

# Technical Appendix

## Does Exchange Rate Stability Increase Trade and Welfare?

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This technical Appendix discusses some welfare results in sections 3 and 4 of the paper that are not formally proven there. We will show three results:

1. When the money supplies have the same unconditional distribution under a float as under a fixed exchange rate, welfare is higher under a float when preferences are separable in consumption and leisure, *and*  $u_{ll} < 0$  (risk-aversion with respect to leisure), *and*  $u_{ll}$  positive (decreasing absolute risk-aversion) or not too negative.
2. When utility is  $u(c) + l + \alpha cl$ , a marginal drop in the correlation between  $M$  and  $M^*$  below 1 (fixed exchange rate), holding constant the unconditional distribution of money supplies, leads to higher welfare when  $\alpha > 0$  (complements), and lower welfare when  $\alpha < 0$  (substitutes).
3. Consider the general setup in Section 4 with technology and fiscal shocks.  $\gamma$  is a parameter that describes the monetary policy rules. It does not affect the expected money supplies.  $\gamma = 0$  represents a fixed exchange rate system. Assume that utility is separable and quadratic in consumption and leisure. Without government spending, the welfare impact of a marginal change in  $\gamma$  from  $\gamma = 0$  depends negatively on the change in both the variance of consumption and leisure (measured at constant prices), with weights depending on risk-aversion with respect to consumption and leisure. With government spending, the change in welfare also depends negatively on the covariance between total labor demand and private sector labor demand.

We will refer to these as Results 1, 2 and 3. The result in Section 3.2, that welfare is higher under a cooperative peg than a float when utility is separable and quadratic, is not explicitly proven here as the proof is analogous to that of Result 1.

### Proof of Result 1

We will write utility as  $u(c) + v(l)$ . Under separable preferences we know that  $p_H = p_{H^*}$ , so that  $c = M/P$  and  $l = 1 - \frac{1}{2} \frac{M+M^*}{P}$ . Denote as  $U(P, \rho)$  expected

utility as a function of the price level and the correlation  $\rho$  between  $M$  and  $M^*$ . The proof will be in two steps:

$$\text{Step 1: } U(P_{fixed}, 1) < U(P_{fixed}, \rho_{float})$$

$$\text{Step 2: } U(P_{fixed}, \rho_{float}) < U(P_{float}, \rho_{float})$$

Subscripts refer to the respective exchange rate system. We now first prove step 1. For a given price level the expected value of  $u(c)$  is clearly the same under both systems because the unconditional distribution of  $M$  is the same. It is therefore sufficient to show that at a given price level  $E(v(l))$  is higher when  $\rho = \text{corr}(M, M^*)$  is less than one. As in the proof of Lemma 1, we assume without loss of generality that the state space is  $[0, 2Z]$  and that (because of symmetric distributions)  $M(z) = M^*(z + Z)$ ,  $M^*(z) = M(z + Z)$ ,  $\pi(z) = \pi(z + Z)$  for  $z \leq Z$ , where  $\pi(\cdot)$  is the probability density function. For a given price level  $P$ , it follows that

$$E(v(l))_{float} = 2 \int_0^Z \pi(z) v\left(1 - \frac{1}{2} \frac{M(z) + M(z + Z)}{P}\right) dz$$

$$E(v(l))_{fixed} = \int_0^Z \pi(z) \left[ v\left(1 - \frac{M(z)}{P}\right) + v\left(1 - \frac{M(z + Z)}{P}\right) \right] dz.$$

Since  $v(\cdot)$  is a concave function,  $v(0.5x + 0.5y) > 0.5v(x) + 0.5v(y)$ . Substituting  $x = 1 - 0.5M(z)/P$  and  $y = 1 - 0.5M(z + Z)/P$ , this proves that  $E(v(l))$  is larger under a float, which completes the proof of step 1.

In order to prove step two, it is sufficient to show that:

$$(i) \ P_{float} < P_{fixed}$$

$$(ii) \ \frac{\partial U(P, \rho_{float})}{\partial P} < 0 \text{ for } P \geq P_{float}$$

We show (ii) first.  $P^2 \partial U(P, \rho) / \partial P = -Eu_c M + 0.5Ev_l(M + M^*)$ . At  $P = P_{float}$  we can write this as  $-(1/\mu)Eu_c M$ , using that from the price equations (14) and (15)  $0.5\mu Ev_l(M + M^*) = (\mu - 1)Eu_c$  when  $p_H = p_H^* = P$ . This expression is thus negative.  $\partial U(P, \rho) / \partial P$  is also negative for  $P > P_{float}$  because  $\frac{\partial}{\partial P}[-Eu_c M + 0.5Ev_l(M + M^*)] = (1/P^2)[Eu_{cc}M^2 + (1/4)Ev_{ll}(M + M^*)^2] < 0$ .

In order to prove (i), define the function  $g(P) = (\mu - 1)Eu_cM - \mu Ev_lM$ . From price equation (14) (using  $p_H = P$ ), in equilibrium  $g(P) = 0$ . Also  $\partial g(P)/\partial P > 0$ . It is therefore sufficient to show that for any given  $P$ ,  $g(P)$  is lower under a fixed exchange rate system than under a float. Since for a given price level  $Eu_cM$  is the same under both systems, it is sufficient to show that  $Ev_lM$  is lower under a float for any  $P$ . Define  $h(M) = v_l(1 - M/P)M$ . Then

$$(Ev_lM)_{fixed} = \int_0^Z \pi(z)(h(M(z)) + h(M(z + Z)))dz$$

$$(Ev_lM)_{float} = 2 \int_0^Z \pi(z)h(0.5M(z) + 0.5M(z + Z))dz.$$

$h(\cdot)$  is a convex function:  $h_{MM} = v_{ll}M/P^2 - 2v_{lu}/P > 0$  under the assumptions made about  $v_{ll}$  and  $v_{lu}$ . Therefore  $0.5h(M(z)) + 0.5h(M(z + Z)) > h(0.5M(z) + 0.5M(z + Z))$ . This proves (i), which completes the proof of Result 1.

## Proof of Result 2

Utility is  $u(c) + l + \alpha cl$ . Write expected utility as  $U(\rho)$ , where  $\rho = corr(M, M^*)$ . We consider a marginal drop in  $\rho$  at  $\rho = 1$ . Consumption is equal to  $c = M/P$ , while leisure is  $l = 1 - 0.5(p_H/P)^{-\mu}M/P - 0.5(p_H^*/P)^{-\mu}M^*/P$ . At  $\rho = 1$ ,  $p_H = p_H^* = P$  and  $\partial P/\partial \rho = 0.5\partial p_H/\partial \rho + 0.5\partial p_H^*/\partial \rho$ . Using this,  $\partial Eu(c)/\partial \rho = -(1/P^2)E(u_cM)\partial P/\partial \rho$ ,  $\partial E(l)/\partial \rho = (1/P^2)E(M)\partial P/\partial \rho$ , and  $\partial E(cl)/\partial \rho = -0.5E(M^2)/P^2 - (1/P^2)E(M)\partial P/\partial \rho + 2(1/P^3)E(M^2)\partial P/\partial \rho$ .

Therefore

$$\frac{\partial U}{\partial \rho} = -0.5\alpha \frac{1}{P^2}E(M^2) + \frac{1}{P^2}[(1 - \alpha)E(M) - Eu_cM + 2\alpha \frac{1}{P}E(M^2)]\frac{\partial P}{\partial \rho}. \quad (A1)$$

From price equation (14),  $E(u_c + \alpha l)M = (\mu/(\mu - 1))E(1 + \alpha c)M$  at  $\rho = 1$  (since  $p_H = P$  under a fixed exchange rate system). Using this, the expression above simplifies to

$$\frac{\partial U}{\partial \rho} = -0.5\alpha \frac{1}{P^2}E(M^2) - \frac{1}{(\mu - 1)P^2}[E(M) + \alpha \frac{1}{P}E(M^2)]\frac{\partial P}{\partial \rho}. \quad (A2)$$

Note that  $E(M) + \alpha E(M^2)/P$  is always positive, even for  $\alpha$  negative. This is because the marginal utility of leisure must be positive for all states of the world, so that  $1 + \alpha c = 1 + \alpha M/P$  must always be positive. Utility therefore depends negatively on an increase in the price level.

In order to compute  $\partial P/\partial \rho$ , we fully differentiate the following equation, obtained by adding price equations (14) and (15):

$$(\mu - 1)(p_H + p_H^*)E(u_c + \alpha l)M = \mu PE(1 + \alpha c)(M + M^*).$$

Doing so yields

$$\frac{\partial P}{\partial \rho} = \frac{0.5\alpha P(2\mu - 1)E(M^2)}{-(\mu - 1)E(u_{cc}M^2) + (2\mu - 1)\alpha E(M^2)}. \quad (\text{A3})$$

The denominator is positive except for  $\alpha$  very negative. We can rule this case out as it corresponds to “unstable” price setting. If the denominator were negative,  $\partial p_H/\partial P$  in the price equation (14) would be larger than one. This leads to explosive price setting behavior. If, for example due to a change in  $\rho$ , individual firms wish to raise their price for a given aggregate price index, the aggregate index will rise as well, leading to a further increase in prices charged by individual firms, etc. When  $\partial p_H/\partial P > 1$ , this process explodes.

It is now clear from (A2) and (A3) that a drop in  $\rho$  lowers the price level and raises welfare when  $\alpha > 0$ , while it raises the price level and lowers welfare when  $\alpha < 0$ . This completes the proof of Result 2.

### Proof of Result 3

Write utility as  $u(c, l) = a_c c - b_c c^2 + a_l l - b_l l^2$ . In order to be as general as possible, we will consider simultaneously uncertainty about money supplies, government spending  $G$  and technology  $A$ . In that case

$$c = \frac{M - G}{P}$$

$$l = 1 - L^p - L^G = 1 - 0.5(p_H/P)^{-\mu} \frac{M - G}{PA} - 0.5(p_H^*/P)^{-\mu} \frac{M^* - G^*}{PA} - \frac{G}{p_G}. \quad (\text{A4})$$

where  $L^p$  and  $L^G$  represent private and public sector labor demand. Total labor demand is  $L = L^p + L^G$ . We now marginally differentiate expected utility with respect to  $\gamma$  (parameter of the monetary policy rules), evaluated at  $\gamma = 0$  (fixed exchange rate). It is assumed that  $\gamma$  affects neither the expected level of money supplies nor the public price level  $p_G$ . Let  $U(\gamma)$  be expected utility as a function of  $\gamma$ . Differentiating expected utility at  $\gamma = 0$ , using  $\partial P/\partial \gamma = 0.5\partial p_H/\partial \gamma +$

$0.5\partial p_H^*/\partial\gamma$ , we obtain

$$\frac{\partial U}{\partial\gamma} = -b_c \frac{\partial var(c)}{\partial\gamma} - b_l \frac{\partial var(l)}{\partial\gamma} + \frac{1}{P^2} (-Eu_c(M-G) + Eu_l \frac{M-G}{A}) \frac{\partial P}{\partial\gamma}.$$

Substituting (4) into (20), using that  $p_H = P$  at  $\gamma = 0$ , it follows that  $(\mu - 1)Eu_c(M-G) = \mu Eu_l(M-G)/A$ . Substituting this, we obtain

$$\frac{\partial U}{\partial\gamma} = -b_c \frac{\partial var(c)}{\partial\gamma} - b_l \frac{\partial var(l)}{\partial\gamma} - \frac{1}{\mu P} Eu_c c \frac{\partial P}{\partial\gamma}. \quad (A5)$$

In order to compute  $\partial P/\partial\gamma$ , we use that from the price equations (20) and (21), after substituting (4),

$$(\mu - 1)Eu_c(M-G)(p_H + p_H^*) = \mu P Eu_l \frac{M + M^* - G - G^*}{A}. \quad (A6)$$

Differentiating (A6), holding  $p_G$  constant, using  $\partial P/\partial\gamma = 0.5\partial p_H/\partial\gamma + 0.5\partial p_H^*/\partial\gamma$  and the definitions of  $c$  and  $l$ , we find:

$$\left\{ 2(\mu - 1) \frac{\partial Eu_c(M-G)}{\partial\gamma} - \mu \frac{\partial Eu_l(M + M^* - G - G^*)/A}{\partial\gamma} \right\} d\gamma = \quad (A7)$$

$$\left\{ 2(\mu - 1)Eu_{cc}(M-G)^2 + 2\mu Eu_{ll} \left( \frac{M-G}{A} \right)^2 \right\} dP.$$

Holding prices constant,

$$\frac{\partial Eu_c(M-G)}{\partial\gamma} = \frac{\partial Eu_c c P}{\partial\gamma} = -2b_c P \frac{\partial var(c)}{\partial\gamma}.$$

Similarly, holding prices constant and using labor demand from (A4),

$$\frac{\partial Eu_l(M + M^* - G - G^*)/A}{\partial\gamma} = -\frac{\partial 4b_l P E(1-L)L^p}{\partial\gamma} = 4b_l P \frac{\partial cov(L, L^p)}{\partial\gamma}.$$

Using these results in (A7) gives

$$\frac{\partial P}{\partial\gamma} = P^3 \frac{\mu b_l \frac{\partial cov(L, L^p)}{\partial\gamma} + (\mu - 1) b_c \frac{\partial var(c)}{\partial\gamma}}{(\mu - 1) b_c E(M-G)^2 + \mu b_l E(M-G)^2/A^2}. \quad (A8)$$

In the absence of government spending  $cov(L, L^p) = var(l)$ . It then follows from (A5) and (A8) that the change in expected utility depends negatively on a weighted average of the change in the variance of consumption and leisure, with

weights dependent on the degree of risk aversion with respect to consumption and leisure. With government spending the change in expected utility also depends negatively on the change in the covariance between total labor demand and private sector labor demand, as this affects the price level.

For the specific rules considered in sections 4.2 and 4.3 the variance of leisure decreases under a float and is therefore negatively related to  $\gamma$ . Since the variance of consumption increases with  $\gamma$ , the total welfare impact depends on the weights  $b_c$  and  $b_l$  in (A5) and (A8), which are, respectively, the rates of risk-aversion with respect to consumption and leisure. For example, when  $b_c$  is large compared to  $b_l$ , the impact on the consumption variance dominates so that welfare is lower under a float (when  $\gamma$  increases).