Can Trade Costs in Goods Explain Home Bias in Assets?\textsuperscript{1}

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Abstract

A debate has been raging in the general equilibrium literature on the extent to which trade costs impact portfolio home bias. We look under the hood of these models and show that in all there is a common term—a covariance-variance ratio—that can be easily observed in the data. When computing this term using data on real exchange rates and asset returns for the United States versus the rest of the world (21 other industrialized countries) we find that the resulting portfolio home bias is close to zero. General equilibrium models that create home bias through trade costs are thus not grounded in empirical reality. Our results enable the general equilibrium literature to move forward, but in a way in which the theoretical models are not at odds with an easily observed empirical regularity.
1 Introduction

A debate has been raging in the general equilibrium literature on the extent to which trade costs impact portfolio home bias.\(^1\) Although in theory trade costs may affect portfolio home bias in different ways, the channel that has been the focus of attention is exclusively through a hedge of real exchange rate risk. With trade costs leading different countries to consume different bundles of goods, the real exchange rate fluctuates in response to changes in relative prices. To limit the risk associated with these real exchange rate fluctuations, portfolio home bias may be optimal, as a theoretical partial equilibrium literature dating from the late 1970s and early 1980s first pointed out.\(^2\)

More recently, the debate on the impact of trade costs on portfolio home bias has moved on to general equilibrium models.\(^3\) While the literature is extensive, no consensus has emerged. Even today opinions on the importance of trade costs for portfolio home bias differ markedly.\(^4\) There are two main reasons why the general equilibrium literature has not been able to resolve this issue. First, implications of trade costs for portfolio home bias in these models are very sensitive to assumptions about parameters and more generally to the structure of the model. Second, applications of general equilibrium models to this question have not been grounded

\(^1\)We will use the term trade costs broadly as referring to all goods market frictions that reduce international goods trade. It involves policy barriers, transportation costs as well as a host of informal barriers associated with doing business in another country. It also includes preference for home goods and barriers sufficiently high for goods to become non-traded.


\(^4\)Examples of papers that have argued that trade costs can explain most of the portfolio home bias are Stockman and Della (1989), Hnatkovska (2005), Serrat (2002), Della, Diba, Stockman and Collard (2007), Obstfeld and Rogoff (2000) and Obstfeld (2007). Examples of papers concluding that trade costs cannot account for observed portfolio home bias are Baxter, Jermann and King (1998), Pesenti and van Wincoop (2002) and Coeurdacier (2008).
in empirical evidence on the stochastic properties of asset returns and real exchange rates that are central to the results.

The goal of this paper is to resolve this question at an empirical level. Ultimately, the portfolio home bias generated through trade costs in this literature always takes place through a hedge of real exchange rate risk. The optimal hedge depends on the covariance of asset returns with real exchange rate fluctuations. We can derive this hedge, and the resulting portfolio home bias, with a relatively minimal set of assumptions. When applying the results to data for the United States versus the rest of the world (an equity-market-capitalization-weighted average of 21 other industrialized countries) we find that that the portfolio home bias that results from trade costs is close to zero.

We do not wish to dismiss the general equilibrium literature—which has provided a great deal of insight about the role of exogenous parameter assumptions, different types of trade costs and other modeling features—as being of no use. However, the literature has been a bit like a pendulum. As each paper tries to outdo previous papers in terms of realism, the pendulum swings back. In the early general equilibrium literature, trade costs had an extreme nature, being zero for traded goods and very high for other goods, making them non-traded. Dellas and Stockman (1989) concluded that agents will then hold perfectly diversified claims on traded goods equity while they hold all claims on domestic non-traded goods equity. They argued that this can explain a lot of the observed home bias. Subsequently Baxter, Jermann and King (1998) showed that this conclusion is based on an unrealistic assumption about preferences that leads to separability between traded and non-traded goods. For a more realistic assumption the home bias in claims on non-traded goods equity goes away.

More recently, Obstfeld and Rogoff (2000) have argued that one should introduce trade costs for goods that are traded. Such trade costs are indeed very high. Anderson and van Wincoop (2004) estimate that the ad valorem tax equivalent of international trade costs, not including non-traded goods, is 74%. Obstfeld and Rogoff (2000) find that introducing such trade costs can account for the observed portfolio home bias. But Coeurdacier (2008) finds that this conclusion no longer holds when adopting a more realistic assumption about the elasticity of substitution between goods. In fact, Coeurdacier (2008) finds that this leads to a foreign bias. In response to this, Obstfeld (2007) argues that one can make the model even more realistic by introducing both non-traded goods and trade costs.
for traded goods. He finds that this can lead to realistic home bias for both claims on traded and non-traded goods equity. The back and forth in conclusions emerging from this literature is almost dizzying. It is hard to see how consensus can develop without grounding theoretical properties of real exchange rates and asset returns in the data.

The wide range of results is due to the sensitivity of equilibrium real exchange rates and asset returns to modeling assumptions. Here we avoid this altogether by directly observing these variables in the data. Rather than write down an entire general equilibrium model, we will therefore consider a partial equilibrium portfolio choice problem that takes asset returns and real exchanges rates as given. It is important to emphasize that such a partial equilibrium portfolio choice problem is always nested within general equilibrium models. The rest of a general equilibrium model (market clearing conditions and other optimality conditions) is not relevant; it will determine equilibrium real exchange rates and asset returns, but these can be observed directly in the data. Conclusions about portfolio home bias due to a real exchange rate hedge only make sense to the extent that these equilibrium variables are consistent with the data.

Rather than making assumptions about the entire general equilibrium structure, it will therefore be sufficient to make a much more limited set of assumptions to derive home bias expressions. The main assumption we need to make concerns the nature of the asset market structure. As the puzzle is about equity home bias, most of the general equilibrium literature assumes that only equity claims are traded. We start with a setup with only trade in equity. We consider several extensions to this benchmark asset market structure. First, we introduce a forward market that can be used to hedge nominal exchange rate risk. Second, we introduce trade in nominal bonds. Finally, we introduce a non-traded asset.

In all cases we show that portfolio home bias depends on a covariance-variance ratio: the covariance between the real exchange rate and the excess return on home relative to foreign equity, divided by the variance of the excess return. When introducing a forward market or trade in nominal bonds, we show that the home bias expression still depends on the covariance-variance ratio, although with different conditioning variables to compute the moments. The main implication of introducing a non-traded asset is to introduce another source of home bias, associated with a hedge against the income from the non-traded asset. While this source of bias has been widely analyzed in the literature as well, it is entirely separate from
the real exchange rate hedge that has been the focus of the literature connecting home bias to trade costs.

The remainder of the paper is organized as follows. In section 2 we derive portfolio home bias expressions under various asset market structures. Section 3 provides empirical results on the covariance-variance ratio and the implied home bias. Section 4 links the results to the general equilibrium (GE) literature. Section 5 discusses three possible rejoinders in defense of a link between trade costs and portfolio home bias. Section 6 concludes.

2 Home Bias Measures

In order to derive expressions for portfolio home bias, we need to make only a limited set of assumptions. Since we take asset returns and real exchange rates as given, we can avoid assumptions about the entire GE apparatus that drives equilibrium asset returns and real exchange rates. We also do not need to make specific assumptions about the nature of trade costs. The literature has only considered the impact of trade costs on portfolio home bias through its effect on the real exchange rate, which we take as given by the data. There may be other channels through which trade costs affect portfolio home bias, such as reduced information asymmetries across countries when lower trade costs increase trade. But these have not been the object of the large literature considered here.

We will make four assumptions. First, agents have constant relative risk-aversion preferences. This is not a strong assumption. The entire GE literature on this question makes this assumption. More importantly, our conclusion that home bias cannot be explained by trade costs will not depend on the exact magnitude of risk-aversion. Second, as in a lot of the GE literature on this topic, we will adopt a static framework. This only abstracts from a potential difference across countries in the hedge of changes in future expected returns. While we do not wish to claim that this dynamic hedge term is necessarily unimportant, it has played no role in the literature.\(^5\)

\(^5\)Only Serrat (2002) emphasized the potential importance of the dynamic hedge in generating portfolio home bias, but Kollmann (2006a) showed that Serrat’s assertions are incorrect. Tille and van Wincoop (2008) find that real exchange rate fluctuations can lead to a portfolio bias associated with the dynamic hedge term. Their calibration suggests that this bias is tiny even when the conventional static hedge term associated with real exchange rate fluctuations is large.
The third assumption is already discussed in the introduction. We will consider different asset market structures. Below we start with a benchmark framework in which only equity is traded across countries. After that we successively add a forward market to hedge nominal exchange rate risk, nominal bonds of both countries, and a non-traded asset.

The last assumption we make is also related to the asset market structure. We will not distinguish between different types of equity within a country. Agents can only buy a claim on the market index in a country. The GE literature often allows for different home bias with respect to traded goods equity and non-traded goods equity. But to the extent that the theory implies differences in home bias across these industries, this is often considered to be a weakness of the theory as there is no evidence of this in the data. As Baxter, Jermann and King (1998) put it, “...it does not appear that investors or mutual fund managers attempt to structure portfolios with very different shares in traded good equities and non-traded good equities”. We will therefore abstract from that possibility.

We should finally point out that we also abstract from the myriad of other possible frictions, especially in financial markets, that can give rise to portfolio home bias. Those are not related to trade costs and are not considered in the literature that connects portfolio home bias to trade costs.

*Trade in Equity*

Consider two countries that only trade equity. Without loss of generality we will denote all asset returns, prices and inflation rates in terms of the currency of country 1. The gross nominal return of country $j$ equity is $R_j$. Country $n$ investors face the following portfolio maximization problem. The initial wealth is $A(n)$, of which a fraction $\alpha_j(n)$ is invested in country $j$ equity. The inflation rate is $e^{\pi(n)}$, so that the real portfolio return is

$$R^p(n) = (\alpha_1(n)R_1 + (1 - \alpha_1(n))R_2) e^{-\pi(n)}$$  \hspace{1cm} (1)

Agents have constant relative risk-aversion preferences:

$$EC(n)^{1-\gamma}/(1-\gamma)$$  \hspace{1cm} (2)

where $C(n) = A(n)R^p(n)$. The first order condition for optimal portfolio choice is

$$E(R^p(n))^{-\gamma} (R_1 - R_2)e^{-\pi(n)} = 0$$  \hspace{1cm} (3)

But their calibration is only illustrative and not aimed to match the data.
Indicating log returns with lower case letters, the first order condition becomes

\[ E e^{-\gamma r^p(n) + r_1 - \pi(n)} = E e^{-\gamma r^p(n) + r_2 - \pi(n)} \]  

(4)

Now adopt a first order log-linearization of the real portfolio return\(^6\)

\[ r^p(n) = \alpha_1(n) r_1 + (1 - \alpha_1(n)) r_2 - \pi(n) \]  

(5)

After substituting in (4) and assuming normality of log returns and inflation, some basic algebra yields the following optimal portfolio:

\[ \alpha_1(n) = \lambda + \frac{\gamma - 1}{\gamma} \frac{\text{cov}(r_1 - r_2, \pi(n))}{\text{var}(r_1 - r_2)} \]  

(6)

When \( \gamma \) is one, investors have logarithmic preferences and the optimal portfolio (the so-called logarithmic portfolio) is given by \( \lambda \), which depends on first and second moments of asset returns but is the same for investors in both countries:

\[ \lambda = \frac{E(r_1 - r_2) + 0.5(\text{var}(r_1) - \text{var}(r_2)) + \gamma \text{cov}(r_2 - r_1, r_2)}{\gamma \text{var}(r_1 - r_2)} \]

We adopt a standard definition of home bias: the fraction invested by country \( n \) investors in country \( n \) equity (\( \alpha_n(n) \)) minus the share of country \( n \)'s equity in the world equity supply (\( \beta_n \)). When applying this definition of home bias to each country we get a complex expression. Moreover, it is hard to implement as for example differences in expected asset returns are hard to measure and will vary over time. In order to obtain a clean home bias expression we will do two things. First, we impose asset market equilibrium.\(^7\) Second, we only consider the average home bias across the two countries.

Asset market equilibrium implies

\[ \alpha_i(1) w_1 + \alpha_i(2) (1 - w_1) = \beta_i \]  

(7)

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\(^6\)Engel and Matsumoto (2008) use the same approach to solve optimal portfolios. An alternative, which gives the same solution, is to take a second order approximation of the log portfolio return and then derive the first order condition and optimal portfolio. See, for example, Bacchetta and van Wincoop (2008). One can also use order calculus to reach the same portfolio solution, as in Tille and van Wincoop (2008).

\(^7\)We do not use asset market equilibrium to solve for equilibrium asset prices and returns as in general equilibrium models. We only use it to obtain a simplified home bias expression.
where $w_1$ is the share of country 1 in world wealth. Using this, and $\alpha_2(n) = 1 - \alpha_1(n)$, the average portfolio home bias across the two countries becomes simply\(^8\)

$$\text{home bias} = 0.5 (\alpha_1(1) - \alpha_1(2))$$

(8)

Let $\Delta q = \pi(1) - \pi(2)$ be the change in the real exchange rate. A rise in $\Delta q$ represents a country 1 real appreciation; that is, an increase in prices in country 1 relative to country 2 (when expressed in a common currency). Let $er = r_1 - r_2$ be the excess return. Using (6), the average home bias becomes

$$\text{home bias} = 0.5 \frac{\gamma - 1}{\gamma} \frac{\text{cov}(er, \Delta q)}{\text{var}(er)}$$

(9)

This is a very simple and powerful equation. With log preferences ($\gamma = 1$) there is no home bias. More generally, (9) shows that the average home bias depends on a covariance-variance ratio: the covariance of the excess return and the real exchange rate divided by the variance of the excess return.

While derived from a simple partial equilibrium portfolio maximization problem, the home bias expression (9) will hold in any two-country GE model with constant relative risk-aversion and trade limited to equity. It is therefore key that GE models match the moment

$$\frac{\text{cov}(er, \Delta q)}{\text{var}(er)}$$

in the data. Unfortunately the literature has paid no attention to this moment and has instead focused on the mapping of various key parameters to portfolio home bias.\(^9\)

Adding a Forward Market

Now consider adding a forward market to cover against nominal exchange rate fluctuations. Let next period’s nominal exchange rate be $S$ and the current spot and forward exchange rates be $\bar{S}$ and $F$ (currency 1 per unit of currency 2). When

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\(^8\)Home bias in country 1 and 2 are equal to (8) times respectively $2(1 - w_1)$ and $2w_1$.

\(^9\)We should note that in response to an earlier version of this paper, several authors have started to link the implications of general equilibrium models for this covariance-variance ratio to what we find in the data in the next section. Examples are Coeurdacier (2008) and Coeurdacier, Kollmann and Martin (2007).
country $n$ investors purchase forward $m(n)A(n)/S$ units of currency 2 in exchange for currency 1, the real portfolio return becomes

$$R^p(n) = \left( \alpha_1(n)R_1 + (1 - \alpha_1(n))R_2 + m(n)\frac{S - F}{S} \right)e^{-\pi(n)} \quad (10)$$

Using math similar to that above (see Appendix A for details) and letting $\Delta s$ be the change in the log exchange rate, the optimal fraction invested in country 1 equity is

$$\alpha_1(n) = \mu + \frac{\gamma - 1}{\gamma} \frac{\text{cov}_{\Delta s}(r_1 - r_2, \pi(n))}{\text{var}_{\Delta s}(r_1 - r_2)} \quad (11)$$

Here the second moments $\text{cov}_{\Delta s}$ and $\text{var}_{\Delta s}$ refer to the covariance and variance based on the components of returns and inflation that are orthogonal to $\Delta s$. Only the parts of asset returns and inflation that are orthogonal to changes in the nominal exchange rate matter for equity portfolio allocation since nominal exchange rate risk can be separately hedged through the forward market. The parameter $\mu$ measures the logarithmic portfolio that depends on first and second moments of returns and exchange rates, but is the same for both countries’ investors.

Applying the same home bias formula for the equity market as before, we get

$$\text{home bias} = 0.5 \frac{\gamma - 1}{\gamma} \frac{\text{cov}_{\Delta s}(er, \Delta q)}{\text{var}_{\Delta s}(er)} \quad (12)$$

The only difference is that now the home bias formula is based on the components of the excess return and real exchange rate that are orthogonal to the nominal exchange rate. Introducing a forward market is important because in the data nominal and real exchange rates are highly correlated. When changes in the nominal exchange rate can be hedged through a forward market, only the component of real exchange rate fluctuations that is orthogonal to nominal exchange rate fluctuations matters for home bias.

*Equity and Nominal Bonds*

Finally, consider a setup in which each country’s equity and nominal bonds are traded. This is equivalent to trade in equity, plus a forward market, plus a nominal bond from either country. The nominal interest rate in country $n$ is $i_n$. Let $b(n)$ be the fraction invested in country 2 nominal bonds by investors from country $n$. 

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Then the portfolio return is
\[ R_p(n) = (1 + i_1)e^{-\pi(n)} + \left( \alpha_1(n)(R_1 - (1 + i_1)) + \alpha_2(n)(R_2 - (1 + i_1)) + b(n) \left( (1 + i_2)\frac{S}{S} - (1 + i_1) \right) \right) e^{-\pi(n)} \] (13)

Leaving details of the algebra to the Appendix, the optimal fraction invested in country 1 and 2 equity is
\[ \alpha_1(n) = \kappa_1 + \frac{\gamma - 1}{\gamma} \frac{\text{cov}_{\Delta s,r_2}(er, \pi(n))}{\text{var}_{\Delta s,r_2}(er)} \] (14)
\[ \alpha_2(n) = \kappa_2 - \frac{\gamma - 1}{\gamma} \frac{\text{cov}_{\Delta s,r_1}(er, \pi(n))}{\text{var}_{\Delta s,r_1}(er)} \] (15)

Here \( \text{cov}_{\Delta s,r_j}(x, y) \) denotes the covariance between the components of \( x \) and \( y \) that are orthogonal to both \( \Delta s \) and \( r_j \). These orthogonal components can be obtained from the error term of a regression of \( x \) and \( y \) on both \( \Delta s \) and \( r_j \). The logarithmic portfolios (\( \kappa_1 \) and \( \kappa_2 \)) depend on first and second moments of asset returns and real exchange rates and are the same for both countries’ investors.

The home bias measure (8) used so far can be written as the average fraction invested in domestic equity, \( 0.5(\alpha_1(1) + \alpha_2(2)) \), minus the average share of domestic equity supply, \( 0.5(\beta_1 + \beta_2) = 0.5 \). With trade in nominal bonds, \( \alpha_1(1) \) and \( \alpha_2(2) \) are now shares of total financial wealth, which is larger than equity wealth. We therefore define home bias as
\[ \text{home bias} = \omega \frac{\alpha_1(1) + \alpha_2(2)}{2} - 0.5 \] (16)
where \( \omega \) is the ratio of world wealth to the world equity market. This is consistent with the measure of home bias without trade in bonds, where \( \omega = 1 \).

Using the equity market clearing conditions \( \alpha_i(1)w_1 + \alpha_i(2)(1 - w_1) = \beta_i/\omega \) for \( i = 1, 2 \), implementing this home bias definition yields (see Appendix B)
\[ \text{home bias} = 0.5\omega \frac{\gamma - 1}{\gamma} \left( w_1 \frac{\text{cov}_{\Delta s,r_1}(er, \Delta q)}{\text{var}_{\Delta s,r_1}(er)} + (1 - w_1) \frac{\text{cov}_{\Delta s,r_2}(er, \Delta q)}{\text{var}_{\Delta s,r_2}(er)} \right) \] (17)

There are two changes relative to home bias measures (9) and (12). First, the home bias is scaled upwards because \( \omega > 1 \). Second, the moment
\[ \frac{\text{cov}(er, \Delta q)}{\text{var}(er)} \]
is now replaced by a weighted average of the same moment based on two different orthogonal components of the real exchange rate and excess return, the first with respect to $\Delta s$ and $r_2$ and the second with respect to $\Delta s$ and $r_1$.

A Non-Traded Asset

Finally consider adding a non-traded asset to the benchmark asset structure with trade in equity. In addition to the return on assets, agents in country $n$ receive the payoff $W(n)$ from non-traded assets such as wages and non-corporate business income. Total consumption is then

$$C = R^p(n)A(n) + W(n)$$  \hspace{1cm} (18)

with the portfolio return as defined in (1). The Appendix shows that this results in an average home bias of

$$home\ bias = 0.5 \frac{\gamma - \frac{1}{f} \text{cov}(er, \Delta q)}{\gamma \text{var}(er)} - 0.5 \frac{1 - f \text{cov}(r_1 - r_2, w(1) - w(2))}{f \text{var}(r_1 - r_2)}$$  \hspace{1cm} (19)

where $f$ is the ratio of financial assets to total wealth around which we expand in the log-linearization.

There are two changes relative to the average home bias expression (9) without the non-traded asset. First, there is now an additional source of home bias reflecting the optimal hedge against the non-traded asset income. This is represented by the second term on the right hand side of (19). There will be home bias when the return on the domestic asset tends to be relatively high at times when the payoff on the domestic non-traded asset is relatively low. This source of home bias has been extensively investigated in the literature in the context of human capital.\footnote{See Bottazzi, Pesenti and van Wincoop (1996), Baxter and Jermann (1997), Julliard (2002, 2004), Engel and Matsumoto (2008), Lustig and van Nieuwerburgh (2007) and Chu (2008).}

However, this is unrelated to trade costs that affect portfolio home bias through a real exchange rate hedge. The latter is captured by the first-term on the right hand side of (19).

Second, the difference between the real exchange rate hedge term in (19) and that in the home bias expression (9) without the non-traded asset is that the covariance-variance ratio is now multiplied by $(\gamma - (1/f))/\gamma$ rather than $(\gamma - 1)/\gamma$. Previously we found that for log preferences, where $\gamma = 1$, the real exchange rate
The hedge term disappears. This is because inflation affects log-consumption additively in that case:

\[
\ln(C(n)) = \ln(A(n)) - \pi(n) + \ln(\alpha_1(n)R_1 + (1 - \alpha_1(n))R(2))
\]

The marginal expected utility form changes in portfolio shares is then unaffected by inflation. This is no longer the case when we add income from a non-traded asset, even when this income is not stochastic. Under log-preferences agents now prefer the asset whose return is more negatively correlated with \( \pi_1 \). This would lead to a foreign bias when the variance-covariance ratio is positive. More generally, home bias will now be lower for a positive variance-covariance ratio.

3 Empirics

Data Description

We compute the covariance-variance ratios in (9), (12), and (17) using monthly data for the period 1988-2005. The two-country framework in the previous section is interpreted as a model of the United States and the rest of the world (ROW). For our purposes, ROW will be composed of an equity-market-capitalization-weighted combination of twenty-one industrialized countries that have complete data.\(^{11}\)

The calculation of the home bias expressions require data for U.S. inflation, U.S. equity returns, as well as three ROW market-capitalization-weighted indexes: a nominal dollar index, an index of foreign equity returns, and a foreign inflation index. Inflation (both U.S. and foreign) and equity returns are expressed in dollars. We use identical weighting schemes to compute the ROW indexes; weights are given by the relative weight of each foreign country in total ROW equity market capitalization.\(^{12}\) Equity indexes, which include both capital gains and dividends and are converted into dollars, are as of month end from MSCI Barra. The excess return is computed as the log first difference between the U.S. and ROW equity

\(^{11}\)The countries included in ROW are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, and United Kingdom.

\(^{12}\)The weights are based on closing values as of December 31 of the previous year. Annual updating of weights is in line with the methodology used by the Federal Reserve Board in forming its monthly trade-weighted dollar indexes. See Loretan (2005).
indexes: \( er = r^{US} - r^{ROW} \). Nominal exchange rates are month-end data from Board of Governors of the Federal Reserve System’s G.5 Report (as compiled by Haver Analytics). Consumer price indexes are from the IMF’s International Financial Statistics database. The ROW dollar price index is computed by first multiplying the local currency price indexes of each country with the nominal exchange rate to convert to dollars and then applying the equity market capitalization weights. The change in the real exchange rate \( \Delta q \) is equal to the difference between U.S. and ROW inflation rates, both expressed in dollars.

**Covariance-Variance Calculations**

Calculations of the covariance-variance ratio in the first home bias expression (9) are given in column (1) of Table 1. The ratio is 0.32. This implies that the maximum home bias (for infinite risk-aversion) is one-half of that, or 0.16. For a rate of risk-aversion of 5, the bias would be 0.13. While not negligible, the bias is substantially below existing estimates of home bias.\(^{13}\) Home bias related to the real exchange rate hedge is even smaller when we allow for non-traded assets.\(^{14}\) If for illustrative purposes we use the estimate of financial to total wealth of 0.22 from Bottazzi et.al. (1996), a rate of risk-version of 5 implies a home bias of only 0.01.

The portfolio bias associated with the real exchange rate hedge vanishes almost entirely when we allow for a forward market to hedge nominal exchange rate risk. To the extent that real exchange changes and excess returns are correlated, this is mostly the result of nominal exchange rate fluctuations that affect both. In column (2) we implement the home bias formula (12) in the presence of a forward market, using the components of the real exchange rate and excess return that are orthogonal to the nominal exchange rate. Now the covariance-variance ratio falls to near zero (0.0052).

A similar result applies when introducing both Home and Foreign nominal bonds in addition to trade in equity of both countries. The covariance-variance ratios embedded in home bias formula (17), shown in columns (3) and (4), are very small. Even after scaling these numbers up by any reasonable estimate of total

\(^{13}\)U.S. home bias has ranged between 0.4 and 0.5 over the past decade. See Ahearne, Griever and Warnock (2004) and Kho, Stulz and Warnock (2006).

\(^{14}\)The maximum home bias, setting the rate of risk-aversion at infinity, is exactly the same when adding a non-traded asset.
financial wealth to equity wealth (represented by $\omega$ in the home bias formula), the result remains close to zero. We can conclude that portfolio home bias associated with hedging real exchange rate risk is essentially zero.

These results are further illustrated in Figures 1 and 2. Figure 1 shows a scatter plot of monthly real exchange rate changes against corresponding excess returns. The line in the figure represents a regression of the real exchange rate change on the excess return. These are the raw data, without conditioning on nominal exchange rate fluctuations. The two series are somewhat positively correlated as they are both sensitive to nominal exchange rate fluctuations. The slope of the regression line is equal to the variance-covariance ratio of 0.32.

Figure 2 shows the same data when we condition on nominal exchange rate changes. Two points are noteworthy. First, real exchange rate changes are now much less volatile. This is a result of the well-known high correlation between nominal and real exchange rates. The low volatility of the real exchange rate after conditioning on nominal exchange rate fluctuations represents the low volatility of consumer price inflation measured in local currencies. Second, the resulting real exchange rate is virtually uncorrelated with the excess return. Both factors contribute to making the variance-covariance ratio essentially zero in this case as we can write

$$\frac{\text{cov}(er, \Delta q)}{\text{var}(er)} = \text{corr}(er, \Delta q) \frac{\sigma_q}{\sigma_{er}}$$

The theoretical partial equilibrium literature mentioned in the introduction has long ago pointed out that no portfolio bias can result from this when local prices do not fluctuate. This is the extreme case where $\Delta q$ is constant after controlling for nominal exchange rate changes. In that case the real exchange rate is entirely driven by nominal exchange rate fluctuations, which can be hedged through bond or forward markets. No equity home bias would result.

We finally conduct sensitivity analysis by re-computing the variance-covariance ratio for different data frequencies. We do so in Table 2 for overlapping 12-month cumulative returns, quarterly data, and annual data. The story remains the same: the relationship between relative equity returns and the real exchange rate is too weak to generate substantial home bias. For annual data the covariance-variance ratio is now even much smaller without controlling for nominal exchange rate changes, further reducing the home bias even in that case.

*Related Evidence*
A couple of other papers have adopted a partial equilibrium portfolio approach to evaluate the importance of home bias due to the real exchange rate hedge. Pesenti and van Wincoop (2002) consider a portfolio maximization problem where equity returns are given, but they do not take the real exchange rate as given. Instead they consider a setup with non-traded and traded goods and zero trade costs for the latter. The supply of non-traded goods follows an exogenous process. Home bias depends on the covariance between asset returns and the growth of non-tradables output, which they evaluate empirically. They conclude that this can account for very little of the observed equity home bias. Their conclusion is consistent with ours. However, the question they address is a much narrower one, specific to non-traded goods. They do not consider the role of trade costs more generally and do not use data on real exchange rates to evaluate overall home bias from the optimal real exchange rate hedge.

Perhaps closer to our approach are Adler and Dumas (1983) and Cooper and Kaplanis (1994). These papers consider partial equilibrium portfolio problems taking equity returns and inflation rates as given. They assume that nominal exchange rate risk can be entirely hedged through bonds. Their bottom line conclusion is that equity home bias cannot be explained by inflation differences across countries. This is consistent with our conclusion that equity home bias cannot be explained through a hedge of real exchange rate risk.

Nonetheless these papers have received virtually no attention in the general equilibrium literature. They do not capture the optimal real exchange rate hedge empirically in a simple way that connects to the general equilibrium models. Cooper and Kaplanis (1994) reject deviations from PPP as an explanation for portfolio home bias based on a rather obscure statistical test of the joint hypothesis that risk-aversion and optimal equity holdings are positive. Adler and Dumas (1983) derive the optimal portfolios for a setup with a large number of countries. This does not lead to a home bias expression that can be compared to the two-country general equilibrium models. Moreover, optimal portfolios in that framework are known to be sensitive to measurement as they depend on the inverse of a large variance-covariance matrix of asset returns. For example, Adler and Dumas (1983) find that the optimal logarithmic portfolio implies that all countries invest -675% in the United States.
4 Link to the General Equilibrium Literature

In the preceding sections we have analyzed the portfolio home bias based on optimal portfolio choice in a partial equilibrium setting. But it nonetheless closely connects to the GE literature on home bias due to trade costs. First, we have adopted a two-country model, as is standard in the GE literature. Second, the assumed constant relative risk-aversion utility function is also standard in the GE literature. Third, most of the GE literature assumes that trade is limited to equity. Therefore the home bias equation (9) connects closely to the GE literature. Finally, partial equilibrium portfolio decisions by agents are embedded within GE models with portfolio choice. The resulting home bias expression is exactly the same when asset returns and the real exchange rate are endogenously determined in GE models.

However, the GE home bias literature related to trade costs does not draw the link between home bias and the covariance-variance ratio and therefore also does not link the covariance-variance implied by the theory to that in the data. As a result the GE models can derive home bias results that are not at all constrained by data on asset returns and real exchange rates. The wide range of conclusions in the literature is therefore not surprising.

In order to illustrate these points, we will consider the example of Coeurdacier (2008), which nests Obstfeld and Rogoff (2000) as a special case. The model in Coeurdacier (2008) is a static two-country GE model with the same constant rate of relative risk-aversion preferences as assumed in section 2. There are two goods. Agents in the Home country receive a stochastic endowment of the Home good while agents in the Foreign country receive a stochastic endowment of the Foreign good. The two countries trade claims on the endowments of both goods. The consumption index in the Home country is

\[
C = \left[ C_H^{(\psi-1)/\psi} + C_F^{(\psi-1)/\psi} \right]^{\psi/(\psi-1)}
\]  

(21)

where \( C_H \) and \( C_F \) are respectively consumption of Home and Foreign goods and \( \psi \) is the elasticity of substitution between Home and Foreign goods. There is a trade cost \( \tau \) that is of the iceberg type: for each good shipped, \( 1/(1 + \tau) \) arrive at the destination. The model in Obstfeld and Rogoff (2000) is the same, but adopts the additional restriction \( \gamma = 1/\psi \), so that overall utility is separable in both goods.

The portfolio home bias in Coeurdacier (2008) is exactly that in (9), with the
covariance-variance ratio equal to
\[
\frac{cov(\text{er}, \Delta q)}{var(\text{er})} = \frac{\theta}{1 - \psi + \theta^2 \left( \psi - \frac{1}{\gamma} \right)} \tag{22}
\]
where
\[
\theta = \frac{1 - (1 + \tau)^{1 - \psi}}{1 + (1 + \tau)^{1 - \psi}}
\]
Here we followed the notation of Coeurdacier (2008). A positive trade cost \(\tau\) implies \(0 < \theta < 1\).

First consider the preferred parameterization by Obstfeld and Rogoff (2000): \(\tau = 0.25\), \(\psi = 6\) and \(\gamma = 1/\psi\). This leads to a covariance-variance ratio of about -0.10. This is clearly inconsistent with the data, where we found a positive covariance-variance ratio of 0.32 with trade limited to equity. Obstfeld and Rogoff (2000) generate a positive home bias with a negative covariance-variance ratio because their assumption \(\gamma = 1/\psi\) implies a rate of relative risk-aversion of less than 1 (\(\gamma = 1/6\)). This near-zero rate of relative risk-aversion is at odds with a substantial body of empirical evidence, but together with the negative covariance-variance ratio of -0.1 implies a home bias of 0.25.\(^{15}\)

Coeurdacier (2008) points that that the assumption \(\gamma = 1/\psi\) is not realistic. While he conducts sensitivity analysis, his preferred parameterization is \(\psi = 5\), \(\gamma = 2\) and \(\tau = 0.63\). This generates a home bias of about -0.13, or a foreign bias of +0.13. In this case the covariance-variance ratio is -0.52, also significantly different from that in the data. In fact, even if we vary \(\tau\) from 0 to 1, \(\gamma\) from 1 to 10 and \(1/(1 - \rho)\) from 1 to 10, the model is not able to come anywhere near the observed 0.32 covariance-variance ratio.\(^{16}\) Coeurdacier (2008) also emphasizes the significant sensitivity of equilibrium portfolio shares to parameter assumptions.

Conclusions about home or foreign bias from GE models with trade costs or home bias in preferences should therefore be considered as suspect as they are not firmly grounded in data on the covariance-variance ratio. If GE models are parameterized to match this key feature of the data, and in addition are rich enough to allow for a forward market or trade in nominal bonds, then one must

\(^{15}\)Obstfeld and Rogoff (2000) report a home bias of 0.31 in this case. The small difference is because the iceberg shipping costs \(\tau\) is defined slightly differently in the two papers. In Coeurdacier (2008), for each good shipped \(1/(1 + \tau)\) goods arrive, while in Obstfeld and Rogoff (2000) \(1 - \tau\) goods arrive. We thank Maurice Obstfeld for explaining this discrepancy to us.

\(^{16}\)Positive numbers can be attained, but they are always much greater than one.
conclude that home bias in the goods market cannot account for home bias in financial markets.

5 Possible Rejoinders

It is worth discussing three possible rejoinders. One response in defense of trade costs as an explanation for portfolio home bias may be that we have not literally conducted a test of the impact of trade costs on portfolio home bias. While trade costs affect portfolio home bias through the real exchange rate, there are other factors that drive the real exchange rate. The covariance between the excess return and the real exchange rate is therefore driven by other factors as well. While this rejoinder is technically correct, we do not see how it could realistically recover trade costs as an explanation for portfolio home bias.

First, we believe that it is fair to say that apart from trade costs of various types, the most important driver of the real exchange rate is generally considered to be nominal rigidities that lead to a close relationship between the nominal and real exchange rate. But we have already largely controlled for this factor by using the component of the real exchange rate that is orthogonal to the nominal exchange rate. Second, while in theory there might still be other factors driving the real exchange rate, it would seem highly implausible for trade costs to lead to a large positive covariance between the real exchange rate and excess return and for other factors to lead to the exact opposite covariance so that the overall covariance happens to be zero. Third, even if this were the case, it simply says that there is another factor that has the exact opposite impact on portfolio home bias, so that overall no portfolio home bias is generated through the covariance between the excess return and the real exchange rate. Finally, if someone really believes this, he or she is faced with the nearly impossible task of developing a general equilibrium model where trade costs by itself lead to a substantial covariance between the real exchange rate and excess return while the overall covariance generated by the model is close to zero. This is definitely not the case in existing GE literature linking trade costs to home bias.

A second possible rejoinder in defense of the link between trade costs and portfolio home bias is that portfolio decisions depend on conditional second moments. In the data we have computed the unconditional covariance between the real ex-
change rate and excess return and the unconditional variance of the excess return. It may be the case that when variables are conditioned on information available at the beginning of the period, the covariance-variance ratio is large enough to generate substantial portfolio home bias. In order to evaluate this possibility we have regressed variables at time \( t \) on a set of macro variables available at time \( t - 1 \). We use the error terms from these regressions to compute the conditional covariance-variance ratio. Table 3 reports results for quarterly data with the conditioning variables being 4 lags of each of the following: US relative to ROW real GDP growth, US relative to ROW long-term bond rates, equity returns (US and ROW), and changes in the real and nominal exchange rates. Comparing the covariance-variance ratios to those in Tables 1 and 2 shows that if anything these ratios are even smaller, therefore strengthening our results. Thus, even conditioning on macro variables, the relationship between real exchange rate changes and relative equity returns is too weak to generate a substantial home bias.

A third possible rejoinder is that results may change when we take into account interactions between goods market and financial frictions. Introducing frictions in financial markets can naturally lead to portfolio home bias in a way that is entirely separate from trade costs. That is why the literature linking home bias to trade costs has abstracted from them and we have abstracted from them here as well. However, it may be argued that the optimal real exchange rate hedge could be affected by financial frictions. We already saw that the non-tradability of certain assets affects the optimal real exchange rate hedge, albeit in a way that only reduces it further.

Coeurdacier (2008) is the only one to our knowledge who has emphasized such a link. He introduces a financial friction in the form of a tax on returns of investment abroad. This is meant to broadly capture a wide range of possible financial frictions associated with investment abroad. He finds that this generates additional home bias equal to the tax divided by the product of the rate of relative risk-aversion and the variance of the excess return. The general equilibrium model employed

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\(^{17}\)For the conditional covariance analysis, data limitations restrict the set of countries in the ROW aggregates to 14; omitted are Austria, Finland, Greece, Hong Kong, New Zealand, Portugal, and Singapore. We rely on IFS data, rather than an international real-time data set (which would greatly reduce the number of countries in the ROW aggregate). Faust, Rogers, and Wright (2003) show that the additional explanatory power gained from using real-time rather than revised data is quite small.

\(^{18}\)Tille and van Wincoop (2008) also introduce this friction and obtain the same portfolio bias
by Coeurdacier (2008) implies that higher trade costs reduce the variance of the excess return and therefore increase the impact of the tax on portfolio home bias. However, conditional on the observed variance of the excess return in the data, there is no link. Moreover, the tax does not affect the optimal real exchange rate hedge that has been the focus of the literature linking trade costs to portfolio home bias. While we cannot rule out other ways that financial frictions may affect the optimal real exchange rate hedge, none have been suggested in the literature as far as we know.

6 Conclusion

The impact of trade costs on portfolio home bias has been the subject of debate for at least three decades. Remarkably though, no consensus has been reached. Opinions vary widely. We argue that this is the result of a lack of grounding of general equilibrium models in data on asset returns and real exchange rates, together with the significant sensitivity of home bias expressions to changes in parameter assumptions. In this paper we have resolved the question at an empirical level. The impact of trade costs on portfolio home bias in the literature takes place through a hedge of real exchange rate risk. We have derived the optimal real exchange rate hedge, and the resulting portfolio home bias, with a relatively minimal set of assumptions. The data speak loud and clear. The optimal real exchange rate hedge is close to zero, casting significant doubt on trade costs as an explanation for portfolio home bias. GE models that do produce substantial home bias do so through an implied covariance-variance ratio that is at odds with the data.
Appendix

A Home Bias with a Forward Market and Equity Markets

In this Appendix we derive the home bias formula (12) in the presence of a forward market. There are now two first order conditions, with respect to \( \alpha_1(n) \) and \( m(n) \), which in the log-return notation are

\[
E e^{-\gamma r^p(n)+r_1-\pi(n)} = E e^{-\gamma r^p(n)+r_2-\pi(n)} \\
E e^{-\gamma r^p(n)+\Delta s-\pi(n)} = E e^{-\gamma r^p(n)+(f-\bar{s})-\pi(n)}
\]

where \( \Delta s \) is the change in the log exchange rate. The linearized log-portfolio return is now

\[
r^p(n) = \alpha_1(n)r_1 + (1 - \alpha_1(n))r_2 + m(n)(\Delta s - (f - \bar{s}) - \pi(n))
\]

To solve for the optimal portfolio allocation \( \alpha_1(n) \) and \( m(n) \), first substitute the log portfolio return into the first order conditions to get

\[
\begin{align*}
\lambda_1 - \gamma \alpha_1(n) \text{var}(r_1 - r_2) - (1 - \gamma) \text{cov}(r_1 - r_2, \pi(n)) - \gamma m(n) \text{cov}(r_1 - r_2, \Delta s) &= 0 \\
\lambda_2 - \gamma \alpha_1(n) \text{cov}(r_1 - r_2, \Delta s) - (1 - \gamma) \text{cov}(\Delta s, \pi(n)) - \gamma m(n) \text{var}(\Delta s) &= 0
\end{align*}
\]

where

\[
\begin{align*}
\lambda_1 &= E(r_1 - r_2) + 0.5(\text{var}(r_1) - \text{var}(r_2)) + \gamma \text{cov}(r_2 - r_1, r_2) \\
\lambda_2 &= E(\Delta s - f + \bar{s}) + 0.5 \text{var}(\Delta s) - \gamma \text{cov}(r_2, \Delta s)
\end{align*}
\]

The parameters \( \lambda_1 \) and \( \lambda_2 \) are the same for investors from both countries.

Solving for \( m(n) \) from the second first order condition and substituting into the first one yields

\[
\begin{align*}
\eta - \gamma \alpha_1(n) \text{var}(r_1 - r_2) - (1 - \gamma) \text{cov}(r_1 - r_2, \pi(n)) &+ \gamma \alpha_1(n) \frac{\text{cov}(r_1 - r_2, \Delta s)^2}{\text{var}(\Delta s)} + (1 - \gamma) \frac{\text{cov}(\pi(n), \Delta s) \text{cov}(r_1 - r_2, \Delta s)}{\text{var}(\Delta s)} &= 0
\end{align*}
\]
where
\[ \eta = \lambda_1 - \lambda_2 \frac{\text{cov}(\Delta s, r_1 - r_2)}{\text{var}(\Delta s)} \]

For any variable \( x \), define \( \hat{x} \) as the residual of a regression of \( x \) on \( \Delta s \). Therefore
\[ x = \frac{\text{cov}(x, \Delta s)}{\text{var}(\Delta s)} \Delta s + \hat{x} \]  \hspace{1cm} (27)

Applying this to \( x = r_1 - r_2 \) and \( x = \pi(n) \), (26) becomes
\[ \eta - \gamma \alpha_1(n) \text{var}(\hat{r}_1 - \hat{r}_2) - (1 - \gamma) \text{cov}(\hat{r}_1 - \hat{r}_2, \hat{\pi}(n)) = 0 \] \hspace{1cm} (28)

This implies (11) with
\[ \mu = \frac{\eta}{\gamma \text{var}(\hat{r}_1 - \hat{r}_2)} \]

Using the home bias expression (8) then yields (12).

B Home Bias with Bond and Equity Markets

In this Appendix we derive the home bias formula (17) in the presence of both bond and equity markets.

In log-return notation, the first order conditions for \( \alpha_1(n), \alpha_2(n) \) and \( b(n) \) are
\[
\begin{align*}
E_t e^{-\gamma r^p(n) + r_1 - \pi(n)} &= E_t e^{-\gamma r^p(n) + \ln(1+i_1) - \pi(n)} \hspace{1cm} (29) \\
E_t e^{-\gamma r^p(n) + r_2 - \pi(n)} &= E_t e^{-\gamma r^p(n) + \ln(1+i_1) - \pi(n)} \hspace{1cm} (30) \\
E_t e^{-\gamma r^p(n) + \ln(1+i_1^*) + \Delta s - \pi(n)} &= E_t e^{-\gamma r^p(n) + \ln(1+i_1) - \pi(n)} \hspace{1cm} (31)
\end{align*}
\]

and the log-linearized portfolio return is
\[ r^p(n) = i_1 + \alpha_1(n)(r_1 - i_1) + \alpha_2(n)(r_2 - i_1) + b(n)(i_2 + \Delta s - i_1) - \pi(n) \] \hspace{1cm} (32)

Substituting (32) into the first order conditions (29)-(31), we get
\[
\begin{align*}
\lambda_1 - \gamma \alpha_1(n) \text{var}(r_1) - \gamma \alpha_2(n) \text{cov}(r_1, r_2) - \gamma b(n) \text{cov}(r_1, \Delta s) - (1 - \gamma) \text{cov}(r_1, \pi(n)) &= 0 \\
\lambda_2 - \gamma \alpha_1(n) \text{cov}(r_1, r_2) - \gamma \alpha_2(n) \text{var}(r_2) - \gamma b(n) \text{cov}(r_2, \Delta s) - (1 - \gamma) \text{cov}(r_2, \pi(n)) &= 0 \\
\lambda_3 - \gamma \alpha_1(n) \text{cov}(r_1, \Delta s) - \gamma \alpha_2(n) \text{cov}(r_2, \Delta s) - \gamma b(n) \text{var}(\Delta s) - (1 - \gamma) \text{cov}(\Delta s, \pi(n)) &= 0
\end{align*}
\]

where
\[
\begin{align*}
\lambda_1 &= E r_1 - \ln(1+i_1) + 0.5 \text{var}(r_1) \\
\lambda_2 &= E r_2 - \ln(1+i_1) + 0.5 \text{var}(r_2) \\
\lambda_3 &= E \Delta s + \ln(1+i_2) - \ln(1+i_1) + 0.5 \text{var}(\Delta s)
\end{align*}
\]
The first step towards solving the home bias formula is the same as with a forward market. Again define \( \tilde{x} \) as the component of \( x \) orthogonal to \( \Delta s \), as in (27). Applying this to returns and inflation rates after substituting the solution for \( b(n) \) from the last first order condition into the first two first order conditions, we get

\[
\eta_1 - \gamma \alpha_1(n) \text{var}(\tilde{r}_1) - \gamma \alpha_2(n) \text{cov}(\tilde{r}_1, \hat{r}_2) + (\gamma - 1) \text{cov}(\tilde{r}_1, \hat{\pi}(n)) = 0 \quad (33)
\]

\[
\eta_2 - \gamma \alpha_1(n) \text{cov}(\tilde{r}_1, \hat{r}_2) - \gamma \alpha_2(n) \text{var}(\hat{r}_2) + (\gamma - 1) \text{cov}(\hat{r}_2, \hat{\pi}(n)) = 0 \quad (34)
\]

where \( \eta_1 \) and \( \eta_2 \) depend on first and second moments that are the same from the perspective of investors of both countries.

Substituting (34) into (33) gives

\[
\mu_1 - \gamma \alpha_1(n) \text{var}(\tilde{r}_1) + \gamma \alpha_1(n) \frac{\text{cov}(\tilde{r}_1, \hat{r}_2)^2}{\text{var}(\hat{r}_2)} + (\gamma - 1) \text{cov}(\tilde{r}_1, \hat{\pi}(n)) - (\gamma - 1) \frac{\text{cov}(\hat{r}_2, \hat{\pi}(n)) \text{cov}(\tilde{r}_1, \hat{r}_2)}{\text{var}(\hat{r}_2)} = 0 \quad (35)
\]

where \( \mu_1 \) is the same for investors of both countries.

Now define \( \tilde{r}_1 \) and \( \hat{\pi}(n) \) as the components of \( \tilde{r}_1 \) and \( \hat{\pi}(n) \) that are orthogonal to \( \tilde{r}_2 \). Again applying (27), (35) becomes

\[
\mu_1 - \gamma \alpha_1(n) \text{var}(\tilde{r}_1) + (\gamma - 1) \text{cov}(\tilde{r}_1, \hat{\pi}(n)) = 0 \quad (36)
\]

Since \( \tilde{r}_1 \) and \( \hat{\pi}(n) \) are the same as the residuals of regressions of respectively \( r_1 \) and \( \pi(n) \) on both \( \Delta s \) and \( r_2 \), (14) follows. Equation (15) follows by symmetry.

Using the equity market clearing conditions

\[
\alpha_i(1)w_1 + \alpha_i(2)(1 - w_1) = \beta_i/\omega \quad (37)
\]

for \( i = 1, 2 \), (16) becomes

\[
\text{home bias} = 0.5\omega [(\alpha_1(1) - \alpha_1(2))(1 - \omega_1) + (\alpha_2(2) - \alpha_2(1))\omega_1] \quad (38)
\]

Substituting (14) and (15) yields (17).

C Home Bias with a Non-Traded Asset

Finally consider adding a non-traded asset to the benchmark asset structure. The first-order condition for optimal portfolio choice is now

\[
EC^{-\gamma}(R_1 - R_2)e^{-\pi(n)} = 0 \quad (39)
\]
where consumption is

\[ C = A(n)R^p(n) + W(n) \]  \hspace{1cm} (40)

In terms of logs the first-order condition is

\[ Ee^{-\gamma c+r_1-\pi(n)} = Ee^{-\gamma c+r_2-\pi(n)} \]  \hspace{1cm} (41)

A first-order log-linearization of consumption gives

\[ c = f[\alpha_1(n)r_1 + (1 - \alpha_1(n))r_2 - \pi(n)] + (1 - f)w(n) \]  \hspace{1cm} (42)

where \( f \) is the ratio of financial to total wealth around which we expand. Assuming again log-normality of returns, the first-order condition gives

\[ \alpha_1(n) = \lambda + \frac{\gamma - \frac{1}{\gamma} \text{cov}(r_1 - r_2, \pi(n))}{\text{var}(r_1 - r_2)} - \frac{1 - f \text{cov}(r_1 - r_2, w(n))}{f \text{var}(r_1 - r_2)} \]  \hspace{1cm} (43)

where

\[ \lambda = \frac{E(r_1 - r_2) + 0.5(\text{var}(r_1) - \text{var}(r_2)) + \gamma f \text{cov}(r_2 - r_1, r_2)}{\gamma f \text{var}(r_1 - r_2)} \]

Using (8) then leads to (19).
References


Table 1. Covariance-Variance Ratios: Monthly Data, 1988-2005

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<td>cov(\Delta q_1, (er, \Delta q))</td>
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<td>var(\Delta q)</td>
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<td>(\text{cov}(er, q))</td>
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Notes. The covariance-variance ratios correspond to those in equations (9), (12), and (17). Specifically, in column (1), corresponding to the expression below equation (9), straight excess returns and real exchange rate changes are used; in column (2), corresponding to the expression in equation (12), \(er\) and \(\Delta q\) are orthogonal to changes in the nominal exchange rate; in column (3), corresponding to the first term in equation (17), \(er\) and \(\Delta q\) are orthogonal to changes in the nominal exchange rate and \(r^{ROW}\); and in column (4), corresponding to the second term in equation (17), \(er\) and \(\Delta q\) are orthogonal to changes in the nominal exchange rate and \(r^{US}\). There are 216 monthly observations underlying each calculation.

Table 2. Covariance-Variance Ratios: Different Frequencies

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<tr>
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Notes. The covariance-variance ratios correspond to those in equations (9), (12), and (17); see Table 1 for details. The number of observations in the three rows are 205, 72, and 18, respectively.

Table 3. Conditional Covariance-Variance Ratios

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<td>cov(\Delta q_1, (er, \Delta q))</td>
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<td></td>
<td>var(er)</td>
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Notes. The covariance-variance ratios correspond to those in equations (9), (12), and (17); see Table 1 for details. The number of observations is 68. The conditioning variables are 4 lags of each of the following: US relative to ROW real GDP growth, long-term bond rates, equity returns (relative, US, and ROW), and changes in the real and nominal exchange rates.
Figure 1
The figure shows a scatter plot of monthly real exchange rate changes against corresponding excess returns.

Figure 2
The figure shows a scatter plot of monthly real exchange rate changes against corresponding excess returns, each conditioned on changes in nominal exchange rates.