Personal Bankruptcy and the Insurance of Labor Income Risk*

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March 29, 2010

Abstract

Recent research [Livshits, MacGee, and Tertilt (2007), Chatterjee *et al.* (2007)] has found that the relatively lenient US bankruptcy code is likely to improve *ex ante* welfare relative to more strict forms of debt forgiveness. The welfare gains come from improved consumption insurance provided by the option to not repay debt in some circumstances. However, *all* instances where the literature finds a beneficial role for bankruptcy have been ones with large and transitory shocks directly to household consumption *expenditures*. It is clear therefore that involuntary reductions in net worth are sufficient to justify US personal bankruptcy. The availability of default will be reflected in the pricing on consumer debt, and so will affect households' ability to smooth consumption across dates and states-of-nature. It is therefore important to observe that a significant fraction of risks to lifetime household resources are not those coming from expenses, but rather from persistent shocks to *labor income*. We investigate the extent to which personal bankruptcy alters the ability of households to insure labor income risk. Our main finding is that the personal bankruptcy option very generally *hinders* the ability of households to protect themselves against labor income risk. From a policy perspective, our results suggest that given the rarity and nature of expense shocks relative to the prevalence and importance of labor income risk, the US bankruptcy system may be fairly costly.

Keywords: *Unsecured Credit, Penalties, Bankruptcy*
JEL Classification Codes: *D91, E21.*

*This paper previously circulated under the title "Are Harsh Punishments for Default Really Better?" We would like to thank seminar and conference participants at UT-Austin, the Board of Governors, the Cleveland Fed, Georgetown University, the Philadelphia Fed, and Queen's University for comments on the earlier versions. Tam thanks the John Olin Foundation for financial support. The opinions expressed here do not reflect the opinion of the Federal Reserve System nor the Federal Reserve Bank of Richmond. All errors are the responsibility of the authors.

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1 Introduction

Recent research [Livshits, MacGee, and Tertilt (2007), Chatterjee *et al.* (2007)] has found that the relatively lenient US bankruptcy code is likely to improve *ex ante* welfare relative to more strict forms of debt forgiveness. The welfare gains come from improved consumption insurance provided by the option to not repay debt in some circumstances. However, *all* instances where the literature finds a beneficial role for bankruptcy have been ones with large and transitory shocks directly to household consumption *expenditures*. It is clear therefore that these "expense shocks" which lead to involuntary reductions in net worth are sufficient to justify US personal bankruptcy.

The availability of default will, of course, be reflected in the pricing on consumer debt, and so will affect households' ability to smooth consumption across dates and states-of-nature. It is therefore important to note that a significant fraction, if not the majority, of risks to lifetime household resources are not those coming from expenses, but rather from persistent shocks to *labor income*. To the extent that the primary instrument available to low-wealth households to deal with labor income risk is unsecured borrowing, anything that affects access to credit will matter for risk-sharing. So a natural question is: To what extent does personal bankruptcy alter the ability of household to insure labor income risk?

In this paper, we evaluate in detail the role of bankruptcy in altering the transmission of labor income risk to consumption, and the effect such modifications have on welfare. Our analysis allows for a very wide range of specifications for labor income risk and households' attitudes towards them. Our main finding is that the personal bankruptcy option very generally *hinders* the ability of households to protect themselves against labor income risk. From a policy perspective, our results therefore suggest that allowing households to use the US personal bankruptcy system because of the presence of expense shocks may be costly.

Our results come from comparing allocations arising from two underlying trading environments. First, we study allocations arising from what we will refer to as the textbook, or "standard model" (SM) of consumption and saving in which households face uninsurable earnings risks with persistent and transitory components. In this model, households can only borrow using non-defaultable debt and also face liquidity constraints. Canonical examples of SM include those laid out in Deaton (1992, chapter 7), and Carroll (1997). To be consistent with the view that borrowing limits should be endogenously determined by repayment incentives, under SM, we investigate primarily the so-called "natural borrowing limit" case.[1]

The second trading arrangement we consider is one where, as before, households face a life-cycle consumption/savings problems in which they face identical risks as in SM, but can issue defaultable debt. We will refer to this as the "default model" (DM). Benchmarks in this literature are Livshits et al. (2007), and Chatterjee et al. (2007). Following these papers, default, or bankruptcy, in the DM model will be represented as a procedure whereby those with negative net worth can obtain a discharge of these obligations, subject to any costs that may be present. The two trading arrangements we consider are thus clearly different. Nonetheless, they are related in a simple way: SM is the limiting case of DM as bankruptcy becomes prohibitively costly.

To focus directly on the role of bankruptcy in insuring labor income risk relative to the SM, we take two steps. First, we deliberately set aside expenditure shocks. In particular, the presence of such shocks rules out the comparison of models with default against the standard model as consumption sets would be empty in some dates and states were it not for the possibility of default. Second, we will examine a wider array of household preferences towards these risks, and the form of uncertainty that they face, than has been done in the literature thus far. Specifically, we (i)

[1] See Ljungqvist and Sargent (2004), p.577.

separate risk-aversion from the intertemporal willingness of households to substitute consumption, and (ii) evaluate the role of ambiguity aversion (or uncertainty aversion) when households are unsure of the stochastic environment they populate.

Both the separation of risk-aversion from intertemporal elasticities and the possibility of ambiguity have been previously identified with a beneficial role for debt default. However, neither has been studied formally. The logic for suspecting that they may be important in delivering a welfare-enhancing role for bankruptcy is as follows. First, the tradeoff between intertemporal and intratemporal smoothing is suggested by, among other, Livshits, MacGee, and Tertilt (2007), which was the first to employ a life-cycle model to study personal bankruptcy. These authors have argued that the fundamental tradeoff created by bankruptcy is between intertemporal and intratemporal smoothing. Assessing the relative importance of these motives therefore requires allowing for preferences in which the two attitudes can be distinct, irrespective of the uncertainty surrounding income. However, all prior work has employed CRRA preferences which conflate the two aspects of household preferences. In contrast, we employ Epstein-Zin recursive utility (Epstein and Zin 1989), which we select due both to its tractability and demonstrated ability to improve the performance of asset pricing models, of which defaultable debt is a special case.[2] Second, with respect to the role of ambiguity in determining the value of an option to default, the legal and political history of bankruptcy law suggests that allowing for the release of debtors subject only to modest penalties is a policy that improve welfare if households are not perfectly sure of the probabilistic structure of income risk (see Jackson 2001 for one example).[3] This view is not confined to legal experts. As noted as early as Friedman (1957), agents will typically be unsure about the process that generates their labor income shocks, instead accepting that a family of potential distributions that may be difficult to distinguish are instead possible. Within this class of preferences, an agent who displays ambiguity aversion (Epstein and Schneider 2003) will solve a max-min problem – the agent will choose the member of the class that makes utility lowest and then choose consumption and savings in order to deliver the highest utility in this worst case.[4] It is precisely this feature of the problem that will allow for a more nuanced understanding of how penalties can be "excessive" and thereby welfare-reducing: bankruptcy elimination through harsh penalties may leave the agent unwilling to borrow at all. As a result, such a policy could perversely inhibit both intertemporal and intratemporal consumption smoothing, despite "mechanically" alleviating the limited commitment problem that the young and poor face. US bankruptcy law appears directly predicated on the idea that penalties can indeed be excessive, in the sense that they may leave would-be borrowers unwilling to do so (see Jackson 2001).

The potential role for ambiguity in altering the welfare implications of having defaultable debt is also suggested by the observation that, in all extant work on consumer default, the relative gains seen in the SM relative to DM really depend on the "worst case" for household income. In particular, the large welfare losses in the DM relative to SM stem from the ability of young agents to borrow out to the natural debt limit. The natural debt limit is, however, extremely sensitive to small changes in the value of the worst-possible labor income realization, particularly for (i) young

[2] The work of Backus, Routledge, and Zin (2007) shows that once risk aversion and intertemporal smoothing motives are separated, who bears risk in a dynamic stochastic setting can be very counterintuitive. Specifically, these authors show that efficient allocations can involve infinitely-risk-averse agents holding most of the risky asset, while those unwilling to trade consumption intertemporally hold most of the assets with uneven intertemporal payoffs. There are of course many other preference structures that separate risk aversion and intertemporal substitution – we discuss some of these structures in the conclusion.

[3] Miao and Wang (2009) study the decision to exercise an option under ambiguity. Due to the presence of fixed costs, bankruptcy has option value.

[4] These preferences are a special case of the more general ambiguity-averse preferences axiomatized by Klibanoff, Marinacci, and Mukerji (2009).

agents for whom the annuity value of future labor income is particularly high, and (ii) all agents when the risk-free borrowing rate is low.[5] This lower bound is difficult to estimate accurately (see Deaton 1992 or Pemberton 1998) and the worst-case outcomes are the primary focus of ambiguity-averse agents; thus, it seems important to understand whether the superiority of SM hinges entirely on the lowest value of income.

Our main finding along these dimensions is that even in the presence of very high levels of uninsurable labor income risk, high risk-aversion, an unwillingness to substitute intertemporally, and the presence of ambiguity, US personal bankruptcy may lower welfare. The intuition for our welfare results involves the relationship between the current economic situation of the borrower and the price of debt. When short-term debt is used in a setting with household labor income risk that is persistent, limited commitment to debt repayment will make credit expensive anytime the household experiences a negative shock; pricing "moves against" the unlucky borrower (in Athreya, Tam, and Young (2009) we argue that unsecured credit markets are not insurance markets for precisely this reason). As a result, agents who most "need" debt to smooth consumption are exactly those that find themselves unable to obtain it, because they also pose the highest risk of default. Tam (2009) extends this result to longer-term arrangements; specifically, he finds that competitively priced longer-period debt (in which the pricing function is held fixed over a number of periods) is welfare-dominated by one-period debt.

In contrast, the existence of the welfare gains from making default attractive enough to generate equilibrium default suggested by the work of Dubey, Geanakoplos, and Shubik (2005) arose in a setting where borrowers of differential default risk were pooled together and thereby *did not* pay the individually actuarially fair price for their debt issuance. Due to the stylized nature of their two-period model, it is not suitable for determining whether defaultable debt is welfare-improving in the actual economy. In quantitative settings where pooling is imposed exogenously, Athreya (2002) and Mateos-Planas and Seccia (2006) still find that welfare is higher in SM than DM. More recently, in a setting where private information allows for equilibrium pooling, the findings of Athreya, Tam, and Young (2008,2009) suggest again that as a quantitative matter, short-term defaultable debt is unlikely to be able to function as a form of insurance. Viewing these findings as a whole, they support the notion that slacker borrowing constraints are welfare-improving when they arise from limited commitment to repayment.[6]

Lastly, with respect to political support for a policy allowing debt default, in addition to the welfare gains from having bankruptcy available in the presence of expense shocks, it seems possible that bankruptcy would enjoy support even in their absence. One obvious possibility is that the current regime may simply reflect objectives other than the maximization of the welfare of newborn agents. We therefore ask if *ex post* welfare can account for the evident political support enjoyed by proponents of relatively lax bankruptcy. Specifically, we ask whether model agents would choose to impose a bankruptcy option in an economy where it was not already present (taking into account all changes resulting from the policy change). We find some support for such a change, but it falls

[5] Denoting by $y_{\min} > 0$ the lowest realization of potential labor income and r the risk-free interest rate on debt, the natural borrowing limit for an infinitely-lived agent is given by $\underline{b}_{nat} \equiv -\frac{y_{\min}}{r}$, a function that asymptotes to $-\infty$ as interest rates go to zero. Assuming a credit card interest rate of 14 percent (the modal interest rate in SCF data in 1983 adjusted for a measure of realized inflation), the natural debt limit moves roughly seven times as much as the minimum income level. For good borrowers, for whom interest rate discounts have recently appeared (Furletti 2003, Athreya, Tam, and Young 2008, Livshits, MacGee, and Tertilt 2008), the natural debt limit will be even more sensitive.

[6] Exogenous borrowing constraints do not necessarily deliver the same welfare result. Obiols-Homs (2009) examines a model in which default is not permitted and borrowing constraints are exogenously-specified. He finds that welfare is maximized at an intermediate borrowing constraint, but it is not clear how to connect his paper to the question of whether agents should be allowed to default or not.

well short of a majority. Support for bankruptcy comes from relatively unlucky middle-aged college graduates: these are agents who borrowed a lot when young, in (rational) anticipation of higher income in middle-age. When realized income did not materialize as expected, such households have significant debt as they approach retirement, and so will benefit from having debt obligations removed. Young agents, by contrast, are almost-uniformly opposed to allowing bankruptcy, and less-educated workers do not generally support it either.

2 Model

The general framework follows Athreya, Tam, and Young (2008, 2009). Households in the model economy live for a maximum of $J < \infty$ periods. However, unlike Athreya, Tam, and Young (2008, 2009), we assume that the economy is small and open, so that the risk-free rate is exogenous, while the wage rate is still determined by a factor price condition.[7] As a result, our welfare calculations will be biased toward finding a positive role for bankruptcy, since any lost resources will be ignored.

2.1 Households

Each household of age j has a probability $\psi_j < 1$ of surviving to age $j+1$ and has a pure time discount factor $\beta < 1$. Households value consumption per household member $\frac{c_j}{n_j}$ and attach a negative value $\lambda_{j,y}$ (in terms of a percentage of consumption) to all nonpecuniary costs of filing for bankruptcy, which depend on type y to be defined below. Their preferences are represented by a recursive utility function $U\left(\left\{\frac{c_j}{n_j}\right\}_{j=1}^{J}\right)$ that we detail below. Households retire exogenously at age $j^* < J$.

We follow Chatterjee *et al.* (2007) in allowing for household-level costs from default that are primarily nonpecuniary in nature. The existence of nonpecuniary costs of bankruptcy are also suggested by the calculations and evidence in Fay, Hurst, and White (1998) and Gross and Souleles (2002), respectively. The former paper shows that a large measure of households would have "financially benefited" from filing for bankruptcy but did not, while both papers document significant unexplained variability in the probability of default across households even after controlling for a large number of observables. These results suggest the presence of implicit unobserved collateral that is heterogeneous across households, including (but not limited to) any "stigma" associated with bankruptcy along with any other costs that are not explicitly pecuniary in nature (as in Athreya 2004). We will therefore sometimes refer to $\lambda_{j,y}$ as stigma in what follows, although we intend it to be more encompassing.

The household budget constraint during working age is given by

$$c_j + q\left(b_j, I\right) b_j + \Delta \mathbf{1}\left(d_j = 1\right) \leq a_j + \left(1 - \tau\right) W \omega_{j,y} y e \nu, \tag{1}$$

where q is an individual-specific bond price that depends on bond issuance b_j and a vector of individual characteristics I and a_j is net worth *after* the current-period default decision. Therefore, $a_j = b_{j-1}$ if the household does not default and $a_j = 0$ otherwise. Δ is the pecuniary cost of filing for bankruptcy. The last term is after-tax current labor income (τ is the tax rate). Log labor income is the sum of four terms: the aggregate wage index W, a permanent shock y realized prior

[7] In our previous work we have introduced a class of "special" agents who hold lots of capital for the purpose of endogenously obtaining a low risk-free rate in the presence of low asset holdings for the median agent. Here, we ignore the general equilibrium determination of returns, and thus drop the special households from the model, because their presence is irrelevant to the question at hand.

to entry into the labor market, a deterministic age term $\omega_{j,y}$, a persistent shock e that evolves as an AR(1)

$$\log\left(e'\right) = \varsigma \log\left(e\right) + \epsilon', \tag{2}$$

and a purely transitory shock $\log(\nu)$. Both ϵ and $\log(\nu)$ are independent mean zero normal random variables with variances that are y-dependent.[8] The budget constraint during retirement is

$$c_j + q\left(b_j, I\right) b_j \leq a_j + \mathbf{\Delta 1}\left(d_j = 1\right) + \upsilon W \omega_{j^*-1,y} y e_{j^*-1} \nu_{j^*-1} + \Upsilon W, \tag{3}$$

where for simplicity we assume that pension benefits are composed of a fraction $\upsilon \in (0,1)$ of income in the last period of working life plus a fraction Υ of average income (which is normalized to 1).

The survival probabilities $\psi_{j,y}$ and the deterministic age-income terms $\omega_{j,y}$ differ according to the realization of the permanent shock. We interpret y as differentiating between non-high school, high school, and college education levels, as in Hubbard, Skinner, and Zeldes (1994), and the differences in these life-cycle parameters will generate different incentives to borrow across types. In particular, college workers will have higher survival rates and a steeper hump in earnings; the second is critically important as it generates a strong desire to borrow early in the life cycle. Less importantly they also face slightly smaller shocks than the other two education groups. The life-cycle aspect of our model is key – in the data, defaults are skewed toward young households (who borrow at least in part for purely intertemporal reasons), particularly for those who do not report medical expenses as a main contributor to their bankruptcy.[9]

Non-pecuniary costs, λ, follow a two-state Markov chain with realizations $\{\lambda_{L,y}, \lambda_{H,y}\}$ that are independent across households, but serially dependent with transition matrix

$$\Pi_\lambda = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix}.$$

Due to data limitations, we assume that the transition probability matrix is symmetric and type-invariant, so the only difference across types in terms of stigma costs are their realizations. Our parametrization is more flexible than we used in previous work (Athreya, Tam, and Young 2008, 2009) so that we can match the default rates across education groups. As we show in a subsequent section, the process is still not flexible enough to match all the targets of interest, although it does a reasonable job. Households cannot borrow or save during the period in which they declare bankruptcy; however, every household has the ability to declare bankruptcy in the periods following the default.[10]

2.2 Loan Pricing

We focus throughout on competitive domestic lending. There exists a competitive market of intermediaries who offer one-period debt contracts and utilize available information to offer individualized credit pricing. Let I denote the information set for a lender and $\widehat{\pi} : b \times I \to [0,1]$ denote the function that assigns a probability of default to a loan of size b given information I. $\widehat{\pi}(b)$ is identically zero for positive levels of net worth and is equal to 1 for some sufficiently large debt

[8]We approximate both e and ν with finite-state Markov chains. This approximation has the convenient property that income is bounded.

[9]See Sullivan, Warren, and Westbrook (2000).

[10]This period of exclusion is considered part of public policy, and is intended to reflect the fact that bankruptcy proceedings take some time; any saving attempted during this process would likely be garnished by the court to pay creditors. The exclusion from borrowing is endogenous, however.

level. The break-even pricing function satisfies

$$q(b, I) = \begin{cases} \frac{1}{1+r} & \text{if } b \geq 0 \\ \frac{(1-\widehat{\pi}(b))\psi_j}{1+r+\phi} & \text{if } b < 0 \end{cases} \tag{4}$$

given $\widehat{\pi}(b)$. r is the exogenous risk-free saving rate and ϕ is a transaction cost for lending, so that $r + \phi$ is the risk-free borrowing rate; the pricing function takes into account the automatic default by those households that die at the end of the period.[11] We assume I contains the entire state vector for the household: $I = (a, y, e, \nu, \lambda, j)$. Zero profit for the intermediary requires that the probability of default used to price debt must be consistent with that observed in the stationary equilibrium, implying that

$$\widehat{\pi}(b) = \sum_{e',\nu',\lambda'} \pi_e(e'|e)\, \pi_\nu(\nu')\, \pi_\lambda(\lambda'|\lambda)\, d\left(b(a, y, e, \nu, \lambda, j), e', \nu', \lambda'\right). \tag{5}$$

Since $d(b, e', \nu', \lambda')$ is the probability that the agent will default in state (e', ν', λ') tomorrow at debt level b, integrating over all such events *tomorrow* produces the relevant default risk. This expression also makes clear that knowledge of the persistent component e is critical for predicting default probabilities; the more persistent e is, the more useful it becomes in assessing default risk.

2.3 Government

The only purpose of government in this model is to fund pension payments to retirees. The government budget constraint is

$$\tau W \int y\omega_{j,y} e\nu \Gamma(a, y, e, \nu, \lambda, j < j^*) = W \int (\upsilon\omega_{j^*-1,y} y e_{j^*-1}\nu_{j^*-1} + \Upsilon) \Gamma(a, y, e, \nu, \lambda, j \geq j^*).$$

2.4 Price Determination

We assume that the risk-free rate r is exogenous and determined by the world market for credit. Given r, profit maximization by domestic production firms implies that

$$W = (1-\alpha)\left(\frac{r}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}$$

where α is capital's share of income in a Cobb-Douglas aggregate production technology. Our assumption that the risk-free rate is exogenous deserves defense. It is certainly reasonable to assume that the US capital market is open, so empirically it is not implausible. Furthermore, if we close the economy we confront the high concentration of wealth puzzle directly – the median-wealth agent in the US has little or no wealth and thus cares about default policy, since they may borrow in the future if unlucky, while the mean agent holds substantial wealth and is unlikely to be concerned with the bankruptcy policy in place. Matching the wealth distribution in our model would be immensely costly and the endogenous adjustment of the interest rate is unlikely to be large.[12]

2.5 Preferences

Here we present the recursive representations of the preferences we study.

[11] We assume any savings of households who die is taxed at 100 percent and used to fund wasteful government spending.

[12] Chatterjee *et al.* (2007) calibrate their model to match the wealth distribution in the US in a dynastic setting. As we have argued, life cycle considerations are important for assessing the welfare effects of bankruptcy.

2.5.1 Constant Relative Risk Aversion

The agent's problem is standard under CRRA preferences, with the Bellman equation for a household of age j given by

$$\begin{aligned} v(a,y,e,\nu,\lambda,j) &= \max_{b,d(e',\nu',\lambda')\in\{0,1\}} \left\{ \frac{n_j}{\rho}\left(\frac{c_j}{n_j}\right)^{\rho} + \beta\psi_{j,y}(EU) \right\} \\ EU &= \sum\nolimits_{e',\nu',\lambda'} \pi_e(e'|e)\,\pi_\nu(\nu')\,\pi_\lambda(\lambda'|\lambda)\,V(b,y,e',\nu',\lambda',j+1) \\ V(b,y,e',\nu',\lambda',j+1) &= (1-d(e',\nu',\lambda'))\,v(b,y,e',\nu',\lambda',j+1) + \\ & \quad d(e',\nu',\lambda')\,v^D(0,y,e',\nu',\lambda',j+1). \end{aligned} \tag{6}$$

The value function for a household that defaulted in the current period is given by

$$\begin{aligned} v^D(0,y,e,\nu,\lambda,j) &= \left\{ \frac{n_j}{\rho}\left(\lambda\frac{c_j}{n_j}\right)^{\rho} + \beta\psi_{j,y}(EU) \right\} \\ EU &= \sum\nolimits_{e',\nu',\lambda'} \pi_e(e'|e)\,\pi_\nu(\nu')\,\pi_\lambda(\lambda'|\lambda)\,v(0,y,e',\nu',\lambda',j+1). \end{aligned} \tag{7}$$

$1-\rho \geq 0$ is the coefficient of relative risk aversion and also the inverse of the elasticity of intertemporal substitution.

2.5.2 Epstein-Zin

Under Epstein-Zin preferences, a household of age j solves the dynamic programming problem

$$\begin{aligned} v(a,y,e,\nu,\lambda,j) &= \max_{b,d(e',\nu',\lambda')\in\{0,1\}} \left\{ n_j\left(\frac{c_j}{n_j}\right)^{\rho} + \beta\psi_{j,y}(EU)^{\frac{\rho}{1-\sigma}} \right\}^{\frac{1}{\rho}} \\ EU &= \sum\nolimits_{e',\nu',\lambda'} \pi_e(e'|e)\,\pi_\nu(\nu')\,\pi_\lambda(\lambda'|\lambda)\,V(b,y,e',\nu',\lambda',j+1) \\ V(b,y,e',\nu',\lambda',j+1) &= (1-d(e',\nu',\lambda'))\,v(b,y,e',\nu',\lambda',j+1)^{1-\sigma} + \\ & \quad d(e',\nu',\lambda')\,v^D(0,y,e',\nu',\lambda',j+1)^{1-\sigma} \end{aligned} \tag{8}$$

where

$$\begin{aligned} v^D(0,y,e,\nu,\lambda,j) &= \left\{ n_j\left(\lambda\frac{c_j}{n_j}\right)^{\rho} + \beta\psi_{j,y}(EU)^{\frac{\rho}{1-\sigma}} \right\}^{\frac{1}{\rho}} \\ EU &= \sum\nolimits_{e',\nu',\lambda'} \pi_e(e'|e)\,\pi_\nu(\nu')\,\pi_\lambda(\lambda'|\lambda)\,v(0,y,e',\nu',\lambda',j+1) \end{aligned} \tag{9}$$

is the value of default. $\sigma \geq 0$ governs the household's aversion to fluctuations in utility across states of nature while $\rho \leq 1$ controls the substitutability between current and future utility; specifically, σ is the coefficient of relative risk aversion with respect to gambles over future consumption and $\frac{1}{1-\rho}$ is the elasticity of intertemporal substitution in consumption. When $\rho = 1-\sigma$ these preferences generate the same ordering over stochastic streams of consumption as expected utility does.

3 Results

The results are organized into two subsections. First, we study the roles played by aversion to fluctuations in consumption over time and across states-of-nature. We begin with expected utility preferences; as is well-known, these preferences do not permit separate parametrization of agent attitudes toward intertemporal and intratemporal fluctuations. We then relax the tight link between aversion to intra- and intertemporal variability in consumption by employing Epstein-Zin preferences. Throughout this subsection, we consider parameter values that lie near the values implied by the benchmark calibration; these values ensure that model outcomes remain in congruence with cross-sectional facts on consumption and income inequality. We show that welfare under the bankruptcy option is lower, at least *ex ante*. Second, based on this result, we ask the "inverse" question: are there economies in which welfare in the standard model is worse? In this subsection, we no longer restrict ourselves to parameters dictated by US data; rather, our goal is to understand whether any parameterizations within the parametric classes we study are capable of generating lax bankruptcy as welfare improving policy. Specifically we consider shocks with counterfactually large persistent and transitory components and preferences that display ambiguity aversion.

3.1 Does Default Help Insure Labor Income Risk?

In this subsection, we evaluate the implications of bankruptcy relative to the standard model for a variety of empirically plausible values for agent attitudes toward intra- and intertemporal consumption smoothing. Before evaluating these alternatives, we present our argument for why bankruptcy regimes must be a matter of policy rather than an endogenous outcome of decentralized trading arrangements. The most prevalent form of explicitly unsecured credit is that arising from the open-ended revolving debt plan offered by credit card lenders. Credit card lending, in turn, has been (certainly since the mid 1990s) extremely competitive.[13] The relevance of the competitiveness of the US unsecured lending industry is that the credit market cannot be punitive in its treatment of those who default. That is, no single firm would be willing to treat an individual borrower any worse than the current assessment of their state would justify. As a result, a household contemplating default in such a setting can safely rule out being "punished" for it. In the case where default conveys no additional information to a lender than what it was able to observe *ex ante*, there is literally no change in terms that are "caused" by the act of reneging on a payment obligation. Conversely, when default does reveal information, the change in terms is again not "punitive" in nature, but instead reflect an updated assessment of default risk. As a result, "high" *ex post* interest rates following bankruptcy are implausibly ascribed to deadweight-loss-inducing penalties. Given the inability of competitive lenders to commit to punishments, bankruptcy costs capable of sustaining unsecured credit markets are likely to require intervention by policy-making authorities.[14] Thus, in the market for unsecured consumer debt, it is likely that *any costs of bankruptcy filing that are in any way punitive have to be policies.*[15]

[13]The average interest rate on credit card balances is high – currently 14 percent – relative to more secured forms of debt. As Evans and Schmalensee (1998) have pointed out, however, it is straightforward to account for the interest rate after funding costs, transactions costs, and most crucially, default costs are taken into account, without relying on market power distortions.

[14]Most dynamic contracting models of limited borrower commitment, for example, currently use implicit or explicit appeals to public institutions with commitment to punish, in order to motivate penalties for the value of autarky (such as Ábráham and Carceles-Poveda 2008). In recent work, Krueger and Uhlig (2007) show that the inability of the supply side of the credit market to commit to punishments can have severe implications for the existence of the market itself. In the "normal" case, Krueger and Uhlig (2007) show that competition in fact collapses credit and insurance markets completely even without informational frictions.

[15]We want to be clear that what we call "penalties" differs from the usage in Ausubel and Dawsey (2008), where rates imposed after late or missed payments are labeled punitive. They attribute the high values of such rates to a common agency problem. Modeling the bilateral contracting problem that would arise in the presence of noncompetitive intermediation is well beyond the goals for this paper. We are pursuing the endogenous determination

At the outset, we noted that for plausible parameterizations of preferences that admit an expected utility representation, the standard model typically maximizes welfare. Our first step is to understand whether this argument against bankruptcy obtains only because of the restriction to expected utility or is a more fundamental property of models of life-cycle consumption smoothing. To collapse the model to the standard model, the specific quantitative experiment we consider is the imposition of a cost of default Δ that is large enough to eliminate all default on the equilibrium path.[16] Before proceeding, we note the following property of our model.

Proposition 1 *For each* (a, y, e, ν, j) *there exists* Δ *large enough that* $\widehat{\pi}(b) = 0$.

This result relies on the nonnegativity condition for consumption – if Δ exceeds the labor income of the household in the current period, default cannot occur since consumption would have to be negative. Given that total labor income is bounded (by assumption) and borrowing is proscribed in the period of default, we can always impose a cost of filing sufficient to generate zero bankruptcy along the equilibrium path. We then compute the change in lifetime utility for each individual given a Δ that exceeds the maximum required; in the absence of general equilibrium effects, we can compute these changes for each individual, rather than simply for newborns, without the need to track transitional dynamics. We will focus in general on *ex ante* welfare of newborns.

Our results are also computed under an assumption of full and symmetric information between borrowers and lenders. As we showed in Athreya, Tam, and Young (2008), asymmetric information leads to adverse selection effects that greatly increase the welfare costs of bankruptcy, so our results will be robust to the introduction of those features. We are cognizant of the regulatory environment in the US, where certain aspects of an individual's state vector – such as race, gender, and age – are legally proscribed under the Equal Credit Opportunity Act from influencing the pricing of unsecured debt.[17] Our results will be robust to such restrictions, since they work in a similar way to asymmetric information.

3.1.1 Calibration

We consider a benchmark case of expected utility, where $\rho = 1-\sigma = -1$. We choose $(\beta, \lambda_{L,y}, \lambda_{H,y}, \pi)$ to match the default rates of each type y, the measure of negative net worth as a fraction of GDP for each type y, the fraction of borrowers, and the discharge ratio (mean debt removed via bankruptcy divided by mean income at time of filing). Table 1 contains the constellation of parameters that fits best (when viewed as exactly-identified GMM with an identity weighting matrix). Other parameters are identical to those in Athreya, Tam, and Young (2008) – these include the resource cost of default Δ, the income processes faced by each type, the measure of each type, and the parameters of the retirement system (θ, Θ).[18]

It is clear that our model is not capable of exactly matching the entire set of moments – for example, we underpredict default rates and discharge, generally underpredict debt-to-income ratios, and overpredict the measure of borrowers. The model actually places very tight links between some variables, restricting the minimization routine's ability to independently vary them. For example, suppose we attempt to improve the model's prediction for the measure of borrowers by increasing

of interest rate hikes for delinquent borrowers in other work.

[16] Similar results would obtain if the government can impose "shame" on households by choosing values for λ, provided it can make λ large enough to guarantee zero default on the equilibrium path. In our model, the Inada condition on consumption implies that such a λ always exists.

[17] Similar regulations are present in the UK and the EU.

[18] Specifically, we set $\upsilon = 0.35$, $\Upsilon = 0.2$, $\phi = 0.03$, $\Delta = 0.03$, $\varsigma = 0.95$, $\sigma^2_{n,\epsilon} = 0.033$, $\sigma^2_{n,\nu} = 0.04$, $\sigma^2_{h,\epsilon} = 0.025$, $\sigma^2_{h,\nu} = 0.021$, $\sigma^2_{c,\epsilon} = 0.016$, and $\sigma^2_{c,\nu} = 0.014$.

β. Holding all other parameters constant, we would reduce default rates and debt-to-income ratios for all types (and these variables are generally already too small). To counteract this effect, we need to move λ for each type and each state. Consider first increasing both λ_i^H and λ_i^L for one type i. While this change would increase the default rate – bankruptcy becomes less costly – it would via a supply side effect tend to reduce debt levels (see Athreya 2004). Suppose instead we increase λ_i^H and decrease λ_i^L; this change has countervailing effects on both default rates and debt levels. For example, default rates could rise because it becomes cheaper for H types, but it also becomes more expensive for L types; thus, whether we can adjust the parameters to increase default depends on whether low-types or high-types respond more elastically. A similar tension exists for debt-to-income ratios – driving it up for one type tends to drive it down for the other. The minimization routine we used to calibrate the model resolved these tensions as well as possible, resulting in some good results – for example, we do a good job at generating discharge – and some poor ones (college types in our model default much less than in the data).

With these shortcomings in mind, we note that the qualitative findings from our analysis do not depend on our specification of the stochastic process for λ; welfare is higher without bankruptcy in cases with (i) a single, constant λ, (ii) constant, but type-specific λ, and (iii) stochastic but not type-specific default costs. Therefore, while our specification allows us to match the salient features of debt and default, it does not play a critical role in the welfare implications of default policy.[19]

3.1.2 Expected Utility and *Ex Ante* Welfare

We consider two environments – one environment with the calibrated value for Δ and one with a cost Δ sufficient to eliminate default on the equilibrium path. Table 2 contains the welfare gain from the standard model in which it is infeasible for any household to declare bankruptcy. Consistent with our previous work, we find that welfare is higher in the standard model *ex ante* for every newborn (independent of type). College types benefit the most from the change, and their welfare gain is substantial (1.2 percent of lifetime consumption). To aid the discussion in subsequent sections where we alter preference parameters, we quickly summarize the reasons for the welfare gains here.

In the standard model, the loss of resources generated by the filing cost is not present. Since we do not impose an economy-wide resource constraint, these lost resources are not important. Instead, the welfare gain is driven by an improved allocation of consumption. Consider the following decomposition of the variance of consumption over the life-cycle:

$$V(\log(c)) = V(E[\log(c)\,|\text{age}]) + E[V(\log(c)\,|\text{age})].$$

We label the first term the "intertemporal" component of consumption smoothing; it represents how expected consumption differs across time periods. The second term is the "intratemporal" component; it measures how much consumption varies across agents of a given age. Roughly speaking, how costly the first component is in terms of welfare depends on the elasticity of intertemporal substitution, because it measures the deterministic variance of consumption over time, whereas the welfare cost of the second part is governed by static risk aversion. In Figure 1 we see that the standard model, or "No-Bankruptcy" (NBK) case, improves intertemporal smoothing (the curve gets flatter) because all lending becomes risk-free. Thus, as we noted in the introduction, the only debt limit that is relevant is the natural debt limit, which is very large in our model for newborn

[19] In the real world "stigma" may also be a function of aggregate default rates (an agent cares less about default if everyone else is defaulting), in which case this invariance may break. To analyze this case would be of interest, but it poses some challenges with respect to calibration. We therefore defer it to future work.

agents. Turning to the intratemporal part, in Figure 2 we see that NBK improves this part as well, restating the analysis in Athreya, Tam, and Young (2009) that unsecured credit markets do not provide insurance. Here, bad shocks trigger tightening of credit constraints, making consumption smoothing across states of nature more difficult. As a result, young agents are unable to respond effectively to bad income realizations when they can declare bankruptcy, causing their consumption to be highly volatile. Under NBK the natural debt limit is sufficient to protect them against adverse shocks; by middle age bankruptcy has ceased to be relevant and thus the two cases largely coincide.[20]

The differences in consumption across the BK and NBK cases are driven by changes in the pricing functions that agents face. In Figures 3-5, we show the pricing functions in the low costs of default environment facing a young college agent across realizations of the persistent shock e. The initial flat segment is driven by Δ and is increasing in the current realization of the persistent shock e. As debt increases, more realizations of e' would trigger default, causing q to decline until it reaches zero; looking across e values we see that higher e realizations permit more borrowing. Of course, higher e realizations in our model are typically associated with less, not more, borrowing, so these increased debt limits are not particularly valuable; instead, the tightening of credit limits when e is low generates substantial costs for poor agents. In contrast, under NBK pricing is flat out to the natural debt limit. Crucially, transitory shocks do not impact pricing; because ν' cannot be predicted using ν, the current transitory shock has no effect on the default decision tomorrow conditional on b (b is changed by the transitory shock, however).

The potential tradeoff between the two components of smoothing motivated the life-cycle analysis of Livshits, MacGee, and Tertilt (2007) and Athreya (2008), so why doesn't bankruptcy generate this tradeoff? As discussed in Athreya, Tam, and Young (2009), bankruptcy can either help or hinder intratemporal smoothing, depending on which agent you ask. An agent facing an income process with low intertemporal variance but high intratemporal variance – that is, tomorrow's expected income is close to current income but tomorrow's income has substantial risk – may benefit from bankruptcy; the intertemporal distortion is minimal while the potential to truncate the consumption distribution at the low end conveys significant benefits (even once pricing is taken into account). In contrast, an agent facing the opposite process – income that grows over time and is relatively safe – generally does not benefit; default is not used because pricing prevents it and the intertemporal distortion is substantial, leading to significant welfare losses. In our model, a young agent is of the second type, especially a college-educated one, while older households are members of the first type. From the perspective of a newborn, it is not surprising that allowing bankruptcy generates a net welfare loss.

3.1.3 *Ex Post* Welfare – Voting over Bankruptcy Policy

Because we study an small open-economy model in which the risk-free rate is fixed, but also allow all pricing to be individualized, there are no "pecuniary" externalities. We can therefore compute the welfare consequences of policy changes for any agent at any point in the state space; since the distribution plays no role in pricing (and therefore no role in welfare), we do not need to calculate the transitional dynamics of the model to get the welfare changes. We ask agents of a given age and type whether, conditional on their current state, they would be in favor of eliminating the option to declare bankruptcy. Figure 6 displays the measure of each type, conditional on age, that would support retaining bankruptcy with the calibrated Δ. A substantial portion of college types oppose elimination, but they are all middle-aged and have experienced histories of bad shocks; the

[20]The figures are drawn for the aggregate, since the results are the same for each type qualitatively. Figures decomposed by type are available from the authors upon request.

peak in opposition occurs earlier for high-school types and later for non-high-school types, with correspondingly fewer such households opposing overall. For the convenience of the reader, Table 3 presents the aggregate measures of each type that oppose eliminating bankruptcy (the column labeled "BK Regime"); they are small for each education group. Furthermore, as is clear from the figures, almost no newborns oppose eliminating the option. In this sense, the statement that eliminating bankruptcy is welfare improving for newborns almost holds even when made conditional on the initial shock realization.

We now consider the inverse of the preceding experiment: agents of different ages and types are asked if they would prefer to *introduce* bankruptcy (again, with Δ set to its calibrated value) into a setting in which it is currently prohibited. As seen in Figure 7, a nontrivial fraction of agents would like to introduce bankruptcy. The intuition here is that the no-bankruptcy case allows significant borrowing at the risk-free rate. As a result, many households, especially the college-educated, borrow when young in anticipation of higher earnings. The relatively unlucky among them then find themselves indebted by middle age and thereby will benefit from the discharge of debts. Moreover, by virtue of being middle-aged, these households place relatively low value on being able to access the cheap unsecured debt later in life. This effect is especially strong for the college-educated, for whom purely intertemporal consumption smoothing motives dictate a strong effort to save for retirement beyond middle age. As a result, a substantial proportion of high-school and college-educated household groups would support the introduction of bankruptcy when they reach middle age. In contrast, those who have not completed high school support the introduction of bankruptcy only late in working-life, when the subsequent increase in borrowing costs is not long-lasting. However, as Table 3 shows (the column "NBK Regime"), the aggregate number of agents who vote in favor of introduction falls well short of majority status.

3.1.4 Separating Risk Aversion from Intertemporal Substitution

As discussed above, the two pieces of the variance decomposition have welfare costs that depend (mainly) on different aspects of preferences. Our benchmark case using CRRA expected utility restricted these two aspects of preferences to be reciprocals of each other. Here, we relax that requirement by using the Epstein-Zin preference structure, and consider two particular deviations. First, we make households more tolerant of intertemporal variance than in the expected utility benchmark by employing a high value for ρ. Second, since the bankruptcy option may shrink the volatility of intratemporal consumption, at least for some ages, making intratemporal variance more costly may help us explain the presence of low bankruptcy costs; we therefore select a relatively high value for σ. It is important to note that this particular combination of insensitivity to the timing of consumption and sensitivity to the income state in which it occurs is the arrangement that gives bankruptcy its best chance of improving *ex ante* welfare and does not lie within the class of expected utility preferences.

The specific experiments we investigate involve changing ρ and σ without recalibrating the entire model. This type of change generates two effects – an effect conditional on borrowing (which we call the price effect) and an effect caused by changes in the number of borrowers (the extensive effect). We then compare the results with cases where the model is recalibrated (to the extent that is possible) in an attempt to isolate the two effects.

We first consider changes in ρ. To understand how this change affects welfare, it is helpful to first consider the extreme case of $\rho = 1$, making the household infinitely willing to move consumption deterministically through time. As $\rho \to 1$, the Bellman equation converges to the form

$$v\left(a, y, e, \nu, \lambda, j\right) = c_j + \beta\psi_{j,y}\left(\sum\nolimits_{e',\nu',\lambda'} \pi_e\left(e'|e\right)\pi_\nu\left(\nu'\right)\pi_\lambda\left(\lambda'|\lambda\right)V\left(b, y, e', \nu', \lambda', j+1\right)\right)^{\frac{1}{1-\sigma}}$$

Here, the household will either completely frontload or backload consumption, depending on the relationship between the discount factor and the interest rate. For the parametrization we use, the effective discount factor (β times the survival probability) lies between the risk-free saving and borrowing rates for almost every age, meaning that households are hand-to-mouth and, critically, do not value the default option at all *no matter how risk averse* they are. For some older households the effective discount factor is sufficiently low that they want to borrow and "frontload" their consumption; the option to default makes complete frontloading impossible and therefore reduces the welfare of these households – since they face no uncertainty default is either probability zero or one and pricing therefore eliminates it. Obviously such extreme consumption behavior is inconsistent with US cross-sectional facts; in particular, the model with $\rho = 1$ would miss very badly on the life cycle pattern of consumption inequality, which in the data is substantially smaller than income inequality.

Returning to less extreme values, Figure 8 displays the pricing function across several different values of ρ and demonstrates the effect on loan prices. As ρ increases, the pricing function shifts downward – at any given level of debt an agent with a higher ρ is more willing to default. The intuition for this result is not straightforward. When ρ increases, the household is more willing to accept deviations in consumption across time; in particular, the household values mean consumption only when $\rho = 1$. If a household enters the current period with some debt there are two options: (i) borrow more if possible or (ii) default and void those obligations. Borrowing more is only feasible if there is a reasonable commitment to repay; since a bad shock would lead to low mean consumption bankruptcy becomes attractive. The cost of strong default incentives – the distortion of intertemporal smoothing – is relatively unimportant.

Consider next an experiment where σ, the risk aversion with respect to gambles over future utility, is increased. Again, turning first to the polar case, let $\sigma \to \infty$, so that the household becomes infinitely risk averse. In this case, the limiting household Bellman equation takes the form

$$v\left(a, y, e, \nu, \lambda, j\right) = \left\{ n_j \left(\frac{c_j}{n_j}\right)^{\rho} + \beta \psi_{j,y} \min_{e', \nu', \lambda'} \left\{ V\left(b, y, e', \nu', \lambda', j+1\right) \right\}^{\rho} \right\}^{\frac{1}{\rho}}.$$

When households are infinitely risk-averse they choose not to borrow, for the reasons outlined in Athreya, Tam, and Young (2009) – unsecured credit markets do not provide insurance and thus agents will be unwilling to pay the transaction cost to borrow. As a result, there is a welfare gain to living in the standard model, as no household has negative net worth. Again, extreme preferences render the model grossly inconsistent with cross-sectional facts; here, consumption inequality would be essentially zero over all ages.

Returning again to more intermediate cases, we see that changes in risk-aversion generates two effects. The extensive margin effect is similar to increasing ρ, but for different reasons. When σ is large, households have a strong demand for precautionary savings; for $\sigma = 5$, for example, we see a clear decline in the measure of total borrowers, again making bankruptcy overall less damaging. The pricing effect is also similar; by increasing risk aversion, we make the household less willing to have consumption differ across states of the world tomorrow. Conditional on borrowing the pricing functions reveal a stronger desire to default – for any given b, the price of debt is decreasing in σ (see Figure 9). As above, there are only two options for a household with debt; since even a moderately bad outcome will cause a highly risk-averse agent to default, commitment is not possible, leaving default as the only option for smoothing consumption across states.[21] Combining

[21] Our model satisfies the conditions noted in Chatterjee *et al.* (2007) that imply default occurs only if current debt cannot be rolled over: If $d\left(\epsilon', \nu', \lambda'\right) > 0$ for some $\epsilon', \nu', \lambda'$ then there does not exist b such that $a + y - q\left(b, Y\right) b > 0$ for total income Y.

these results into one statement, we see that no combination of (ρ, σ) leads to bankruptcy being a welfare-improving policy, although for extreme cases it will be nearly innocuous.

Table 3 shows that welfare is higher (for newborns) in the standard model, but that the gains from (imposing the high Δ) decline with risk aversion and EIS. $\rho > 1 - \sigma$ – which is satisfied when either parameter increases – implies the household has a preference for early resolution of uncertainty; thus, bankruptcy appears to be least damaging when households prefer to resolve their risk early rather than late.

All of these results are obtained without recalibrating the model. To ensure that our findings are not particularly sensitive to this strategy, we also recalibrate the model for different values of ρ and σ, to the extent that this recalibration is possible; Table 1 contains the new parameter settings that best fit the targets under alternative settings. By doing so, we attempt to shut off the extensive margin, although we are not completely successful. When we recalibrate, we find that with high EIS all welfare gains from eliminating bankruptcy are substantially reduced, with both noncollege types now barely benefiting at all (see Table 4), while for high risk aversion the welfare gains increase slightly. As noted above, this welfare change is entirely due to the shifts in the pricing function that higher EIS and/or higher risk aversion engender. Thus, for no parameter combination that we consider do we observe welfare gains from retaining the bankruptcy option.

A summary of findings thus far is that bankruptcy significantly worsens allocations for income risk and preference parameters that are empirically plausible for US data, as well as for more extreme values of preference parameters within the class of Epstein-Zin non-expected utility preferences. We turn now to the question of whether such policies continue to remain desirable under two additional (and more substantial) departures from the settings studied so far.

3.2 Is the Standard Model Ever Worse?

We begin this section by allowing for the underlying volatility of income to be driven by relatively more and less persistent income shocks. For this experiment, we hold the unconditional variance of labor income fixed and vary the relative contributions of the persistent component e and the transitory component ν. We then ask whether a relaxation in the household's understanding of the probabilistic structure of earnings risk can open the door for welfare-improving default. For this experiment, we allow for households to display ambiguity aversion in the sense of Klibanoff, Marinacci, and Mukerji (2009).[22]

3.2.1 The Roles of Persistent and Transitory Income Risk

It has long been known that self-insurance, and therefore also the benefit of insurance markets, hinges critically on the persistence of the risks facing households. As a general rule, the more persistent are shocks, the more difficult they are to deal with via the accumulation of assets in good times and decumulation and borrowing in bad times. In contrast, purely transitory income shocks can typically be smoothed effectively. In a pure life-cycle model, however, there are additional impediments to self-insurance: young households are born with no wealth and often face incentives to borrow arising from purely intertemporal considerations. In particular, those with relatively high levels of human capital, especially the college-educated, can expect age-earnings profiles with a significant upward slope into late middle-age. As a result, such households would like to borrow

[22] There are connections between ambiguity aversion and the concept of Knightian uncertainty from Bewley (2002), although the latter concept does not permit preferences to be represented by a utility function and is therefore hard to analyze quantitatively. There are also connections between ambiguity aversion and robust decision-making as defined by Hansen and Sargent (2007).

even in the absence of any shocks to income, often substantially, against their growing expected future income. In contrast, those households with low human capital face a far less income-rich future, and as a result borrow primarily to deal with transitory income risk.

In order to understand the role that the persistence of income risk plays in the welfare gains or losses arising from US-style bankruptcy, we now evaluate the effects of changes in the persistent component of household income risk for all three classes of households. However, in order to avoid conflating persistence and overall income volatility, we adjust the variance of transitory income volatility such that the overall variance of log labor income remains constant.[23] Figure 10 and Tables 5 and 6 present the welfare and consumption smoothing implications of the standard model under varying income shock persistence. The first column of each table documents the fraction of total variance contributed by the persistent component.

Normatively, three findings are noteworthy. First, and perhaps most importantly, the standard model displays higher welfare irrespective of the nature of shocks accounting for observed income volatility. This result strengthens our findings thus far, and it further suggests that defaultable debt is simply unlikely to be useful to households. It is also a particularly important form of robustness, given both the general importance of persistence for the efficacy of self-insurance and borrowing and because estimates of income shock persistence vary dramatically (see Guvenen 2007, Hryshko 2008, or Guvenen and Smith 2009 for discussions of the debate between "RIP" and "HIP" and their impact on persistence estimates) and appear to vary over the life cycle (Karahan and Ozkan 2009).

Second, the effect of the contribution of persistent shocks to income volatility depends on the education level of households. In particular, when volatility is driven primarily by persistent shocks, the relatively well-educated benefit from the elimination of default substantially more than their less-educated counterparts. Conversely, when most income variability is driven by large but transitory shocks, it is the relatively less-educated who benefit most from the elimination of the default option. The intuition for this result comes from the nature of borrowing: college types borrow primarily to use future expected income today while less-educated types borrow to smooth shocks.

Third, within each educational class, the welfare losses from bankruptcy decline monotonically as the relative contribution of the persistence of the shock grows; default on debt is least (most) useful when income volatility is driven primarily by shocks which are transitory (persistent). What is surprising, but in keeping with the main theme of our results, is that in *no* case is it true that US-style bankruptcy is *ex ante* more desirable than allocations obtaining under the standard model. Moreover, even in the case where essentially all income risk is delivered in the form of persistent shocks where credit markets are least useful in dealing with income risk, outcomes that allow bankruptcy are worse for agents than those arising in the standard model. The welfare in the standard model is non-trivially higher, at up to 1.24 percent of consumption for college-educated households (as seen in Figure 10).

In Figures 11 and 12, we display the measure of borrowers at each age and the conditional mean of debt among those who borrow for two levels of the importance of persistent income risk.[24] The fact that the losses from allowing bankruptcy rise for all agent-types with the importance of transitory shocks is a consequence of the increased usefulness of credit in dealing with transitory income risk. Conversely, when shocks are primarily persistent, a negative realization requires a more frequent borrowing and more debt in middle age; the combination is ultimately unable to stem the transfer of income risk to consumption volatility. In Tables 5 and 6, we see that irrespective of

[23] Athreya, Tam, and Young (2009) is primarily concerned with the role of income variance in models of default.

[24] From the perspective of a newborn, the measure of borrowers of a given age equals the probability of the newborn borrowing at that age.

bankruptcy policy, persistence translates into higher consumption volatility, and that the presence of lax default policy seen in Table 6 does little to stem the flow of income risk into consumption risk (echoing our previous result in Athreya, Tam, and Young 2009). In fact, by comparing Table 5 with Table 6, it is clear that for all the persistence levels we consider, total unconditional consumption volatility within each permanent-income group is lower under DM than SM. This result is of course central in explaining the uniformly positive welfare gains available under SM.

We turn next to the relationship between shock persistence and equilibrium default rates, displayed in Figure 13. Default is "U-shaped," with high default rates at both ends. To understand this shape, consider first the case where the labor income shocks are nearly all transitory (the left side of the graph). Here, agents can generally manage their risk effectively via saving and dissaving, but they choose to augment the self-insurance mechanism with default at higher rates than they do in the benchmark setting. The reason they do so is that risk-based pricing is not effective here, because there is no useful information contained in the current labor income of the borrower that would identify future bad risks. In contrast, the case where labor income is driven entirely by the persistent component (the right side of the graph), high default is the result of agents being generally unable to smooth consumption; persistent shocks are hard to smooth using assets alone (and if permanent are in fact impossible). As a result, despite the pricing effects borrowers will use default relatively often (and pay the costs to do so). The middle parts of the graph, where default is lowest, balance these two effects.

Intuitively, in the standard model, borrowers realize that debt must be repaid, and under high persistence, heavy borrowing in response to a negative shock makes low future consumption relatively likely. Nonetheless, credit markets are willing to lend to such households at the risk-free rate (adjusted for any transactions costs of intermediation), making total debt rise. When bankruptcy is available, borrowing today to deal with persistent income risk does not expose the borrower to severe consumption risk in the long term as default offers an "escape valve," but it does expose lenders to severe credit risk in the near term. Creditors then price debt accordingly; as seen in Figure 14, when shocks are primarily persistent, as the current shock deteriorates so do the terms at which borrowers can access credit. Moreover, under a bad current realization of income, households facing persistent risk see a disproportionate decline in the price of any debt they may issue, while the reverse occurs in the event of a good current realization of income; the pricing functions essentially "switch places." Yet, despite the increased sensitivity of loan pricing to the borrower's current income state under relatively high persistence, the welfare gains under SM, though still positive, fall. This result obtains because of the reduction in the ability of self-insurance, inclusive of borrowing, to prevent income fluctuations from affecting consumption. To sum up, income risk is quantitatively relevant in governing the losses conferred by bankruptcy, but irrelevant for altering the qualitative welfare property that in the absence of expense shocks, the bankruptcy option lowers welfare.

3.2.2 Ambiguity Aversion

We turn next to the question of whether bankruptcy can improve outcomes when households are not perfectly certain about the probabilistic structure of income risk. As mentioned at the outset, the idea is that allowing for default can potentially encourage households to borrow to deal with is hopefully a temporary fall in income, knowing that if income does not rebound, then the debt can be discharged. We formalize this idea as in Klibanoff, Marinacci, and Mukerji (2009) by assuming agents are averse to ambiguity. In this formulation, a household of age j solves the dynamic programming problem

$$v\left(a,y,e,\nu,\lambda,j\right) = \max_{b,d(e',\nu',\lambda')\in\{0,1\}} \left\{ \begin{array}{c} \frac{n_j}{1-\sigma}\left(\frac{c_j}{n_j}\right)^{1-\sigma} + \\ \beta\psi_{j,y}\sum_{e',\nu'} p\left(e',\nu'|e,\nu\right)\Phi\left(EU\right) \end{array} \right\} \quad (10)$$

$$EU = \sum_{e',\nu',\lambda'} \pi_e\left(e'|e\right)\pi_\nu\left(\nu'\right)\pi_\lambda\left(\lambda'|\lambda\right)V\left(b,y,e',\nu',\lambda',j+1\right)$$

where

$$\Phi\left(x\right) = \begin{cases} \frac{1-\exp(-\eta x)}{1-\exp(-\eta)} & \text{if } \eta > 0 \\ x & \text{if } \eta = 0 \end{cases}$$

determines preferences over ambiguity. $\eta \geq 0$ controls the attitude toward ambiguity; as η increases, the household becomes more averse to ambiguous stochastic processes. The restrictions on the choices of $p\left(e',\nu'|e,\nu\right)$ are that they must sum to 1 for each (e,ν) and every element must lie in some set $\mathcal{P} \subset [0,1]$; we nest the standard model by setting the $\mathcal{P}$ set to be an arbitrarily small interval around the objective probabilities.[25] We use π to denote objective probabilities and p to denote subjective ones; note that households are assumed to be uncertain only about the distribution of income shocks, not the process for λ.

Because we are interested in these preferences only to the extent that they may provide an environment in which relatively low-cost bankruptcy and debt-discharge are welfare-enhancing, we will deliberately take the most extreme case of $\eta = \infty$, yielding the max-min specification from Epstein and Schneider (2003):

$$v\left(a,y,e,\nu,\lambda,j\right) = \max_{b,d(e',\nu',\lambda')\in\{0,1\}} \left\{ \frac{n_j}{1-\sigma}\left(\frac{c_j}{n_j}\right)^{1-\sigma} + \beta\psi_{j,y} \min_{p(e',\nu'|e,\nu)} EU \right\} \quad (11)$$

$$EU = \sum_{e',\nu',\lambda'} p\left(e',\nu'|e,\nu\right)\pi_\lambda\left(\lambda'|\lambda\right)V\left(b,y,e',\nu',\lambda',j+1\right)$$

$$V\left(b,y,e',\nu',\lambda',j+1\right) = \left(1-d\left(e',\nu',\lambda'\right)\right)v\left(b,y,e',\nu',\lambda',j+1\right) + d\left(e',\nu',\lambda'\right)v^D\left(0,y,e',\nu',\lambda',j+1\right)$$

where

$$v^D\left(0,y,e,\nu,\lambda,j+1\right) = \left\{ \frac{n_j}{1-\sigma}\left(\lambda\frac{c_j}{n_j}\right)^{1-\sigma} + \beta\psi_{j,y} \min_{p(e',\nu'|e,\nu)} EU \right\} \quad (12)$$

$$EU = \sum_{e',\nu',\lambda'} p\left(e',\nu'|e,\nu\right)\pi_\lambda\left(\lambda'|\lambda\right)v\left(0,y,e',\nu',\lambda',j+1\right)$$

is the value of default.

The min operator that appears in front of the summation reflects the agent's aversion to uncertainty – as shown by Epstein and Schneider (2003), a household who is infinitely-uncertainty-averse chooses the subjective distribution of future events that is least favorable and then makes their decisions based on that subjective distribution. The size of the set of possible processes $\mathcal{P}$ measures the amount of ambiguity agents face; a typical p_{ij} element lies in the interval $\left[\mathbf{p}_1^{ij},\mathbf{p}_2^{ij}\right] \subset [0,1]$.[26]

Standard ambiguity aversion models imply that households will learn over time and reject stochastic processes that are inconsistent with observed data (for example, a household who initially entertains the possibility of permanently receiving the worst-possible income level forever will

[25]We do not require that the household assume that the probabilities of the independent events are independent in every distribution that is considered. That is, the household may be concerned that the independence property is misspecified and therefore select a worst-case distribution in which the events are correlated.

[26]Hansen and Sargent (2007) provide an interpretation of $\mathcal{P}$ in terms of detection probabilities.

dismiss this process as soon as one non-worst realization occurs). We will focus our attention on a special case of extreme ambiguity aversion in which this learning does not occur; if bankruptcy is not useful in this environment, it is unlikely to be optimal when households face less uncertainty over time.

Given the qualifications and considerations discussed above, we now evaluate outcomes in the standard model in the case where $\mathcal{P} = [0, 1]$, the most extreme case possible (households behave as if the minimum income draw will be realized with probability 1 next period). The intuition is that such a case offers the possibility, discussed at the outset, that lax penalties for default might actually encourage the use of credit for consumption in a setting where the agent's aversion to ambiguity would otherwise preclude becoming indebted. And in fact, we *do* find that this case delivers bankruptcy as welfare-improving for some agents (see Table 7). However, this finding is very limited: benchmark default costs improve welfare for only the college type and the welfare gain is tiny (under 0.2 percent of consumption). As a result, unconditional *ex ante* welfare is negative since college types are not a large enough group to overcome the losses to the remainder of the population. It is interesting to see, however, that the welfare changes from allowing bankruptcy are now reversed – the largest gains are experienced by the most educated, while the least educated suffer more. Part of the intuition for this result is that it is the best educated who face the steepest mean age-earnings profiles. Therefore, these agents would have the strongest purely intertemporal motives to borrow, absent any ambiguity. Low default costs mitigate the effect of ambiguity and allow for states in which a temporarily unlucky college-educated agent would find borrowing desirable.

Pricing is presented in Figures 16 and 15. Notice that for the low realization of e, the pricing function under ambiguity aversion is everywhere below the baseline expected utility case, but for the higher realization they switch places; ambiguity-averse agents with high income actually pose *less* of a default risk. The difference in pricing stems only from a difference in the households' willingness to default next period for a given b. Since default has a fixed cost component (Δ), households want to time their usage of bankruptcy; in particular, households must balance the gains from defaulting tomorrow from those arising from waiting until additional shocks have been realized. This fact places the expectations of income in periods after tomorrow at the heart of the timing of default decisions, and here households who face ambiguity about the income process act quite differently from those in the benchmark economy.[27]

Take first the household with low e. For a "rational expectations" household income in the distant future is expected to be better than whatever is realized tomorrow, as e is persistent but mean-reverting; for the household facing ambiguity, however, income is actually expected to be no better, or even worse, than tomorrow's realization. Since ambiguity-averse households do not think the future will be better, they may as well default next period if the realization of income is bad; lenders must therefore offer them higher interest rates to break even. In contrast, the ambiguity-averse household with higher e views a realization near the mean for next period as unexpectedly *good*, but does not expect better times in the more-distant future. Default in the next period is therefore not as valuable as waiting for a future period when those bad states are expected to occur. In contrast, without ambiguity a bad realization will induce the household

[27]The exposition is simpler if we refer to the expectations of the households facing ambiguity as coinciding with the choice of p, because the ambiguity-averse agents act *as if* those probabilities were the objective ones. Of course, if one were to ask ambiguity-averse agents about their forecasts of future income, they would use the true objective probabilities; they just do not use these probabilities for decisions. The proper phrasing of our statement "ambiguity-averse agents expect low future income" would be the more cumbersome "ambiguity-averse agents act as if they expect low future income." We abuse the notion of expectation slightly as a result, and beg for the reader's indulgence on this matter.

to substantially revise their future expectations downward, making default today more attractive (the decline in future income makes the fixed cost of default worth paying).[28] The result is that ambiguity-averse households with high current income obtain better terms.

Is such extreme ambiguity aversion "reasonable?" It seems highly unlikely that households entertain a stochastic process in which they receive the worst possible outcome forever with probability one as reasonable, at least not for long – after all, they need only observe the fact that their income is occasionally higher than the lower bound to discard this process empirically. As we noted above, we could introduce this learning into the model – since the households are simply learning about an exogenous process, it can be done "offline" – but it is computationally quite burdensome to condition the set of permissive stochastic processes on the history of observations.[29] It is also the case that this extreme ambiguity leads to a discrepancy between model and data in terms of borrowing patterns; there is far too little debt, which lessens our interest in making this economy "more realistic." If we consider smaller limits for $\mathcal{P}$ – such as 10 percent above or below the objective value – we find that bankruptcy is welfare-reducing for all education levels. Thus, while ambiguity aversion provides a theoretical foundation for bankruptcy options, it does not appear to provide an empirically tenable one.

4 Concluding Remarks

We have studied the efficacy of default in helping households better insure labor income risk in a large range of settings in which risk aversion, intertemporal smoothing motives, income risk, and uncertainty–and attitudes to uncertainty–over income risk itself were all varied. Our findings here suggest that within the broad class of models used thus far to develop quantitative theory for unsecured consumer credit and default, relatively generous US-style bankruptcy does not appear to be capable of providing protection against labor income risk.

Despite the fact that we find that labor income risk is not well hedged from the ex-ante perspective, we also show that there are *ex post* beneficiaries from allowing bankruptcy as it currently is; specifically, we show that the standard model generates a positive measure of agents *ex post* who would vote to introduce default. Our calibrated model predicts that these agents do not constitute a majority, though, since they are primarily college-educated middle-aged households who have been unlucky enough to still have significant debt. This result warrants further investigation since it may help explain why bankruptcy penalties are becoming less stringent over time (with the exception of some aspects of the most recent reform).

Our results also suggest that "expense" shocks or catastrophic movements in net worth are likely to be essential to justify the view of bankruptcy as a welfare-improving social institution. To the extent that uninsured, catastrophically large, and "involuntary" expenditures are indeed a feature of the data, a natural question is whether consumer bankruptcy is the best way to deal with such events. Given the nature of resource transfers created by bankruptcy and the constraints that it imposes on the young, who disproportionately account for *both* the income-poor and uninsured, this statement seems unlikely.

With respect to future work, in light of the findings of this paper and the larger quantitative theory of consumer default, three directions seem particularly useful. First, even from an *ex ante* perspective, DM may still be preferable to SM. A more "normative" approach, for example, which

[28] The median e has the pricing functions crossing, so that agents who face ambiguity are more likely to default on small debts but less likely to default on large ones.

[29] Since this learning is not Bayesian, it can be quite difficult to write recursively, and in any case learning about discrete processes generally involves a large number of states. Campanale (2008) investigates non-Bayesian learning in a two-state model where it introduces only one additional state.

asks if observed default procedures can arise an optimal arrangement under plausible frictions, may yield different conclusions. One interesting example of the latter approach is the theoretical work of Grochulski (2009), where bankruptcy is shown to be one method for decentralizing a constrained Pareto optimum in the presence of private information. Quantifying models with default and endogenously-derived asset market structures may lead to better understanding of policy choices in this area (such as why Europe has chosen to make bankruptcy available under very strict conditions, and social insurance generous, while the US has chosen the opposite).

Second, with respect to the experiments we studied, we were led to allow for two specific preference extensions beyond CRRA expected utility in order to accurately assess the particular tradeoffs created by default. While we emphatically did not attempt to turn the paper into a survey of any larger variety of non-expected utility preferences, some further extensions seem potentially important: disappointment aversion (Gul 1991 or Routledge and Zin 2008), deviations from geometric discounting (Laibson 1997), habit formation (Constantinides 1990), and loss aversion (Barberis, Huang, and Santos 2001). Why these preferences specifically? In each case, the more general preference structure breaks the link between risk aversion and intertemporal substitution (and generally make risk aversion state-dependent), and some (such as nongeometric discounting and loss aversion) provide arguments for government intervention; there is also extensive empirical work supporting many of them. A recent contribution to this literature is Nakajima (2008), who investigates whether the temptation preferences of Gul and Pesendorfer (2001) alter the consequences of bankruptcy reform.[30] We suspect other work will follow.

Finally, we have modeled default here as the formal legal process of voiding debts. In practice, bankruptcy is generally preceded by a period of delinquency in which obligations are not met (for example, missed minimum payments on credit cards). In a world where delinquency is an option, bankruptcy policy may have quite limited effects on borrowing limits, as banning it would not guarantee repayment; households may never file for bankruptcy but also never repay their debts. Athreya, Sánchez, and Young (2010) study this question.

5 Appendix: Computational Considerations

We make some brief points here regarding the computation of the model. The model is burdensome to calibrate, and all programs are implemented using Fortran95 with OpenMP messaging.

In all the models we study, the objective function (the right-hand-side of the Bellman equation) is not globally concave, since the discrete nature of the bankruptcy decision introduces convex segments around the point where the default option is exercised (we find that, as in Chatterjee *et al.* 2007, the default decision encompasses an interval and in our case it extends to $b = -\infty$ as Δ is smaller than even the worst income realization). The nonconcavity poses a problem for local optimization routines, so we approach it using a global strategy. We use linear splines to extend the value function to the real line and a golden section search to find the optimum, with some adjustments to guarantee that we bracket the global solution rather than the local one. It is straightforward to detect whether we have converged to the local maximum at any point in the state space, as the resulting price function will typically have an upward jump.

For the ambiguity aversion case we have a saddlepoint problem to solve. By the saddlepoint theorem we can do the maximization and minimization in any order; the minimization (conditional on b and d) is a linear program that we solve using a standard simplex method conditional on some b (as in Routledge and Zin 2009). We then nest this minimization within our golden section

[30] Nakajima (2009) finds that increasing borrowing constraints in a model with quasi-geometric discounting is not always welfare-improving, similar to Obiols-Homs (2009).

search, again with adjustments to deal with the presence of the local maximum. For our model, this linear program turns out to be extremely simple to solve – the household puts as much weight as allowed on the worst possible outcome, then as much weight as allowed on the next worst, and so on. This simplicity arises because the worst-case outcome does not depend on b.

To impose boundedness on the realizations of income, we approximate both e and ν by Markov chains using the approach in Flodén (2008). Having income be bounded above is convenient since it implies that there always exists a cost of default Δ such that bankruptcy is completely eliminated because it becomes infeasible. Quite naturally, bankruptcy is also likely not to occur when Δ is high enough even if filing is feasible for some types; in general, households with high income are not interested in the default option in our model.[31]

Figure 17 shows a typical objective function for a household in our benchmark case (expected utility with $\sigma = \rho^{-1} = 2$). The objective function has three distinct segments. The first segment is at the far right, where the values for both the low and high cost types coincide. In this region, default is suboptimal because borrowing either does not or barely exceeds Δ. The second segment is at the other end, where $q(b) = 0$; although impossible to see in the picture, the low cost default experiences slightly more utility in this region since default is less painful. The action is all in the middle segment. For this particular individual, the high cost type (λ_L) borrows significantly more than the low cost type; this extra borrowing reflects primarily the pricing function (as seen in the lower panel) and not any particular desire to borrow. High cost types have more implicit collateral and are less likely to default at any given debt level, so they face lower interest rates. As a result, high type borrowers today who become low type borrowers tomorrow are a main source of default in our model – they both have debts and are not particularly averse to disposing of those debts through the legal system. Since type is persistent, low type borrowers today will not generally make the same choice – the supply side of their credit market will contract.

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Table 1

Calibration

Case	$\rho=-1,\sigma=2$		$\rho=-0.5,\sigma=2$		$\rho=-1,\sigma=5$	
Parameter,Target	Parameter	Outcome	Parameter	Outcome	Parameter	Outcome
$\lambda^h_{nhs},\pi_{nhs}=1.03\%$	0.8972	0.31%	0.8668	1.24%	0.9376	0.51%
$\lambda^l_{nhs},E(\frac{b}{y}\|b<0)_{nhs}=0.1552$	0.7624	0.2071	0.6929	0.2104	0.7538	0.1561
$\lambda^h_{hs},\pi_{hs}=1.11\%$	0.8832	0.97%	0.8064	1.29%	0.8872	1.31%
$\lambda^l_{hs},E(\frac{b}{y}\|b<0)_{hs}=0.5801$	0.7135	0.1835	0.6933	0.1825	0.6236	0.2553
$\lambda^h_{coll},\pi_{coll}=0.57\%$	0.7067	0.63%	0.7136	0.79%	0.7055	0.76%
$\lambda^l_{coll},E(\frac{b}{y}\|b<0)_{coll}=0.7251$	0.5698	0.1504	0.6352	0.1506	0.4205	0.2194
$\beta,\Pr(b<0)=12.5\%$	0.9765	17.5%	0.9895	13.3%	0.9532	12.5%
$\rho_\lambda,\frac{E(b\|d=1)}{E(y\|d=1)}=0.56$	0.8597	0.3986	0.6655	0.4073	0.7658	0.4630

Table 2

Welfare Gains (w/o Recalibration)

$\sigma = 2$ & $EIS = 0.5$	Coll	HS	NHS
$BK \to NBK$	1.21%	0.54%	0.52%
$\sigma = 2$ & $\mathbf{EIS = 0.67}$	Coll	HS	NHS
$BK \to NBK$	0.58%	0.21%	0.13%
$\boldsymbol{\sigma = 5}$ & $EIS = 0.5$	Coll	HS	NHS
$BK \to NBK$	0.47%	0.16%	0.13%

Table 3

Measure of Agents in Favor of Bankruptcy

Education	BK Regime	NBK Regime
Coll	6.45%	4.09%
HS	4.05%	3.26%
NHS	0.16%	0.24%
Total	4.05%	2.98%

Table 4

Welfare Gains (w/ Recalibration)

$\sigma = 2$ & $EIS = 0.5$	Coll	HS	NHS
$BK \rightarrow NBK$	1.21%	0.54%	0.52%
$\sigma = 2$ & $\mathbf{EIS = 0.67}$	Coll	HS	NHS
$BK \rightarrow NBK$	0.28%	0.05%	0.04%
$\boldsymbol{\sigma = 5}$ & $EIS = 0.5$	Coll	HS	NHS
$BK \rightarrow NBK$	1.28%	0.57%	0.56%

Table 5

Consumption Smoothing (BK)

	Intra			Inter			Total		
	Coll	HS	NHS	Coll	HS	NHS	Coll	HS	NHS
1.0%	0.0306	0.0462	0.0575	0.0359	0.0364	0.0386	0.0665	0.0826	0.0961
10.0%	0.0377	0.0561	0.0872	0.0343	0.0367	0.0357	0.0720	0.0938	01229
20.0%	0.0459	0.0807	0.1092	0.0336	0.0347	0.0325	0.0795	0.1154	0.1417
30.0%	0.0538	0.0884	0.1367	0.0327	0.0327	0.0297	0.0865	0.1211	0.1664
40.0%	0.0619	0.1013	0.1472	0.0316	0.0301	0.0280	0.0925	0.1314	0.1752
50.0%	0.0700	0.1146	0.1613	0.0305	0.0284	0.0263	0.1005	0.1430	0.1876
60.0%	0.0779	0.1280	0.1797	0.0294	0.0264	0.0241	0.1065	0.1544	0.2038
70.0%	0.0859	0.1413	0.1992	0.0283	0.0247	0.0224	0.1141	0.1660	0.2216
80.0%	0.0946	0.1543	0.2182	0.0272	0.0231	0.0211	0.1218	0.1774	0.2393
90.0%	0.1053	0.1681	0.2368	0.0258	0.0212	0.0199	0.1311	0.1893	0.2567
99.0%	0.1248	0.1863	0.2566	0.0235	0.0187	0.0180	0.1483	0.2050	0.2680

Table 6

Consumption Smoothing (NBK)

	Intra			Inter			Total		
	Coll	HS	NHS	Coll	HS	NHS	Coll	HS	NHS
1.0%	0.0196	0.0307	0.0474	0.0318	0.0314	0.0120	0.0514	0.0621	0.0594
10.0%	0.0271	0.0397	0.0577	0.0315	0.0298	0.0124	0.0586	0.0695	0.0801
20.0%	0.0360	0.0541	0.0771	0.0311	0.0290	0.0131	0.0671	0.0831	0.0902
30.0%	0.0444	0.0683	0.0971	0.0306	0.0284	0.0137	0.0750	0.0967	0.1108
40.0%	0.0524	0.0820	0.1173	0.0300	0.0277	0.0144	0.0824	0.1097	0.1317
50.0%	0.0600	0.0951	0.1364	0.0295	0.0271	0.0151	0.0895	0.1222	0.1515
60.0%	0.0673	0.1076	0.1550	0.0291	0.0267	0.0158	0.0964	0.1343	0.1708
70.0%	0.0743	0.1197	0.1729	0.0288	0.0262	0.0164	0.1031	0.1495	0.1893
80.0%	0.0811	0.1314	0.1903	0.0285	0.0258	0.0171	0.1096	0.1627	0.2075
90.0%	0.0878	0.1428	0.2072	0.0282	0.0255	0.0178	0.1160	0.1638	0.2250
99.0%	0.0935	0.1528	0.2218	0.0280	0.0253	0.0182	0.1215	0.1781	0.2400

Table 7

Welfare Effects under Ambiguity Aversion

$\mathcal{P} = [0, 1]$	NHS	HS	Coll
BK $\rightarrow$ NBK	0.215%	0.189%	−0.185%
$\mathcal{P} = \min(1, \pi + 0.5)$	NHS	HS	Coll
BK $\rightarrow$ NBK	0.296%	0.219%	0.044%

Figure 1: Intertemporal Consumption Smoothing, Expected Utility

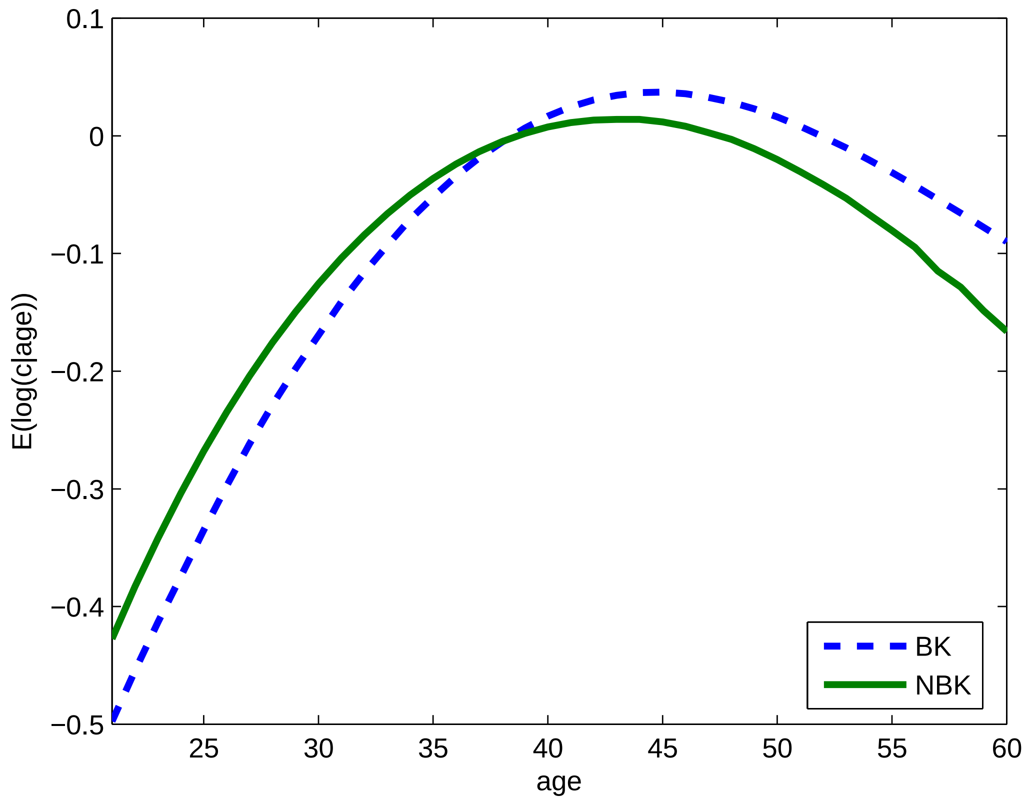

Figure 2: Intratemporal Consumption Smoothing, Expected Utility

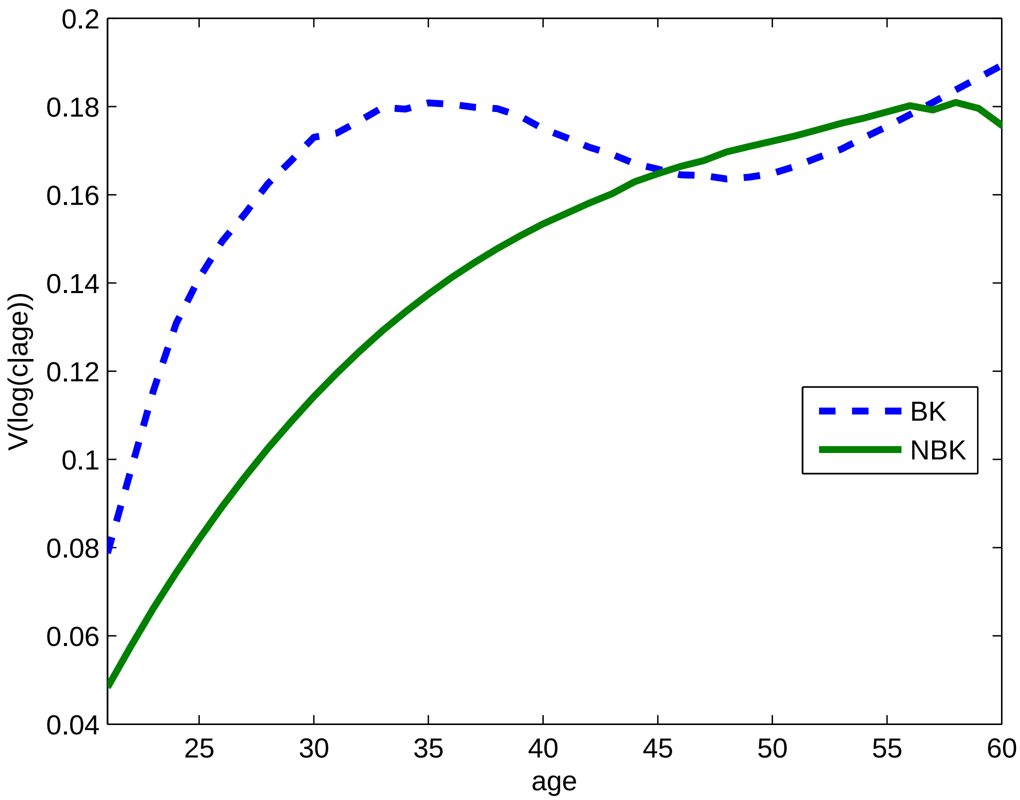

Figure 3: Pricing, Expected Utility

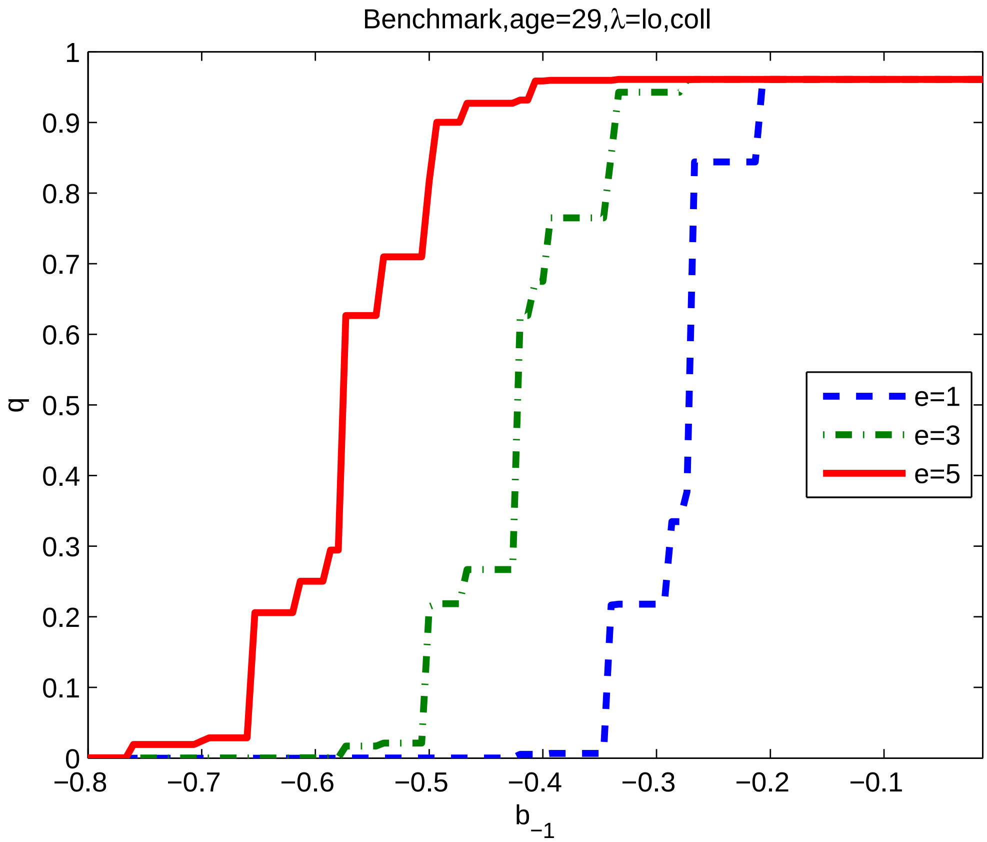

Figure 4: Pricing, Expected Utility

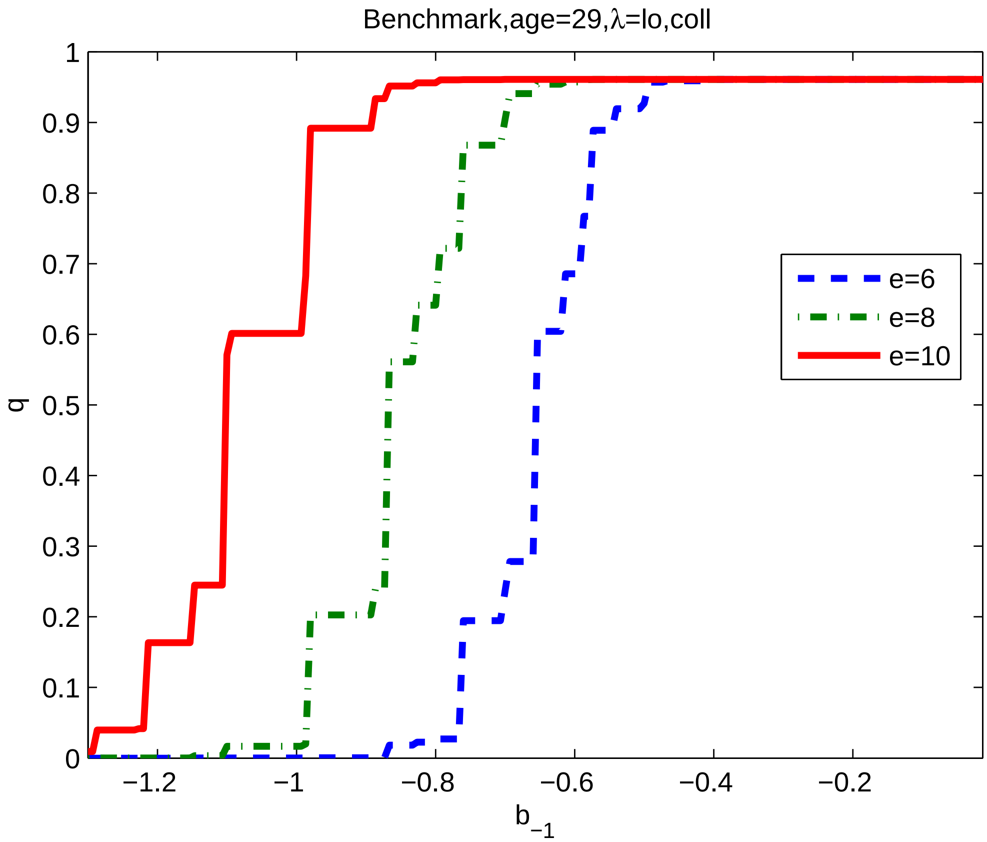

Figure 5: Pricing, Expected Utility

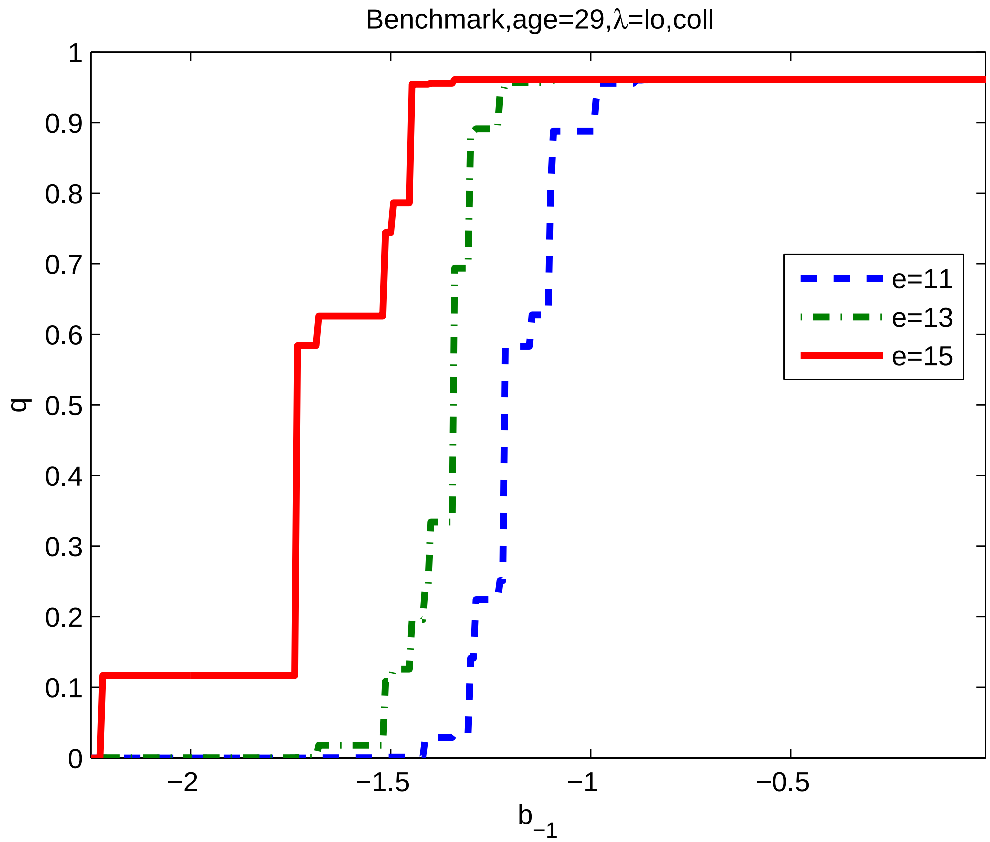

Figure 6: Fraction Supporting Bankruptcy, BK Regime

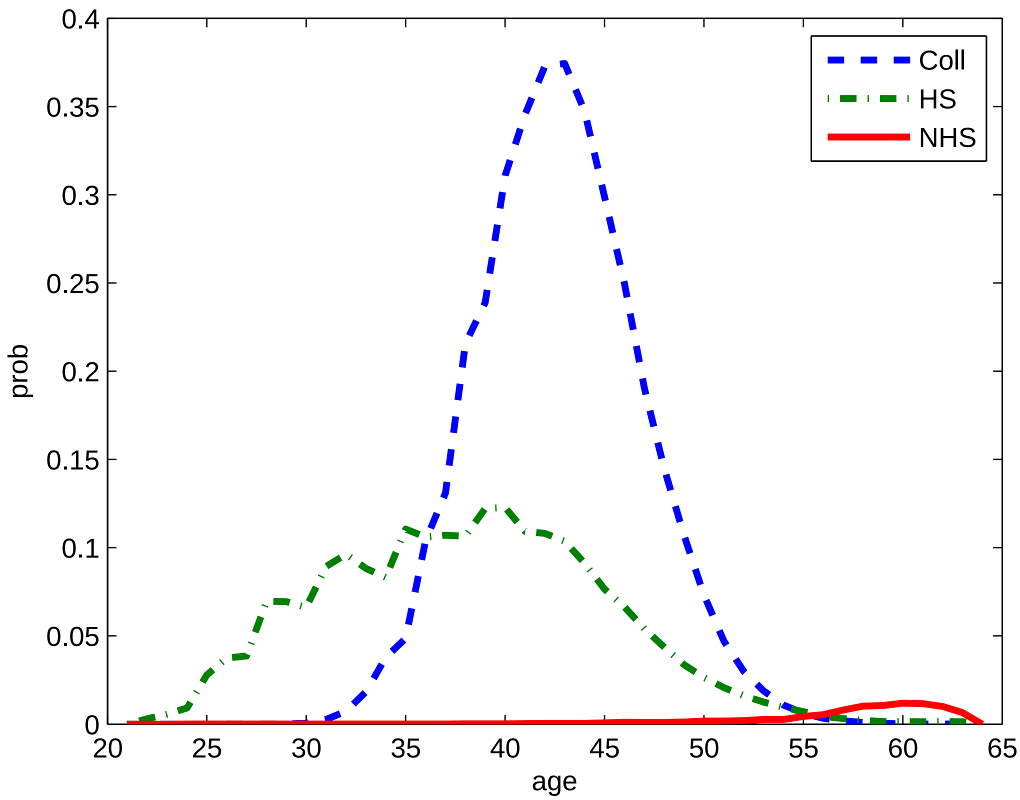

Figure 7: Fraction Supporting Bankruptcy, NBK Regime

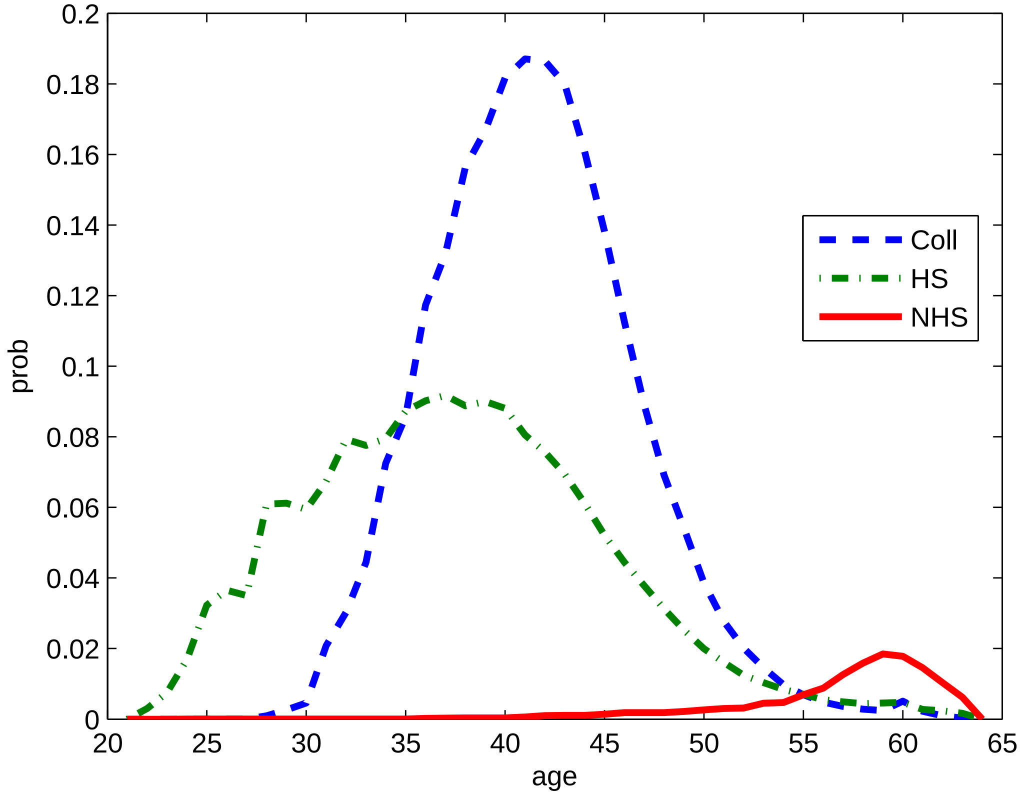

Figure 8: Pricing, Epstein-Zin with Different EIS

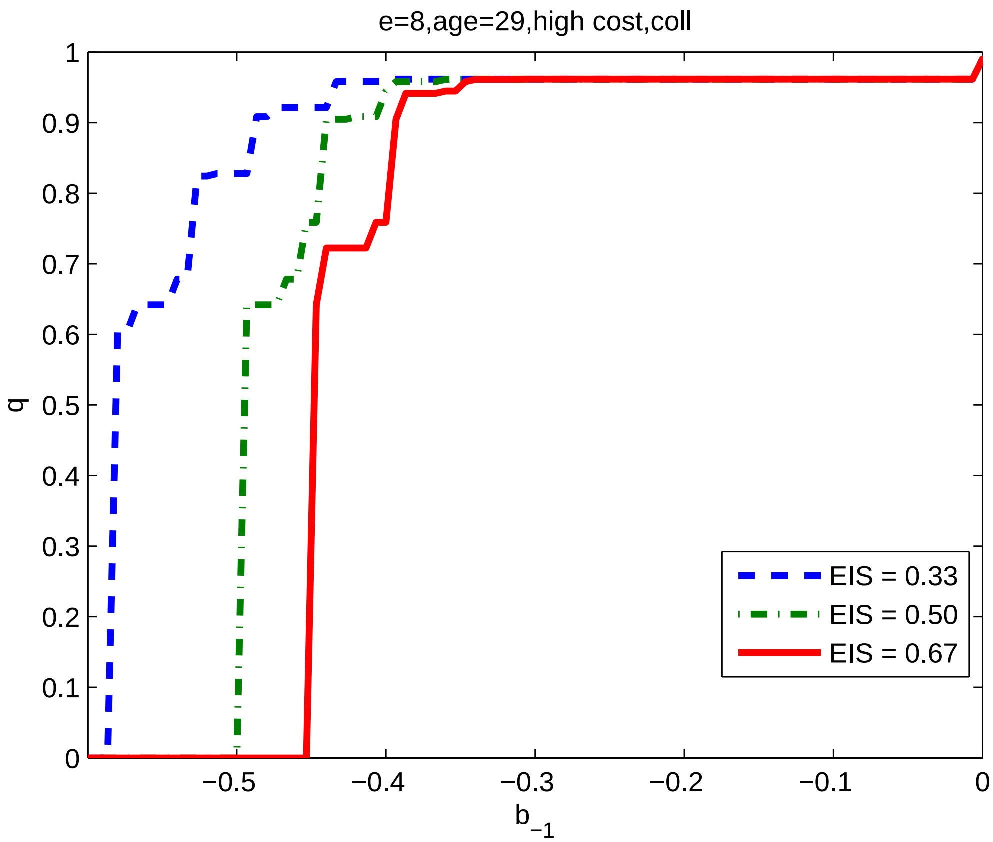

Figure 9: Pricing, Epstein-Zin with Different Risk Aversion

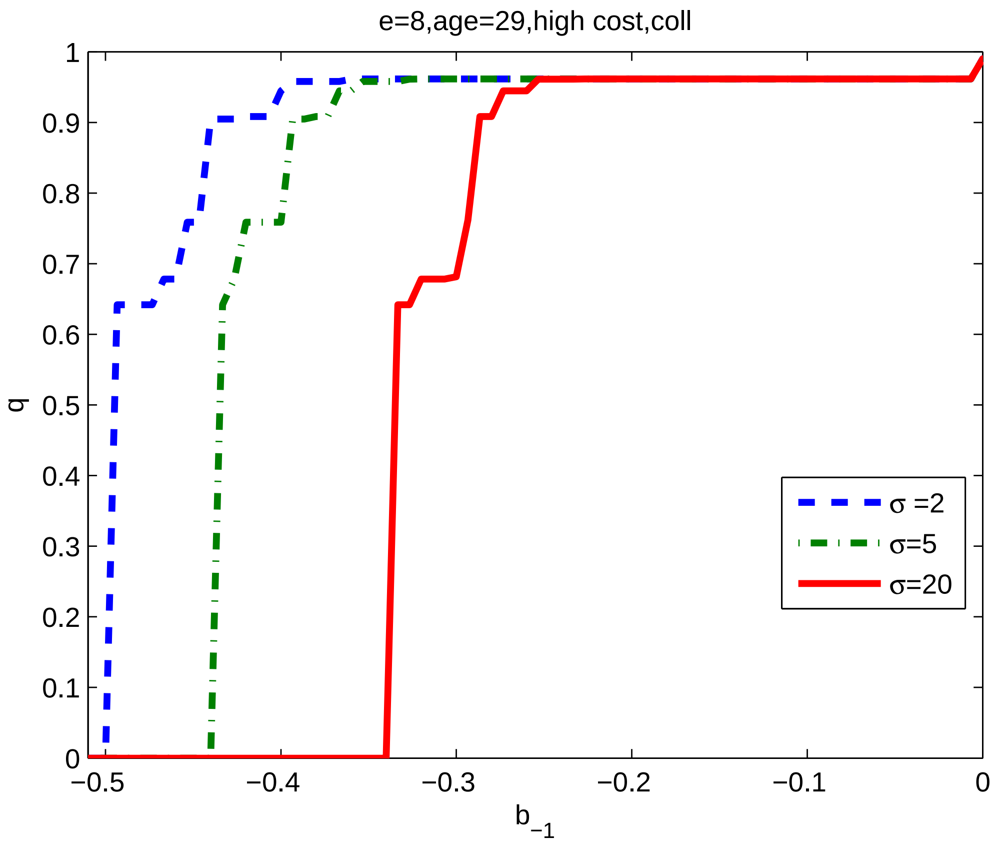

Figure 10: Welfare Gains from Eliminating Bankruptcy

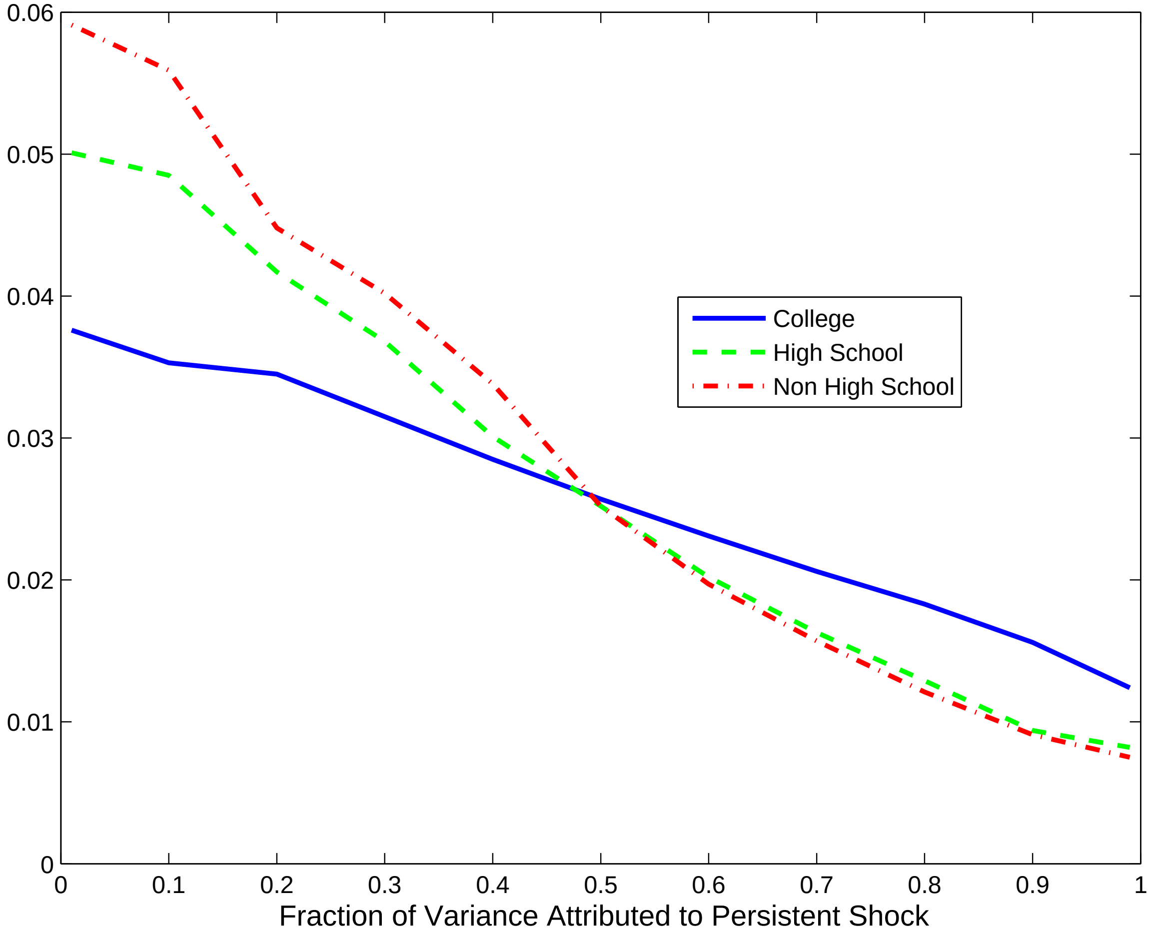

Figure 11: Fraction of Borrowers

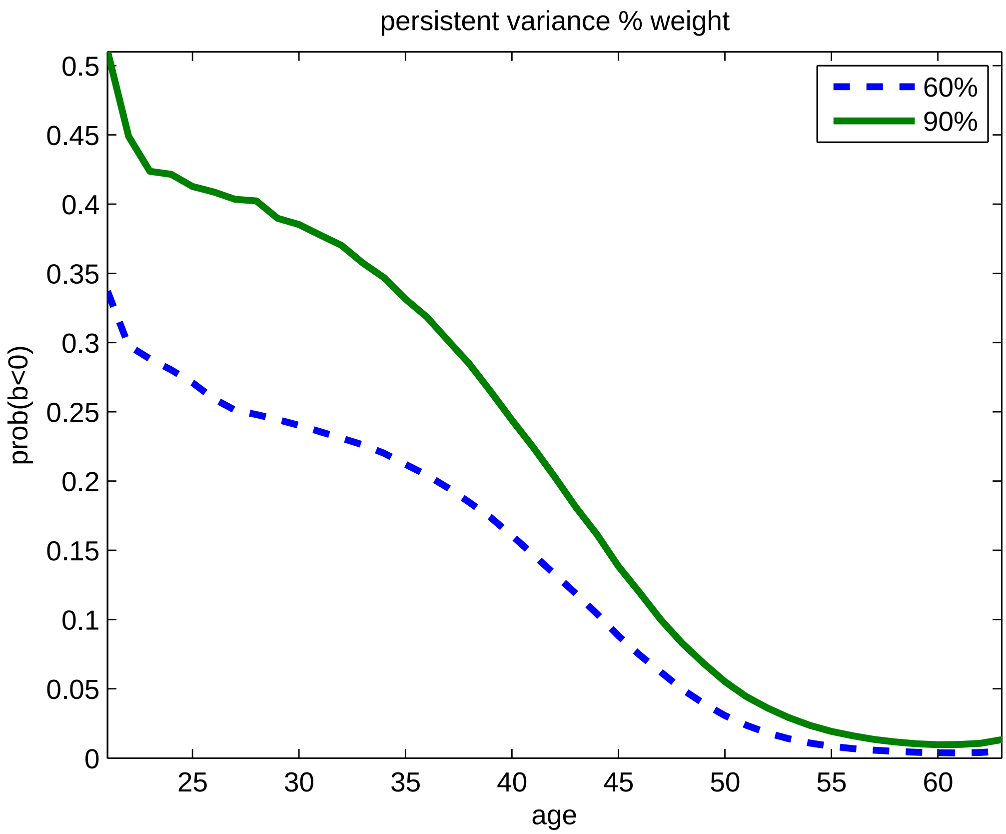

Figure 12: Mean Debt of Borrowers

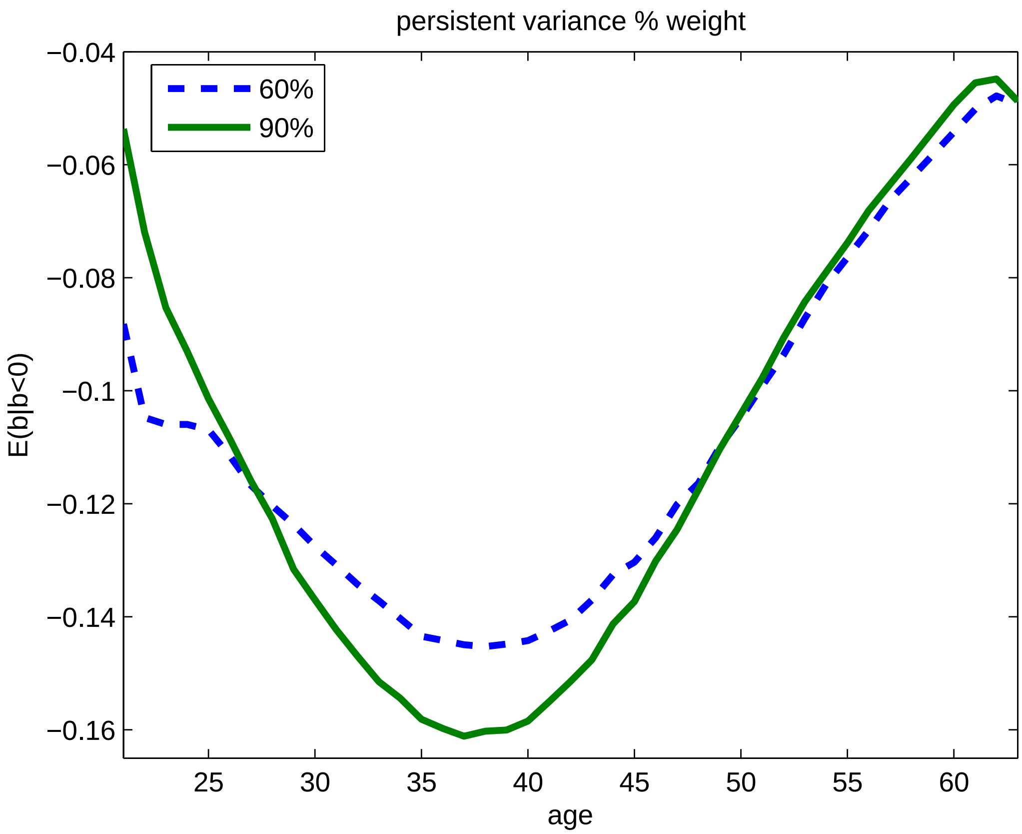

Figure 13: Default Rates

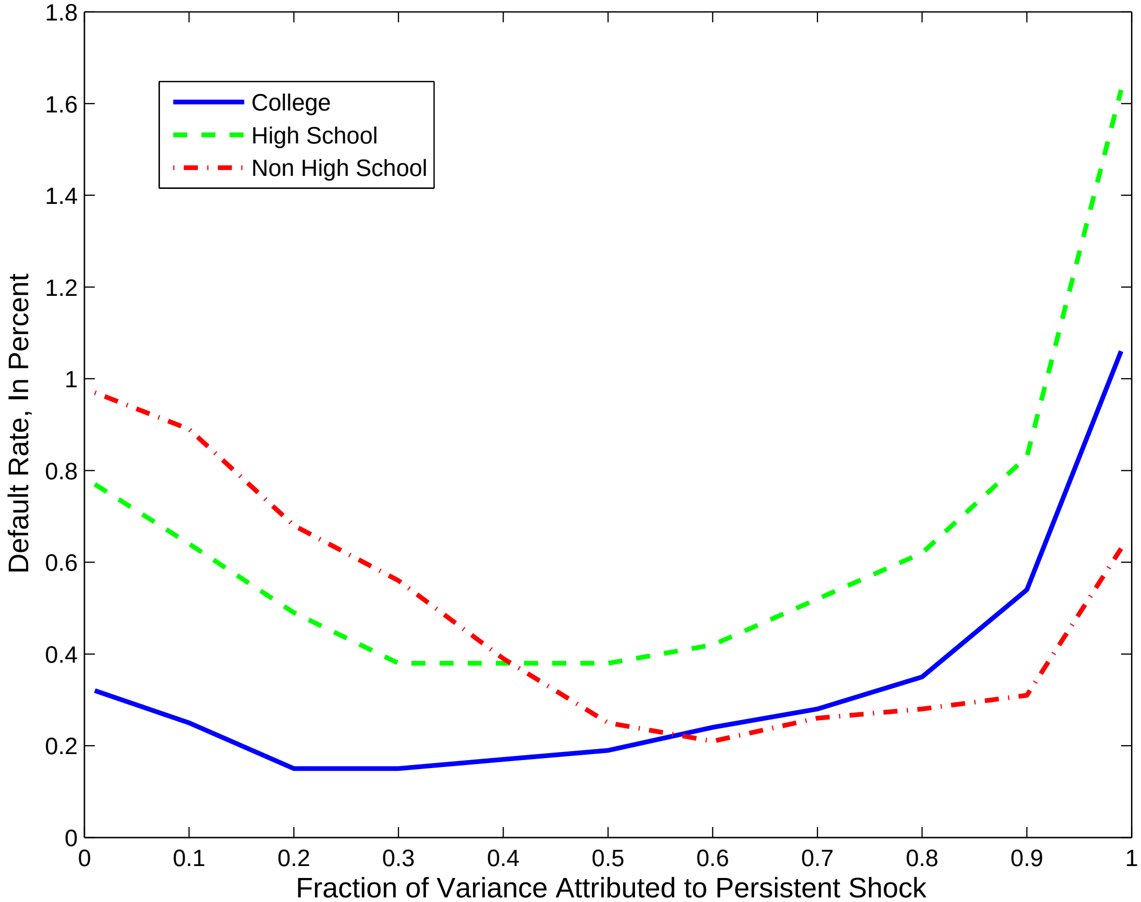

Figure 14: Pricing Functions

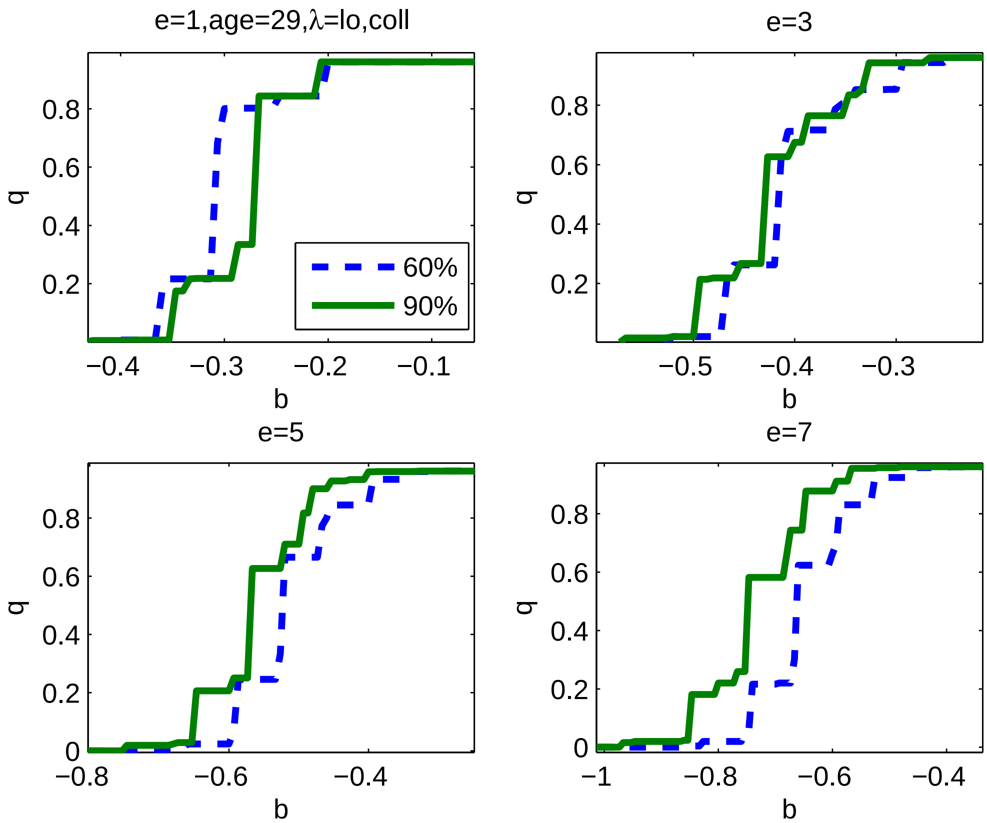

Figure 15: Pricing, Ambiguity Aversion, Low e

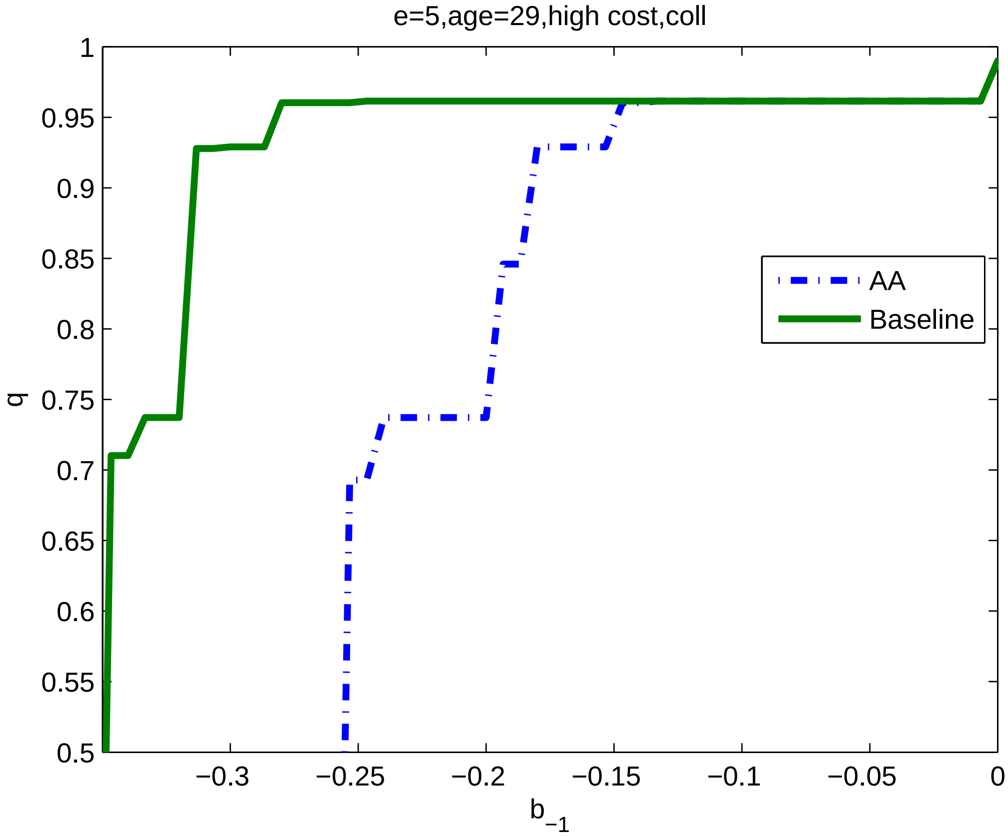

Figure 16: Pricing, Ambiguity Aversion, High e

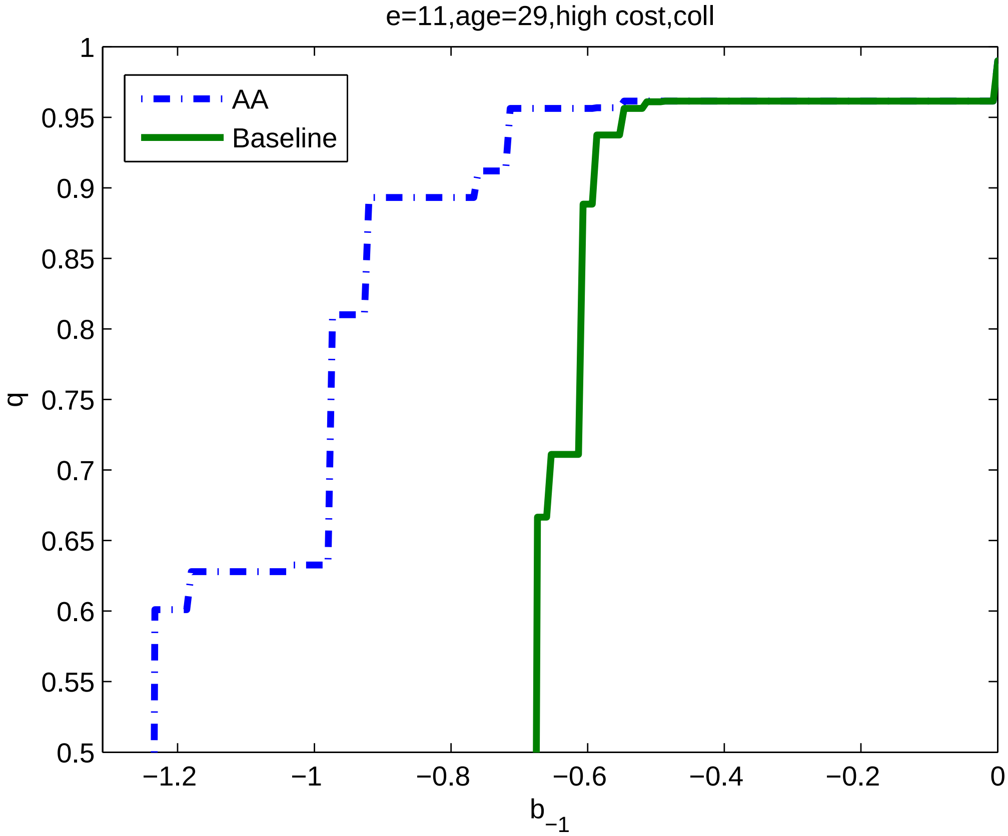

Figure 17: Optimal Choice of b given q