

International Capital Flows¹

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Abstract

The surge in international asset trade since the early 1990s has led to renewed interest in models with international portfolio choice. We develop the implications of portfolio choice for both gross and net international capital flows in the context of a simple two-country dynamic stochastic general equilibrium (DSGE) model. We focus on the time-variation in portfolio allocation following shocks, and resulting capital flows. Endogenous time-variation in expected returns and risk, which are the key determinants of portfolio choice, affect capital flows in often subtle ways. The model is consistent with a broad range of empirical evidence. An additional contribution of the paper is to overcome the technical difficulty of solving DSGE models with portfolio choice by developing a broadly applicable solution method.

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1 Introduction

In his Ohlin lecture, Obstfeld (2007) emphasizes the “explosion in international asset trade” since the early 1990s and calls for the development of a general equilibrium portfolio-balance model to analyze its implications, echoing his earlier statement that “at the moment we have no integrative general-equilibrium monetary model of international portfolio choice, although we need one” (Obstfeld, 2004). The recent surge in international financial integration¹ has indeed lead to a broad consensus on the need to include portfolio choice in such models, an aspect that has been cast aside since ad-hoc portfolio balance models of the 1970s fell out of favor. Typical of current views, Gourinchas (2006) writes “Looking ahead, the next obvious step is to build general equilibrium models of international portfolio allocation with incomplete markets. I see this as a major task that will close a much needed gap in the literature... ”.

The goal of this paper is to fill this gap. We develop the implications of portfolio choice for both gross and net international capital flows in the context of a simple two-country dynamic stochastic general equilibrium (DSGE) model. We show how endogenous time-variation in expected returns and risk, which are the key determinants of portfolio choice, affect capital flows. We deliberately consider a simple model to focus squarely on portfolio choice. While we do not formally test the model, we show that it is consistent with a broad range of empirical evidence. An additional contribution of the paper is to overcome the technical difficulty of solving DSGE models with portfolio choice by developing a solution method that is broadly applicable beyond our simple setup.

The analysis reveals some surprising results regarding the role of time-varying risk and expected returns. First, we show that even though the variance of the shocks in the model is held constant, the second moments that drive portfolio choice endogenously vary in response to movements in the state space. Second, these fluctuations in second moments affect capital flows only to the extent that they affect the portfolio choice of domestic and foreign investors differently. We show that they can be an important driving force behind international capital flows and generate a positive co-movement between gross capital inflows and outflows that is consistent with the data. Third, while endogenous changes in the difference in expected returns across assets affect portfolio choice, the link between capital flows

¹See for instance Gourinchas and Rey (2007), Lane and Milesi-Ferretti (2005) and Tille (2008).

and expected returns is weak. This is both because expected return differences are small in equilibrium and because several important factors driving expected return differences have no bearing on capital flows. This suggests caution when conducting empirical work on the link between capital flows and expected returns.

The model also sheds light on the mechanisms through which international imbalances are financed. The traditional view is that an external debt is financed through the present value of future trade surpluses. Recently however, Gourinchas and Rey (2007) have emphasized that it can also be financed through higher expected returns on external assets than liabilities, and find a significant role for this expected asset return channel for the United States. Our analysis confirms that expected asset price changes (both exchange rates and equity prices) play a role in financing the external debt. Nonetheless, the income streams on the various assets (e.g. dividends) adjust to equalize the predictable overall returns across assets (dividends plus expected asset price changes). This implies that to a first-order an external debt is fully financed by future trade surpluses.²

One of the main reasons that portfolio choice has been largely absent from open economy DSGE models is the difficulty in solving such models. The standard method for solving DSGE models is to simply linearize around the deterministic steady state, and then solve the resulting set of linear difference equations. This can be extended to compute the second-order solution of the model. These methods can be easily be implemented in large models. However, this standard approach breaks down in the presence of portfolio choice. For example, the allocation around which the model is expanded cannot be the deterministic steady state as portfolio choice is undetermined in such an environment.

The solution method we develop stays very close to the familiar first and second order approximation techniques. Indeed, the model is solved using them for all equations except the optimality conditions for portfolio choice, which need to be approximated to higher orders. Intuitively, this reflects the fact that portfolio allocation is about risk, a dimension that is not captured by a first-order approximation. Specifically, solving for portfolio choice in the allocation around which the model is expanded requires a second-order expansion of optimality conditions for portfolio choice in order to capture the risk at the core of portfolio choice. In order to capture the time-variation of portfolio allocation a third-order expansion

²Pavlova and Rigobon (2007) independently develop a similar point.

of the optimality conditions for portfolio choice is needed. While third-order terms are usually seen as irrelevant due to their small magnitude, this is not the case in the context of portfolio choice where third-order changes in expected returns and risk induce first-order changes in portfolio shares.³ An important point is that the method is not limited to the simple model that we consider, but is applicable to a broad range of richer settings.

The remainder of the paper is organized as follows. Section 2 puts our work in the context related literature on the role of portfolio choice. Our simple model is described in Section 3, with Section 4 discussing the solution method. We develop various implications for international capital flows in Section 5. Section 6 illustrates our results using a numerical example and relates various implications of the model to a broad range of empirical evidence. Section 7 concludes.

2 Related Literature

Portfolio balance models of the late 1970s and early 1980s modeled asset demand as a function of expected asset returns, income, wealth and prices. A nice review can be found in Branson and Henderson (1985). A clear drawback of this literature is that asset demand specifications were not derived from optimization. A number of papers at the time did consider the micro foundations of asset demand in the context of open economy models with optimal portfolio choice.⁴ However, these were partial equilibrium models and the portfolio choice elements were never fully integrated in the general equilibrium models that followed.

Since the early 1980s these models were gradually replaced by general equilibrium models with optimizing agents. Portfolio choice was overlooked as most of these models assume that either only a riskfree bond is traded or that financial markets are complete. The one-asset models have nothing to say about gross international capital flows or asset positions. This is a problem because asset trade, just

³For example, in a simple two-asset portfolio choice problem portfolio shares will depend on the expected excess return (the difference in return between the two assets) divided by the variance of the excess return. Since the denominator (variance of excess return) is second-order, a third-order change in the expected excess return leads to a first-order change in the portfolio share.

⁴See for example Adler and Dumas (1983), Braga de Macedo (1983), Braga de Macedo, Goldstein and Meerschman (1984), Kouri (1976), Kouri and Braga de Macedo (1978) and the review in Branson and Henderson (1985).

like goods trade, is two-way trade with similar assets (e.g. equity) both imported and exported. Moreover, changes in expected returns in one country relative to another, or changes in risk characteristics of assets, play no role. At the other extreme, in models where financial markets are complete, Obstfeld and Rogoff (1996) argue that capital flows are “...merely an accounting device for tracking the international distribution of new equity claims foreigners must buy to maintain the efficient global pooling of national output risks.” Capital flows are rarely even considered in these models as the real allocation can be computed independent of the exact structure of asset markets that implements asset market completeness.⁵

Portfolio choice was not completely cast aside, as a large and still growing literature has considered the issue of portfolio home bias. However, this literature is concerned with the steady state portfolio allocation rather than with time-variation in portfolio allocation that gives rise to capital flows. Some recent contributions along this line include Engel and Matsumoto (2006), Heathcote and Perri (2007), Kollman (2006), Coeurdacier (2007) and Coeurdacier, Kollmann and Martin (2007).

The impact of portfolio choice on capital flows was analyzed explicitly in the important contributions by Kraay and Ventura (2000,2003). While they consider partial equilibrium small open economy models, with an exogenous return on investment abroad and only one-way capital flows (from the small to the large country), these papers are nonetheless important as they are the first ones to develop a portfolio perspective on international capital flows. They make a key distinction between portfolio growth (investment of saving at steady state portfolio shares) and portfolio reallocation (change in optimal portfolio shares), which we will adopt here as well.

In terms of the solution method the paper is most closely related to Devereux and Sutherland (2007). They independently and simultaneously developed a solution method for DSGE models with portfolio choice that is essentially the same as ours, as discussed in Section 4.⁶ Their contribution focuses only on the methodological aspects though and does not discuss implications for international capital flows. The work by Evans and Hnatkovska (2007b) is important as well. Their method combines a variety of discrete time approaches (perturbation and

⁵In a setup where a full set of Arrow Debreu securities covering all possible future contingencies is traded in an initial period, subsequent capital flows will simply be zero.

⁶Their work builds on the previous papers Devereux and Sutherland (2006, 2008).

projection methods) with continuous time approximations (of portfolio return and second-order dynamics of the state variables), making it hard to compare to the one developed here.⁷ By contrast, our method stays closer to standard first and second-order solution methods.

Most recently Rigobon and Pavlova (2007) develop an analytical solution to a DSGE model with portfolio choice and discuss various issues associated with external debt financing. The model and solution method are elegant, but analytical solutions can only be achieved for particular parameterizations (e.g. log utility). The same can also be said of Devereux and Saito (2007), who also derive an analytical solution for portfolio choice in a DSGE model. The focus in that paper is on the stationarity of the international wealth distribution.

3 A two-country, two-good, two-asset model

We consider a deliberately minimalist model that focuses squarely on the optimal international allocation of portfolios and abstracts from all other decisions. Our approach adopts almost the exact opposite modeling strategy as in standard DSGE open economy models, which encompass decisions about consumption, leisure and investment, but ignore portfolio choice. Our choice of a simple setup is solely to focus on the key element that has been missing from dynamic stochastic open economy models. As we will discuss later, the solution method can easily be applied to richer and more complex models.

3.1 Two goods: production and consumption

There are two countries of equal size, Home and Foreign, that each produce a different good that is consumed worldwide. Production uses a constant returns to scale technology combining labor and capital:

$$Y_{i,t} = A_{i,t} K_{i,t}^{1-\theta} N_{i,t}^{\theta} \quad i = H, F$$

where H and F denote the Home and Foreign country respectively. Y_i is the output of the country i good, K_i is the capital input and N_i the labor input. A_i is an

⁷Evans and Hnatkovska (2005, 2007a) and Hnatkovska (2006) apply the solution method to discuss implications for issues such as the volatility of asset prices, the implications of international financial integration and portfolio home bias.

exogenous stochastic productivity term which is the only source of shocks in the model. A share θ of output is paid to labor, with the remaining going to capital. The capital stocks and labor inputs are fixed and normalized to unity. Outputs therefore simply reflect the levels of productivity, which follow an exogenous autoregressive process:

$$Y_{i,t} = A_{i,t} \quad ; \quad a_{i,t+1} = \rho a_{i,t} + \epsilon_{i,t+1} \quad (1)$$

where lower case letters denote logs and $\rho \in (0, 1)$. The productivity innovations in both countries are iid, with a $N(0, \sigma^2)$ distribution.

We consider home bias in preferences, with consumers putting a higher weight on locally-produced goods. The resulting difference in the consumption baskets and consumer prices generate a role for movements in the real exchange rate. The consumption baskets are standard CES aggregates, and are presented in the first column of the table below. C is the overall consumption of the Home consumer, C_H denotes her consumption of Home goods and C_F denotes her consumption of Foreign goods. The corresponding variables for the Foreign consumer are denoted by asterisks. λ is the elasticity of substitution between Home and Foreign goods, and α captures the relative preference towards domestic goods, with $\alpha > 0.5$ corresponding to home bias in consumption. We take the Home good as the numeraire and write the relative price of the Foreign good as P_F . The consumer price indexes in the two countries are shown in second column.

Consumption indices	Price indices
$C_t = \left[(\alpha)^{\frac{1}{\lambda}} (C_{H,t})^{\frac{\lambda-1}{\lambda}} + (1-\alpha)^{\frac{1}{\lambda}} (C_{F,t})^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}$	$P_t = \left[\alpha + (1-\alpha) [P_{F,t}]^{1-\lambda} \right]^{\frac{1}{1-\lambda}}$
$C_t^* = \left[(1-\alpha)^{\frac{1}{\lambda}} (C_{H,t}^*)^{\frac{\lambda-1}{\lambda}} + (\alpha)^{\frac{1}{\lambda}} (C_{F,t}^*)^{\frac{\lambda-1}{\lambda}} \right]^{\frac{\lambda}{\lambda-1}}$	$P_t^* = \left[(1-\alpha) + \alpha [P_{F,t}]^{1-\lambda} \right]^{\frac{1}{1-\lambda}}$

3.2 Two assets: rates of return

Two assets are traded, namely claims on the Home capital stock K_H and claims on the Foreign capital stock K_F . We refer to these as Home and Foreign equity. The price at time t of a unit of Home equity is denoted by $Q_{H,t}$. The holder of this claim gets a dividend in period $t+1$, which is a share $1-\theta$ of output (1), and can sell the claim for a price $Q_{H,t+1}$. The overall return on a Home equity is then:

$$R_{H,t+1} = 1 + (Q_{H,t+1} - Q_{H,t})/Q_{H,t} + (1-\theta)A_{H,t+1}/Q_{H,t} \quad (2)$$

Similarly, the price at time t of a unit of Foreign equity is denoted by $Q_{F,t}$, and the return on Foreign equity is:

$$R_{F,t+1} = 1 + (Q_{F,t+1} - Q_{F,t})/Q_{F,t} + (1 - \theta)P_{F,t+1}A_{F,t+1}/Q_{F,t} \quad (3)$$

All prices and returns in (2)-(3) are measured in terms of the numeraire Home good. (2)-(3) show that the returns consist of a capital gain or loss due to movements in equity prices and a dividend yield.

While agents can invest in equity abroad, this entails a cost. Specifically, the agent receives only the returns in (2)-(3) times an iceberg cost $e^{-\tau} < 1$. We introduce this cost for two reasons. First, it allows us to capture the hurdles of investing outside the domestic country in a simple way, reflecting the cost of gathering information on an unfamiliar market for instance.⁸ Second, it implies that financial markets are incomplete, despite the presence of two assets and two shocks.⁹ We assume that the cost is small enough to ensure a well-behaved portfolio allocation (specifically, τ is "second-order", i.e. proportional to σ^2).

At the end of period t a Home agent invests a fraction $k_{H,t}^H$ of her wealth in Home equity and a fraction $1 - k_{H,t}^H$ in Foreign equity. The overall real return on her portfolio, expressed in terms of the Home consumption basket, is then:

$$R_{t+1}^{p,H} = [k_{H,t}^H R_{H,t+1} + (1 - k_{H,t}^H) e^{-\tau} R_{F,t+1}] P_t / P_{t+1} \quad (4)$$

Similarly, a Foreign agent invests a fraction $k_{H,t}^F$ of her wealth in Home equity, and a fraction $1 - k_{H,t}^F$ in Foreign equity, leading to an overall real return in terms of the Foreign consumption basket of:

$$R_{t+1}^{p,F} = [k_{H,t}^F e^{-\tau} R_{H,t+1} + (1 - k_{H,t}^F) R_{F,t+1}] P_t^* / P_{t+1}^* \quad (5)$$

3.3 Wealth dynamics

A pitfall of models with incomplete financial markets is that they can imply a non-stationary distribution of wealth, as even transitory shocks can have a permanent effect on the distribution of wealth. A country will then eventually own

⁸It is by now quite common to introduce such exogenous financial frictions. Other examples, with more detailed motivating discussions, are Martin and Rey (2004), Coeurdacier (2007) and Coeurdacier and Guibaud (2005).

⁹Even in the absence of this friction, financial markets are incomplete when $\lambda\gamma \neq 1$, where γ is the rate of relative risk-aversion discussed below. See the discussion in Obstfeld and Rogoff (2000), page 364. Their model is one with trade costs, but that is observationally equivalent to home bias in preferences.

the entire world, so that the long run wealth distribution is not determined. This indeterminacy rules out the use of the standard approximation methods as there is no allocation to which the economy will converge.

We get around this problem by considering investors with finite lives, following the framework of Caballero, Fahri and Gourinchas (2008). Specifically, agents die with constant probability ψ each period, and new agents are born at the same rate to keep the population constant. Keeping with our focus on portfolio choice, we abstract from any other decision. Newborn agents inelastically supply one unit of labor and never work thereafter. Dying agents liquidate their entire wealth and consume. The other agents re-invest their wealth in the two equities, possibly altering their portfolio allocation. This is their only decision. Since the probability of death is the same for all agents, total consumption is simply equal to aggregate wealth times the probability of death.

The wealth of a particular Home investor j accumulates according to

$$W_{t+1}^j = W_t^j R_{t+1}^{p,H} \quad (6)$$

where $R_{t+1}^{p,H}$ is given by (4).¹⁰ Aggregate wealth accumulation differs from (6) for three reasons. First, only the agents that will be alive next period participate in asset markets. They account for a fraction $1 - \psi$ of wealth. Second, the labor income of the newborns raises aggregate wealth. Third, the cost of investment abroad, τ , does not affect the dynamics of aggregate wealth. Specifically, we assume that it does not represent lost resources, but instead is a fee paid to a broker, which we take to be the newborn agents. We denote the aggregate wealth of the Home and Foreign countries, measured in terms of their respective consumption baskets, by W_t and W_t^* . Their dynamics are given by:

$$W_{t+1} = (1 - \psi) [k_{H,t}^H R_{H,t+1} + (1 - k_{H,t}^H) R_{F,t+1}] \frac{P_t}{P_{t+1}} W_t + \frac{\theta A_{H,t+1}}{P_{t+1}} \quad (7)$$

$$W_{t+1}^* = (1 - \psi) [k_{H,t}^F R_{H,t+1} + (1 - k_{H,t}^F) R_{F,t+1}] \frac{P_t^*}{P_{t+1}^*} W_t^* + \frac{\theta P_{F,t+1} A_{F,t+1}}{P_{t+1}^*} \quad (8)$$

¹⁰The portfolio return will be the same for all Home investors as they all choose the same portfolio in equilibrium.

3.4 Markets clearing

Using (1) and the allocation of consumption between Home and Foreign goods, the two goods market clearing conditions are

$$A_{H,t} = \alpha (P_t)^\lambda \psi W_t + (1 - \alpha) (P_t^*)^\lambda \psi W_t^* \quad (9)$$

$$A_{F,t} = (1 - \alpha) (P_{F,t})^{-\lambda} (P_t)^\lambda \psi W_t + \alpha (P_{F,t})^{-\lambda} (P_t^*)^\lambda \psi W_t^* \quad (10)$$

Turning to asset markets, the total values of Home and Foreign equity supply are equal to $Q_{H,t}$ and $Q_{F,t}$ since the capital stocks are normalized to 1. The amounts invested by Home and Foreign agents at the end of period t , measured in Home goods, are $(1 - \psi) W_t P_t$ and $(1 - \psi) W_t^* P_t^*$ respectively. The market clearing conditions for Home and Foreign asset markets are then

$$Q_{H,t} = (1 - \psi) [k_{H,t}^H W_t P_t + k_{H,t}^F W_t^* P_t^*] \quad (11)$$

$$Q_{F,t} = (1 - \psi) [(1 - k_{H,t}^H) W_t P_t + (1 - k_{H,t}^F) W_t^* P_t^*] \quad (12)$$

3.5 Portfolio allocation

The only decision faced by agents is the allocation of their investment between Home and Foreign equity. A Home agent j who dies in period $t + 1$ consumes her entire wealth and gets utility

$$U_{t+1}^j = (W_{t+1}^j)^{1-\gamma} / (1 - \gamma) \quad \gamma > 1$$

We denote the value of wealth in period t by $V(W_t^j)$. As agents face a probability ψ of dying the next period, the Bellman equation is

$$V(W_t^j) = \beta(1 - \psi) E_t V(W_{t+1}^j) + \beta \psi E_t (W_{t+1}^j)^{1-\gamma} / (1 - \gamma) \quad (13)$$

where β is the discount rate.

We conjecture the following form for the value of wealth:

$$V(W_t^j) = e^{v+f_H(S_t)} (W_t^j)^{1-\gamma} / (1 - \gamma) \quad (14)$$

where v is a constant, S_t is the state space discussed below and the function $f_H(S_t)$ captures time variation in expected portfolio returns, which endogenously vary with the state. For given wealth, utility is higher ($f_H(S_t)$ is lower) the larger

are expected future portfolio returns. For Foreign investors the function $f_H(S_t)$ is replaced by $f_F(S_t)$.

Agent j of the Home country chooses the portfolio allocation to maximize (13), subject to (6) and (4). The first-order conditions for Home and Foreign investors are:

$$E_t \Lambda_t (R_{H,t+1} - e^{-\tau} R_{F,t+1}) = 0 \quad ; \quad E_t \Lambda_t^* (e^{-\tau} R_{H,t+1} - R_{F,t+1}) = 0 \quad (15)$$

where

$$\begin{aligned} \Lambda_t &= ((1 - \psi)e^{v+f_H(S_{t+1})} + \psi) \left(R_{t+1}^{p,H} \right)^{-\gamma} P_t / P_{t+1} \\ \Lambda_t^* &= ((1 - \psi)e^{v+f_F(S_{t+1})} + \psi) \left(R_{t+1}^{p,F} \right)^{-\gamma} P_t^* / P_{t+1}^* \end{aligned}$$

are the asset pricing kernels of the Home and Foreign investors respectively. The optimality conditions for portfolio choice (15) show that investors equalize the expected discounted return on each asset. Therefore the expected product of the asset pricing kernel and the excess return is equal to zero.

Using (14), the Bellman equation (13) for a representative investor in country i is

$$e^{v+f_i(S_t)} = \beta E_t \left((1 - \psi)e^{v+f_i(S_{t+1})} + \psi \right) \left(R_{t+1}^{p,i} \right)^{1-\gamma} \quad i = H, F \quad (16)$$

which gives an implicit solution to the function $f_i(S_t)$.

4 Solution of the model

The model consists of 11 independent equations that are listed in Appendix A. The solution method is made clearer by defining two measures of portfolio shares. The average share invested in Home equity is denoted by $k_t^A = 0.5(k_{H,t}^H + k_{H,t}^F)$. The difference in portfolio shares captures the extent to which Home investors are more heavily invested in Home equity than Foreign investors are, and is written as $k_t^D = k_{H,t}^H - k_{H,t}^F$. It is therefore a measure of portfolio home bias.

The solution method builds on standard linear and quadratic approximation methods around an allocation. While this allocation is usually computed as the deterministic steady state, the introduction of portfolio choice raises a complexity. As portfolio choice is driven by risk, it is not well-defined in a deterministic environment. The problem arises specifically for the difference across countries in

portfolio shares. Even in a deterministic environment, the average portfolio share, k_t^A , is simply determined by the asset market clearing conditions (11)-(12). These conditions show how much Home and Foreign equity needs to be held by investors in equilibrium, but shed no light on which investor should hold it. The difference in portfolio shares, k_t^D , is not well-defined in a deterministic equilibrium. The solution method therefore gives special treatment to the difference in portfolio shares and the difference in portfolio Euler equations (15) between Home and the Foreign investors. No special treatment is required for any of the remaining equations and variables, which we refer to as the “other equations” and “other variables” for brevity. As discussed above, the average portfolio share, k_t^A , is one of these “other variables”.

A central aspect of our approach is to split the various variables across components of different orders. A variable x_t can be written as the sum of its zero-order, first-order and higher-order components, namely: $x_t = x(0) + x_t(1) + x_t(2) + \dots$. The zero-order component, $x(0)$, is the value of x_t when the volatility of shocks becomes arbitrarily small ($\sigma \rightarrow 0$). The first-order component, $x_t(1)$, is proportional to σ or model innovations. The second-order component, $x_t(2)$, is proportional to σ^2 or the product of model innovations, and so on. This decomposition also applies to portfolio shares, and in particular to the difference k_t^D . For instance, the zero-order portfolio difference, $k^D(0)$, is independent of the value of σ , as shown below. Note that it is only defined as long as σ is positive, even if infinitesimally small.

In order to compute first and higher-order components of model equations we expand around zero-order components of all variables. The zero-order components of the “other variables” follow directly from the zero-order components of the “other equations”.¹¹ Computing the zero-order component of the difference in portfolio shares is more complex. We now turn to the description of the solution method, which proceeds in two steps. We keep the description as non-technical as possible, focusing on the methodology. The technical details are outlined in Appendices B and C for the first and second-order components of Bellman equations and the third-order components of Euler equations for portfolio choice, with a full

¹¹In particular, dropping country subscripts due to symmetry, we have $W(0) = 1/\psi$, $R(0) = (1 - \psi\theta) / (1 - \psi)$, $Q(0) = (1 - \psi) / \psi$, $A(0) = P_F(0) = 1$, $v(0) = \ln(\psi) - \ln(R(0)^{\gamma-1} / \beta - 1 + \psi)$ and $k^A(0) = 0.5$.

description of all the algebra left to a Technical Appendix available on request.¹²

4.1 The first step

The first step involves jointly solving the first-order component of the “other variables” and the zero-order component of the difference in portfolio shares k_t^D .

First-order solution of “other variables”

Conditional on a value for the zero-order component of the difference in portfolio shares, $k^D(0)$, the first-order component of the “other variables” is solved from the first-order component of the “other equations” using the entirely standard first-order solution method based on a log-linear approximation around the zero-order values of the variables. We define the difference and average of variables across countries with superscripts D and A ($x_t^D = x_t^H - x_t^F$ and $x_t^A = 0.5(x_t^H + x_t^F)$). The model boils down to 5 control variables and 3 state variables.¹³

$$cv_t = (w_t^A, p_{F,t}, k_t^A, q_{H,t}, q_{F,t})' \quad (17)$$

$$S_t = (a_t^D, w_t^D, a_t^A)' \quad (18)$$

The standard first-order solution technique applied to the first-order components of the log-linearized equations then provides a solution of the following form:

$$cv_t(1) = BS_t(1) \quad ; \quad S_{t+1}(1) = N_1S_t(1) + N_2\epsilon_{t+1} \quad (19)$$

where B , N_1 and N_2 are matrices and $\epsilon_{t+1} = (\epsilon_{H,t+1}, \epsilon_{F,t+1})'$ are the model innovations. The first-order component of the Bellman equations (16) gives the first-order components of the functions $f_H(S_t)$ and $f_F(S_t)$, denoted by $H_{1,H}S_t(1)$ and $H_{1,F}S_t(1)$, and also implies $v(1) = 0$.

Zero-order solution of portfolio share difference

The first-order solution (19) is conditional on the unknown $k^D(0)$, which is solved by taking the second-order component of the difference across countries of

¹²In the working paper version, Tille and van Wincoop (2007), we provide a more general description of the solution method that applies to any order of approximation.

¹³The average wealth level is not a separate state variable as the first-order components of w_t^A and a_t^A are identical.

the portfolio Euler equations (15). Abstracting from the algebraic details, we get

$$k^D(0) = 2 \frac{\tau}{\gamma \text{var}(er_{t+1}(1))} + \frac{\gamma - 1}{\gamma} \frac{\text{cov}(p_{t+1}(1) - p_{t+1}^*(1), er_{t+1}(1))}{\text{var}(er_{t+1}(1))} \quad (20)$$

$$+ \frac{(1 - \psi') \text{cov}((H_{1,H} - H_{1,F}) S_{t+1}(1), er_{t+1}(1))}{\gamma \text{var}(er_{t+1}(1))}$$

where $er_{t+1} = r_{H,t+1} - r_{F,t+1}$ is the excess return on Home equity, and $\psi' = 1 - \beta(1 - \psi)R(0)^{1-\gamma}$. Each of the three terms on the right-hand side of (20) is a ratio of second-order variables (proportional to σ^2). This illustrates why the second-order components of portfolio Euler equations are necessary to compute the zero-order component of portfolio shares.

In terms of economic intuition, a positive value of (20) implies portfolio home bias, while a negative value implies foreign bias. (20) shows three sources of portfolio bias. The first reflects the cost of investing abroad, τ , with a higher cost making investing in domestic equity more attractive. The second reflects the comovements of the real exchange rate and excess return. Assuming $\gamma > 1$, it is attractive for Home investors to invest in the Home equity if the excess return on Home equity is high in states where the Home price index is relatively high, i.e. Home equity is a good hedge of real exchange rate risk. The final source reflects a hedge against changes in future expected portfolio returns, which are captured by the functions $H_{1,H}S_{t+1}(1)$ and $H_{1,F}S_{t+1}(1)$ in the value function of Home and Foreign investors next period. A high value of these functions indicates a future state with low expected returns. It is attractive for Home investors to invest in Home equity when the excess return on Home equity is high in such states.

Fixed point problem

With the exception of τ , all the second-order components in the three ratios in (20) are based on variances and covariances of first-order components of model variables. These can be computed from the first-order solution (19). In turn, the first-order solution is conditional on $k^D(0)$. Intuitively, when $k^D(0)$ is non-zero Home and Foreign agents choose different portfolios and therefore their overall portfolio return responds differently to return innovations. This affects the first-order component of relative wealth, which in turn affects relative consumption and asset demand. We therefore have a fixed point problem: $k^D(0)$ maps into the first-order solution (19), which maps into $k^D(0)$ in (20). By substituting the

solution for $k^D(0)$ from the fixed point problem into (19), we have fully solved the first-order component of “other variables” as well.

In terms of capital flows, the solution so far only allows us to compute net capital flows, not gross flows.¹⁴ To a first order, net capital flows only depend on the average portfolio share k_t^A , which is one of the “other variables”. A reduction in $k_t^A(1)$ implies that overall investors are selling Home equity, leading to a net capital flow out of the Home country. By contrast, gross capital flows also depend on the first-order component of the difference in portfolio shares, $k_t^D(1)$, which we have not yet solved for.

4.2 The second step

The second step provides us with the first-order component of the difference in portfolio shares, $k_t^D(1)$. Conceptually it is identical to the first step but one order higher. We now combine the third-order component of the difference in portfolio Euler equations (15) with the second-order components of all “other equations” to jointly solve for $k_t^D(1)$ and the second-order component of all “other variables”.

Second-order solution of “other variables”

We start by conjecturing a solution for $k_t^D(1)$ that is linear in the state variables:

$$k_t^D(1) = k_s S_t(1) \tag{21}$$

where k_s is a 1 by 3 vector. Conditional on (21), the second-order component of the “other variables” is solved from the second-order component of the “other equations” using the standard second-order solution method.¹⁵ The solution of the second-order component of control variables, for example $p_{F,t}(2)$, takes the following form:

$$p_{F,t}(2) = p_s S_t(2) + S_t(1)' p_{ss} S_t(1) + k_p \sigma^2 \tag{22}$$

where p_s is a vector, p_{ss} a matrix and k_p a scalar. The second-order solution for

¹⁴This can be checked from the expressions (26)-(27) in section 5.

¹⁵For descriptions of second-order solutions see Kim et.al. (2003), Schmitt-Grohe and Uribe (2004) and Lombardo and Sutherland (2007). The Technical Appendix provides all details in the context of the present model.

state space accumulation takes the form

$$S_{t+1}(2) = N_1 S_t(2) + \begin{bmatrix} S_t(1)' N_{3,1} S_t(1) + \epsilon'_{t+1} N_{4,1} \epsilon_{t+1} + S_t(1)' N_{5,1} \epsilon_{t+1} \\ S_t(1)' N_{3,2} S_t(1) + \epsilon'_{t+1} N_{4,2} \epsilon_{t+1} + S_t(1)' N_{5,2} \epsilon_{t+1} \\ S_t(1)' N_{3,3} S_t(1) + \epsilon'_{t+1} N_{4,3} \epsilon_{t+1} + S_t(1)' N_{5,3} \epsilon_{t+1} \end{bmatrix} + N_6 \sigma^2 \quad (23)$$

where $N_{3,i}$, $N_{4,i}$ and $N_{5,i}$ are matrices and N_6 is a vector. The second-order component of the Bellman equations (16) yields the second-order components of the functions $f_H(S_t)$ and $f_F(S_t)$.

An important implication of (22) is that the second-order solution of the “other variables” naturally leads to endogenous variation over time in second moments, even though the standard deviation of shocks is constant. This is best illustrated by considering a simple case where there is only one state variable, s_t , which simply evolves according to $s_{t+1} = s_t + \epsilon_{t+1}$, where ϵ_{t+1} is a shock with variance σ^2 . Assuming that $p_t^F = s_t + s_t^2$, it follows that $p_{F,t+1}$ will depend on the time $t + 1$ innovation in the form $\epsilon_{t+1} + \epsilon_{t+1}^2 + 2s_t \epsilon_{t+1}$. The last term shows that the conditional variance of $p_{F,t+1}$ depends on s_t and is therefore time-varying. It also shows that one needs to distinguish between second moments and second-order. In this example the variance of $p_{F,t+1}$ has a second-order component of σ^2 and a third-order component of $4\sigma^2 s_t$. The third-order component of the variance therefore captures movements in the variance driven by the state variable.

More generally, consider two variables x and y for which the expected first-order components are zero. The third-order components of the variance and covariance are then $\hat{var}(x) = 2Ex(1)x(2)$ and $\hat{cov}(x, y) = Ex(1)y(2) + Ex(2)y(1)$.¹⁶ Third-order components of variances and covariances involving the “other variables” of the model therefore depend on the first and second-order solution for these variables. They take the form $\sigma^2 \kappa S_t(1)$, where κ is a 1 by 3 vector.

First-order solution of portfolio share difference

The second-order solution of the “other variables” described above is conditional on $k_t^D(1)$, which is solved by taking the third-order component of the difference across countries of the portfolio Euler equations (15). Following steps outlined

¹⁶For example, $var(x) = E(x^2) - (Ex)^2$. Substituting $x = x(0) + x(1) + x(2) + \dots$ and using $Ex(1) = 0$, the third-order component of $var(x)$ is $2Ex(1)x(2)$.

in Appendix C we get

$$k_t^D(1) = -k^D(0) \frac{\hat{var}(er_{t+1})}{var(er_{t+1}(1))} + \frac{\gamma - 1}{\gamma} \frac{\hat{cov}(p_{t+1} - p_{t+1}^*, er_{t+1})}{var(er_{t+1}(1))} + \quad (24)$$

$$\frac{\hat{cov}(f_{Ht+1} - f_{Ft+1}, er_{t+1}) + 0.5\psi' E_t [(f_{Ht+1}(1))^2 - (f_{Ft+1}(1))^2] er_{t+1}(1)}{\gamma var(er_{t+1}(1))/(1 - \psi')}$$

The denominator of each ratio in (24) is second-order, while the terms in the numerator are all third-order, so that the ratios are all first-order. The first term on the right-hand side of (24) shows that an increase in the variance of the excess return reduces the relevance of the financial friction for the portfolio decision, which translates into a smaller home bias. The interpretation of the second and thirds terms in (24) parallels that of the corresponding terms in (20). For example, if Home equity becomes a better hedge against real exchange risk following a shift in the state variables ($\hat{cov}(p_{t+1} - p_{t+1}^*, er_{t+1}) > 0$), the Home investor responds by shifting into Home equity. The last term shows that a similar shift occurs when Home equity becomes a better hedge against changes in future expected portfolio returns.

Fixed point problem

We solve for the vector k_s in (21) by solving the fixed point of a function that maps k_s into itself. Given a vector k_s we solve the second-order components of the “other” model variables. Together with the first-order components of the “other” model variables, this allows us to solve the third-order components of the moments \hat{var} and \hat{cov} in (24), in turn yielding a new vector k_s .

4.3 General applicability and accuracy

Our method combines first, second and third-order expansions of various equations. This degree of complexity, and our focus on a minimalists model, should not be interpreted as limiting the potential use of the method. Indeed, Devereux and Sutherland (2007), who independently and simultaneously developed a solution method that is essentially the same as the one described above, show that the method is easy to implement even for large scale DSGE models with portfolio choice. They show in the context of a quite general setup that one can obtain simple analytical solutions to the fixed point problems in both steps of the solution. Since the rest of the solution involves standard first and second-order solutions for

the “other variables”, the solution is no more involved than for models without portfolio choice, and the model can be solved using standard software packages for the first and second-order solutions.

Another potential issue relates to the accuracy of the solution method. It is important to note that the issue of accuracy is no different than for DSGE models without portfolio choice that are solved with first or second-order solution methods. It is certainly possible to write down models where local approximations can be far off. An example is a model with large idiosyncratic income shocks, such as associated with unemployment, which can lead to large deviations from the point of approximation. However, this is a general limitation of local solution methods and has little to do with the introduction of portfolio choice per se. The accuracy issue is not more pronounced for our method than for standard local approximation methods applied to models without portfolio choice.

5 Implications for International Capital Flows

We now turn to the linkages between portfolio choice and international capital flows. We start by defining the passive portfolio share, a useful concept for our analysis. It captures the direct impact of movements in asset prices on the composition of portfolios. For instance, an increase in the price of Home equity, relative to Foreign equity, raises the weight of Home equity in investors’ portfolios. This does not entail any action from investors. The first-order component of the passive portfolio share is given by:

$$k_t^p(1) = k(0)(1 - k(0))q_t^D(1) \tag{25}$$

where $q_t^D(1) = q_{H,t}(1) - q_{F,t}(1)$ is the first-order component of the difference between the Home and Foreign equity prices, and $k(0) = k_H^H(0) = k_F^F(0)$ is the zero-order component of the fraction of wealth that is invested domestically. The passive portfolio share is the same for Home and Foreign investors.

Turning to capital flows, we distinguish between capital outflows and inflows. The former correspond to flows that are initiated by Home investors, while the latter are initiated by Foreign investors. Using standard balance of payments accounting we derive the following expressions for the first-order components of

capital outflows and inflows:

$$outflows = (1 - k(0))s_t(1) - \frac{1 - \psi}{\psi} (\Delta k_{H,t}^H(1) - \Delta k_t^P(1)) \quad (26)$$

$$inflows = -(1 - k(0))s_t(1) + \frac{1 - \psi}{\psi} (\Delta k_{H,t}^F(1) - \Delta k_t^P(1)) \quad (27)$$

where s_t is the ratio of Home national saving to GDP, and Δx_t denotes $x_t - x_{t-1}$. A positive value of (26) indicates that Home investors send money to the Foreign country, while a positive value of (27) shows that Foreign investors send money to the Home country. Net capital flows are simply the difference between (26) and (27).

(26) and (27) show that two components drive capital flows, namely portfolio growth and portfolio reallocation.¹⁷ Portfolio growth captures the capital flows that occur when additional saving is invested according to the zero-order portfolio shares. As the zero-order fraction invested abroad is $1 - k(0)$ for both countries, a first-order movement in Home saving by $s_t(1)$ translates into first-order outflows of $(1 - k(0))s_t(1)$. As we abstract from investment in physical capital, Home and Foreign saving add up to zero, hence the portfolio growth components in (26) and (27) exactly mirror each other.

The portfolio reallocation component of capital flows is the second term in (26) and (27). It reflects the first-order component of portfolio shares. Two key points emerge. First, it is changes in portfolio shares that drive capital flows rather than their level. (26) shows that outflows increase when the share of Home equity in Home investors' portfolio decreases, $\Delta k_{H,t}^H(1) < 0$. Second, capital flows are driven by the gap between the optimal portfolio shares, $k_{H,t}^H(1)$ and $k_{H,t}^F(1)$, and the passive portfolio share. Intuitively, investors do not need to shift money across countries when the impact of asset prices on portfolios delivers the desired allocation. Consider for instance a shock that increases the optimal portfolio share of Home equity for Foreign investors ($\Delta k_{H,t}^F(1) > 0$) and boosts the passive share of Home equity ($\Delta k_t^P(1) > 0$). Even though investors want to shift their portfolios towards Home equity, this does not imply that they move funds to the Home country.

¹⁷The distinction between portfolio growth and portfolio reallocation has also been made by Kraay and Ventura (2000, 2003) and Guo and Jin (2007). There is a subtle but important difference though. These papers define portfolio growth as domestic saving times the ratio of the net foreign asset position to total wealth. This ratio is interpreted as a portfolio choice. Our definition allows for a more general equilibrium approach where decisions about the external equity liabilities of each country correspond to portfolio decisions by the other country.

If movements in equity prices raise the passive portfolio share beyond its desired level ($\Delta k_t^p(1) > \Delta k_{H,t}^F(1)$), investors will undo the excess passive portfolio shift by selling Home equity and reallocating funds to Foreign equity. This corresponds to a negative inflow in (27). These points are summarized in our first result:

Result 1 *Capital inflows and outflows can be broken into a portfolio growth and a portfolio reallocation component. The portfolio growth component captures capital flows that result when saving is invested in line with the zero-order portfolio shares. The portfolio reallocation component captures capital flows driven by first-order changes in portfolio shares, relative to the passive portfolio share.*

In order to understand the determinants of the portfolio reallocation component in (26)-(27), we need to derive expressions for the optimal portfolio shares. The first-order components of the portfolio shares for each country can be computed from the average and difference of portfolio shares across countries, $k_t^A(1)$ and $k_t^D(1)$.¹⁸ (24) shows that the first-order component of the difference in portfolio shares, $k_t^D(1)$, is entirely driven by the third-order component of second moments. These $\hat{v}ar$ and $\hat{c}ov$ terms reflect time-variation in second moments associated with changes in the state space. It is important to note that these terms arise endogenously even though the variance of shocks is kept constant.

Result 2 *Even when the standard deviation of model innovations is constant, the second moments affecting portfolio choice endogenously vary over time with changes in the state space.*

The expression (24) for $k_t^D(1)$ was computed using the third-order component of the difference in portfolio Euler equation across countries. We can similarly compute $k_t^A(1)$ from the third-order component of the average of portfolio Euler equations across countries. Following steps outlined in Appendix C we write:¹⁹

$$k_t^A(1) = \frac{E_t er_{t+1}(3)}{\gamma var(er_{t+1}(1))} + \frac{\hat{c}ov_t^A}{var_t(er_{t+1}(1))} \quad (28)$$

¹⁸Specifically: $k_{H,t}^H(1) = k_t^A(1) + 0.5k_t^D(1)$ and $k_{H,t}^F(1) = k_t^A(1) - 0.5k_t^D(1)$.

¹⁹ $k_t^A(1)$ is solved in the first step of the solution. That solution was a result of a supply perspective, using the fact that the average portfolio share is related to relative asset supplies through asset market clearing. Equation (28) follows from a demand (or portfolio choice) perspective. In equilibrium the expected excess return adjusts to reconcile these two perspectives.

where:

$$\begin{aligned} \hat{c}ov_t^A &= \frac{\gamma - 1}{2\gamma} [\hat{c}ov(p_{t+1} + p_{t+1}^*, er_{t+1}) - \hat{v}ar(r_{H,t+1}) + \hat{v}ar(r_{F,t+1})] \\ &\quad + \frac{1 - \psi'}{2\gamma} \hat{c}ov(f_{Ht+1} + f_{Ft+1}, er_{t+1}) \\ &\quad + \frac{\psi'(1 - \psi')}{4\gamma} E_t [(f_{Ht+1}(1))^2 + (f_{Ft+1}(1))^2] er_{t+1}(1) \end{aligned}$$

(28) shows that the first-order component of $k_t^A(1)$ depends on time-varying second moments, denoted by $\hat{c}ov_t^A$. In addition, $k_t^A(1)$ is affected by the time-varying expected excess return on Home equity, $E_t er_{t+1}(3)$. The latter does not enter $k_t^D(1)$ as Home and Foreign investors respond to expected returns in the same way.

Result 3 *Changes over time in optimal portfolio shares are associated with time variation in expected excess returns and second moments. Second moments that affect portfolio choice involve asset returns, goods prices and future expected portfolio returns. The third-order component of expected excess returns and second moments affects the first-order component of portfolio shares.*

It would be tempting to conclude from the results so far that capital flows are driven by portfolio growth, as well as time-varying expected returns and second moments that enter (24) and (28). This inference is not accurate however, as capital flows (26)-(27) are driven by the gap between the optimal and passive portfolio shares.

It is useful to derive the equilibrium expected excess return on Home equity to shed further light onto the determinants of capital flows. Using the first-order component of the asset market equilibrium (11)-(12), we write:

$$\Delta k_t^A(1) - \Delta k_t^p(1) = -\frac{1}{2} \frac{\psi}{1 - \psi} k^D(0) s_t(1) \quad (29)$$

Intuitively, an increase in Home saving (and therefore drop in Foreign saving) raises the demand for Home equity in the presence of portfolio home bias ($k^D(0) > 0$). Asset market clearing requires either an increase in the relative supply of Home equity through a higher asset price, which raises the passive portfolio share $\Delta k_t^p(1)$, or a decrease in the demand for Home equity through a shift in the world portfolio away from Home equity ($\Delta k_t^A(1) < 0$).

Combining (28) and (29) gives an expression for changes in expected excess return:

$$\Delta E_t er_{t+1}(3) = \Delta E_t er_{t+1}(3)^S + \Delta E_t er_{t+1}(3)^P + \Delta E_t er_{t+1}(3)^{TVM} \quad (30)$$

where:

$$\begin{aligned} \Delta E_t er_{t+1}(3)^S &= -\gamma \text{var}_t(er_{t+1}(1)) \frac{1}{2} \frac{\psi}{1-\psi} k^D(0) s_t(1) \\ \Delta E_t er_{t+1}(3)^P &= \gamma \text{var}_t(er_{t+1}(1)) \Delta k_t^p(1) \\ \Delta E_t er_{t+1}(3)^{TVM} &= -\Delta \hat{cov}_t^A \end{aligned}$$

(30) shows that there are three determinants of expected excess return changes. The first, $\Delta E_t er_{t+1}(3)^S$, reflects saving. Intuitively, higher saving in the Home country boost the relative demand for Home equity due to home bias ($k^D(0) > 0$). The expected excess return on Home equity needs to fall to switch asset demand toward Foreign equity and clear asset markets. The second determinant, $\Delta E_t er_{t+1}(3)^P$, is associated with changes in the passive portfolio share. An increase in the relative price of Home equity raises the relative supply of Home equity. Asset market clearing requires a shift of asset demand towards Home equity, which is achieved through an increase in the expected excess return on Home equity. The last determinant, $\Delta E_t er_{t+1}(3)^{TVM}$, reflects time-varying second moments. When these moments boost the world demand for Home equity ($\Delta \hat{cov}_t^A > 0$), asset market clearing requires an offsetting reduction in asset demand through a lower expected excess return on Home equity.

In addition, we can show from the first and second-order component of the average of portfolio Euler equations (15) that both the first and second-order components of the expected excess return are zero ($E_t er_{t+1}(1) = E_t er_{t+1}(2) = 0$). (30) therefore captures the total change in the expected excess return up to third-order accuracy. Our results for the expected excess return are summarized as:

Result 4 *Changes in expected excess returns are associated with (i) saving, (ii) changes in relative asset prices, and (iii) time-varying second moments that affect the average portfolio share. Changes in expected excess returns are small (third-order).*

We now turn to the link between the expected excess returns and capital flows. Since Home and Foreign investors have the same expectations, expected excess

returns affect capital flows only through the average portfolio share, not the difference in portfolio shares. Combining (28) and (30) the difference between the change in the average portfolio share and the passive portfolio share is

$$\Delta k_t^A(1) - \Delta k_t^P(1) = \frac{\Delta E_t er_{t+1}(3)^S}{\gamma var(er_{t+1}(1))} \quad (31)$$

Intuitively, an increase in Home saving boosts the relative demand for Home equity because of portfolio home bias. Asset market clearing requires an offsetting reduction in the demand through a lower expected excess return on Home equity. Investors across the world reallocate their portfolio towards Foreign equity, leading to a capital flow out of the Home country ($\Delta k_t^A(1) - \Delta k_t^P(1) < 0$).

(31) shows that the average portfolio reallocation is not driven by the overall change in the expected excess return, but only by the component associated with saving. By contrast, the components associated with asset price changes, $\Delta E_t er_{t+1}(3)^P$, and time-varying second moments, $\Delta E_t er_{t+1}(3)^{TVM}$, entail no capital flows.

Result 5 *There is no straightforward link between changes in the expected excess return and capital flows. Changes in the equilibrium expected excess return associated with changes in relative asset prices, or with time-varying second moments, do not generate capital flows. Only the changes associated with saving lead to portfolio reallocation that affects capital flows.*

We are now ready to present the drivers of capital flows. Using (31), capital flows (26)-(27) are written as:

$$outflows = (1 - k(0))s_t(1) - \frac{1 - \psi}{\psi} \frac{\Delta E_t er_{t+1}(3)^S}{\gamma var(er_{t+1}(1))} - \frac{1 - \psi}{\psi} \frac{\Delta k_t^D(1)}{2} \quad (32)$$

$$inflows = -(1 - k(0))s_t(1) + \frac{1 - \psi}{\psi} \frac{\Delta E_t er_{t+1}(3)^S}{\gamma var(er_{t+1}(1))} - \frac{1 - \psi}{\psi} \frac{\Delta k_t^D(1)}{2} \quad (33)$$

As described above, the portfolio growth component reflects the investment of saving based on the zero-order portfolio allocation. Movements in expected excess returns that are linked to saving affect capital inflows and outflows with opposite signs. Finally, time-varying second moments that affect the difference in portfolio shares, $k_t^D(1)$, affect capital inflows and outflows with the same sign.

Result 6 *Capital outflows and inflows are driven by three factors. The first is a portfolio growth component. The second is associated with time-varying expected excess return on Home equity, but only a component of it. The third is associated with time-varying second moments, but only to the extent that they affect Home and Foreign portfolio allocation differently.*

(32) and (33) also have implications for net capital flows and for the co-movement between capital inflows and outflows. The difference between (32) and (33) shows that net flows only reflect the first two components and are therefore not affected by $k_t^D(1)$. By contrast, the sum of gross capital flows only reflects the difference in portfolio shares $k_t^D(1)$. Changes in second moments that drive $k_t^D(1)$ lead to a positive co-movement between capital inflows and capital outflows. For instance, an increase in the portfolio share difference ($\Delta k_t^D(1) > 0$) leads to both negative outflows and inflows as all investors repatriate money towards their own country.²⁰

Result 7 *Capital inflows and outflows are positively correlated when there is sufficient time-variation in second moments that affect Home and Foreign portfolios differently. This is the only element in the model that leads to a positive co-movement between capital inflows and outflows.*

6 A numerical illustration

6.1 Parameterization

We illustrate the implications of our simple model through a numerical example. The parameterization we adopt is for illustrative purposes only, not to match the data of any particular country. Nonetheless, we discuss the empirical relevance of the model at a qualitative level at the end of this section.

We assume a labor share of output, θ , of 0.7. Productivity shocks are assumed to be highly persistent, with $\rho = 0.99$. Productivity innovations have a standard deviation of $\sigma = 5\%$. Turning to consumers' preferences, we assume home bias

²⁰The negative co-movement between inflows and outflows due to portfolio growth is an artifact of the absence of investment in the model. More generally Home and Foreign saving could move together when there are corresponding changes in global investment since world saving equals world investment.

in preferences by setting $\alpha = 0.8$. The elasticity of substitution between Home and Foreign goods is set at $\lambda = 2$. The rate of relative risk-aversion, γ , is set at 10 and $\beta = 1$. Agents face a probability of death of $\psi = 0.05$, leading to a consumption-wealth ratio of 5%. The transaction cost on investing abroad, τ , is set at 0.419%. These parameters generate a sizable home bias in equity holdings, with the zero-order component of the fraction invested in domestic equity equal to 0.8.²¹ We consider the dynamic response of the economy to a 5% increase in Home productivity.

6.2 Real exchange rate and equity prices

Chart 1 illustrates the dynamic response of the relative price of the Foreign good. The increase in Home productivity boosts the supply of the Home good, leading to an immediate increase in the relative price of the Foreign good (a Home real depreciation). This is followed by a gradual reduction (Home real appreciation) as the shock dissipates. Chart 2 shows the dynamic response of equity prices, depicting the Home equity price in units of the Home good and the Foreign equity price in units of the Foreign good. The persistent Home productivity shock immediately raises the Home equity price. It also leads to a small increase in the Foreign equity price as some of the additional wealth stemming from the higher productivity is invested in Foreign equity. While the increase in Foreign equity prices is larger when expressed in Home goods, Home equity prices still increase by more on impact. Equity prices subsequently move back to the steady state, which implies a larger expected drop in the Home equity price than in the Foreign equity price.

6.3 Capital flows

Chart 3 shows the response of gross and net capital flows from the perspective of the Home country, expressed as a fraction of initial GDP. The positive income shock in the Home country boosts income more than wealth and consumption,

²¹This implies that agents invest 30% more in the domestic country than under perfect diversification. Of this, there is a bias of +67% invested in the domestic country due to the financial friction τ , a negative bias of -40% due to a negative correlation between the real exchange rate and excess return (this is a foreign bias) and a positive home bias of +3% due to the hedge against changes in expected portfolio returns.

leading to a rise in saving and a net capital outflow. Initially both capital inflows and outflows go down, with investors from both countries selling assets abroad and bringing the money to their own country. By contrast, in subsequent periods investors from both countries shift funds to the Foreign country, so that the Home country experiences positive capital outflows and negative capital inflows.

Portfolio reallocation

We know from (26) and (27) that capital flows are driven by portfolio growth and portfolio reallocation. The latter is associated with changes in portfolio shares in deviation from the passive portfolio share. Chart 4 shows the impact of the shock on both the passive portfolio share invested in Home equity and the optimal portfolio shares of Home and Foreign investors invested in Home equity. The increase in Home equity prices automatically boosts the value of investors' holdings of Home equity and raises the passive portfolio share. The optimal portfolio share differs across investors, with Home investors choosing a larger share of Home equity in their portfolio than Foreign investors ($k_t^D(1) > 0$). The Home portfolio share exceeds the passive portfolio share, leading Home investors to actively reallocate their portfolio by selling Foreign equity to buy Home equity. By contrast, the Foreign portfolio share is lower than the passive portfolio share, so that Foreign investors actively reallocate their portfolio towards Foreign assets. This retrenchment towards domestic assets accounts for the negative capital inflows and outflows in Chart 3 in the immediate response to the shock. After the initial shock the gradual reduction in the price of Home equity leads to a gradual decrease in the passive portfolio share. As the reduction in the optimal portfolio shares of both Home and Foreign investors is more pronounced, they both actively reallocate their portfolio towards Foreign assets, accounting for the positive capital outflows and negative capital inflows in Chart 3.

Components of capital flows

Charts 5 and 6 show the three components of capital flows that are represented in (32) and (33). The portfolio growth component leads to positive capital outflows and negative capital inflows (thin line). The productivity increase in the Home country boosts Home saving. This leads to increased capital outflows as 20% of the increase in Home saving is invested in the Foreign country. The drop in Foreign saving leads to a drop in capital inflows.

The other two components reflect portfolio reallocation. The dotted lines represent changes in the expected excess return. As a result of portfolio Home bias, the rise in Home saving leads to an excess demand for Home equity. A drop in the expected excess return on Home equity is then needed to clear asset markets. This leads to a reallocation of portfolios towards Foreign equity, so that capital outflows are positive and capital inflows are negative.

The last component is associated with time-varying second moments and is represented by the thick lines. We have seen that changes in second moments only affect capital flows to the extent that they affect portfolios of Home and Foreign investors differently. This is reflected in the immediate increase in k_t^D at the time of the shock. An increase in the volatility of the excess return at the time of the shock leads to a reduction in home bias. This is however more than offset by an increase in the covariances between the excess return on the one hand and the real exchange rate and the hedging component on the other hand. This leads to increased home bias: $k_t^D(1) > 0$. From (32) and (33) this translates into negative capital inflows and outflows.

While time-varying second moments have a very large transitory impact on capital flows, portfolio growth and changes in expected excess returns have a much more persistent impact. The difference can be understood as follows. Persistence in the state variables, due to the highly persistent technology shocks, leads to long-lasting changes in the level of saving and second moments. The portfolio growth and expected excess return components of capital flows both depend on the *level* of saving and therefore inherit its persistence. By contrast, the third component is driven by *changes* in second moments, which have very little persistence. Time-varying second moments therefore have a large impact at the time of shock, when they change substantially, but not thereafter as they change only very gradually. This can be seen in Chart 4 where the home bias $k_t^D(1)$ remains stable after the initial increase.

Disconnect between capital flows and expected return changes

(30) and (31) show a disconnect between capital flows and changes in the expected excess return, as only one component of the latter affects capital flows. This is illustrated by Chart 7 which shows the equilibrium expected excess return as well as its three components. The overall expected excess return on Home equity rises in response to the shock, while the only component that affects capital

flows—associated with the level of saving—drops. It is the drop in the expected excess return due to higher Home saving that leads to increased capital outflows and lower capital inflows, as documented by the broken lines in Charts 5 and 6.

The other two components of the equilibrium expected excess return are both positive and much larger. The higher relative price of Home equity boosts the relative supply of Home equity. In order to clear financial markets, the demand for Home equity needs to be raised through higher expected returns. As the passive portfolio involves no asset trade, there are no capital flows associated with this. Finally, the average portfolio share k_t^A invested in Home equity drops due to time-varying second moments, cov_t^A . To clear asset markets this is offset by higher expected returns on Home equity, with no implication for capital flows.

Capital flows under a simulation of the model

Our analysis so far focuses on the impulse responses to a particular shock. We can draw implications for the co-movements and volatility of capital flows by simulating the model over 30 periods. The three components of capital flows in (32) and (33) are presented in Chart 8 and 9 for capital outflows and inflows, respectively. Both charts clearly show that time-varying second moments drive the volatility of capital flows, while expected excess returns and portfolio growth drive their persistence. In addition, the time-varying second moments lead to a positive correlation between outflows and inflows, while the other two components imply a negative correlation. Total capital outflows and inflows are presented in Chart 10. Under our parameterization, the dominant influence of time-varying second moments leads to a positive correlation (0.34) between capital inflows and outflows. A lower persistence of technology shocks would lead to an even higher correlation between capital inflows and outflows as the portfolio growth and expected excess return components of capital flows would be less persistent.

6.4 Valuation effects and return differentials

Recent contributions point to substantial valuation effects that capture the impact of changing asset prices and exchange rates on the value of external assets and liabilities. In addition, expected valuation effects can play a role on the financing

of external imbalances. We now turn to the role of these channels in our model.

Impact on the value of external assets and liabilities

As illustrated in Charts 11 and 12, unexpected valuation effects have large effects on the value of outstanding external assets and liabilities. Chart 11 shows the paths of gross external assets and liabilities of the Home country, as well as its net external asset position, while Chart 12 presents the net external position together with cumulative net capital flows. The initial response of both gross assets and liabilities is almost entirely due to unexpected valuation effects. Specifically, the increase in the price of Home equity boosts the value of liabilities to Foreign investors. In addition, the real depreciation of the Home currency raises the value of Home investors' holdings in the Foreign country. The first effect is larger and the Home country becomes a net debtor. After the initial shock gross liabilities drop much faster than gross assets and soon the Home country becomes a net creditor. Chart 12 shows that this is driven to a large extent by cumulative net capital outflows. In addition the Home country benefits from fully expected valuation gains, primarily reflecting the gradual fall in the price of the Home equity that reduces the value of external liabilities. The positive expected valuation effects are illustrated by the decreasing gap between cumulative capital outflows and the net external position in Chart 12.

External adjustment

Gourinchas and Rey (2007) show that expected valuation effects may play an important role in financing a positive external debt. An expected depreciation of the dollar boost the dollar return on the U.S. investors' foreign holdings, a channel that could contribute to financing the U.S. external debt. Using standard balance of payments accounting within the context of our model we write:

$$-nfa_t(1) = \sum_{s=1}^{\infty} \frac{E_t tb_{t+s}(1)}{R(0)^s} + GA(0) \sum_{s=1}^{\infty} \frac{E_t (r_{F,t+s}(1) - r_{H,t+s}(1))}{R(0)^{s-1}} \quad (34)$$

where nfa is the net foreign asset position, tb is the trade balance and $GA(0)$ is the zero-order component of gross assets. (34) shows that a net external debt can be financed by either expected future trade surpluses or by more favorable expected future returns on external assets (Foreign equity) than external liabilities (Home equity).

The expected return differential in (34) reflects three factors: valuation changes associated with the real exchange rate, valuation changes associated with equity prices, and dividend payments. A net external debt can then be repaid through four channels: (i) the present value of expected future trade surpluses, (ii) the present value of expected valuation gains associated with the real exchange rate, (iii) the present value of expected valuation gains associated with equity prices and (iv) the present value of expected differences between Home and Foreign dividend yields.

Under our parametrization, a 5% increase in Home productivity leads to a net external debt of 6.2% of GDP on impact. Chart 13 shows how this debt is financed by the four factors listed above. Expected valuation effects are large: the expected real appreciation of the Home currency generates valuation losses that amount to 5.1% of GDP in present value terms. On the other hand, the expected fall in Home equity prices, relative to Foreign equity prices, translates into an expected gain for the Home country equal to 7.2% of GDP in present value terms. Overall valuation effects therefore amount to a net gain of 2.1%, which represents a third of the net external debt.

While valuation effects are important, they do not translate into a predictable return differential as they are exactly offset by differences in expected dividend yields. As Home productivity is persistently higher, the expected dividend yield is larger for Home than Foreign equity. The resulting net dividend payment to Foreign investors amount to 2.1% of GDP in present value terms. The three factors that enter expected returns then cancel out and the net external debt is entirely repaid through future trade surpluses.

This result is driven by the fact that the expected first-order component of the excess return is zero, a general implication of arbitrage in portfolio choice. Taking the first-order component of (15) shows that the expected first-order component of expected returns must be the same for all assets. This condition can only be relaxed by introducing elements that break the arbitrage across various assets.²²

²²One example is Bacchetta and van Wincoop (2007), who introduce a portfolio decision making cost (or asset management cost), leading to infrequent portfolio decisions.

6.5 Relation to the Data

Our numerical illustration is meant to merely outline the main implications of the model. While we do not undertake a formal empirical test of the model, many of its qualitative features are consistent with a broad set of empirical evidence.

In terms of capital flows data, both Kraay and Ventura (2003) and Guo and Jin (2007) find that in the short-run portfolio reallocation is a far more important source of capital flow fluctuations than portfolio growth, in line with our model. Also relevant is the evidence reported in Hau and Rey (2007). They use data on stock holdings of thousands of global mutual funds and find that funds actively reallocate towards Foreign stock following an increase in the relative return on Home stock relative to Foreign stock. This is consistent with the active reallocation towards Foreign stock in our model by both Home and Foreign investors after the initial period of the shock.²³ Hau and Rey (2007) refer to this aspect as portfolio rebalancing, an aspect that is exactly the same as our definition of portfolio reallocation. In addition the model can account for a positive correlation between capital inflows and outflows. For the United States the unconditional correlation between capital inflows and outflows (share of GDP) is 0.83 for annual data from 1970 through 2006 and 0.64 for quarterly data.²⁴

The model illustrates the links between time-varying second moments and capital flows and between time-varying expected returns and capital flows. To our knowledge, no empirical work has yet been done on the link between time-varying second moments and capital flows. This will be an important area of future research given the central role of time-varying risk in the model. The finance literature has extensively documented the time-varying nature of both variances and covariances of asset returns, with changes in second moments being quite persistent. This aspect is captured by a variety of statistical models, including ARCH or GARCH models and stochastic-volatility models.²⁵ In our model, the time-varying nature of second moments is entirely endogenous and follows naturally from the solution method. Also consistent with the data, changes in second-moments are highly

²³Even though portfolio reallocation differs significantly between Home and Foreign investors during the period of the shock, the model implies that even then Home and Foreign investors on average reallocate towards Foreign stock.

²⁴For changes in inflows and outflows these correlations are respectively 0.80 and 0.84.

²⁵For a brief overview of this large literature, see Campbell, Lo and MacKinlay (1997), chapter 12.

persistent as they inherit the persistence of the state variables.

In a recent paper, Didier and Lowenkron (2007) set out to illustrate the link between time-varying expected returns and net capital flows. Using data for the United States and Japan, they estimate a VAR involving US and Japanese asset returns and other financial variables. The resulting time-varying expected excess returns are applied to a simple portfolio choice model to develop the implications for net capital flows. While the implied theoretical net capital flows are positively correlated with actual net capital flows, they are much more volatile (by a factor 300) than net capital flows in the data. This suggests that either actual expected excess returns are far smaller or that components of expected excess returns do not affect capital flows. This is consistent with the theory, which tells us that equilibrium expected excess returns that drive net capital flows are third-order and that a large component of expected excess returns do not impact capital flows. The recent evidence by Curcuru, Dvorak and Warnock (2008) is also consistent with small expected excess returns. While they do not consider the time-varying nature of expected excess returns, they find that the average difference between the return on U.S. assets and liabilities is negligible within asset classes (bonds or stocks).²⁶

The model implies that unexpected valuation effects have a large impact on the value of external assets and liabilities. In the short-run these can significantly outweigh the impact of net capital flows on the net foreign asset position. This is consistent with recent evidence on the importance of valuation effects, such as Gourinchas and Rey (2007), Lane and Milesi-Ferretti (2005) and Tille (2008). Valuation effects have received significant attention in recent years as they interact with rapidly increasing levels of external assets and liabilities.²⁷ The model also implies that expected valuation effects can be very large. However, the model implies that predictable differences in expected returns cannot contribute to financing a net external debt to the first-order, in contrast to the empirical findings by Gourinchas and Rey (2007). We cannot rule out that the third-order component of the expected excess return plays this role, but these third-order components are very small.

²⁶Based on data from 1986 to 2005 they find that the average returns on US assets and liabilities are respectively 13.1% and 13.8% for stock and 9.3% and 8.7% for bonds.

²⁷For industrialized countries the sum of external assets and liabilities as a fraction of GDP has increased from less than 50% in the 1970s to about 350% today.

7 Conclusion

We develop a simple two-country DSGE model with portfolio choice in order to shed light on the implications of portfolio choice for international capital flows. We show that capital flows are driven by three factors: portfolio growth, portfolio reallocation associated with time-varying expected returns, and portfolio reallocation associated with time-varying second moments, with the last two playing a sizable role. We also find that several important factors that determine equilibrium expected return differences have no effect on capital flows. We stress the relevance of endogenous variations in second moments, even though the standard deviation of model innovations is constant. These changing second moments affect capital flows only to the extent that they affect domestic and foreign investors differently, and lead to positive co-movements between capital inflows and outflows.

An additional contribution of the paper is to develop a widely applicable method for solving DSGE models with portfolio choice. The solution method stays very close to standard first and second-order solution methods. While our focus here has been on international capital flows, the method can be used to address many other types of questions where portfolio choice may be relevant. Open economy DSGE models no longer need to be restricted to a single traded bond or complete markets.

While the model is qualitatively consistent with a broad range of empirical evidence, an important agenda for future research is to more formally connect general equilibrium portfolio choice models to the empirical evidence on capital flows and international asset holdings. The main goal of this research should be to match some key aspects of the data, such as the level of external (and bilateral) asset holdings, the volatility of gross and net capital flows, the co-movement between capital inflows and outflows and the relationship between capital flows and asset prices. The large empirical finance literature on time-varying second moments can potentially shed light on the link between capital flows and changing second moments. Another relevant line of research focuses on the empirical evidence for the different components of capital flows, such as different asset categories and portfolio growth versus portfolio reallocation. In parallel to the international trade literature, gravity models for bilateral asset positions can be developed that inform us of the magnitude and determinants of international financial frictions.

The model discussed in this paper was kept simple in order to focus on the

role of portfolio choice while abstracting from other choice variables that are not key to the analysis here. Future work will need to introduce consumption and investment decisions as well. Other natural extensions are to introduce monetary elements, fiscal policy, additional assets and multiple countries. A potentially rewarding strategy may also be to introduce information asymmetries as in the noisy rational expectations literature in finance. All these extensions will put us in a better position to confront the model to data on gross and net capital flows and external asset holdings, analyze policy questions related to capital flows, and make meaningful predictions related to the external adjustment process faced by countries with large external imbalances.

Appendix

A Equations of the model

The model can be summarized by 11 equations. Denoting the log of variables by lower case letters (except for portfolio shares), we write these equations as:

$$a_{H,t+1} = \rho a_{H,t} + \epsilon_{H,t+1} \quad (35)$$

$$a_{F,t+1} = \rho a_{F,t} + \epsilon_{F,t+1} \quad (36)$$

$$e^{w_{t+1}+p_{t+1}} = (1 - \psi) [k_{H,t}^H e^{r_{H,t+1}} + (1 - k_{H,t}^H) e^{r_{F,t+1}}] e^{w_t+p_t} + \theta e^{a_{H,t+1}} \quad (37)$$

$$e^{w_{t+1}^*+p_{t+1}^*} = (1 - \psi) [k_{H,t}^F e^{r_{H,t+1}} + (1 - k_{H,t}^F) e^{r_{F,t+1}}] e^{w_t^*+p_t^*} + \theta e^{p_{F,t+1}+a_{F,t+1}} \quad (38)$$

$$e^{a_{H,t}} = \alpha \psi e^{w_t+\lambda p_t} + (1 - \alpha) \psi e^{w_t^*+\lambda p_t^*} \quad (39)$$

$$e^{q_{H,t}} = (1 - \psi) [k_{H,t}^H e^{w_t+p_t} + k_{H,t}^F e^{w_t^*+p_t^*}] \quad (40)$$

$$e^{q_{F,t}} = (1 - \psi) [k_{F,t}^H e^{w_t+p_t} + k_{F,t}^F e^{w_t^*+p_t^*}] \quad (41)$$

$$E_t \left((1 - \psi) e^{v+f_H(S_{t+1})} + \psi \right) e^{-\gamma r_{t+1}^{p,H}} (e^{r_{H,t+1}} - e^{r_{F,t+1}-\tau}) e^{p_t-p_{t+1}} = 0 \quad (42)$$

$$E_t \left((1 - \psi) e^{v+f_F(S_{t+1})} + \psi \right) E^{-\gamma r_{t+1}^{p,F}} (e^{r_{H,t+1}-\tau} - e^{r_{F,t+1}}) e^{p_t^*-p_{t+1}^*} = 0 \quad (43)$$

$$e^{v+f_H(S_t)} = \beta E_t \left((1 - \psi) e^{v+f_H(S_{t+1})} + \psi \right) e^{(1-\gamma)r_{t+1}^{p,H}} \quad (44)$$

$$e^{v+f_F(S_t)} = \beta E_t \left((1 - \psi) e^{v+f_F(S_{t+1})} + \psi \right) e^{(1-\gamma)r_{t+1}^{p,F}} \quad (45)$$

(35) and (36) are the autoregressive processes for productivity. (37)-(38) are the wealth dynamics in the Home and Foreign countries. (39) is the Home goods market clearing condition (we can omit the Foreign goods market clearing condition due to Walras's law). (40)-(41) are the market clearing conditions for Home and Foreign equities. (42)-(43) are the optimal portfolio conditions for Home and Foreign investors. Finally, (44)-(45) are the Bellman equations for Home and Foreign investors.

These equations depend on consumer price indices, as well as asset and portfolio

returns, which in logarithmic form can be written as

$$e^{(1-\lambda)p_t} = \alpha + (1 - \alpha) e^{(1-\lambda)p_{Ft}} \quad (46)$$

$$e^{(1-\lambda)p_t^*} = (1 - \alpha) + \alpha e^{(1-\lambda)p_{Ft}} \quad (47)$$

$$e^{r_{H,t+1}} = e^{q_{H,t+1} - q_{H,t}} + (1 - \theta) e^{a_{H,t+1} - q_{H,t}} \quad (48)$$

$$e^{r_{F,t+1}} = e^{q_{F,t+1} - q_{F,t}} + (1 - \theta) e^{p_{F,t+1} + a_{F,t+1} - q_{F,t}} \quad (49)$$

$$e^{r_{t+1}^{p,H}} = [k_{H,t}^H e^{r_{H,t+1}} + (1 - k_{H,t}^H) e^{r_{F,t+1} - \tau}] e^{p_t - p_{t+1}} \quad (50)$$

$$e^{r_{t+1}^{p,F}} = [k_{H,t}^F e^{r_{H,t+1} - \tau} + (1 - k_{H,t}^F) e^{r_{F,t+1}}] e^{p_t^* - p_{t+1}^*} \quad (51)$$

(46)-(47) define the consumer prices indexes. (48)-(49) define the rates of return on Home and Foreign equity. Finally, (50)-(51) define the rates of return on the portfolios of Home and Foreign investors.

B Expansions of the Bellman equation

The state variables of the model are given by (18). The zero-order components of the logs of productivity are zero. Similarly, wealth is identical in both countries to a zero-order, implying that the zero-order component of the cross-country difference in (logs) of wealth is zero. This implies $S(0) = 0$. The elements of the Bellmann equation for the Home investor (44) are solved by taking a second-order expansion around $S(0)$. The resulting expression contains both first- and second-order components. The first-order components are:

$$v(1) + H_{1,H} S_t(1) = (1 - \psi') [v(1) + H_{1,H} E_t S_{t+1}(1)] + E_t (1 - \gamma) r_{t+1}^{p,H}(1) \quad (52)$$

where $v(1)$ is the first-order component of v , $H_{1,H}$ is a 1x3 vector with the first derivative of $f_H(S)$, evaluated at $S(0)$, and $\psi' = 1 - \beta(1 - \psi)R(0)^{1-\gamma}$. (52) is solved by $v(1) = 0$ and:

$$H_{1,H} = (1 - \gamma) r_s (I_3 - (1 - \psi') N_1)^{-1}$$

where r_s is a 1x3 matrix taken from the first-order solution of the portfolio return for the Home investor. From the first-order solution of control variables (19), we can write $r_{t+1}^{p,H}(1) = r_s S_{t+1}(1)$. I_3 is a 3x3 identity matrix and N_1 is the 3x3 matrix from (19).

The second-order components of (44) are:

$$\begin{aligned}
H_{1,H}S_t(2) + \frac{1}{2} \left[[H_{1,H}S_t(1)]^2 + 2v(2) + S_t(1)' H_{2,H}S_t(1) \right] = & \quad (53) \\
(1 - \psi') H_{1,H}E_t S_{t+1}(2) + (1 - \gamma) E_t r_{t+1}^{p,H}(2) + \frac{\psi'}{2} E_t \left[(1 - \gamma) r_{t+1}^{p,H}(1) \right]^2 + \\
\frac{1 - \psi'}{2} E_t \left[\left[H_{1,H}S_{t+1}(1) + (1 - \gamma) r_{t+1}^{p,H}(1) \right]^2 + 2v(2) + S_{t+1}(1)' H_{2,H}S_{t+1}(1) \right]
\end{aligned}$$

where $v(2)$ is the second-order component of v and $H_{2,H}$ is a 3x3 matrix with the second derivative of $f_H(S)$, evaluated at $S = 0$.

(53) entails cross-products of the first-order components of the state variables, $S_{t+1}(1)$, and the portfolio return, $r_{t+1}^{p,H}(1)$. These terms are taken from the first-order solution (19). (53) also includes the second-order components of the state variables, $S_{t+1}(2)$, which are taken from (23), as well as the second-order component of the expected portfolio return, $E_t r_{t+1}^{p,H}(2)$, which takes a form similar to (22):

$$E_t r_{t+1}^{p,H}(2) = r_s S_t(2) + S_t(1)' r_{ss} S_t(1) + \hat{r} \sigma^2 \quad (54)$$

where r_{ss} is a 3x3 matrix and \hat{r} is a scalar.

We use (53), along with the solution for $S_{t+1}(1)$, $S_{t+1}(2)$, $r_{t+1}^{p,H}(1)$ and $E_t r_{t+1}^{p,H}(2)$ to solve for $H_{2,H}$.²⁸ The 9x1 vector $H_{2,H}^{vec}$ is the "vectorized" form of the 3x3 matrix $H_{2,H}$. Specifically, the first three elements of $H_{2,H}^{vec}$ are the first row of $H_{2,H}$, the next three elements are the second row of $H_{2,H}$ and the last three elements are the third row of $H_{2,H}$. $H_{2,H}^{vec}$ is solved from (53) as:

$$H_{2,H}^{vec} = (I_9 - (1 - \psi') \hat{N})^{-1} H_3^{vec}$$

where I_9 is a 9x9 identity matrix. \hat{N} is a 9x9 matrix that consists of cross-products of various elements of the N_1 matrix from (19). The 9x1 vector H_3^{vec} is the "vectorized" form of a 3x3 matrix H_3 . The matrix H_3 includes cross-products of the matrices $H_{1,H}$ and r_s , as well as the matrix r_{ss} in the second-order component of the expected portfolio return (54), specifically:

$$\begin{aligned}
H_3 &= -H_{1,H}' H_{1,H} + 2F_1 + 2(1 - \gamma) r_{ss} + (1 - \psi') N_1' H_{1,H}' H_{1,H} N_1 \\
&\quad + 2(1 - \psi') (1 - \gamma) N_1' H_{1,H}' r_s + (1 - \gamma)^2 r_s' r_s \\
F_1 &= (1 - \psi') \sum_{v=1}^3 H_{1,H}(v) N_{3,v}
\end{aligned}$$

²⁸We also solve for $v(2)$, but this element does not affect portfolio choice.

where $H_{1,H}(v)$ is the v 'th element of the 1×3 vector $H_{1,H}$ and the 3×3 matrices $N_{3,v}$ are the same as in (23).

The corresponding matrices for the Foreign investor, $H_{1,F}$ and $H_{2,F}$, are computed analogously.

C First-order difference in portfolio shares

The solution of the first-order component of the portfolio share difference k_t^D relies on the third-order components of the optimal portfolio conditions (42)-(43). The expansion of the condition for the Home investor (42) leads to:

$$\begin{aligned} & E_t er_{t+1}(3) + E_t er_{t+1}(1) r_{t+1}^A(2) + E_t er_{t+1}(2) r_{t+1}^A(1) \\ & + E_t er_{t+1}(1) \left[(1 - \psi') f_{Ht+1}(2) - \gamma r_{t+1}^{p,H}(2) + p_t(2) - p_{t+1}(2) \right] \\ & + E_t er_{t+1}(2) \left[(1 - \psi') f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1) \right] \\ & + \tau E_t \left[r_{t+1}^A(1) + (1 - \psi') f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1) \right] + O_3 = 0 \end{aligned} \quad (55)$$

where $er_{t+1}(i) = r_{H,t+1}(i) - r_{F,t+1}(i)$, $r_{t+1}^A(i) = 0.5[r_{H,t+1}(i) + r_{F,t+1}(i)]$, and τ is second-order. The first term in (55) is the third-order component of the expected excess return. The next two terms are the third-order components of the cross-product between excess returns and the average return, and consists of products of first- and second-order terms. Similarly, the fourth and fifth terms are the third-order components of the cross-product between excess returns and the pricing kernel. The sixth term reflects the friction in investing abroad, τ . The last term in (55) consists of cubic-products of first-order elements:

$$\begin{aligned} O_3 &= \frac{1}{6} E_t \left[(r_{H,t+1}(1))^3 - (r_{F,t+1}(1))^3 \right] \\ &+ E_t \left[(1 - \psi') f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1) \right] r_{t+1}^A(1) er_{t+1}(1) \\ &+ \frac{1}{2} (1 - \psi') E_t \left[f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1) \right]^2 er_{t+1}(1) \\ &+ \frac{1}{2} \psi' E_t \left[-\gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1) \right]^2 er_{t+1}(1) \end{aligned}$$

The various components of O_3 are solved using the first-order solution (19). We can show that the resulting expression is:

$$O_3 = 2r_{DE}B_H A_H \sigma^2 S_t + \frac{\psi'(1 - \psi')}{2} E_t [f_{Ht+1}(1)]^2 er_{t+1}(1)$$

where A_H is a 1x3 vector and r_{DE} and B_H are scalars. r_{DE} reflects the sensitivity of the first-order excess return to innovations:

$$er_{t+1}(1) = r_{DE}\epsilon_{t+1}^D$$

where $\epsilon_{t+1}^D = \epsilon_{H,t+1} - \epsilon_{F,t+1}$. A_H and B_H reflect the first-order solution of a combination of the average rate of return $r_{t+1}^A(1)$ and the pricing kernel:

$$r_{t+1}^A + (1 - \psi')f_{Ht+1}(1) - \gamma r_{t+1}^{p,H}(1) + p_t(1) - p_{t+1}(1) = A_H S_t + B_H \epsilon_{t+1}^D + C_H \epsilon_{t+1}^A$$

where C_H is a scalar and $\epsilon_{t+1}^A = 0.5(\epsilon_{H,t+1} + \epsilon_{F,t+1})$.

We undertake similar steps using the condition for the Foreign investor (43). Taking the difference between (55) and its equivalent for the Foreign investor, we write:

$$\begin{aligned} & \frac{\psi'(1-\psi')}{2} E_t [[f_{Ht+1}(1)]^2 - [f_{Ft+1}(1)]^2] er_{t+1}(1) \\ & + E_t er_{t+1}(1) \left[\begin{aligned} & (1-\psi')[f_{Ht+1}(2) - f_{Ft+1}(2)] - \gamma(r_{t+1}^{p,H}(2) - r_{t+1}^{p,F}(2)) \\ & + (p_t(2) - p_t^*(2)) - (p_{t+1}(2) - p_{t+1}^*(2)) \end{aligned} \right] \\ & + E_t er_{t+1}(2) \left[\begin{aligned} & (1-\psi')[f_{Ht+1}(1) - f_{Ft+1}(1)] - \gamma(r_{t+1}^{p,H}(1) - r_{t+1}^{p,F}(1)) \\ & + (p_t(1) - p_t^*(1)) - (p_{t+1}(1) - p_{t+1}^*(1)) \end{aligned} \right] = 0 \end{aligned} \quad (56)$$

The first-order component of the difference in portfolio shares, $k_t^D(1)$, enters (56) through the second-order components of the portfolio returns. Taking the second-order components of (50)-(51) leads to:

$$\begin{aligned} r_{t+1}^{p,H}(2) - r_{t+1}^{p,F}(2) &= k^D(0)er_{t+1}(2) + (p_t(2) - p_t^*(2)) \\ &- (p_{t+1}(2) - p_{t+1}^*(2)) + k_t^D(1)er_{t+1}(1) \end{aligned}$$

Similarly, taking the first-order component of (50)-(51) leads to:

$$r_{t+1}^{p,H}(1) - r_{t+1}^{p,F}(1) = k^D(0)er_{t+1}(1) + (p_t(1) - p_t^*(1)) - (p_{t+1}(1) - p_{t+1}^*(1))$$

Using this result, (56) becomes:

$$\begin{aligned} & \frac{\psi'(1-\psi')}{2} E_t [[f_{Ht+1}(1)]^2 - [f_{Ft+1}(1)]^2] er_{t+1}(1) \\ & + (1-\psi')c\hat{v}(f_{Ht+1} - f_{Ft+1}, er_{t+1}) - \gamma k^D(0)v\hat{a}r(er_{t+1}) \\ & + (\gamma-1)c\hat{v}(p_{t+1} - p_{t+1}^*, er_{t+1}) - \gamma k_t^D(1)var(er_{t+1}) = 0 \end{aligned} \quad (57)$$

where $\hat{c}ov(x_{t+1}, y_{t+1}) = E_t x_{t+1}(1) y_{t+1}(2) + E_t x_{t+1}(2) y_{t+1}(1)$ and $\hat{v}ar(x_{t+1}) = \hat{c}ov(x_{t+1}, x_{t+1})$ and $\hat{v}ar(er_{t+1}) = E_t [er_{t+1}(1)]^2$. (24) follows simply from (57).

The elements of (57) are computed by using the first-order solution (19), the second-order dynamics of the state variables, (23), and the second-order solution for the control variables, which are of the form of (22). For instance, the excess returns are:

$$er_{t+1}(1) = r'_\epsilon \epsilon_{t+1} \quad er_{t+1}(2) = S_t(1)' M \epsilon_{t+1}$$

where r'_ϵ is a 1x2 vector, $\epsilon_{t+1} = [\epsilon_{H,t+1}, \epsilon_{F,t+1}]'$ and M is a 3x2 matrix. Using these expression, we write:

$$\hat{v}ar(er_{t+1}) = 2E_t er_{t+1}(1) er_{t+1}(2) = 2\sigma^2 r'_\epsilon M' S_t(1) \quad (58)$$

(58) shows that the third-order components of the variances and covariances in (57) reflect the second-order variance of the innovations, σ^2 , along with the first-order state variables, $S_t(1)$. Solving for all the third-order components of the variances and covariances in (57) along similar lines we compute the first-order difference in portfolio shares as a function of the first-order components of state variables:

$$k_t^D(1) = k_s S_t(1)$$

where k_s is a 1x3 vector.

Taking the average of (55) and its equivalent for the Foreign investor, we write:

$$\begin{aligned} 0 &= E_t er_{t+1}(3) + \frac{1}{4} \psi'(1 - \psi') E_t [f_{H,t+1}(1)^2 + f_{F,t+1}(1)^2] er_{t+1}(1) \\ &+ \hat{c}ov_t \left[\begin{array}{c} er_{t+1}, r_{t+1}^A + (1 - \psi') \frac{1}{2} [f_{H,t+1} + f_{F,t+1}] \\ -\gamma \frac{1}{2} (r_{t+1}^{p,H} + r_{t+1}^{p,F}) + \frac{1}{2} (p_t - p_t^*) - \frac{1}{2} (p_{t+1} + p_{t+1}^*) \end{array} \right] \end{aligned}$$

Using the first and second-order components of (50)-(51) this becomes:

$$\begin{aligned} 0 &= E_t er_{t+1}(3) + \frac{1}{4} \psi'(1 - \psi') E_t [f_{H,t+1}(1)^2 + f_{F,t+1}(1)^2] er_{t+1}(1) \\ &+ \frac{\gamma - 1}{2} \hat{c}ov_t(p_{t+1} + p_{t+1}^*, er_{t+1},) \\ &- \frac{\gamma - 1}{2} E_t [2r_{H,t+1}(1)r_{H,t+1}(2) - 2r_{F,t+1}(1)r_{F,t+1}(2)] \\ &+ \frac{1}{2} (1 - \psi') \hat{c}ov_t(f_{H,t+1} + f_{F,t+1}, er_{t+1}) - \gamma k_t^A(1) E_t (er_{t+1}(1))^2 \end{aligned}$$

where we used the fact that $E_t (er_{t+1}(1))^3 = 0$. (28) follows from a simple rearrangement of terms.

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