

On the Unstable Relationship between Exchange Rates and Macroeconomic Fundamentals¹

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Abstract

It is well known from anecdotal and survey evidence that the relationship between the exchange rate and macro fundamentals is highly unstable. We argue that large and frequent variations in the relationship between the exchange rate and macro fundamentals naturally develop when structural parameters in the economy are unknown. We show that the reduced form relationship between exchange rates and fundamentals is then driven not by the structural parameters themselves, but rather by expectations of these parameters. These expectations can be highly unstable as a result of perfectly rational “scapegoat” effects. This mechanism can also be relevant in other contexts of forward looking variables and could explain the widespread evidence of parameter instability found in macroeconomic and financial data.

1 Introduction

“The dollar’s resilience in the wake of recent dire US economic data has raised the prospect that the currency market may be experiencing one of its periodic changes in focus” (Financial Times, February 11, 2008)

“The dollar’s latest stumble ... came despite optimistic economic data from the US. But analysts said the movement of the US currency was no longer driven by growth fundamentals. All the focus is on the deficit now...” (Financial Times, February 11, 2003)

As reflected in these quotes, foreign exchange traders regularly change the weight they attach to different macro indicators. Cheung and Chinn (2001) have documented these changes through a survey of U.S. foreign exchange traders. More recent and systematic surveys confirm this feature. Fratzscher et al. (2011) use monthly survey data for a set a of 12 currencies over 9 years and find large time variation in the importance that FX traders attach to different macro fundamentals as determinants of exchange rates.¹ Such changes naturally lead to an unstable relationship between exchange rates and macro fundamentals.

The main goal of this paper is to show that large and frequent variations in the relationship between the exchange rate and macro fundamentals occur naturally when there is uncertainty about the structural parameters in the economy. We show that with parameter uncertainty, the relationship between a forward looking variable like the exchange rate and macro fundamentals is determined not by the structural parameters themselves, but rather by the expectations of structural parameters. Moreover, we show that these expectations can vary significantly over time, giving rise to a highly unstable reduced form relationship between exchange rates and fundamentals. This happens even though agents are perfectly rational Bayesian learners.

In addition to the usual learning process, whereby parameter expectations con-

¹The survey data on “Economic factors affecting exchange rates” is collected by Consensus Economics. As factors they consider relative growth, inflation differential, trade/current account, interest rate differential, and equity flows. On their website, Consensus Economics writes “... the importance of each [factor] varies both from country to country and, for a given currency, over time.”

verge to their actual values, expectations can exhibit large short-term fluctuations. These are due to a mechanism that we refer to as a “scapegoat” effect. Some information about the nature of structural parameters can be derived by analyzing macroeconomic data and exchange rates. But these data are also driven by shocks to *unobserved* fundamentals. Such unobserved fundamentals can generate considerable confusion in the short to medium run. When the exchange rate fluctuates as a result of an unobserved macroeconomic shock, it can be optimal for agents to blame this on an observed macro fundamental by giving it more weight and therefore making it a “scapegoat”.² For example, when the dollar depreciates it is natural to attribute it to a large current account deficit. This happens even when the depreciation is unrelated to the current account deficit.

In illustrating the importance of such scapegoat effects, and their role in the unstable reduced form relationship between exchange rates and fundamentals, we slightly generalize the “canonical” exchange rate model. This is actually a broad class of exchange rate models that can be reduced to a single stochastic difference equation, derived from an interest rate parity equation and an equation that relates the interest differential to observed fundamentals. The latter can be obtained either from monetary policy specifications or money market equilibrium in a standard monetary model. We assume that the underlying parameters in this second equation, such as monetary policy or money demand parameters, or the relationship between policy targets and observed fundamentals, are not perfectly known. Moreover, we assume that some macro fundamental is not observable.

Our paper is not the first to introduce parameter uncertainty and Bayesian learning in a standard exchange rate model. Lewis (1989) assumes the existence of a one-time change in the constant term of the money demand equation. Kaminsky (1993) assumes that money growth is equal to a drift term that can switch between

²In a previous short paper, Bacchetta and van Wincoop (2004), we developed the idea of such a scapegoat effect in the context of a simple static noisy rational expectations model in which some parameters are unknown. We showed that excessive weight could be given to a variable depending on the correlation between the noise shock and the fundamental shock. However, since that model is static it could not be used to address the unstable dynamic relationship between exchange rates and fundamentals and its implications. Apart from the dynamic setup, the model in this paper also differs in that there is no private information as in Bacchetta and van Wincoop (2004). Scapegoat effects naturally develop as long as there is incomplete information about parameters; the information does not need to be private.

two values based on a Markov process. In both cases agents learn about the unknown parameters through Bayesian updating. Tabellini (1988) emphasized that such a framework can lead to increased exchange rate volatility relative to the case where parameters are known.³ However, these papers do not consider uncertainty about parameters multiplying fundamentals and the role of unobserved fundamentals. Consequently, the scapegoat effect is not present in this prior literature.

To examine the quantitative relevance of the effects we describe, we calibrate the model to data for 5 industrialized countries, matching moments related to interest rates and exchange rates and the explanatory power of observed fundamentals. We find that the derivative of the exchange rate with respect to fundamentals can be very volatile when structural parameters are unknown. However, we show that this instability in the reduced form relationship between exchange rates and macro fundamentals is not detected by standard parameter instability tests.

The next section presents the model. It shows that the relationship between the exchange rate and fundamentals depends on expectations of structural parameters. We then go on to derive how these expectations are updated with a Kalman filter. Section 3 calibrates the model based on data on interest rates and exchange rates and presents numerical results for the relationship between exchange rates and fundamentals based on simulations. Section 4 discusses the empirical relevance of the scapegoat effect, an extension to time-varying structural parameters and the broader role of the mechanism in macroeconomics and finance. Section 5 concludes.

2 A Model with Unknown Parameters

The underlying framework is a standard exchange rate model with constant parameters.⁴ We first describe the model when parameters are known. Then we show how the exchange rate is affected by parameter expectations when parameters are unknown.

³There is also a vast literature on learning and exchange rates with deviations from rational expectations (e.g. Goldberg and Frydman, 1996, Gourinchas and Tornell, 2004, or Lewis and Markiewicz, 2009).

⁴In this paper we only consider constant parameters. In a previous version of this paper, Bacchetta and van Wincoop (2009), we examined the case where parameters vary over time. While more realistic, that case significantly complicates the analysis and its exposition. With constant parameters, agents eventually learn their true value. But we will show that the rate of learning is very slow when standard calibrations are used.

In deriving parameter expectations, we show that in addition to a standard learning process, there is a mechanism that we call a scapegoat effect. This mechanism leads to an unstable relationship between fundamentals and exchange rates.

2.1 Basic Framework

We consider the class of fundamental-based exchange rate models that can be reduced to a single stochastic difference equation. The equilibrium value of the exchange rate in these models depends on the present value of expected future fundamentals. We start with the usual case of known parameters. We follow Engel and West (2005) and slightly rewrite their equation (1):

$$s_t = (1 - \lambda) \left[F_t + b_t + \sum_{j=1}^{\infty} \lambda^j E_t (F_{t+j} + b_{t+j}) \right] - \lambda \left[\phi_t + \sum_{j=1}^{\infty} \lambda^j E_t \phi_{t+j} \right] \quad (1)$$

where s_t is the log nominal exchange rate (domestic per foreign currency), E_t is the expectation of the representative investor, ϕ_t is the risk premium and $0 < \lambda < 1$. We denote by F_t a linear combination of observed macro fundamentals: $F_t = \mathbf{f}'_t \boldsymbol{\beta}$ where $\mathbf{f}_t = (f_{1t}, f_{2t}, \dots, f_{Nt})'$ is the vector of N observed macroeconomic fundamentals and $\boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_N)'$ is the vector of associated parameters. Finally, b_t represents unobserved macro fundamentals.

Engel and West (2005) present several models that lead to this equation.⁵ A core element of these models is an interest rate parity condition:

$$E_t s_{t+1} - s_t = i_t - i_t^* + \phi_t \quad (2)$$

where i_t and i_t^* represent the domestic and foreign nominal one-period interest rates. The other basic element is an equation that relates the interest rate differential to the exchange rate and to observed and unobserved fundamentals. This equation represents for example the reduced form of a monetary model or the differential in interest rate rules. This equation can be written in the following form:

$$i_t - i_t^* = \mu s_t - \mu (F_t + b_t) \quad (3)$$

⁵See also Nason and Rogers (2008) who derive this equation from a DSGE model.

Combining equations (2) and (3), integrating forward and assuming no bubble gives equation (1), where $\lambda = 1/(1 + \mu)$. Notice that in practice we know that λ is very close to 1 (e.g. see Engel and West, 2005).⁶

Throughout the paper we will assume that the observed fundamentals \mathbf{f}_t follow a random walk and that the unobserved fundamental b_t follows an AR process:

$$b_t = \rho_b b_{t-1} + \varepsilon_t^b \quad (4)$$

where the innovation has mean zero and variance σ_b^2 . If the parameters β are known, agents know the value of b_t even if it is not directly observable. They can infer its value from (3) as agents can observe the exchange rate, interest rates and F_t .

Assuming for now that the risk premia ϕ_t are zero, when the parameters β are known these assumptions imply that (1) becomes

$$s_t = \mathbf{f}_t' \beta + \theta b_t \quad (5)$$

where

$$\theta = \frac{1 - \lambda}{1 - \rho_b \lambda} \quad (6)$$

It is immediate in this case that the derivative of the exchange rate with respect to fundamentals is

$$\frac{\partial s_t}{\partial f_{nt}} = \beta_n \quad (7)$$

The derivative of the exchange rate with respect to fundamentals is therefore equal to a known constant, which is equal to the structural parameter β_n .

2.2 Exchange Rate when Parameters are Unknown

We now turn to the case where the parameters β are unknown. Specifically, assume that during an initial period 1 all parameters are drawn from a distribution with mean $\bar{\beta}$ and standard deviation σ . Agents can learn over time about the value of the parameters from the observation equation (3). In particular, as the interest

⁶The key point in Engel and West (2005) is that when λ is close to 1, the exchange rate will be close to a random walk. This holds as long as either F_t , b_t or ϕ_t is I(1). This conclusion does not depend on whether the parameters β are known or unknown. In the numerical application in section 3 we assume that both F_t and ϕ_t are I(1).

rate and exchange rate are observable, agents can observe $F_t + b_t = \mathbf{f}'_t \boldsymbol{\beta} + b_t$. Here both the parameters $\boldsymbol{\beta}$ and the fundamental b_t are unknown. Agents use these observations each period to update their beliefs about the parameters and b_t .

When parameters are unknown, equation (1) no longer implies (5). Instead, we have

$$s_t = (1 - \lambda)\mathbf{f}'_t \boldsymbol{\beta} + \lambda \mathbf{f}'_t E_t \boldsymbol{\beta} + (1 - \lambda)b_t + \lambda \theta \rho_b E_t b_t \quad (8)$$

Since agents know the value of $\mathbf{f}'_t \boldsymbol{\beta} + b_t$, for a given expectation $E_t \boldsymbol{\beta}$ of the parameters we have

$$E_t b_t = \mathbf{f}'_t \boldsymbol{\beta} + b_t - \mathbf{f}'_t E_t \boldsymbol{\beta} = \mathbf{f}'_t (\boldsymbol{\beta} - E_t \boldsymbol{\beta}) + b_t \quad (9)$$

The exchange rate can then be written as

$$s_t = \mathbf{f}'_t (\theta \boldsymbol{\beta} + (1 - \theta) E_t \boldsymbol{\beta}) + \theta b_t \quad (10)$$

The only difference in comparison to equation (5) is that the coefficients multiplying the fundamentals are not equal to the structural parameters $\boldsymbol{\beta}$, but to a weighted average of the structural parameters and of expectations of these parameters. This significantly changes the relationship between the exchange rate and fundamentals as the weight on the true parameters is small: since λ is close to one, θ is close to zero. Moreover, as we will see, even though the parameters themselves are constant, expectations of parameters can change significantly over time. This affects the impact of fundamentals on the exchange rate.

The derivative of the exchange rate with respect to fundamentals is now

$$\frac{\partial s_t}{\partial f_{nt}} = \theta \beta_n + (1 - \theta) E_t \beta_n + (1 - \theta) \mathbf{f}'_t \frac{\partial E_t \boldsymbol{\beta}}{\partial f_{nt}} \quad (11)$$

First consider the sum of the first two terms, which is the weighted average of the parameter β_n and its expectation. Again, since θ is close to zero almost all the weight is on the expectation rather than the parameters themselves. As we will see below, changes in fundamentals also change the expectations of parameters. In the last term of (11) the derivative of the expected parameters with respect to the fundamental interacts with the level of fundamentals. This last term turns out to be particularly volatile.

2.3 Expectation of Parameters

We now turn to the computation of the expected parameters. For illustrative purposes we describe the case where $N = 1$, so that there is only one fundamental. After that we briefly describe how the results generalize to multiple fundamentals, leaving the derivation to the Appendix.

Each period agents observe $y_t = f_t\beta + b_t$. In period 1 the parameter β is drawn from a distribution with mean $\bar{\beta}$ and standard deviation σ . We assume that $f_1 = b_1 = 0$, so that in period 1 itself the signal y_1 is simply zero and provides no information about the value of β . Starting in period 2, agents gradually learn through the signals $f_t\beta + b_t$. It is useful to rewrite the new information that comes available at time t , starting at $t = 2$, as

$$\tilde{\Delta}y_t = y_t - \rho_b y_{t-1} = \tilde{\Delta}f_t\beta + \varepsilon_t^b \quad (12)$$

where $\tilde{\Delta}f_t = f_t - \rho_b f_{t-1}$. Written in this way, the signal $\tilde{\Delta}y_t$ depends on the unknown parameter β , multiplied by the fundamental $\tilde{\Delta}f_t$, plus an idiosyncratic noise ε_t^b .

In period 1 the expectation of β is equal to $\bar{\beta}$ and the perceived standard deviation is equal to σ . After that expectations can be updated using the Kalman filter. It is useful to first consider the Kalman updating formulas for uncertainty about the parameter. Let p_t be the variance of the expectation of β at time t .

Starting with $p_1 = \sigma^2$, the Kalman updating formulas give

$$p_t = p_{t-1}\alpha_t \quad (13)$$

where

$$\alpha_t = \frac{\sigma_b^2}{(\tilde{\Delta}f_t)^2 p_{t-1} + \sigma_b^2} \quad (14)$$

Note that $0 < \alpha_t < 1$ and in fact α_t is close to 1. The difference from one is second-order as $(\tilde{\Delta}f_t)^2 p_{t-1}$ is fourth order. This means that agents learn slowly. While uncertainty about the parameter declines over time, this happens at a slow rate. The logic behind this is that in the signal $\tilde{\Delta}f_t\beta + \varepsilon_t^b$, the parameter β is multiplied by a small first-order variable $\tilde{\Delta}f_t$, especially when ρ_b is close to one.

Using this result we have

$$p_t = g_t\sigma^2 \quad (15)$$

where

$$g_t = \prod_{i=2}^t \alpha_i \quad (16)$$

Here g_t represents cumulative learning. It is the remaining uncertainty about the parameter as a fraction of the initial uncertainty σ^2 . As α_t is close to 1, g_t declines only slowly with time.

Next consider the updating formula for the parameter β . Starting with $E_1\beta = \bar{\beta}$, for $t > 1$ we have

$$E_t\beta = \alpha_t E_{t-1}\beta + (1 - \alpha_t)\beta + z_t \tilde{\Delta} f_t \varepsilon_t^b \quad (17)$$

where

$$z_t = \frac{p_{t-1}}{(\tilde{\Delta} f_t)^2 p_{t-1} + \sigma_b^2} \quad (18)$$

The first two terms on the right hand side of (17) reflect the speed of learning. If the last term is zero, the expectation of β at time t is a weighted average of the expectation at time $t - 1$ and the actual value of the parameter β . We have already seen that agents learn slowly when α_t is close to 1. Here this is reflected in a high weight of the current expectation on last period's expectation.

The last term on the right hand side of (17) is key to our results. It reflects what we call a scapegoat effect. It depends on the product $\tilde{\Delta} f_t$ and ε_t^b . In order to understand this result, recall that through the exchange rate and the interest rate differential agents each period observe $\tilde{\Delta} f_t \beta + \varepsilon_t^b$. Assume that $\varepsilon_t^b > 0$ and $\tilde{\Delta} f_t > 0$. Agents do not know whether the larger than expected value of the signal is a result of a positive innovation ε_t^b or $\tilde{\Delta} f_t > 0$ combined with a larger than previously expected value of β . They will at least give some weight to the latter by raising their expectation of β . In this case the fundamental f_t becomes the scapegoat when the true shock is in the noise ε_t^b . Making the fundamental the scapegoat for the noise shock is perfectly rational.

Integrating (17) forward, we have

$$E_t\beta = g_t \bar{\beta} + (1 - g_t)\beta + g_t \sum_{i=2}^t \frac{z_i}{g_i} \tilde{\Delta} f_i \varepsilon_i^b \quad (19)$$

The first two terms are a weighted average of the initial estimate $\bar{\beta}$ and the actual parameter itself. As learning is slow, g_t remains close to 1 for a long time, so

that the expectation of β remains more influenced by $\bar{\beta}$ than by β for a long time. Moreover, the expectation of the parameter remains disconnected from its actual value because of the last term, which reflects the scapegoat effect. It depends on the product of current and past values of $\tilde{\Delta}f_t$ and ε_t^b . While the fundamentals and the noise shocks have nothing to do with the value of the parameter itself, the scapegoat effect that results from their current and past products can have a significant effect on the expectation of the parameter.

Finally, to determine the last term in equation (11) we need to compute the derivative of $E_t\beta$ with respect to f_t . From (17) we see that:

$$\frac{\partial E_t\beta}{\partial f_t} = \frac{\partial \alpha_t}{\partial f_{nt}}(E_{t-1}\beta - \beta) + \left(\frac{\partial z_t}{\partial f_t} \tilde{\Delta}f_t + z_t \right) \varepsilon_t^b \quad (20)$$

The last term on the right hand side depends on the innovation ε_t^b , which is a transitory shock. This term therefore leads to a transitory component in the derivative of the exchange rate with respect to fundamentals.

The Appendix derives analogous expressions for the case of N fundamentals. For uncertainty, the extension to the multivariate case gives

$$\mathbf{P}_t = \mathbf{G}_t \sigma^2 \quad (21)$$

where \mathbf{P}_t is now the variance of the entire vector $\boldsymbol{\beta}$ of parameters. \mathbf{G}_t is defined in the Appendix, representing cumulative learning in a way that is analogous to g_t for the single fundamental case.

The generalization of (19) to the multivariate case is

$$E_t\boldsymbol{\beta} = \mathbf{G}_t \bar{\boldsymbol{\beta}} + (I_N - \mathbf{G}_t)\boldsymbol{\beta} + \mathbf{G}_t \sum_{i=2}^t \mathbf{G}_i^{-1} \mathbf{Z}_i \tilde{\Delta}\mathbf{f}_i \varepsilon_i^b \quad (22)$$

where I_N is an identity matrix of size N and \mathbf{Z}_t is a multivariate generalization of z_t that is defined in the Appendix. Notice that the scapegoat effect is again associated with the interaction between current and past fundamentals $\tilde{\Delta}\mathbf{f}_t$ and noise innovations ε_t^b . In the multivariate case the extent to which a fundamental becomes a scapegoat depends on the product $\tilde{\Delta}f_{nt}\varepsilon_t^b$ for other fundamentals as well, which determines the plausibility of other fundamentals as scapegoats.

Finally, in analogy to (20), the Appendix derives an expression of the derivative of $E_t\boldsymbol{\beta}$ with respect to fundamentals that depends on $E_{t-1}\boldsymbol{\beta} - \boldsymbol{\beta}$ and a term that

is proportional to ε_t^b . The latter again leads to an entirely transitory component in the derivative of the exchange rate with respect to fundamentals.

3 Numerical Analysis

To evaluate the potential significance of the scapegoat effect, we calibrate the model to monthly data for exchange rates, interest rates and observed fundamentals. We then compute the derivative of the exchange rate with respect to fundamentals, as well as the expectation of parameters, during a simulation of the model. We show that both can be very volatile, while the learning process about the structural parameters is slow.

3.1 Calibration

We use data on 5 currencies relative to the U.S. dollar: Swiss Franc, British pound, Canadian dollar, Japanese yen and German mark (Euro since 1999) with monthly data from September 1975 to September 2008. The macro fundamentals are the differential of money supply growth, industrial production growth and unemployment rate growth relative to the U.S., the growth in the oil price and the lagged interest rate differential relative to the U.S. A description of the data can be found in Bacchetta et al. (2010). Consistent with the data, we treat one period in the model as one month.

For calibration purposes we re-introduce time-varying risk premia in order to match the observed properties of exchange rates.⁷ Let v_t be the present discounted value of the risk premium in the last term of (1):

$$v_t = \sum_{k=0}^{\infty} \lambda^k E_t \phi_{t+k}$$

To match the observed volatility and autocorrelation of Δs_t , we assume that v_t follows the process

$$v_{t+1} - v_t = \psi_1(v_t - v_{t-1}) - \psi_2 v_t + \varepsilon_{t+1}^v \tag{23}$$

⁷These risk premia do not directly affect the relationship between the exchange rate and fundamentals discussed in the previous section. But they affect other parameter choices below that do affect this derivative.

where $\varepsilon_{t+1}^v \sim N(0, \sigma_v^2)$.

In calibrating the model, we first generate a history of 10 years, so that agents have already had 10 years to learn about the values of the structural parameters. After that we simulate the model for 397 months (just over 33 years), the same sample as in the data. We repeat this 1000 times and compute the average moments across these simulations.

Table 1 reports the parameters adopted for the benchmark parameterization. The first 5 parameters are associated with the process for b_t and v_t . These are set to match the average across the 5 currencies of four moments related to exchange rates and interest rates: the standard deviation of Δs_t , the standard deviation of $i_t - i_t^*$, the first-order autocorrelation of Δs_t and the first-order autocorrelation of $i_t - i_t^*$. The next two parameters relate to the fundamentals. Consistent with the data, we set the number of fundamentals equal to $N = 5$. The standard deviation σ_f of fundamental innovations is set to match the explanatory power of the fundamentals. This is the average across the currencies of the R^2 of a regression of Δs_t on the fundamentals. It is a very low 0.023 in the data, consistent with the well-known limited explanatory power of macro fundamentals for the exchange rate. Finally, we set λ equal to 0.97, consistent with values for λ reported in Engel and West (2005) that are close to 1.

We set $\bar{\beta} = 5$ and $\sigma = 2.5$ in the benchmark parameterization.⁸ This means that a 2 standard deviation confidence band lies between 0 and 10. We do not calibrate this uncertainty, but will consider the impact of changing the degree of uncertainty about the parameter.

3.2 Results

Learning

Before considering a particular simulation, it should be noted that agents learn very slowly. The uncertainty about the parameters, as measured by their standard deviation, is 2.5 in period 1. At the end of the 10 year history that we generate, it is on average 2.3. This is the uncertainty at the start of the sample of 397 months.

⁸The important parameter is the ratio $\sigma/\bar{\beta}$. The levels of σ and $\bar{\beta}$ are just a matter of standardization: e.g., doubling them leads to identical results if we half the volatility of fundamentals.

At the end of the sample the standard deviation is on average 1.9. Overall the reduction in risk is therefore minimal. While we only simulate the model over 33 years, we find that it takes 145 years for this uncertainty to be reduced by half.

Unstable Relationship Between Exchange Rate and Fundamentals

Figure 1 reports the derivative of the exchange rate with respect to the 5 fundamentals for a particular simulation of a 397 month period. Again the simulation starts with a history of 10 years prior to the reported results. We set the actual values of all 5 parameters equal to their unconditional mean of 5. But of course, agents do not know this.

Figure 1 shows a highly unstable derivative of the exchange rate with respect to each of the fundamentals. If the parameters were known, the derivative would be constant and equal to 5 for each of the parameters. Instead we see substantial fluctuations in the derivative. There are both large high frequency fluctuations and lower frequency fluctuations. The weight attached to fundamental 1 is mostly below 5, while that of fundamental 5 is mostly above 5. For the other fundamentals it fluctuates between levels well above and well below 5. The range over which the weights vary is about 3 to 4 for all fundamentals.

To provide further insight into what is driving this unstable relationship between exchange rates and fundamentals, Figure 2 shows the expectation of the 5 parameters. Not surprisingly, this expectation is also very volatile, with both low and high frequency fluctuations. Fundamental 5 is clearly a systematic scapegoat, while fundamental 1 is not. But more importantly, the expectation of the parameters varies significantly over time.

One striking difference between Figures 1 and 2 is that the very sharp high frequency fluctuations in Figure 1 are absent from Figure 2. The derivative of the exchange rate with respect to fundamentals does not just depend on the expectation of the parameters, but also on the derivative of expected parameters with respect to the fundamentals. As we saw in (20), the latter has a transitory component that depends on ε_t^b . This generates the large high frequency fluctuations in Figure 1 that are absent from Figure 2.

Figure 3 further illustrates how this unstable relationship between the exchange rate and the fundamentals is a result of the uncertainty about the level of the parameters. It shows the average standard deviation of $\partial s_t / \partial f_{nt}$ and $E_t \beta_n$ as a

function of the standard deviation σ that represents the initial uncertainty about the parameters. The average is computed over 1000 simulations and over the 5 fundamentals. We vary σ from 0 to 5 (twice the value in the benchmark). As expected, the unstable relationship between the exchange rate and fundamentals, as measured by the standard deviation of $\partial s_t / \partial f_{nt}$, is closely related to the initial uncertainty about the parameters. Except for very low levels of risk, the relationship between the volatility of $\partial s_t / \partial f_{nt}$ and σ is almost linear.

Parameter Instability Test

An interesting question is whether the instability generated by the model is consistent with the data. This might also allow us to determine the magnitude of parameter uncertainty. Unfortunately, the type of parameter instability we consider is difficult to detect with statistical tests. For example, we conduct a Quandt-Andrews breakpoint test. The test statistic, due to Andrews (1993), is the average F statistic over multiple breakpoint tests. All breakpoints from 60 months to 338 months into the sample are considered. The average F statistic is 1.0 in the model under the benchmark parameterization. This is well below what is needed to reject the null of a constant relationship between the exchange rate and parameters. When we apply the same test to the data, the average over the 5 currencies for the average F statistics is 1.3, which also cannot reject stability. Thus, the results of stability tests in the model are consistent to what we find in the data, but no instability is detected.

This lack of evidence of instability may be surprising, especially in the model where we know from Figure 1 that the relationship between the exchange rate and fundamentals is highly unstable. Results do not change fundamentally if we consider multiple breakpoints at once. There are two reasons why it is hard to reject the null of stable coefficients. First, the high frequency fluctuations in the derivative will not be captured by breakpoint tests. Second, and more importantly in the context of exchange rate regressions, fundamentals play very little role in driving the exchange rate. The R^2 of the regression of the change in the exchange rate on fundamentals is only 0.023 in both the data and the model. This means that 98% of the variance of the exchange rate is unrelated to fundamentals. Even if we doubled the R^2 in the model by increasing the standard deviation of the

fundamentals, there is little change in the breakpoint test results.⁹

Closely related to this, Table 2 shows that the exchange rate and interest rate moments used for the calibration of the model remain virtually unaffected when we vary σ from 0 (no parameter uncertainty) to 5 (twice the uncertainty as in the benchmark). Again, this is because observed fundamentals are not the main driver of exchange rate fluctuations.

4 Discussion

In this section we address three issues. First, we consider the empirical relevance of the scapegoat theory. Second, we discuss how results are affected when the structural parameters are not just unknown but also time-varying. Finally, we discuss broader implications for parameter instability in macroeconomics.

Empirical Relevance

In response to an earlier version of this paper, Fratzscher et al. (2011) have developed a test of the scapegoat theory. They use monthly survey data for 40-60 FX traders on the importance of six macro variables as determinants of exchange rates. Three findings are of interest. First, the importance of the variables (on a scale of 0 to 10) varies significantly over their 9-year sample. Second, introducing an interaction term between fundamentals and survey weights they provide strong evidence that the coefficient on a fundamental in the exchange rate equation is indeed larger when agents believe that a fundamental is more important. Finally, they regress the survey weights on an interaction term similar to $\varepsilon_t^b f_{nt}$ in our model, which captures the scapegoat effect. They measure the unobserved macro shock using order flow data. They present strong evidence that the survey weight on a fundamental rises with $\varepsilon_t^b f_{nt}$, therefore giving strong support to the scapegoat

⁹The evidence of instability in exchange rate models is mixed. Rossi (2006) provides some evidence of parameter instability in reduced form exchange rate models. But the evidence is not very strong. She considers 4 models. There is no evidence of parameter instability in the first two models. In the last two models rejection of the null of constant parameters is test and currency specific. Rejections of the null occur at most for 1 or 2 out of 5 currencies. On the other hand, in a forecasting exercise Sarno and Valente (2009) find that “(exchange rate) models that optimally use the information in the fundamentals change often and this implies frequent shifts in the parameters”.

theory.

Time-Varying Structural Parameters

We have assumed that the structural parameters are constant but unknown. One can also generate a time-varying relationship between the exchange rate and macro fundamentals by making the exact opposite assumption: structural parameters that are known and time-varying. In this case the derivative of the exchange rate with respect to fundamental n would simply be equal to β_{nt} . Large time-variation in β_{nt} would generate a very unstable relationship between exchange rates and fundamentals. It could also be consistent with the first two types of evidence in Fratzscher et al. (2011) discussed above and we could think of β_{nt} as a measure of the importance that FX traders attach to different macro fundamentals as determinants of exchange rates.

But this does not seem very plausible. There is widespread uncertainty about the levels of parameters even among economists. In addition, while structural parameters are likely to change over time, it is hard to see what would generate continuous large jumps in such structural parameters from month to month and year to year. Perhaps most convincing, the last test conducted by Fratzscher et al. (2011) addresses this issue head on. It concludes that the reduced form parameter instability is the result of scapegoat effects as opposed to exogenous time-variation in structural parameters.

A more plausible assumption is to combine parameter uncertainty with time-varying parameters. Even though structural parameters probably do not jump back and forth from month to month, economic structures evolve over time. In a previous version of this paper, Bacchetta and van Wincoop (2009), we explicitly introduced slowly time-varying structural parameters. The parameters are unknown and agents gradually learn about their level.

The implication of such a setup is very much like the one in this paper and Figure 1 of the earlier version of the paper looks very much like Figure 1 in this paper. The key to the results is that there is significant uncertainty about the level of parameters. In this paper we have not taken a stand on where that uncertainty comes from. In a setup with slowly time-varying parameters the uncertainty is generated as parameters can move significantly over a very long period of time. In such a setup with slowly time-varying parameters agents will never learn the true

values of parameters as they are chasing a moving target.

However, there are many other ways that parameter uncertainty can be generated. One possibility is that there are infrequent very large jumps in parameters, perhaps because of a revolution, a technological breakthrough or a fundamental change in the nature of the government or monetary policy. Our results suggest that even if such changes occur only once in a century, the uncertainty about the magnitude of the parameters may remain very large at all times. Finally, even if parameters never changed, we need data to learn about their values. We currently have less than 40 years of data since the switch to a floating exchange rate system in 1973. Our results suggest that this may not nearly be enough to learn the values of the structural parameters or even significantly reduce the uncertainty about their values.

Parameter Instability in Macro and Finance

While the focus of this paper is on exchange rates, our explanation for the unstable reduced-form relationship could apply similarly to other forward looking financial or macroeconomic variables. As first shown by Stock and Watson (1996), and since then by many others, the phenomenon of reduced form parameter instability in macroeconomic data is widespread.¹⁰ There has also been great interest in the impact of parameter or model uncertainty on optimal monetary policy.¹¹ The same is the case for financial data. In a survey, Pastor and Veronesi (2009) point out that “parameter uncertainty is ubiquitous in finance” and “many facts that appear baffling at first sight seem less puzzling once we recognize that parameters are uncertain and subject to learning”.¹²

The application to exchange rates in this paper is just one illustration of how uncertainty about structural parameters can generate significant instability in reduced form parameters. More generally, the mechanism that we have highlighted occurs whenever agents have difficulty distinguishing between unobserved macro fundamentals and unobserved structural parameters. This applies not just to other

¹⁰Recent contributions include Cogley and Sargent (2005), Del Negro and Otrok (2007), or Primiceri (2005).

¹¹See for example contributions by Hansen and Sargent (2008), Onatski and Williams (2003) or Levin et al. (2006).

¹²For example, Cogley (2005) and Piazzesi and Schneider (2007) introduce uncertainty about time-varying parameters to explain the term spread.

asset prices, such as equity prices, but to macro models in general in which agents need to learn about the structure of the model or its parameters.

5 Conclusion

Anecdotal and survey evidence suggest that the relationship between the exchange rate and macro fundamentals is highly unstable. In order to explain this, we have developed a model where structural parameters are unknown. We have shown that the relationship between a forward looking variable like the exchange rate and macro fundamentals is determined not by the structural parameters themselves, but rather by the expectations of these structural parameters. These expectations can vary significantly over time due to perfectly rational scapegoat effects as agents have difficulty distinguishing unobserved fundamentals and unobserved structural parameters. Such effects can be expected to hold more broadly in macro and finance models beyond the particular application to exchange rates discussed here.

Appendix: Kalman Filter with N Fundamentals

While in 2.3 we focused on a single fundamental, in this Appendix we describe the expectation of parameters $E_t\boldsymbol{\beta}$ in the case of N fundamentals. In period 1, we start from $P_1 = \sigma^2 I_N$, where I_N is the identity matrix. We also start from $E_1\boldsymbol{\beta} = \bar{\boldsymbol{\beta}}$.

The uncertainty is updated as

$$P_t = \mathbf{A}_t P_{t-1} \quad (24)$$

where

$$\mathbf{A}_t = I_N - \mathbf{P}_{t-1} \tilde{\Delta} \mathbf{f}_t (\tilde{\Delta} \mathbf{f}_t' \mathbf{P}_{t-1} \tilde{\Delta} \mathbf{f}_t + \sigma_b^2)^{-1} \tilde{\Delta} \mathbf{f}_t' \quad (25)$$

It follows that

$$P_t = \mathbf{G}_t \sigma^2 \quad (26)$$

where

$$\mathbf{G}_t = \mathbf{A}_t \mathbf{A}_{t-1} \dots \mathbf{A}_2 \quad (27)$$

We have

$$E_t \boldsymbol{\beta} = \mathbf{A}_t E_{t-1} \boldsymbol{\beta} + (I_N - \mathbf{A}_t) \boldsymbol{\beta} + \mathbf{Z}_t \tilde{\Delta} \mathbf{f}_t \varepsilon_t^b \quad (28)$$

where

$$\mathbf{Z}_t = \mathbf{P}_{t-1} (\tilde{\Delta} \mathbf{f}_t' \mathbf{P}_{t-1} \tilde{\Delta} \mathbf{f}_t + \sigma_b^2)^{-1} \quad (29)$$

It follows that

$$E_t \boldsymbol{\beta} = \mathbf{G}_t \bar{\boldsymbol{\beta}} + (I_N - \mathbf{G}_t) \boldsymbol{\beta} + \mathbf{G}_t \sum_{i=2}^t \mathbf{G}_i^{-1} \mathbf{Z}_i \tilde{\Delta} \mathbf{f}_i \varepsilon_i^b \quad (30)$$

We finally compute $\partial E_t \boldsymbol{\beta} / \partial f_{nt}$. Define $\boldsymbol{\eta}_n$ as a vector of size N with a 1 in space n and zeros otherwise. Using (28), we have:

$$\frac{\partial E_t \boldsymbol{\beta}}{\partial f_{nt}} = \frac{\partial \mathbf{A}_t}{\partial f_{nt}} (E_{t-1} \boldsymbol{\beta} - \boldsymbol{\beta}) + \left(\frac{\partial \mathbf{Z}_t}{\partial f_{nt}} \tilde{\Delta} \mathbf{f}_t + \mathbf{Z}_t \boldsymbol{\eta}_n \right) \varepsilon_t^b \quad (31)$$

Defining the scalar:

$$\pi_t = \tilde{\Delta} \mathbf{f}_t' \mathbf{P}_{t-1} \tilde{\Delta} \mathbf{f}_t + \sigma_b^2 \quad (32)$$

we can write:

$$\mathbf{Z}_t = \frac{\mathbf{P}_{t-1}}{\pi_t} \quad (33)$$

This implies that:

$$\frac{\partial \mathbf{Z}_t}{\partial f_{nt}} = -2 \frac{\tilde{\Delta} \mathbf{f}'_t \mathbf{P}_{t-1} \boldsymbol{\eta}_n \mathbf{P}_{t-1}}{\pi_t^2} \quad (34)$$

Similarly we can write:

$$\mathbf{A}_t = I_N - \frac{\mathbf{P}_{t-1} \tilde{\Delta} \mathbf{f}_t \tilde{\Delta} \mathbf{f}'_t}{\pi_t} \quad (35)$$

Therefore:

$$\frac{\partial \mathbf{A}_t}{\partial f_{nt}} = -\frac{\mathbf{P}_{t-1} (\boldsymbol{\eta}_n \tilde{\Delta} \mathbf{f}'_t + \tilde{\Delta} \mathbf{f}_t \boldsymbol{\eta}'_n)}{\pi_t} + 2 \frac{\tilde{\Delta} \mathbf{f}'_t \mathbf{P}_{t-1} \boldsymbol{\eta}_n \mathbf{P}_{t-1} \tilde{\Delta} \mathbf{f}_t \tilde{\Delta} \mathbf{f}'_t}{\pi_t^2} \quad (36)$$

We can just substitute (34) and (36) into (31).

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Table 1 Benchmark Parameter Assumptions*

Parameters	
σ_b	0.022
ρ_b	0.9
σ_v	0.0282
ψ_1	0.1
ψ_2	0.1
N	5
σ_f	0.000252
λ	0.97
$\bar{\beta}$	5
σ	2.5

Table 2 Moments: Data and Model

	Data	Benchmark $\sigma = 2.5$	$\sigma = 0$	$\sigma = 5$
Standard Deviation Δs_t in %	2.91	2.90	2.87	2.92
$\text{Corr}(\Delta s_t, \Delta s_{t-1})$	0.04	0.04	0.05	0.04
Standard Deviation $i_t - i_t^*$ in %	0.22	0.22	0.22	0.22
$\text{Corr}(i_t - i_t^*, i_{t-1} - i_{t-1}^*)$	0.92	0.90	0.90	0.90
R^2 regression Δs_t on fundamentals	0.023	0.024	0.023	0.028

Figure 1 Derivative Δs_t with respect to Δf_{nt}

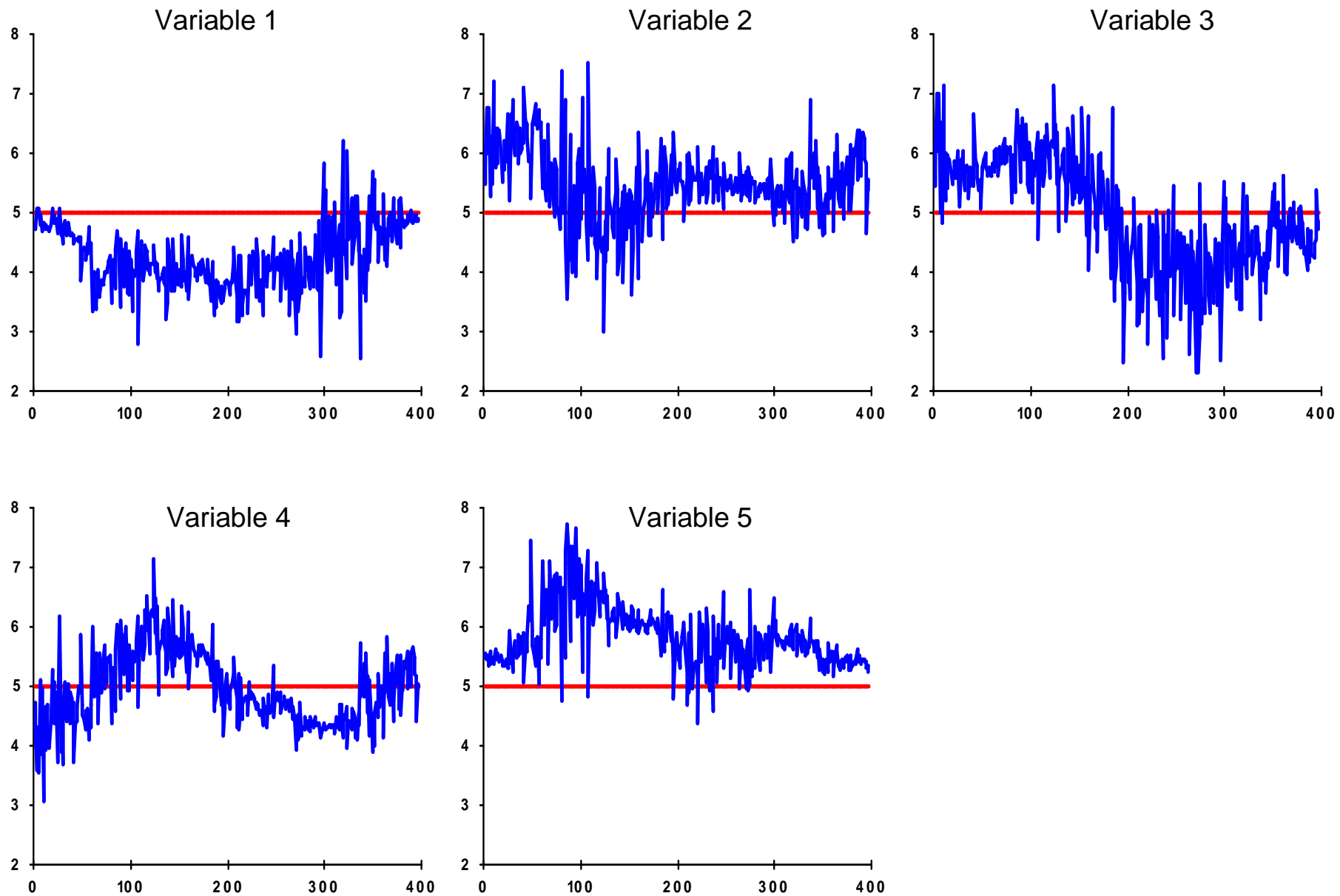


Figure 2 Expectation of Parameters

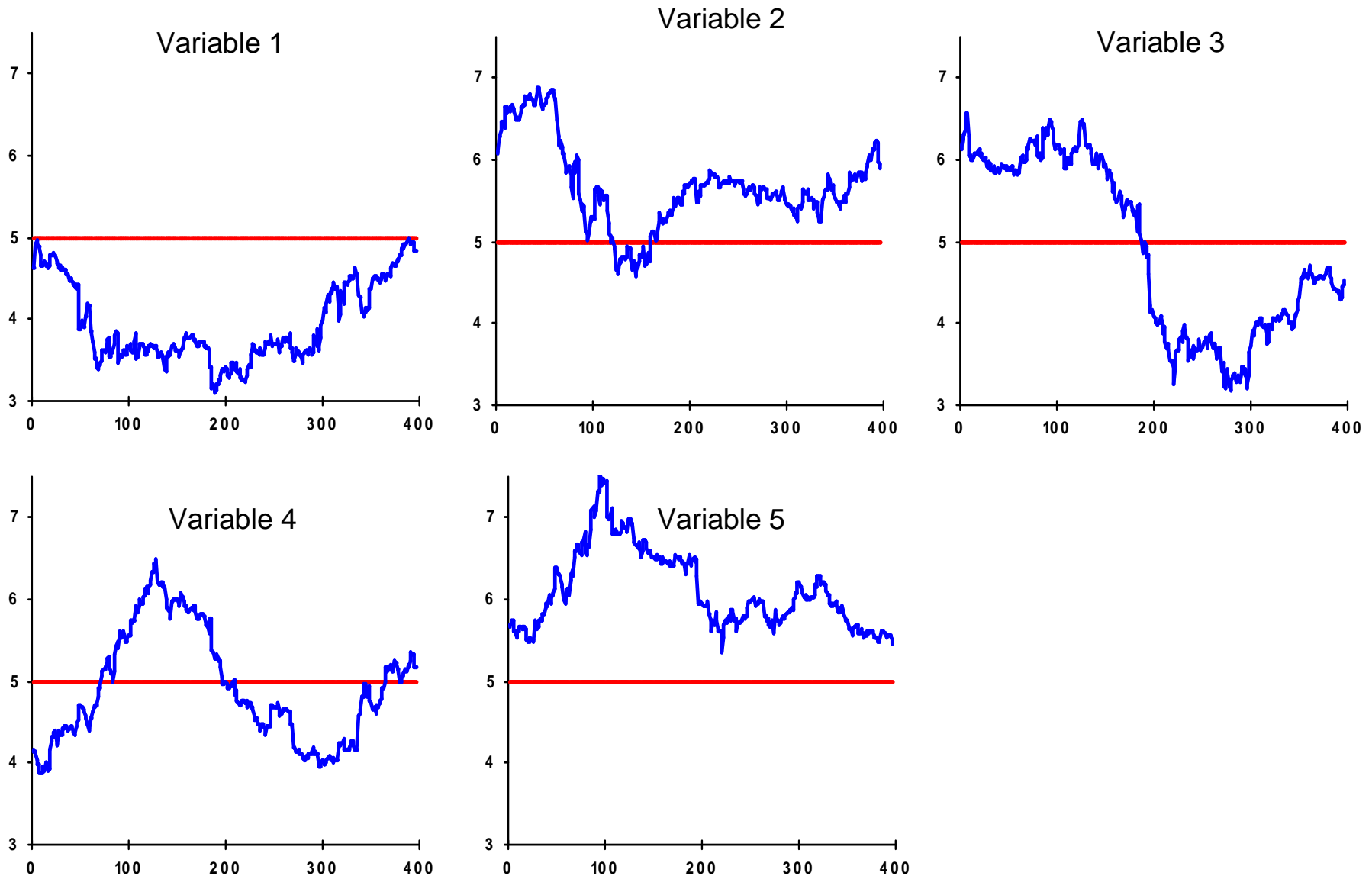


Figure 3 Role of Parameter Uncertainty

