_{1}^{H} \equiv \frac{P_{H,1}}{P_{H,0}}$ denoting gross inflation from period 0 to period 1. $P_{H,0}$ is the aggregate price index for the home produced goods given by

$$\left( P_{H,0}^{H} \right)^{1-\sigma} = (1 - \alpha) \tilde{p}_0(h)^{1-\sigma} + \alpha \left( P_{-1}^{H} \right)^{1-\sigma},$$

that can be rewritten as

$$\left( \frac{1 - \alpha \left( \Pi_{0}^{H} \right)^{\sigma-1}}{1 - \alpha} \right)^{1-\sigma} = \frac{\tilde{p}_0(h)}{P_{H,0}^{H}}, \quad (17)$$

with $\Pi_{1}^{H} \equiv \frac{P_{H,1}}{P_{H,0}}$.

A similar problem arises in period 1 in which only a fraction of firms $(1 - \alpha)$ can reset prices. Since prices can be reset for every firm in period 2, the pricing problem in period 1
is the same as in the flexible price case:

\[ \tilde{p}_1(h) = \frac{\sigma}{\sigma - 1} \frac{W_1}{z_1^t} \]  

(18)

with the aggregate price index for the home produced goods in period 1 given by

\[ (P^H_1)^{1-\sigma} = (1 - \alpha)\tilde{p}_1(h)^{1-\sigma} + \alpha (P^H_0)^{1-\sigma} \]

that can be rewritten as:

\[ \left( \frac{1 - \alpha (\Pi^H_1)^{\sigma-1}}{1 - \alpha} \right)^{\frac{1}{\sigma - 1}} = \frac{\tilde{p}_1(h)}{P^H_1} \]

(19)

It is now useful to examine how the credit constraint can affect firm behavior and vice versa in the presence of nominal rigidities. The interaction between the credit constraint and nominal rigidities is direct in period 0 and indirect in period 1 and 2, since in period 1 and 2 firms reset prices at the flexible price level. In period 0, a binding constraint, or an expected binding constraint in period 1, reduces aggregate demand and tends to lower domestic producer inflation other things being equal, compared to an economy in which there is no borrowing constraint. In period 1 and 2, the effect is indirect through the endogenous state variable \( B^*_t \) that determines the household debt position at the beginning of period \( t \). Indeed, the lower (higher) the level of debt accumulated in the previous period \( (B^*_t \text{ more negative/positive}) \), the lower (higher) are the resources available to household for spending in the current period, given the level of other variables. Thus, other things being equal, higher (lower) debt implies lower (higher) demand and lower (higher) domestic producer inflation.

Inflation, in turn, also determines an inefficient allocation of resources between tradable and non-tradable goods that can influences the tightness of the borrowing constraint. To see this, note that pricing decisions in period 0, 1 and 2 can be summarized in terms of the following equations:

\[ \left( \frac{1 - \alpha (\Pi^H_0)^{\sigma-1}}{1 - \alpha} \right)^{\frac{1}{\sigma - 1}} = \frac{E_0 \left( \frac{P^N_0 z_0^N \delta(L_0^N)^{\delta-1}}{z_0^N P^H_0^0} Y_{H,0} + \beta \alpha Q_{0,1} \frac{P^N_1 z_1^N \delta(L_1^N)^{\delta-1}}{z_1^t P_{H,1}} \right) (\Pi^H_1)^{1+\sigma} Y_{H,1}}{E_0 \left[ Y_{H,0} + \beta \alpha Q_{0,1} (\Pi^H_1)^{\sigma} Y_{H,1} \right]} \]

(20)

for period 0;

\[ \left( \frac{1 - \alpha (\Pi^H_1)^{\sigma-1}}{1 - \alpha} \right)^{\frac{1}{\sigma - 1}} = \frac{\sigma}{\sigma - 1} \frac{P^N_1 z_1^N \delta(L_1^N)^{\delta-1}}{P^H_1 z_1^t} \]

(21)
for period 1, and

\[ 1 = \frac{\sigma}{\sigma - 1} \frac{P_2^N z_2^N \delta \left( I_2^N \right)^{\delta-1}}{P_2^H z_2^H} \tag{22} \]

for period 2, where \( Q_{0,1} = \frac{1}{1+g} \).

Note now that, from (21), positive inflation determines an inefficient allocation of resources between tradable and non-tradable goods. Indeed, inflation create a wedge between the relative price of tradable goods over non-tradable goods and their marginal rate of transformation. When inflation is positive resources tend to shift towards the non-tradeables sector, implying a decline in tradable production, other things being equal. Through this effect, inflation affect the possibility that the borrowing constraint binds by increasing the amount agents need to borrow in order to enjoy a given level of tradable consumption. Note however that, positive inflation might also imply higher nominal interest rate depending on the design of monetary policy and by this channel, as we described above, affects the borrowing capacity of agents through the effects on asset prices and the nominal exchange rates.

### 2.3 Monetary and prudential policies

We model monetary policy with a simple interest rate that reacts only to domestic producer inflation:

\[ (1 + i_t^{TR}) = \beta^{-1} \bar{\Pi} \left( \frac{\Pi_t^H}{\bar{\Pi}} \right)^{\phi_n} \tag{23} \]

in which the target inflation \( \bar{\Pi}_t \) is time invariant and set equal to zero.\(^6\) Macro-prudential policy is modeled in two different ways, consistent with alternative proposals in the ongoing policy debate.

First, we consider a second interest rate rule with an explicit macro-prudential argument in period zero.\(^7\) While the model would allow for several possibilities, in addition to the inflation term, we include the level of borrowing from an aggregate perspective as a share of total consumption expenditure with a somewhat arbitrarily small coefficient. More formally,

\(^6\)There is an issue here in terms of which measure of inflation to target. Here we have included PPI inflation. An alternative is to include the CPI inflation rate that indirectly includes also changes in the nominal exchange rate. This, however, in our model, might have prudential effects to the extent to which the exchange rate enters also the leverage constraint.

\(^7\)As we shall see, the fact that the second argument in the interest rate rule is active only in period zero is crucial for its performance.
the alternative rule is:

\[
(1 + i_t) = \beta^{-1} \Pi_t \left( \frac{\Pi_t}{\Pi} \right)^{\phi_s} \left( 1 - \frac{S_t B_{t+1}^{B^*}}{P_tC_t} \right)^{\phi_{B^*}} \text{ for } B_{t+1}^* < 0 \tag{24}
\]

\[
= (1 + i_t^{TR}) \left( 1 - \frac{S_t B_{t+1}^{B^*}}{P_tC_t} \right)^{\phi_{B^*}}
\]

where \( (1 + i_t^{TR}) = \beta^{-1} \Pi_t \left( \frac{\Pi_t}{\Pi} \right)^{\phi_s} \) is the hypothetical level of the interest rate that would prevail if \( \phi_{B^*} = 0 \), which is used below for the purpose of explaining how macro-prudential policy works in our model. This rule says that, all else being equal, the nominal interest rate in period \( t \) is higher the higher the level of aggregate borrowing in foreign currency as a share of consumption spending. When \( \phi_{B^*} = 0 \) the nominal interest rate will be the same as in (24). When instead \( \phi_{B^*} \neq 0 \) nominal interest rates are higher than in the standard rule for a given amount of debt, and as such the interest payment on debt increases, constraining current spending.

The relatively higher current interest rate also provides an incentive to contain current period borrowing that determine how much debt will be carried forward into the next period. From our set of equilibrium conditions, in fact, we can see that (24) affects the intertemporal margin in (6) and (7) by tilting the profile of consumption towards future consumption as opposed to present consumption and reducing the amount that agents want to borrow other things being equal. In fact the Euler equation in period 0 becomes:

\[
\frac{1}{(1 + i_0^{TR}) \left( 1 - \frac{S_0 B_1^{B^*}}{P_0 C_0} \right)^{\phi_{B^*}}} = E_t \left[ \beta \frac{C_1^{-\rho} P_0}{P_1 C_0^{-\rho}} \right] \tag{25}
\]

In this case, the international parity condition becomes

\[
\left( 1 - \frac{S_0 B_1^{B^*}}{P_0 C_0} \right)^{\phi_{B^*}} E_t \left[ \frac{C_1^{-\rho}}{P_1} (1 + i_0^{TR}) \right] = \left[ \mu_0 + E_t \left[ \frac{C_1^{-\rho} S_1}{P_1 S_0} (1 + i^*) \right] \right]
\]

since the augmented rule is based on aggregate debt and agents take it as given when they allocate their wealth between home and foreign currency bonds. Thus, macro-prudential monetary policy makes domestic borrowing relatively more expensive compared to foreign one and affects directly the intertemporal allocation of consumption of households (see (25)).

Second, we also consider a separate macro prudential policy rule which is a tax on the amount that the economy borrows in the aggregate. This second tool acts simultaneously and independently from the interest rate tool. Like in the previous case, we allow for this
macro-prudential tool only in period 0 since in our three-period economy, the constraint might be binding only in period 1. In this case, the budget constraint in period 0 becomes:

\[ Q_0 A_1 + P_0 C_0 + B_1 + S_0 B_1^*(1 - \tau_0^*) = B_0 (1 + i_{-1}) + S_0 B_0^*(1 + i_{-1}) + A_0 (D_0 + Q_0) + W_0 L_0 + F_0 + T_0 \]

where \( B_1^*(1 - \tau_0^*) \) is the after-tax borrowing proceeding available for consumption, and \( T_0 \) is a lump-sum transfer from the government (with the government that follows a balanced budget rule \( T_0 = -S_0 B_1^* \tau_0^* \)). Our macro-prudential tax rule is then given by:

\[
(1 - \tau_0^*) = \left(1 - \frac{S_t B_{t+1}^*}{P_t C_t}\right)^{-\phi_H^*} \text{ for } B_{t+1}^* < 0,
\]

which implies that after-tax borrowing proceeds decreases with the level of debt.

Similarly to the case of the augmented interest rate rule above, this tax applies when the economy is borrowing from the rest of the world. The intertemporal margin that now is distorted is the Euler equation for foreign bonds:

\[
S_0 \frac{C_0^{-\rho}}{P_0} (1 - \tau_0^*) = S_0 \mu_1 + \beta E_t \left[ S_1 \frac{C_1^{-\rho}}{P_1} (1 + i^*) \right]
\]

and the international parity condition becomes similar to the one in the augmented Taylor rule case:

\[
(1 - \tau_0^*) E_t \left[ \frac{C_1^{-\rho}}{P_1} (1 + i_0) \right] = \left[ \mu_0 + E_t \left[ \frac{C_1^{-\rho}}{P_1} S_1 (1 + i^*) \right] \right].
\]

Here macro-prudential policy alters the relative return of domestic asset versus foreign asset by making foreign currency return relatively more expensive compared to the case in which monetary policy is augmented by a macro-prudential component. The main difference with respect to the previous case is that now there is no intertemporal distortion in the consumption profile across time. In fact in this case (6) holds:

\[
\frac{1}{(1 + i_0^{R})} = E_t \left[ \beta \frac{C_1^{-\rho}}{P_1} \frac{P_0}{C_0^{-\rho}} \right].
\]

So in our formulation an independent macro-prudential policy acts directly on the quantity that agents borrow and reduces the net amount that they borrow but it does not distort the intertemporal consumption choice.


3 Model parametrization and solution

Despite its relative simplicity, the model we set up has no closed form solution and is solved numerically. The model is parameterized in the simplest possible manner as we do not attempt to use it quantitatively but rather to provide examples of the possible interactions between the two frictions in the models and the alternative policies we consider. In fact, the three-period structure of the model imposes terminal conditions for the net foreign position and asset prices that require sharp and unrealistic movements in most endogenous variables between periods. Moreover, the exercises we run requires that, for given initial conditions, the structural parameters stay constant across different policy regimes. But alternative policy regimes might have very different properties, making it difficult to find a common set of structural parameter values for which we can solve the model and run a large set of policy experiments. For instance, while in our baseline case the borrowing constraint binds only in one state in period 1, alternative specifications of monetary and macro-prudential policy result in the constraint binding in neither states or both states. The highly non-linear nature of the model that is fully taken into account by the solution method also adds a degree of complexity in finding a suitable parametrization. Nonetheless, to the extent possible, in doing our numerical examples, we borrow parameter values from the literature.

Table 1 reports the chosen parameter values of the model, the shocks’ process, and the initial conditions. The tradeable sector technology shock $z_T$ is a two-state Markov process that can take two values, either 0.9 or 1.1 (bad and good state, respectively) with transition matrix:

$$
\mathbf{p} = \begin{bmatrix}
0.4 & 0.6 \\
0.4 & 0.6
\end{bmatrix}.
$$

The shock hits the economy in period 0 and in period 1, so the economy has two possible states in period 1 and four states in period 2.

The elasticity of substitution between tradable and non-tradable goods and between home and foreign tradable goods is set to one for simplicity. The relative weight of non-tradable goods is set to 0.5. As a result, tradable and non-tradable consumption are the same in units of consumption. The size parameter $n = 0$ and the degree of openness $\gamma$ is .25, which together yields a value for the relative weight of home tradable goods $\nu$ of .75. The elasticity of substitution within home tradables goods is set to 6 to yield a mark up of 20%, which is a conventional value. The labor share parameter $\delta$ is set to 0.75, slightly higher than usually assumed but not outside a plausible range of values if we consider self-employment. The intertemporal substitution and risk aversion are set $\rho = 1$, as in Jeanne
and Korinek (2010).

The nominal rigidity parameter is set somewhat arbitrarily to $\alpha = 0.5$. This is below the typical value around .75 used in infinite horizon models and implies that half of the firms can adjust prices in period 0 and also in period 1. The coefficient in the interest rate rule on domestic producer inflation is set to $\phi_\pi = 1.5$. We also use a more aggressive reaction parameter toward domestic producer inflation which is $\phi_\pi = 2$. The coefficients on macro prudential policy are set at $\phi_{B^*} = 0.02$ in all cases.

The parameter $\psi$ of the collateral constraint is set to a value such that the constraint is never binding in period 0, and to 2.267 in period 1, so that the constraint binds in at least one state in the baseline case. We then keep the structural parameters of the model constant across experiments and change only the policy rules. When we change the policy rule, the constraint may bind in both states or in neither state, and the value of the credit multipliers, when they are positive, indicate the severity of the crisis. Financial stability, therefore, varies endogenously with alternative policy rules in the models. Note however that, because the Markov shock process has only two states, the probability at time 0 that the constraint binds at time 1 is exogenous in the model and coincides with the probability that the economy switches from the bad state in period 0 (in which it is initialized) to state in which the constraint binds in period 1. Therefore, the probability of the crisis cannot be used as a measure of financial stability in the model.\(^8\)

The exogenous dividend process is constant in nominal terms over time and set to $D_0 = D_1 = D_2 = 0.5$. The foreign interest rate and the discount rate are constant and such that $\beta = 1/(1 + i^*) = 1$, like Jeanne and Korinek (2010). Foreign prices are also constant and normalized to 1: $P^* = P_0^F = P_1^F = P_2^F = 1$. The terminal exchange rate level is $S_2 = 1$. All allocations are initialized with $B_0^* = -3.76$ in the negative state (state 1). Note however that the value of initial debt in either domestic currency or unit of consumption will differ across experiments endogenously playing an important role in behavior of the economy under alternative policy rules. In fact, all else being equal, the higher the value of debt entering period $t$, the smaller the amount of resources available for consumption in period $t$ and $t+i$ ($i=1,2$).

The model’s core non-linear equilibrium conditions (including the resource constraint of the tradable sector derived in appendix) are solved for all states of the economy simultaneously with the Matlab function $fsolve$, for given initial and terminal conditions and the state of the tradeable sector technology shock $Z^T$. Like Benigno et al. (2011), we convert the complementary slackness conditions for the collateral constraint into a single nonlinear

\(^8\)Benigno et al (2011) show that the probability of the crisis is not a good measure of policy effectiveness as it might be very low and yet be associated with welfare reducing policies.
equation following Garcia and Zangwill (1981). In a few cases in which the default initial condition does not yield a solution we employ a homotopy method to generate a better initial conditions that lead to the solution of the model—see again Garcia and Zangwill (1981).

We evaluate alternative policy rules by comparing welfare. This is computed from as the ex ante value of the expected utility:

$$V = \frac{C_0^{1-\rho}}{1-\rho} + p_{11}C_{1.1}^{1-\rho} + p_{12}C_{1.2}^{1-\rho} + p_{11}p_{11}C_{2,11}^{1-\rho} + p_{11}p_{12}C_{2,12}^{1-\rho} + p_{12}p_{21}C_{2,21}^{1-\rho} + p_{12}p_{22}C_{2,22}^{1-\rho},$$

where $C_0$ is total consumption at time zero, $C_{1,i}$ is the total consumption in period 1 in state $i$ with $i = 1, 2$, $C_{2,ij}$ is the total consumption in period 2 if state $i$ realized in period 1 and state $j$ realizes in period 2, and $p_{ij}$ with $i, j = 1, 2$ are the transition probabilities of the Markov process above, in which state 1 is the negative one.

### 4 Alternative policy rules

In this section we study the impact of alternative specifications for monetary and macro-prudential policy rules. We first discuss briefly three well known benchmarks. The first is a frictionless small open economy that, while not necessarily a Pareto efficient economy, helps to provide intuition of how the two frictions work and interact in our three period model. The second is a flexible price economy with the financial friction that is comparable to the competitive equilibrium allocation of the models in the Neo-Fisherian literature on financial stability—e.g., Benigno et al (2011), Jeanne and Korinek (2011) and Bianchi and Mendoza (2011)). The third is a sticky price economy without the financial friction that corresponds to the traditional New Keynesian framework. We then consider alternative monetary and macro-prudential policy regimes for economies with both nominal and financial frictions.

We examine three policy regimes. The first is a traditional interest rate rule that responds only to inflation in a more or less aggressive way. The second is a two-instrument regime with the same traditional interest rate rule and a tax on debt as a macroprudential policy rule. The third is an interest rate rule that responds to both inflation and debt. As a general caveat to our analysis, we note from the outset that the results we report and discuss below should not be seen as general properties of our model economy as they depend heavily on both the model specification adopted and the parameter values chosen, but rather as examples illustrating the rich and complex interaction between asset prices, consumption and production decisions on the one hand and monetary and prudential policies on the other. On the other hand, a robust and common feature of the different cases we discuss is
that welfare enhancing policies work by supporting the borrowing (and hence consumption) capacity of the economy, and thus by relaxing the borrowing constraint of our production economy.

Table 2 summarizes the allocations in all cases. The first three columns report the allocations for a flexible price economy with an interest rate rule that responds only to inflation (with a 1.5 coefficient), with and without the collateral constraint, and with the constraint and a tax rule on debt. As we can see the borrowing constraint reduces lifetime utility, while adding the tax on debt in period zero along with the interest rate rule increases utility slightly in the case of flexible prices (even if traditional monetary policy is relatively less aggressive or more accommodative).

The presence of the borrowing constraint decreases utility by hampering consumption smoothing over time and across states. The model has three periods, and there is initial debt (constant across experiments in units of foreign currency) that needs to be repaid in full in period 2 (i.e., $B_3^* = 0$). This implies that, as we can see from experiment 1 in Table 2, the frictionless economy is on a debt repayment path with a current account surplus in all periods and states, and tradable consumption that is roughly constant over time and across states. With the borrowing constraint instead (experiment 2), consumption is not constant over time and across states, and is much lower in both periods and states than without the constraint.

Note that the borrowing constraint binds in the good state of the tradable productivity process in this production economy with flexible prices. With the realization of the positive state in period 1, the physical amount of tradable output increases and puts downward pressure on producer price inflation and the nominal exchange rate. The interest rate falls driven by the monetary rule, but not enough to support the asset price that in equilibrium falls in this case. As a result the borrowing capacity of the economy meets its limit. Interestingly, however, the level of borrowing at which the constraint binds in period 1 is much higher than the equilibrium level in the frictionless economy. This is because the interest rate (and the asset price) in the constrained economy are much lower (higher) than in the unconstrained one allowing for a much larger borrowing capacity. At the same time, the exchange rate is more depreciated in the constrained economy requiring a larger borrowing capacity.

The introduction of a tax rule on borrowing along with a traditional interest rate rule increases welfare in the constrained economy with flexible prices (experiment 3 in Table 2). This is achieved by inducing a relatively less procyclical allocation of borrowing over time and across states relative to the economy with constraint and without the tax rule on debt. In fact, in equilibrium, tradable consumption (and the current account surplus) are higher
(lower) in period zero and in the bad state in period 1. Higher interest rates in period 0, other things being equal, also induces a relative more appreciated nominal exchange rate in period 0 that makes the initial debt burden easier to repay over time and hence also permitting to consume more in period 0. Note here also that the borrowing multiplier in the good state, when the constraint binds, is lower than the case without the tax rule on debt, suggesting that the crisis is less costly in this case when it does occurs.

Consider now a set of economies with sticky prices, both with and without leverage constraint (experiments 4 to 9 in Table 2), and with the constraint and a tax rule on debt. The first three economies have a pure inflation targeting rule with a 1.5 coefficient on inflation, with inflation measured by the PPI index, i.e., \((\Pi_t^P)\) like in the first three experiments with flexible prices. The second three ones have a more aggressive reaction to inflation with a 2 coefficient on inflation.

The economy with price rigidity without constraint (Table 2, experiment 4) has a similarly smooth path of tradable consumption compared to the flexible price economy without constraint (experiment 1). Period 1 inflation, which stems from higher marginal costs associated with higher domestic production necessary to repay the initial debt, is lower than in the frictionless case. The nominal interest rate is also lower in this case. The exchange rate is more depreciated and the initial debt burden is consequently larger. As a result, the overall profile of tradable consumption is lower than with flexible prices. Thus, the nominal rigidity friction reduces welfare by requiring a more depreciated exchange rate with accommodative policy relative to the flexible price allocation.

When we examine economies with sticky prices and the borrowing constraint but no tax rule on debt, however, we note that a relatively less aggressive monetary policy can induce allocations that may have higher welfare ranking than the corresponding flexible prices ones (experiment 5 and 8 in Table 2). One interpretation is that, with higher inflation, there is less pressure on the current nominal exchange rate to adjust by depreciating because interest rates are higher and support it relative to the relevant case with flexible prices. A relatively less depreciated currency increases agents borrowing capacity in period 0 allowing them to consume more. This shows that, with multiple distortions, inflation can be welfare-enhancing.

In sharp contrast, with a more aggressive traditional monetary policy (with a 2 coefficient on inflation), there is a trade off between macroeconomic and financial stability. This is in the sense that, while stabilizing inflation more aggressively in response to shocks, this policy can makes financial crises more frequent and more severe (as measured by the values of the borrowing multipliers that are now positive in both states and one order of magnitude larger in the positive one). The asset price is much higher in this economy in
period zero and in period 1 in the good state, while the nominal exchange rate tends to be relatively more depreciated. As a result, the economy experiences a higher current account surplus and lower consumption in period 0 and is much more volatile in period 1 because the constraint ends up binding in both states of the world. By comparison, there is no such a trade off in the sticky price economies without borrowing constraint (experiments 4 and 7), and the more aggressive reaction to inflation induces higher welfare in those cases, consistent with traditional New-Keynesian priors.

Alternative prudential policies also interact with monetary policy in a rich and non-linear way (experiments 6 and 9 in Table 2). For instance, conducting macro-prudential policies with a tax rule on debt is welfare increasing when monetary policy is aggressive because it helps resolve the trade off we described above (experiment 9), but it is welfare decreasing with a more accommodative policy (experiment 6). When the traditional component of monetary policy is more accommodative (aggressive), a tax rule on debt will make foreign borrowing relatively more (less) expensive compared to domestic ones in period 0, leading to a more (less) depreciated currency that worsens (improves) the initial debt position, and has negative (positive) impact on consumption.

Perhaps surprisingly, the trade off between monetary and financial stability seemingly disappears when prudential policy is implemented adding debt to the interest rate rule (experiments 10 and 11). With both aggressive and accommodative reaction to inflation, this policy rules remove the constraint in both states and achieve a level of welfare just below the level in the corresponding unconstrained economies. Key to interpreting this result is to note that augmenting the rules with debt is active only in normal times (in period zero) in the experiments, and does not interfere with the interest rate setting when the occasional crisis occurs in period 1. An aggressive traditional monetary policy, instead, would react strongly to inflation in both normal and crisis times and, in equilibrium, induces excessively low interest rates in both period 0 and 1 (experiment 8). This in turns lead to a pattern of asset price behavior in period zero that is inferior compared to the one with the prudential component in the interest rule (experiment 11) from a welfare perspective; thus, suggesting that the non-linear feature of the prudential interest rate rule is the key to its good welfare properties.

5 Conclusions

In this paper we set up a model with both a nominal rigidity and a financial friction and we study their general equilibrium interaction. Both frictions are specified in a manner that is consistent with two separate strands of literature which have focused on macroeconomic
and financial stability separately. While acknowledging that analysis and the results are explorative and illustrative rather than definitive, we find that in a model with financial friction a policy of price stability may be dominated by a policy in which inflation is positive (in producer inflation terms) and that there might be a trade off between macroeconomic and financial stability when the traditional component of monetary policy is aggressive. We also find that a separate tax rule on debt is welfare improving only when there is such a trade off between macroeconomic and financial stability, which is (not) the case only when the traditional component of monetary policy is aggressive (accommodative). Adding a debt component to a traditional interest rate rule is always welfare increasing when both model frictions are active regardless of the nature of the traditional component of monetary policy.

The analysis in the paper highlights the difficulties of reaching general and definitive conclusions on the design of macro-prudential policies as the interactions with the traditional component of monetary policy are complex and highly non-linear. From a practical perspective, our results suggest that booming small open economies might need macro prudential policies if they run a monetary policy with an aggressive traditional component but would not stand to gain much if the traditional component of their monetary policy is accommodative.
References


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[16] Lambertini, Mendicino and Punzi (2011), "Leaning Against Boom-Bust Cycles in Credit and Housing Prices" manuscript.


A Appendix: the equilibrium conditions

In this appendix we list the equilibrium conditions for our general model. Equilibrium in non-tradeable goods market requires:

\[ z_t^N (L_t^N)^\delta = C_t^N \]  \hspace{1cm} (27)

In determining the goods market equilibrium condition for tradeable we use the assumptions that labor is in fixed labor supply and is normalized to 1, that domestic bonds are traded only among domestic household and the asset that serves as a collateral is in fixed supply. Firms’ profits are given by

\[ F_t = P_t^N z_t^N (L_t^N)^\delta - W_t L_t^N + \frac{1}{n} \int_0^n \left( p_t(z) y_t(z) - \frac{W_t y_t(z)}{z_t^N} \right) dz \]
\[ = P_t^N C_t^N - W_t L_t^N + P_t^H Y_t^H - W_t \frac{1}{n} \int_0^n l_t^T(z) dz. \]

As we also have a fixed total labor supply

\[ L_t = L_t^N + \frac{1}{n} \int_0^n l_t^T(z) dz = 1. \]  \hspace{1cm} (28)

Assuming that domestic-currency denominated bonds are trades only among domestic households we have

\[ \int_0^n B(i) di = 0. \]

As the asset \( A \) is in fixed supply \( (A_{t+1} = A_t = 1) \). So the resource constraint in the tradeable sector becomes:

\[ P_t^H C_t^H + P_t^F C_t^F + S_t B_{t+1}^* = S_t B_t^* (1 + i_{t-1}^*) + D_t + P_t^H Y_t^H, \]  \hspace{1cm} (29)

where \( D_t \) is the dividend flow from holding the fixed asset and it is assumed to be exogenously given. By using the demand equation for tradeable goods, we can then rewrite the resource constraint for tradeables by as:

\[ P_t^T C_t^T + S_t B_{t+1}^* = S_t B_t^* (1 + i_{t-1}^*) + D_t + P_t^H Y_t^H. \]
### Table 1. Model Parameters, Initial Conditions, And Stochastic Process

#### Structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of substitution between tradable and non-tradable goods</td>
<td>$\kappa = 1$</td>
</tr>
<tr>
<td>Relative weight of tradable and non-tradable goods</td>
<td>$\omega = 0.5$</td>
</tr>
<tr>
<td>Elasticity of substitution between home and foreign tradable goods</td>
<td>$\theta = 1$</td>
</tr>
<tr>
<td>Relative weight of home tradable goods</td>
<td>$v = 0.75$</td>
</tr>
<tr>
<td>Size</td>
<td>$n = 0.0$</td>
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<tr>
<td>Openess</td>
<td>$\gamma = 0.25$</td>
</tr>
<tr>
<td>Elasticity of substitution within home tradables</td>
<td>$\sigma = 6$</td>
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<tr>
<td>Labor share in production</td>
<td>$\delta = 0.75$</td>
</tr>
<tr>
<td>Credit constraint parameter</td>
<td>$\psi = 2.267$</td>
</tr>
<tr>
<td>Share of firms resetting prices</td>
<td>$\alpha = 0.5$</td>
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<tr>
<td>Intertemporal substitution and risk aversion</td>
<td>$\rho = 1$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 1$</td>
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<tr>
<td>Inflation coefficient</td>
<td>$\phi_\pi = 1.5$</td>
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<tr>
<td>Debt coefficient in augmented interest rate rule</td>
<td>$\phi_B = 0.01$</td>
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<tr>
<td>Debt coefficient in tax rule</td>
<td>$\phi_B = -0.01$</td>
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</table>

#### Exogenous variables

<table>
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<tr>
<th>Variable</th>
<th>Value</th>
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<tr>
<td>World real interest rate</td>
<td>$i^* = 0$</td>
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<tr>
<td>Technology levels</td>
<td>$z^N = z^T = 1$</td>
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<tr>
<td>Dividend</td>
<td>$D_1 = D_2 = D_3 = 0.5$</td>
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<tr>
<td>Initial debt position</td>
<td>$B^*_0 = -3.76$</td>
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<tr>
<td>Terminal exchange rate level</td>
<td>$S_2 = 1$</td>
</tr>
<tr>
<td>Foreign prices</td>
<td>$P^* = P_0^{F*} = P_1^{F*} = P_2^{F*} = 1$</td>
</tr>
</tbody>
</table>

#### Tradable Productivity Markov Process

<p>| States | ${0.9, 1.1}$ |
| Transition probabilities | ${0.4, 0.6; 0.4, 0.6}$ |</p>
<table>
<thead>
<tr>
<th>Prices</th>
<th>Flexible</th>
<th>Sticky</th>
</tr>
</thead>
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<tr>
<td>Borrowing Constraint</td>
<td>Unconst.</td>
<td>Const. 1.5</td>
</tr>
<tr>
<td>Monetary policy (Inflation coefficient in interest rate)</td>
<td>None (1)</td>
<td>None (2)</td>
</tr>
<tr>
<td>Prudential policy</td>
<td>None (3)</td>
<td>None (4)</td>
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<tr>
<td>Experiment number</td>
<td>None (5)</td>
<td>None (6)</td>
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<td></td>
<td>Unconst. 1.5</td>
<td>Const. 2</td>
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<tr>
<td>Utility</td>
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<tr>
<td>Credit multiplier in bad state</td>
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<td>0.0000</td>
</tr>
<tr>
<td>Credit multiplier in good state</td>
<td>0.0000</td>
<td>0.0784</td>
</tr>
<tr>
<td>Consumption</td>
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<td>0.1430</td>
</tr>
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</tr>
<tr>
<td>Period 1, bad state</td>
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<tr>
<td>Period 1, good state</td>
<td>0.0507</td>
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<tr>
<td>Tradable resource constraint in period 0 (in units of consumption)</td>
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</tr>
<tr>
<td>Tradable consumption</td>
<td>0.2540</td>
<td>0.0710</td>
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<tr>
<td>Current account</td>
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<td>0.5550</td>
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<td>Tradable resource constraint in period 1, good state (in units of consumption)</td>
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<td>0.4360</td>
</tr>
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<td>Tradable consumption</td>
<td>0.2550</td>
<td>0.1670</td>
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<tr>
<td>Current account</td>
<td>0.0670</td>
<td>0.2690</td>
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<tr>
<td>Tradable resource constraint in period 1, bad state (in units of consumption)</td>
<td>0.3450</td>
<td>0.6560</td>
</tr>
<tr>
<td>Tradable consumption</td>
<td>0.2730</td>
<td>0.0550</td>
</tr>
<tr>
<td>Current account</td>
<td>0.0720</td>
<td>0.6010</td>
</tr>
<tr>
<td>PPI</td>
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<tr>
<td>Period 0, level</td>
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<tr>
<td>Period 1, % change, bad State</td>
<td>57.80</td>
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<td>Nominal Interest Rate (% per period)</td>
<td>140.50</td>
<td>20.20</td>
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<td>Period 0, level</td>
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<td>Period 1, level, good state</td>
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<td>Period 1, % change, bad state</td>
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<td>Period 1, % change, good state</td>
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<tr>
<td>Nominal Asset Price</td>
<td>0.2230</td>
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</tr>
<tr>
<td>Period 1, level, bad state</td>
<td>0.2520</td>
<td>0.6680</td>
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<tr>
<td>Period 1, % change, bad state</td>
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<td>-65.90</td>
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<tr>
<td>Period 1, % change, good state</td>
<td>-18.00</td>
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<tr>
<td>Borrowing</td>
<td>-0.2700</td>
<td>-2.2840</td>
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<tr>
<td>Period 0 (nominal level, domestic currency--S0*B1)</td>
<td>-0.0960</td>
<td>-1.0530</td>
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<tr>
<td>Period 0 (units of consumption--S0*B1/P0))</td>
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<td>Period 0 (share of cons. exp.--(S0*B1)/P0C0))</td>
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<td>-16.00</td>
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<td>In good state</td>
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<td>Period 1 (share of cons. exp.--(S1*B2)/P1))</td>
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<tr>
<td>Period 1 (share of cons. exp.--(S1*B2)/P1C1))</td>
<td>-1.50</td>
<td>-145.90</td>
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</table>