Optimal Stabilization Policy in a Model with Endogenous Sudden Stops:

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Abstract

We develop a framework to study optimal stabilization policy in an economy with an occasionally binding credit constraint. The objective of the paper is to provide a framework to understand both the optimal response to a ‘sudden stop,’ as well as the behavior of optimal policy outside of the crisis period. In the model, the policy instrument of the government is a distortionary tax wedge on consumption of non-tradable goods. We find that, for a plausible calibration of the model, the optimal policy is highly nonlinear. If the constraint is not binding, the optimal tax rate is zero, as in an economy without credit constraint. If the constraint is binding, the optimal tax rate is negative, meaning that the government subsidizes non tradable consumption.
1 Introduction

Much attention has been devoted to understanding the causes of the periodic crises that haunt emerging market countries. Some of these episodes, labeled ‘sudden stops’ (see Calvo, 1998), are characterized by a sharp reversal in private capital flows, large drops in output and consumption coupled with large decline in asset prices and the real exchange rates. Progress has been made in understanding optimal policy responses in models in which the economy is in a sudden stop.\footnote{See Braggion, Christiano, Roldos (2007), Caballero and Panageas (2007), Christiano, Gust, and Roldos (2004), Cúrdia (2007), Caballero and Krishnamurthy (2005) and Devereaux and Poon (2004) on the monetary policy response to these crisis periods.}
The focus of this paper is to address the issue of optimal stabilization policy for an economy that might be subject to a sudden stop, so as to provide direction on how stabilization policy should be designed both for the tranquil periods in which emerging markets spend most of the time, as well as periods when a crisis is looming on the horizon or the economy is in a sudden stop. That is, we investigate optimal stabilization policy in an environment in which the access to international capital markets is not only incomplete but might also be suddenly curtailed.\footnote{Durdu and Mendoza (2005) analyze broad alternative policy strategies in such an environment. Adams and Billi (2006) study optimal monetary policy in a very simple new Keynesian model in which the zero bound constraint is occasionally binding. Benigno, Otrok, Rebucci and Young (2007) compare the welfare properties of alternative interest rate rules in a model in which there is also a nominal rigidity.}

In modelling the possibility of a sudden stop, we follow the contributions by Mendoza (2000, 2002) and consider a two-sector (tradable and non-tradable) small open economy in which financial markets are not only incomplete, but also imperfect because access to foreign financing is intermittent and occasionally constrained. We assume that international borrowing cannot be made state-contingent, because the asset menu is restricted to a one period risk-less bond paying off the exogenously given foreign interest rate. In addition, we assume that the international borrowing is constrained by a fraction of households’ total income. Therefore, the actual credit limit is endogenous since domestic agents’ ability to borrow from foreigners is limited by the endogenous evolution of income and prices. In this setting, the occurrence of a ‘sudden stop’ (i.e. the situation in which the international borrowing constraint becomes binding) is an endogenous outcome of the model depending
on the history and state of the economy.

The simpler version of our model considers only one source of shocks (i.e. to the tradable endowment), abstract from capital accumulation, and allow for a non-distortionary financing of the policy through lump-sum taxes. We then extend our analysis to the case of endogenous capital accumulation in which the financing of the optimal policy is distortive through capital income tax.

In this class of models, agents self insure against the low-probability but high cost possibility of a sudden stop generated by the occasionally binding credit constraint. This is through precautionary saving and associated accumulation of net foreign assets. Our goal is to explore both the optimal policy response to the sudden stop and how this precautionary savings motive affects the design of the policy rules in tranquil times.

In the optimal policy analysis, we focus on a tax wedge on non-tradable consumption, with a balanced budget rule by lump sum transfers. We first compute the optimal policy and then compare it to alternative simple, non-contingent tax rules. One important result of the analysis is that the optimal stabilization policy is highly non-linear. If the credit constraint is not binding, optimal policy would mimic the one that would arise in an economy without a credit constraint (zero tax rate in our simple model). If the liquidity constraint is binding, the optimal tax rate is negative, meaning that the government subsidizes non-tradables consumption, thereby supporting both the demand and the supply of non-tradable goods in the economy.

The paper is most closely related two broad strands of literature. The first strand focuses on financial frictions that may help replicate the main features of the business cycle in emerging market economies—e.g., Mendoza (1991, 2002), Neumeyer and Perri (2005) and Oviedo (2006). The second strand focuses on the analysis of optimal fiscal policy in dynamic general equilibrium models (see, for example, Chari and Kehoe, 1999; Schmitt-Grohé and Uribe, 2004). While studies of emerging market business cycles can provide a realistic description of the economic environment in which these economies operate, the question of how policy should be set in such environments remain open. With respect to this literature our contribution consists in studying the optimal policy in the presence of a general form of
financial friction.

Our analysis is also related to the growing literature on the interaction between house prices, borrowing constraints and the role of monetary policy (see for example Iacoviello, 2005). Most of these works assume that the collateral constraint expressed in terms of the value of the house stock is always binding, while the solution methods we implement would allow for examining the situation in which a borrowing constraint might not bind in equilibrium.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses its calibration and solution. Section 4 presents the optimal policy results. Section 5 reports an extensive sensitivity analysis. Section 6 concludes. Technical details, including on the numerical algorithm we use to solve the model, are in appendix.

2 Model

This section simplifies the two-good, small open, production economy, with a form of liability dollarization and an occasionally binding credit constraint, originally proposed by Mendoza (2002). Compared to that model, we consider only one source of disturbance, to aggregate productivity in the tradable sector of the economy, and we allow for distortionary tax rate on non-tradable consumption. The specification of endogenous discounting is also simplified by assuming that the agents’ discount rate depends on aggregate consumption as opposed to the individual one, as in Schmitt-Grohé and Uribe (2003).³

2.1 Households

There is a continuum of households \( j \in [0, 1] \) that maximize the utility function

\[
U^j \equiv E_0 \left\{ \sum_{t=0}^{\infty} \exp (-\theta_t) u \left( C_t^j - z (H_t^j) \right) \right\},
\]

³So, our formulation corresponds to what Schmitt-Grohé and Uribe (2003) call “endogenous discount factor without internalization. Due to precautionary savings it may not be necessary in the stochastic model.
with $C^j$ denoting the individual consumption basket and $H^j$ the individual supply of labor. We assume that the expected utility includes an endogenous discount factor as:

$$\theta_t = \theta_{t-1} + \beta \ln \left( 1 + C \left( C^T_t, C^N_t \right) - z \left( H_t \right) \right)$$

$$\theta_0 = 1,$$

with $C$ denoting aggregate per capita consumption that the individual household takes as given.\(^4\)

The functional form of the period utility function is,

$$u \left( C \left( C^T_t, C^N_t \right) - z \left( H_t \right) \right) = \frac{1}{1-\rho} \left( C_t - \frac{H^\delta_t}{\delta} \right)^{1-\rho}, \quad (2)$$

omitting for simplicity the superscript $j$, and where $\delta$ is the elasticity of labor supply with respect to the real wage, and $\rho$ is the coefficient of relative risk aversion. The consumption basket $C$ is a composite of tradable and non-tradable goods:

$$C_t = \frac{1}{\kappa} \left( C^T_t \right)^{\frac{\kappa-1}{\kappa}} + \frac{1-\omega}{\kappa} \left( C^N_t \right)^{\frac{\kappa-1}{\kappa}} \left( C^N_t \right)^{\frac{\kappa-1}{\kappa}}. \quad (3)$$

The parameter $\kappa$ denotes the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while $\omega$ represents a weighting factor. The corresponding aggregate price index is given by

$$P_t = \left[ \omega + (1 - \omega) \left( P_N^t \right)^{1-\kappa} \right]^\frac{1}{1-\kappa};$$

the price of tradables is normalized to 1.

Households maximize utility subject to the following period budget constraint expressed in units of tradable consumption (where again for simplicity we omit the superscript $j$):

$$C^T_t + \left( 1 + \tau^N_t \right) P_t^N C^N_t = \pi_t + W_t H_t - B_{t+1} - (1 + i) B_t - T^T_t - P_t^N T^N_t, \quad (4)$$

where $W_t$ is the real wage, $B_{t+1}$ denotes the amount of bonds issued with gross real return $1+i$, $\tau^N_t$ is a distortionary taxes on non-tradables consumption, and $T^T$ and $T^N$ are lump

\(^4\)Endogenous discounting pins down a well defined net foreign asset position in the deterministic steady state of the model.
sum taxes in units of tradables and non-tradables, respectively. $\pi_t$ represents per capita firm profits and $W_t H_t$ represents the household labor income.

International financial markets are incomplete and access to them is also imperfect. International borrowing cannot be made state-contingent, because the asset menu includes only a one period bond denominated in units of tradable consumption, paying off the exogenously given foreign interest rate. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geq - \frac{1 - \phi}{\phi} [\pi_t + W_t H_t].$$

(5)

This constraint (5) depends endogenously on the current realization of profits and wage income. One important feature of (5), is that it captures the effects of liability dollarization since foreign borrowing is denominated in units of tradables while part of the income that serves as a collateral comes from the nontradables sector. Roughly speaking, this constraint assumes that only a fraction of current income can be effectively claimed in the event of default, so lenders are unwilling to permit borrowing beyond that limit.

Our borrowing constraint is not derived as the outcome of an optimal credit contract. As emphasized in Mendoza (2002), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, that would be the outcome of the interaction between a borrower and a risk-neutral lender in a contracting framework as in Eaton and Gersovitz (1981).

Households maximize (1) subject to (4) and (5) by choosing $C_t^N, C_t^T, B_{t+1},$ and $H_t$. The first order conditions of this problem are the following:5

$$\frac{C_{C_t^N}}{C_{C_t^T}} = (1 + \tau_t^N) P_t^N,$$

(6)

$$u_{C_t} C_{C_t^T} = \mu_t,$$

(7)

5We denote with $C_{C_t^N}$ the partial derivative of the consumption index $C$ with respect to non-tradable consumption. $u_C$ denotes the partial derivative of the period utility function with respect to consumption and $z_H$ denotes the derivative of labor disutility with respect to labor.
\[
\mu_t + \lambda_t = \exp\left(-\beta \ln\left(1 + C\left(C_t^T, C_t^N\right) - z(H_t)\right)\right) (1 + i) E_t[\mu_{t+1}],
\]  
(8)

and

\[
z_H(H_t) = C_{C_t^T} W_t \left[1 + \frac{\lambda_t}{\mu_t} \frac{1 - \phi}{\phi}\right].
\]  
(9)

\[\mu_t \text{ and } \lambda_t \text{ are the multipliers on the budget and liquidity constraint, respectively. As usual, the relevant transversality conditions are assumed to be satisfied. Equation (6) determines the optimal allocation of consumption across tradable and nontradable goods by equating the marginal rate of substitution between } C_t^N \text{ and } C_t^T \text{ with the relative price of non-tradable and the ratio of distortionary taxation in the two sectors. (7) determines the multiplier } \mu_t \text{ in terms of the marginal utility of tradable consumption. Equation (8) is obtained from the optimal choices of domestic and foreign denominated bonds. Note when the multiplier on the international borrowing constraint is positive (i.e. the constraint is binding), the standard Euler equation incorporates a term } \lambda_t \text{ that can be interpreted as country-specific risk premium on external financing. Finally (9) determines the optimal supply of labor as a function of the relevant real wage and the multipliers. It is important to note that the presence of the international borrowing constraint increases the marginal benefit of supplying one unit of labor since this improves agents’ borrowing capacity.}

\subsection{2.2 Firms}

Our small open economy is endowed with a stochastic stream of tradable goods, \(\exp(\varepsilon_t^T)Y_t^T\), where \(\varepsilon_t^T\) is a random Markov disturbance, and produces non-tradable goods, \(Y_t^N\). Unlike Mendoza (2002), we assume that \(\varepsilon^T\) follows a standard autoregressive process of the first order, AR(1). However, we abstract from other sources of macroeconomic uncertainty, such as shocks to the technology for producing non-tradables, the world interest rate, and the tax rate for simplicity.

Firms produce non-tradables goods \(Y_t^N\) based on the following Cobb-Douglas technology

\[
Y_t^N = AK^\alpha H_t^{1-\alpha},
\]
where $K$ is a constant level of capital stock, and $A$ is a scaling factor. Since capital stock is given, the firm’s problem is static and current-period profits ($\pi_t$) are:

$$\pi_t = \exp\left(\varepsilon^T_t\right) Y^T + P_t^N AK^\alpha H_{1-\alpha} - W_t H_t.$$ 

The first order condition for labor demand, in equilibrium, gives:

$$W_t = (1 - \alpha) P_t^N AK^\alpha H_t^{-\alpha},$$

so the real wage ($W_t$) is equal to the value of the marginal product of labor.

### 2.3 Government

The government runs a balanced budget in each period, so that the consolidated government budget constraint is given by

$$\exp(G^T_t) + P_t^N \exp(G^N_t) = \tau_t^N P_t^N C_t^N + T_t^T + P_t^N T_t^N.$$ 

Stabilization policy is implemented by means of a distortionary tax rate $\tau_t^N$ on private domestic non-tradables consumption.

Movements in the primary fiscal balance are offset via lump-sum rebates or taxes. Specifically, we assume that the government keeps a constant level of non-tradable expenditure financed by a constant lump-sum tax (i.e. $\exp(G^N_t) = T^N$). Thus, changes in the policy variable $\tau^N$ are financed by a combination of changes in the lump-sum transfer on tradables, $T_t^T$, and the endogenous response of the relative prices, for given public expenditures on tradable and non-tradables. This simplifying assumption implies that we abstract from the important practical issue of how to finance changes in the tax rate in the case in which they are negative (i.e., $\tau^N < 0$ is a subsidy). This allows us to focus on the implications of the occasionally binding constraint for the design of the tax policy abstracting from other optimal tax policy considerations. We study the implication of distortionary financing of the optimal policy in the last section of the paper.

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6 We analyze a model with endogenous capital accumulation in the last section of the paper.

7 Mendoza and Uribe (2000) emphasize how movements in this tax rate can approximate some of the effects induced by currency depreciation in monetary models of exchange rate determination. As such, it captures one important aspect of monetary policy in emerging markets, which is distinct from the more conventional role of monetary policy in the presence of nominal rigidities.
2.4 Aggregation and equilibrium

We now consider the aggregate equilibrium conditions. Combining the household budget constraint, government budget constraint, and the firm profits we have that the aggregate constraint for the small open economy can be rewritten as

\[ C^T_t + P^N_t C^N_t + B_{t+1} = \exp(\varepsilon^T_t) Y^T_t + P^N_t Y^N_t + (1 + i) B_t - \exp(G^T_t) - P^N_t \exp(G^N_t), \]

and the equilibrium condition in the non-tradeable good sector is

\[ C^N_t + \exp(G^N_t) = Y^N_t = AK^\alpha H_1^{1-\alpha}. \tag{11} \]

Combining these two equations we have

\[ C^T_t = Y^T_t - \exp(G^T_t) - B_{t+1} + (1 + i) B_t, \tag{12} \]

that represents the evolution of the net foreign asset position as if there were no international borrowing constraint. In this model though, using the definitions of firm profit and wages, the liquidity constraint implies that the amount that the country as a whole can borrow is constrained by a fraction of the value of its GDP:

\[ B_{t+1} \geq -\frac{1 - \phi}{\phi} \left[ \exp(\varepsilon^T_t) Y^T_t + P^N_t Y^N_t \right]. \tag{13} \]

3 Calibration and solution

In this section we discuss the calibration of model parameters, as well as the solution method. The calibration of the model is reported in Table 1 and largely follows Mendoza (2002), who calibrates his model to the Mexican economy. We set \( Y_T = 1 \) as a normalization of output and set \( i = 0.0159 \), which yields an annual real rate of interest of 6.5 percent. The three elasticities are set to typical values for these parameters, \( \kappa = 0.760 \), \( \rho = 2 \) and \( \delta = 2 \). In a later section we show that our results are not sensitive to the choice of these parameters over a reasonable range. The credit constraint is set at \( \phi = 0.74 \), so that the constraint is almost binding in the deterministic steady state of the model. Government spending is set
as 1.7 percent of output in tradable sector and 14.1 percent of output in the nontradable sector. The tax rate on non tradable consumption in the steady state and in the model without optimal policy is fixed at $\tau = 0.0793$. The optimal policy is then compared to the competitive equilibrium allocation in a model with set to $\tau = 0.0793$ at every date and state. The labor share of production in the non tradable output sector is $\alpha = 0.636$. We then set $\beta$, $\omega$, and $AK^\alpha$ to obtain the steady state foreign borrowing to GDP ratio of 35 percent, a steady state ratio of tradable to non-tradables output of 64.8 percent, and a steady state relative price of non-tradables equal to one. This results in slightly lower value of the discount rate, $\beta = 0.0177$, than in Mendoza (2002).

The stochastic shock in the model is a shock to tradeable output, which we model as an AR(1) process. Specifically, the shock process is $\varepsilon_t$ is

$$\varepsilon_t = \rho \varepsilon_{t-1} + \sigma_n n_t,$$

where $n_t$ is an iid $N(0,1)$ innovation, and $\sigma_n$ is a scaling factor. The parameters of the AR(1) process are chosen to match the standard deviation and serial correlation of tradeable output in Mexico of 3.36 percent and 0.553, respectively. These are the same moments that Mendoza (2002) matched with a discrete two state Markov chain.

In order to compute the competitive equilibrium of the economy without optimal policy (i.e., $\tau_t^N = 0$ for all $t$), we solve a quasi-planner problem that satisfies the following Bellman equation:

$$V(B_t, \varepsilon^T_t) = \max_{B_{t+1}} \left\{ u(C_t - z(H_t)) + \exp (-\beta \ln (C_t - z(H_t))) E[V(B_{t+1}, \varepsilon^T_{t+1})] \right\}.$$  

(14)

The constraints on this problem are the competitive equilibrium conditions (??)-(??), the aggregate consumption definition (3), and the credit constraint (5). To solve the constrained problem, we use a spline parameterization for the value function, solve the maximization using feasible sequential quadratic programming methods, and solve for the fixed point using value iteration with Howard improvement step.\(^9\)

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\(^8\)To maintain consistency with the model that has capital accumulation later in the paper, a fixed fraction of output in every period is assumed to be investment.

\(^9\)The algorithm for solving the problem is described in Appendix B.
Figure 1 plots the equilibrium decision rule (or policy function) for gross foreign borrowing, $B_{t+1} = g\left(B_t, \varepsilon_t^T\right)$. This intersects the 45° line at the boundary of the constrained region; that is, if the economy perpetually received this realization of the shock, it would converge to a level of external debt for which the credit constraint is just binding. If the economy happened to find itself in the interior of the constrained region, it would diverge to $B = -\infty$, violating the implicit transversality condition that requires long-run solvency. Therefore the decision rules must be truncated at the boundary. This divergence would occur in any state in which there exists a positive probability of entering the interior of the constrained region; with an AR(1) process this probability is always positive.

Figure 2 compares the equilibrium decision rules with and without the credit constraint for the real wage ($W_t$), the relative price of non tradable ($P_t^N$), aggregate consumption ($C_t$), employment ($H_t$). The first point to observe is that both employment and consumption are lower in the presence of the credit constraint, reflecting precautionary savings motives driven by the possibility of hitting the credit constraint. Consumption is lower because households save a higher fraction of their income to accumulate foreign assets. The reduction in consumption drives down the relative price of non-tradables, reducing the real wage and lowering labor supply. These decision rules also illustrate the sharp decline in aggregate activity generated by a sudden stop in the model: as the NFA position of the economy approaches the constraint, both consumption and employment drop dramatically, dragging the relative price of non-tradables and the real wage lower. The shape of these functions is important when discussing the cause and the consequences of a sudden stop in this model. Sudden stops are caused by a succession of bad shocks in the model, not by one single and large bad draw, but the effects are felt before the economy reaches the constraint’s binding region.

\(^{10}\) All decision rules reported are conditional on the value of the shock which corresponds to the negative of the standard deviation of its marginal distribution. As there is only one asset, gross and net foreign liabilities or assets (NFA) coincide.

\(^{11}\) With the preferences assumed here, there is no wealth effect on labor supply, so the substitution effect induced by a lower cost of leisure is the only effect operative.

\(^{12}\) Concave consumption functions are a standard prediction of buffer stock models of precautionary savings (e.g., Carroll, 2004).
Precautionary saving induced by the constraint is quantitatively significant in the model. For instance, the average NFA position in the ergodic distribution of the economy with no collateral constraint is \( B = -3.0 \) (or about -30 percent of annual GDP), while in the economy with the constraint \( B = -2.37 \) (or about -20 percent of annual GDP). This difference is large, considering the small shocks that hit this economy and the relatively low degree of risk aversion. In contrast, Aiyagari (1994) finds that measured uninsurable idiosyncratic earnings risk, which is an order of magnitude larger than the shocks considered here, generates only a 3 percent increase in the aggregate capital stock. The main reason why precautionary savings is larger here is that the return on saving is fixed in our model (i.e. the foreign interest rate is exogenous). Additional saving does not reduce its return, a mechanism that tends to weaken precautionary balances in a closed economy setting, such as the one studied by Aiyagari (1994).

4 Optimal stabilization policy

To solve the optimal tax problem, the only change needed is to introduce a second control into the problem to (14) – the tax rate – yielding the functional equation

\[
\tilde{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}, \tau_t^N} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t)))E \left[ V^n(B_{t+1}, \varepsilon_{t+1}^T) \right] \right\}. 
\]

The decision rule for \( \tau^N \), as well as for the lump sum transfer over GDP \( (P_t^N T_t^N / Y) \), which illustrates the potential financing requirements of the optimal policy, are plotted in Figure 3. Figure 4 plots the same decision rules reported in Figure 2, with and without optimal policy for \( \tau^N \). Figure 5 plots the decision rule for foreign borrowing with and without the optimal policy, while Figure 6 reports the ergodic distribution of foreign borrowing for different model economies.

The optimal policy is highly non-linear (Figure 3). The optimal policy in response to a sudden stop is a subsidy, and its effect is to “lift” the decision rules for any level of the endogenous state, and hence permitting more borrowing. With such a subsidy, demand and to a lesser extent supply for non-tradable goods increases, as a result the relative price of
non-tradables goods rises, yielding higher non-tradables output and hence collateral for the 
credit constraint (Figure 4). And more collateral permits the economy to borrow more than 
it otherwise could do in the absence of a policy response.

The optimal policy in tranquil times is to set $\tau^N = 0$, like in the unconstrained economy. 
In the model without collateral constraint, there is no policy trade off, and setting $\tau^N = 0$ is 
always optimal, despite the incompleteness of the international asset market. The tax wedge 
$\tau^N$ does not affect the intertemporal decisions in the model and hence has no role to play to 
mitigate the consequence of market incompleteness. In the model with the constraint, there 
is a trade-off between efficiency (i.e., to minimize marginal distortions by setting $\tau^N = 0$), 
and the need to reduce the probability of hitting the constraint or relax it when it is binding. 
Therefore, when the constraint (??) is not binding, the optimal policy is $\tau^N = 0$ as in the 
unconstrained economy.

Figure 5 shows that the optimal policy of $\tau^N$ is such that the liquidity constraint becomes 
“just binding”; that is, the policy function for $B_t$ is tangent to the binding region and the 
corresponding multiplier $\lambda_t$ of the liquidity constraint remains 0. The goal of optimal policy 
is to distort the economy as little as possible, and any deviation of the shadow price of 
foreign borrowing from zero is costly. Therefore the planner relaxes the constraint just 
enough to make it non-binding. But the constraint is not relaxed beyond this, because that 
involves additional distortions that are welfare-reducing if the constraint is not restricting 
consumption possibilities further.

To see the intuition for these results, note that optimal policy has two roles in the model. 
The first role for policy is related to the occasionally binding credit constraint, and there are 
two goals for policy in the presence of the this credit constraint (though not exclusive)—to 
reduce the probability of reaching the constraint and to minimize the effects when it binds. 
We find that optimal policy actually achieves both of them. It lowers the likelihood of 
entering the binding region and it mitigates the effects of the binding constraint. It does so 
by “moving” the constraint outward, so as to make the multiplier exactly zero in all periods, 
in both binding and nonbinding states. Second, as in other incomplete market models (such 
as Aiyagari 1995), it may be possible to increase welfare by choosing policies that reduce
agents precautionary savings. This role for policy is independent of the presence of the credit constraint and relies only on the general inefficiency of incomplete market models.\textsuperscript{13} We find that policy is not affected by this second motive.

The average foreign borrowing position in the ergodic distribution of the economy is not affected significantly by optimal policy. As Figure 6 shows, relative to the no-stabilization case (with the credit constraint), average foreign debt and total GDP increases by about 3 percent under the optimal policy, to yield a constant foreign debt position of about -20 percent of GDP. However, the probability of hitting the constraint in the ergodic distribution decreases by about 15 percent, from 0.6 percent without optimal tax policy to 0.5 percent with it.

The optimal policy is state-contingent, requiring knowledge of the unobservable shocks for its implementation. We therefore also explore the impact of simple, constant subsidy rules that are not state contingent, and can be easily financed (meaning relatively small). Table 2 shows the welfare gains from moving from the economy without policy to the ones with non-state contingent policy that we consider (\(\tau = 0, -0.05, -0.10\)). This shows that the fixed tax rule moves the economy in the direction of the optimal policy. Interestingly, this suggests that a small overvaluation, which effectively subsidizes consumption of non-tradable goods, may be a desirable policy option.

5 Sensitivity analysis

In this section we explore the robustness of the optimal policy results to alternative values of key structural parameters, as well as of the stochastic process for the tradable endowment. From the outset, it is important to mention that none of these changes affect the main result, namely the absence of a precautionary component in the optimal policy. This suggests that the result is a robust qualitative feature of the model. The inner working of the model, as illustrated by the decision rules for the main endogenous variables, is also fairly robust to

\textsuperscript{13}Our model also features an externality—the endogenous discount factor depends on aggregate consumption, and therefore agents do not internalize the effect of current consumption and labor supply on discounting. But this effect should be minor since the discount factor is nearly inelastic.
alternative parameterization.

As the optimal policy hinges on the labor effort behavior and the substitutability between tradable and non-tradable goods in consumption, it is important to consider alternative values for $\kappa$ and $\delta$. A second set of parameters potentially affecting the working of the model include the degree of risk aversion ($\rho$), the tightness of the credit constraint in the deterministic steady state ($\phi$), and finally the parameters governing the stochastic process for the tradable endowment ($\rho_\varepsilon$ and $\sigma_n$ respectively).

We consider four alternative cases for $\kappa$ and $\delta$, two higher values and two lower values than assumed in the baseline, changing only one parameter at a time. Specifically, we consider the following alternative cases: $\kappa = 0.3$ or $\kappa = 0.9$ (less or more substitutability between tradable and non tradable goods in consumption than in the baseline) and $\delta = 1.2$ or $\delta = 5$ (higher and lower labor elasticity than in baseline); and four cases for $\rho = 5$ (more risk aversion), $\phi = 0.5$ (looser constraint in the deterministic steady state and less likely to be occasionally binding), $\rho_\varepsilon = 0.95$ and $\sigma_n = 0.05$ (more persistent or more volatile AR1 process).

The results are summarized in Figure 7. The results are broadly robust, except in the case of a lower labor supply elasticity. When tradables and non-tradables goods become closer substitutes ($\kappa = 0.9$), optimal policy would cut taxes less aggressively compared to the baseline specification. The general principle of optimal policy is to relax the borrowing constraint by increasing the value of collateral when the constraint becomes binding (i.e. by raising $P_t^N Y_t^N$). When the intratemporal elasticity of substitution between tradables and non tradables is higher, it is more efficient to do so by increasing the relative price of non-tradables and decreasing non tradables production. Indeed, for a given relative price of non tradables and a given subsidy, a higher substitutability between tradables and non tradables will push the demand for tradable goods higher. Since tradable output is exogenously given, demand needs to be decreased in order to clear the tradable goods market if the economy cannot borrow from abroad. For a relatively higher $\kappa$ this could be achieved with a relatively lower subsidy. Non-tradables demand will rise relatively more than with a lower $\kappa$ so that the relative price of non tradables is higher, real wages are lower, and non-tradables production is lower since agents will decrease their labor supply (there is only the substitution effect
here determined by the decrease in real wages). The opposite logic applies in the case in which $\kappa = .3$.

When labor supply becomes more elastic ($\delta = 1.2$), optimal policy would cut taxes more aggressively compared to the baseline specification. In this case it is efficient to relax the borrowing constraint by increasing non-tradables production and decreasing the relative price of non-tradables. Indeed, for a given real wage the more elastic is labor supply the higher is production of non-tradables. Equilibrium in the non-tradables goods market is achieved by decreasing the relative price of non-tradables and increasing demand by subsidizing non-tradables consumption more aggressively than in the baseline parametrization. The opposite logic applies in the case in which labor supply is less elastic ($\delta = 5$).

When the constraint is looser ($\phi = 0.5$), the probability that the constraint becomes binding is higher for a given value of $B$, and it is optimal to cut taxes more aggressively. A higher value of the collateral is reached by increasing both the relative price of non-tradables and non-tradables production compared to the baseline specification.

6 Capital accumulation and distortionary financing

This section investigates the role that capital may have in affecting the decision rules and the optimal government policy. The model we use has a fixed capital stock. Allowing for investment and endogenous capital accumulation may require an optimal tax response even when the credit constraint is not binding, and hence introduce a precautionary component in the optimal policy.

The introduction of capital also allows us to investigate alternative specifications of the budget rule and the financing of the optimal policy. In equilibrium, the amount of financing needed to implement the optimal policy is potentially large, and in our model the financing entailed by the (large) consumption subsidy is costless. Hence, a large subsidy can be applied right when the constraint binds. If the budget is balanced by a distortionary capital tax rather than a lump-sum transfer, the government may find it optimal to start using the consumption subsidy before the credit constraint is hit. If there is an increasing cost of the
consumption subsidy, however, the government may want to start with a smaller subsidy away from the constraint that increases as the constraint is approached. Financing the subsidy through a distortionary capital tax is one way to capture this increasing cost.\footnote{In practice, the financing of the expansionary government policy in response to a sudden stop is likely to come from the drawdown of accumulated official reserves, thereby supporting the real exchange rate and the non-tradable sector of the economy. Vice versa, a small precautionary component would result in the accumulation of official reserves over time. However, studying the optimal level of official reserves requires and multiple asset framework, which is behind the scope of this paper.}

6.1 Model changes

To incorporate capital accumulation and capital income taxation, we modify the household and firm problem as follows. Households continue to maximize (1), subject to a modified period budget constraint (still expressed in units of tradable consumption, and omitting the superscript \( j \) for simplicity):

\[
C_t^T + (1 + \tau_t^N) P_t^N C_t^N + \left( K_t - \left( 1 + \Phi \left( \frac{x_t}{K_{t-1}} \right) \right) K_{t-1} \right) \\
= (1 - \tau^K_t) (r_t - \delta) K_{t-1} + W_t H_t - B_{t+1} - (1 + i) B_t - T^T_t - P^N_t T^N
\]

where \( \tau^K_t \) is a distortionary tax on capital income, and \( K_{t-1} \) represents per capita stock of capital that the household owns and rents to firms in period \( t \), \( \delta \) is the depreciation rate, and \( \Phi (.) \) is a cost of installing investment goods, with \( x_t \) representing gross investment.\footnote{The function \( \phi(.) \) is such that, in the deterministic, steady state \( \phi(.) = x/k, \phi'(.) = 1, \) and \( \phi''(.) < 0. \)}

The credit constraint becomes:

\[
B_{t+1} \geq -\frac{1 - \phi}{\phi} \left[ (1 - \tau^K_t) (r_t - \delta) K_{t-1} + W_t H_t \right].
\]

The international borrowing constraint depends on profits as it did in the model without capital accumulation. The difference is that the return to capital is now potentially time varying.

Adding capital accumulation adds a first order condition for \( K_t \), the amount of capital
to carry to next period:

\[
\mu_t \left( 1 + \phi' \left( \frac{x_t}{K_{t-1}} \right) \right) = \exp \left( -\beta \ln \left( 1 + C \left( C_t^T, C_t^N \right) - z \left( H_t \right) \right) \right)
\]

\[
\{ E_t \left[ \mu_{t+1} \left( (1 - \tau_{t+1}^K) (r_{t+1} - \delta) + 1 + \phi \left( \frac{x_{t+1}}{K_t} \right) - \phi' \left( \frac{x_{t+1}}{K_t} \right) \right) \right] \}
\]

where \( \Omega_{t+1} = (1 - \delta) + \phi \left( \frac{x_{t+1}}{K_t} \right) - \phi' \left( \frac{x_{t+1}}{K_t} \right) \left( \frac{x_{t+1}}{K_t} \right) \). All other first order conditions are unchanged.

The firms’ production functions are unchanged, but capital is now endogenous. So, the typical firm maximises its profit by choosing the amount of labor and capital to demand:

\[
\max_{K_t, H_t} \left[ \exp \left( \hat{z}^T T \right) Y^T + P_t^N Y^N - W_t H_t - r_t K_{t-1} \right]
\]

subject to the production function:

\[
Y_t^N = AK_t^\alpha H_t^{1-\alpha}
\]

The corresponding first order conditions are now given by:

\[
W_t = (1 - \alpha) P_t^N A_t \left( \frac{K_{t-1}}{H_t} \right)^\alpha
\]

\[
r_t = \alpha P_t^N A_t \left( \frac{K_{t-1}}{H_t} \right)^{\alpha-1}
\]

As before, firms are wholly owned by domestic households.

Finally the government continues to runs a balanced budget in each period, so that the budget constraint is:

\[
\exp(G_t^T) + P_t^N \exp \left( G_t^N \right) = \tau_t^N P_t^N C_t^N + T_t^T + P_t^N T_t^N + \tau_t^K (r_t - \delta) K_{t-1}.
\]

As before stabilization policy is implemented by means of a distortionary tax rate \( \tau_t^N \) on non-tradables consumption, but now it is financed through capital income taxation (i.e. through changes in \( \tau_t^K (r_t - \delta) K_{t-1} \)), while we keep both lump-sum transfers fixed at their (deterministic) steady state values.
When we calibrate the model with capital we minimize distance between model with and without capital. Specifically, we keep all the calibration settings here the same as in the model without capital, except that we do not impose the size of nontradable investment in the economy. Additionally, we set the capital stock in the steady state to be 1.45 times the annual GDP and the capital depreciation to be 21.7 percent of GDP (Parameter and the steady state values are in appendix.)

6.2 Results
<To be completed>

7 Conclusions

In this paper we studied optimal stabilization policy in a small open economy in which there is the risk of an endogenous Sudden Stop due to the presence of an occasionally binding credit constraint. We find that, for a plausible calibration of the model, the optimal policy is non linear. If the liquidity constraint is not binding, the optimal tax rate is zero, in the absence of other distortions, like in an economy without a credit constraint. This suggests that, to a first approximation, stabilization analysis for the tranquil times can be conducted in more conventional models. If the liquidity constraint is binding, however, the optimal tax rate is negative, meaning that the government should subsidize non-tradables consumption.
A Appendix

This appendix reports the model steady state and provides the details of the numerical algorithm we use to solve the model and compute optimal policy.

A.1 Steady state

The deterministic steady state equilibrium conditions are given by the following set of equations. The first four correspond to the first order conditions for the household maximization problem,

\[
\left(\frac{1 - \omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{C^T}{C^N}\right)^{\frac{1}{\kappa}} = (1 + \tau^N)P^N,
\]

\[
\left(C - \frac{H^\delta}{\delta}\right)\omega^{\frac{1}{\kappa}} \left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}} = \mu,
\]

\[
\left(1 + \frac{\lambda}{\mu}\right) = \exp\left(-\beta \ln\left(1 + C - \frac{H^\delta}{\delta}\right)\right)(1 + i),
\]

\[
H^{\delta-1} = W\omega^{\frac{1}{\kappa}} \left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}} \left[1 + \frac{\lambda}{\mu} \frac{1 - \phi}{\phi}\right],
\]

and the fifth is the definition of the consumption index:

\[
C \equiv \left[\omega^{\frac{1}{\kappa}} \left(\frac{C^T}{C}\right)^{-\frac{1}{\kappa}} + (1 - \omega)\right]^{\frac{\kappa-1}{\kappa}} \left(\frac{C^N}{C} \left(\frac{C^N}{C}\right)^{-\frac{1}{\kappa}}\right)^{\frac{\kappa-1}{\kappa}}.
\]

The other equilibrium conditions are given by the liquidity constraint

\[
B \geq -\frac{1 - \phi}{\phi} [Y^T + P^NY^N]
\]

and the equilibrium condition in the tradable sector that will determine the level of tradable consumption in the case in which the liquidity constraint is binding (i.e. \(\lambda > 0\))

\[
C^T + B = Y^T + (1 + i)B - G^T.
\]

We then have the production function for the non-tradeable sector and the good market equilibrium for non tradeables.

\[
Y^N = AK^\alpha H^{1-\alpha}
\]

\[
Y^N = C^N + G^N.
\]
A.2 Solution algorithm

The solution algorithm we follow is a standard value iteration approach augmented with Howard improvement steps, also known as policy function iteration.\footnote{Judd (1999) and Sargent (1987) contain references for the Howard improvement step, which is also referred to as policy iteration.} We initialize the algorithm by guessing a value function on the right-hand-side of equation (14). This guess consists of a vector of numbers over a fixed set of nodes in the space \((B, e^T)\). We then extend the value function to the entire space for \(B\) by assuming it is parameterized by a linear spline.

To perform the maximization we use feasible sequential quadratic programming. We first proceed by assuming that the collateral constraint (??) is not binding and solving the optimization problem, obtaining the values for all variables other than \(B\), and using the competitive equilibrium equations (3), (??), (??), (??), (10), (??) (this step involves solving one equation numerically, which we do using bisection). After the maximization step has obtained a candidate solution we check whether it violates the credit constraint. If it does not, we have computed the maximum for that value in the state space. If the constraint is violated, we replace (??) with (??) holding with equality and solve as before.\footnote{In some cases, particularly if \(\kappa > 1\), there may exist multiple solutions to the equilibrium conditions for given values of \((B, B')\) when the constraint is binding. In these cases, one can use (??) to compute a value for \(\lambda\) and choose the solution where \(\lambda \geq 0\).} Thus, we have computed

\[
\hat{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t)))E \left[ V^n(B_{t+1}, \varepsilon_{t+1}^T) \right] \right\}.
\]

(17)

We then take several Howard improvement steps, each of which involves the functional equation

\[
\hat{V}_{n+1}(B_t, \varepsilon_t^T) = u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t)))E \left[ \hat{V}_n(B_{t+1}, \varepsilon_{t+1}^T) \right],
\]

(18)
where the difference between this equation and (17) is the absence of a maximization step. After $N$ iterations on this equation, we obtain the updated value function $V^{n+1}(B_t, \varepsilon_t^T) = \hat{V}_N(B_t, \varepsilon_t^T)$, and we continue iterating until the value function converges.

In our implementation we set $N = 40$, although a much smaller number of policy maximization steps is usually sufficient to achieve convergence. The number of nodes on the grid for $B$ is 25, and we place most of them in the constrained region where the value function displays more curvature.

Solving the model with capital is similar – the only change is that we use bilinear splines to parameterize the value function in the space $(K_t, B_t)$. To solve the optimal tax problem, the only change needed is to introduce a second control into the problem – the tax rate – yielding the functional equation

$$
\hat{V}_0(B_t, \varepsilon_t^T) = \max_{B_{t+1}, \varepsilon_{t+1}^T} \left\{ u(C_t - z(H_t)) + \exp(-\beta \ln(1 + C_t - z(H_t)))E \left[ V^n \left( B_{t+1}, \varepsilon_{t+1}^T \right) \right] \right\}.
$$

The process for $\varepsilon^T$ is a continuous state-space AR(1) with normal innovations. We compute the expectation in the right-hand-side of (17) and (18) using Gauss-Hermite quadrature, which converts the integral into a weighted sum where the nodes are the zeros of a Hermite polynomial and the weights are taken from a table in Judd (1998):

$$
E \left[ V^n \left( B_{t+1}, \varepsilon_{t+1}^T \right) \right] = \frac{1}{\sqrt{\pi}} \sum_{i=1}^{m} \omega_i V^n \left( B_{t+1}, \rho \varepsilon_t^T + \sqrt{2} \sigma n_i \right).
$$

Linear splines are used to evaluate the value function at $\varepsilon_{t+1} = \rho \varepsilon_t^T + \sqrt{2} \sigma n_i$ points that are not on the grid.

A.3 Computing consumption equivalents

To compute welfare gains from optimal policy, we consider the functional equations

$$
V_{PO} \left( B_t, \varepsilon_t^T \right) = u(C_{PO} \left( B_t, \varepsilon_t^T \right) - z(H_{PO} \left( B_t, \varepsilon_t^T \right))) + \\
\exp(-\beta \ln(C_{PO} \left( B_t, \varepsilon_t^T \right) - z(H_{PO} \left( B_t, \varepsilon_t^T \right))))E \left[ V_{PO} \left( B_{PO} \left( B_t, \varepsilon_t^T \right), \varepsilon_{t+1}^T \right) \right]
$$

1
and

\[
V_{CE} \left( B_t, \varepsilon_t^T \right) = u(C_{CE} \left( B_t, \varepsilon_t^T \right) - z(H_{CE} \left( B_t, \varepsilon_t^T \right))) + \\
\exp(-\beta \ln(C_{CE} \left( B_t, \varepsilon_t^T \right) - z(H_{CE} \left( B_t, \varepsilon_t^T \right)))) E \left[ V_{CE} \left( B_{CE} \left( B_t, \varepsilon_t^T \right), \varepsilon^T_{t+1} \right) \right]
\]

the first corresponds to the value function in the optimal allocation and the second to the value function in the competitive economy without stabilization policy. We then inflate total consumption in (20) by a fraction \(\chi\), keeping the decision rules fixed, so that

\[
V_{CE} \left( B_t, \varepsilon_t^T ; \chi \right) = u((1 + \chi) C_{CE} \left( B_t, \varepsilon_t^T \right) - z(H_{CE} \left( B_t, \varepsilon_t^T \right))) + \\
\exp(-\beta \ln((1 + \chi) C_{CE} \left( B_t, \varepsilon_t^T \right) - z(H_{CE} \left( B_t, \varepsilon_t^T \right)))) E \left[ V_{CE} \left( B_{CE} \left( B_t, \varepsilon_t^T \right), \varepsilon^T_{t+1} \right) \right].
\]

For each state \((B_t, \varepsilon_t^T)\), we set

\[
V_{PO} \left( B_t, \varepsilon_t^T \right) = V_{CE} \left( B_t, \varepsilon_t^T ; \chi \right)
\]

and solve this nonlinear equation for \(\chi\), which yields the welfare gain from switching the optimal policy conditional on the current state. To obtain the average gain, we simulate using the decision rules from (20) and weight the states according to the ergodic distribution.
References


Table 1. Calibrated parameters and steady state values for the model without capital

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of substitution between tradable and non-tradable goods</td>
<td>$\kappa = 0.760$</td>
</tr>
<tr>
<td>Intertemporal substitution and risk aversion</td>
<td>$\rho = 2$</td>
</tr>
<tr>
<td>Labor supply elasticity</td>
<td>$\delta = 2$</td>
</tr>
<tr>
<td>Credit constraint parameter</td>
<td>$\phi = 0.74$</td>
</tr>
<tr>
<td>Labor share in production</td>
<td>$\alpha = 0.636$</td>
</tr>
<tr>
<td>Relative weight of tradable and non-tradable goods</td>
<td>$\omega = 0.344$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.0177$</td>
</tr>
<tr>
<td>Production factor</td>
<td>$AK^\alpha = 1.723$</td>
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<tr>
<td>Tax rate on non-tradable consumption</td>
<td>$\tau = 0.0793$</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Endogenous variables</th>
<th>Steady state values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per capita NFA</td>
<td>$B = -3.562$</td>
</tr>
<tr>
<td>Relative price of non-tradable</td>
<td>$P_N = 1$</td>
</tr>
<tr>
<td>World real interest rate</td>
<td>$i = 0.0159$</td>
</tr>
<tr>
<td>Tradable government consumption</td>
<td>$\exp(G_T) = 0.0170$</td>
</tr>
<tr>
<td>Nontradable government consumption</td>
<td>$\exp(G_N) = 0.218$</td>
</tr>
<tr>
<td>Per capita tradable consumption</td>
<td>$C_T = 0.607$</td>
</tr>
<tr>
<td>Per capita non-tradable consumption</td>
<td>$C_N = 1.093$</td>
</tr>
<tr>
<td>Per capita consumption</td>
<td>$C = 1.698$</td>
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<tr>
<td>Per capita tradable GDP</td>
<td>$Y_T = 1$</td>
</tr>
<tr>
<td>Per capita non-tradable GDP</td>
<td>$Y_N = 1.543$</td>
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<tr>
<td>Per capita GDP</td>
<td>$Y = 2.543$</td>
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<table>
<thead>
<tr>
<th>Productivity process</th>
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</thead>
<tbody>
<tr>
<td>Persistence</td>
<td>$\rho_e = 0.553$</td>
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<tr>
<td>Volatility</td>
<td>$\sigma_n = 0.028$</td>
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</tbody>
</table>
Table 2. Calibrated parameters and steady state values for the model with capital

<table>
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<tr>
<td>Credit constraint parameter</td>
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<tr>
<td>Labor share in production</td>
<td>$\alpha = 0.509$</td>
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<tr>
<td>Relative weight of tradable and non-tradable goods</td>
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<td>Production factor</td>
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<td>Capital depreciation rate</td>
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<table>
<thead>
<tr>
<th>Endogenous variables</th>
<th>Steady state values</th>
</tr>
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<tr>
<td>Per capita NFA</td>
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<tr>
<td>Relative price of non-tradable</td>
<td>$P_N = 1$</td>
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<td>World real interest rate</td>
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<td>Capital Stock</td>
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<table>
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<th>Productivity process</th>
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</thead>
<tbody>
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<td>Persistence</td>
</tr>
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<td>Volatility</td>
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Table. Welfare Gains of Moving from Benchmark Economy (In Percent) 1/

<table>
<thead>
<tr>
<th>Percent increase in average lifetime consumption</th>
<th>Tax/Subsidy</th>
<th>Credit Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50%</td>
<td>0.079</td>
<td>NO</td>
</tr>
<tr>
<td>3.41%</td>
<td>0</td>
<td>YES</td>
</tr>
<tr>
<td>3.84%</td>
<td>0</td>
<td>NO</td>
</tr>
<tr>
<td>3.60%</td>
<td>OPTIMAL POLICY</td>
<td>YES</td>
</tr>
</tbody>
</table>

1/ Benchmark is economy with fixed tax rate at 7.93 percent and credit constraint.
Figure 1: Policy function of net foreign asset with and without the liquidity constraint and baseline $\tau_t^N \equiv 0.0793$
Figure 2: Policy functions for key endogenous variables with and without the liquidity constraint and baseline \( \tau_t^N \equiv 0.0793 \)
Figure 3: Optimal policy for tax rate and lump-sum tax

(a) $\tau^N$

(b) $\frac{P^N \tau^N}{Y}$
Figure 4: Policy functions for key endogenous variables with and without optimal tax policy

(a) $W$

(b) $P^N$

(c) $C$

(d) $H$
Figure 5: Optimal policy function for net foreign asset with the liquidity constraint.
Figure 6: **Ergodic distribution of net foreign asset**

![Graph showing the ergodic distribution of net foreign asset](image)

- **Unconstrained**
- **Constrained, w/o Optimal Policy**
- **Constrained, w Optimal Policy**

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Figure 7: Sensitivity Analysis