

A New Perspective on “The New Rule” of the Current Account

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Abstract

In an influential series of contributions, Kraay and Ventura (2000, 2003) offer a “new rule” for the current account: in response to a temporary income shock, the change in the current account is equal to the change in saving times the ratio of net foreign assets to wealth. We analyze the impact of a temporary income shock on the current account in the context of a two-country dynamic general equilibrium model of portfolio choice and show that the new rule does not hold. We also show that the cross-section evidence reported by Kraay and Ventura in favor of the new rule is a feature implied by the steady state of the model that is conceptually distinct from the new rule. We argue that the new rule could only hold in a model with one-way capital flows (only inflows or outflows, but not both), a feature that is strongly counterfactual.

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1 Introduction¹

How countries allocate saving between domestic and foreign assets is a central issue in international macroeconomics, especially in light of the surge of international financial integration since the mid-1990's. In a series of influential articles, Aart Kraay and Jaume Ventura (2000, 2003, 2008, hereinafter KV) develop a “new rule” for the dynamic response of the current account to temporary shocks. It states that following a transitory income shock “the current account response equals the saving generated by the shock times the country’s share of (net) foreign assets in total assets.”² KV provide a theoretical explanation for this pattern that emphasizes the need to view international capital flows from a portfolio choice perspective. Their contributions were several years ahead of the recent renewed interest in portfolio choice models in open economy macroeconomics and in analyzing international capital flows from a portfolio choice perspective.

This paper evaluates the validity of the new rule by analyzing the impact of a temporary income shock on the current account in the context of a two-country general equilibrium model with portfolio choice. We find that the new rule does not hold. The intuition is straightforward. Consider an economy with a zero net foreign asset position, but positive gross external assets and liabilities in line with empirical evidence. The new rule predicts that the current account does not change in response to a temporary income shock that raises saving. Our model implies a current account surplus through two channels. First, some of the increase in saving is invested abroad in line with the country’s initial portfolio allocation between domestic and foreign assets. Second, as most of the increase in saving is invested domestically due to portfolio home bias, the marginal product of capital

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²We have inserted “net” in brackets as the term “foreign assets in total assets” in Kraay and Ventura always refers to the ratio of the net foreign asset position (assets minus liabilities) to total financial wealth.

in the home country falls relative to the foreign country. This leads to a portfolio shift towards foreign assets and therefore a further rise in net capital outflows from the home country.

KV emphasize that the new rule would hold when there are very limited decreasing returns to scale or even constant returns to scale. This would remove the second channel outlined above as there would not be any feedback from relative investment to relative asset returns and therefore portfolio allocation. Temporary income shocks then do not affect portfolio shares. But the conclusion by KV that the new rule then holds hinges on the assumption that only home investors invest abroad. In that case the ratio of net foreign assets to wealth corresponds to the portfolio share of foreign assets from the perspective of the domestic country. This correspondence is not valid when investors in both countries hold international assets as is clearly the case in the data. The ratio of net foreign assets to wealth is then no longer a portfolio share of the domestic country as it also depends on the portfolio allocation of the foreign country as well as the relative wealth of the two countries. We show that the new rule could only hold in a model with one-way capital flows, i.e. where the domestic country can invest in foreign assets but not the other way around. This is clearly counterfactual.

The evidence in favor of the new rule presented by KV relies on a panel regression of the current account on a term equal to the share of net foreign assets in total wealth times saving, which gives a coefficient of 1. As they acknowledge, this is driven entirely by the cross-section dimension of the data. An analogous cross-section regression also leads to a coefficient of 1 and an R^2 of about 0.85. We show that such cross-section evidence is a feature implied by the steady state of the model. It is fundamentally distinct from the new rule, which is about the dynamic response to temporary income shocks.

While the main theme of the paper is that the new rule does not hold, this does not call into question the importance of the contributions by KV in terms of highlighting an alternative view of the current account based on a portfolio theory as opposed to the traditional intertemporal theory of the current account. In the intertemporal theory, the current account is determined by intertemporal consumption decisions (which drives saving) and an equality between the marginal product of capital across countries (which drives investment). By contrast, in the portfolio theory of the current account capital flows are driven by portfolio allocation with investment simply accommodating the portfolio demand for assets.

The portfolio theory is correct when there is little feedback from changes in relative investment, and therefore relative capital stocks and returns, to portfolio choice. This would be particularly the case under constant returns to scale.

Which of the portfolio theory and intertemporal theory provides the best account of current account dynamics? We answer this question using an exact numerical solution of the general equilibrium model. The current account response to temporary income shocks is consistent with the intertemporal theory of the current account unless we are extremely close to constant returns to scale. At that point the portfolio theory is more relevant. However, even then the new rule does not hold because of two-way capital flows. When countries invest 20 percent of their wealth abroad in the steady state, a temporary increase in saving by 100 implies capital outflows of 20 under the portfolio theory regardless of the ratio of net foreign assets to wealth.

The remainder of the paper is organized as follows. In section 2 we describe a two-country general equilibrium portfolio choice model with temporary income shocks. Section 3 analyzes the impact of a temporary income shock on the current account. It explains why the new rule does not hold and analyzes the issue more broadly in the context of the intertemporal and portfolio theories of the current account. Section 4 explains why the cross-section evidence in KV is simply a steady state feature of the model that is unrelated to the new rule. Section 5 addresses whether any other type of model exists where the new rule does hold, and section 6 concludes.

2 A Two-Country General Equilibrium Model

We now consider a simple two-country general equilibrium model with endogenous international portfolio choice. The main goal of the model is to evaluate the relevance of the new rule by analyzing the impact of temporary income shocks on net capital flows. The goal of the model is not to generate current account dynamics that matches various aspects of the data. In reality there are many other types of shocks that we will abstract from in order to focus on the impact of temporary income shocks. The message would be the same for any other temporary shock that changes saving without at the same time changing the expected marginal product of capital. For example, the impact of an exogenous shock to saving (e.g. through

a change in the time discount rate) on the current account will be analogous to the impact of a temporary income shock that changes saving without affecting the expected marginal product of capital.

Many shocks, such as persistent productivity shocks, simultaneously affect both saving and the expected marginal product of future capital (which affects investment). Such shocks do not allow us to analyze how a change in saving itself affects investment (and therefore the current account) through various channels. We therefore abstract from such shocks in order to focus on the new rule, which is about the impact of saving on investment (and therefore the current account).

We start by presenting the main blocks of the model and the solution method, leaving a more detailed exposition to the Appendix.³

2.1 Production and Investment

The world consists of two countries, Home and Foreign, denoted with superscripts H and F respectively. They produce the same good, with the production function in country i ($= H, F$) given by:

$$Y_t^i = A_t^i (K_t^i)^{1-\omega} (N_t^i)^\omega \quad (1)$$

where N_t^i is the labor input, K_t^i the capital stock, and A_t^i the stochastic productivity. N_t^i grows at a constant rate g in both countries. We denote the share of the Home labor force to the world one by n , implying that $N_t^H = n(1+g)^t$ and $N_t^F = (1-n)(1+g)^t$. Lower case letters denote variables scaled by the labor force. The capital-labor ratio for instance is $k_t^i = K_t^i/N_t^i$.

Productivity in both countries follows a simple i.i.d. process:

$$A_t^i = 1 + \varepsilon_t^i \quad (2)$$

where ε_t^i has a zero expected value and variance of σ_a^2 . The shocks are uncorrelated across the two countries. Our assumption that productivity shocks are purely transitory facilitates the analysis by abstracting from fluctuations in capital accumulation that reflect predictable productivity movements.

Wages are equal to the marginal product of labor:

$$W_t^i = \omega A_t^i (k_t^i)^{1-\omega} \quad (3)$$

³A more thorough description of the solution is given in a Technical Appendix available on request.

The dynamics of the capital stock reflects investment and depreciation:⁴

$$K_{t+1}^i = (1 - \delta) K_t^i + I_t^i$$

2.2 Two Assets

Investors worldwide trade claims on the capital stock of both countries. The gross return on country i capital consists of the dividend yield and the value of residual capital:

$$R_{t+1}^i = 1 - \delta + (1 - \omega) A_{t+1}^i (k_{t+1}^i)^{-\omega} \quad (4)$$

Portfolio home bias is a central aspect in the analysis. We generate it by assuming that investors receive only a fraction $1 - \tau$ of the return abroad, where τ is a second-order constant (i.e. proportional to the variance of model innovations) that captures the hurdles of investing abroad. This iceberg cost does not generate a loss in resources, and is instead a fee paid to a broker in the investor's country, who consumes it in the same period (Tille and van Wincoop 2008). In period t investors in country i invest a fraction z_t^i of their wealth in Home capital. The portfolio returns from t to $t + 1$ of investors in the two countries are then

$$R_{t+1}^{p,H} = z_t^H R_{t+1}^H + (1 - z_t^H)(1 - \tau) R_{t+1}^F \quad (5)$$

$$R_{t+1}^{p,F} = z_t^F (1 - \tau) R_{t+1}^H + (1 - z_t^F) R_{t+1}^F \quad (6)$$

2.3 Consumption and Portfolio Choice

We minimize the complexity of the model by adopting a simple overlapping generation structure, with agents living for two periods. This ensures a well-defined steady state wealth distribution and simplifies the portfolio choice problem by restricting the investment horizon to one period. Agents borne in period t supply one unit of labor and earn the wage W_t . They consume some of their income when young and invest the rest to finance their consumption when old in period $t + 1$. A young agent in country i at time t maximizes an intertemporal CRRA utility of

⁴We abstract from adjustment costs in investment, so that consumption and capital goods are identical and the relative price of capital is unity. Kraay and Ventura (2003) rely on adjustment costs in order to explain why the new rule does not hold in time series data. By contrast, we show that the new rule does not hold even without adjustment costs.

consumption:

$$\frac{(C_{y,t}^i)^{1-\gamma}}{1-\gamma} + \beta^i E_t \frac{(C_{o,t+1}^i)^{1-\gamma}}{1-\gamma}$$

where β^i is the the time-discount rate in country i . We allow it to differ across the two countries to generate different saving rates in the steady state. Utility is maximized subject to the budget constraint:

$$C_{o,t+1}^i = (W_t^i - C_{y,t}^i) R_{t+1}^{p,i} \quad (7)$$

and the portfolio return (5) or (6). The first order conditions with respect to $C_{y,t}^i$ and z_t^i are

$$(C_{y,t}^i)^{-\gamma} = \beta^i E_t (C_{o,t+1}^i)^{-\gamma} R_{t+1}^{p,i} \quad i = H, F \quad (8)$$

$$E_t \left(R_{t+1}^{p,H} \right)^{-\gamma} (R_{t+1}^H - (1-\tau)R_{t+1}^F) = 0 \quad (9)$$

$$E_t \left(R_{t+1}^{p,F} \right)^{-\gamma} ((1-\tau)R_{t+1}^H - R_{t+1}^F) = 0 \quad (10)$$

(8) is the consumption Euler condition that links the marginal utility of consumption across the two periods. (9) and (10) are the portfolio Euler conditions for Home and Foreign investors, respectively. Both conditions show that the expected product of the asset pricing kernel and asset return are equalized across all assets.

Finally, the asset market clearing conditions in the two countries are:⁵

$$K_{t+1}^H = (W_t^H - C_{y,t}^H) N_t^H z_t^H + (W_t^F - C_{y,t}^F) N_t^F z_t^F \quad (11)$$

$$K_{t+1}^F = (W_t^H - C_{y,t}^H) N_t^H (1 - z_t^H) + (W_t^F - C_{y,t}^F) N_t^F (1 - z_t^F) \quad (12)$$

2.4 Solution Method

We solve the model using the local approximation method recently developed by Devereux and Sutherland (2008) and Tille and van Wincoop (2008). We briefly summarize the method. A useful step is to define the worldwide average of portfolio shares, z_t^A , and the cross country difference, z_t^D , as:

$$z_t^A = \eta z_t^H + (1-\eta) z_t^F \quad z_t^D = z_t^H - z_t^F$$

⁵The budget constraints together with asset market clearing conditions imply that the world goods market equilibrium condition is satisfied as well.

where η is the share of the Home country in the world wealth in the steady state.⁶

The method provides an exact solution to the zero, first and second-order components of control and state variables. A variable x_t can be written as the sum of its components of various orders: $x_t = x(0) + x_t(1) + x_t(2) + \dots$. The zero-order component, $x(0)$, is the level of a variable when standard deviations approach zero (deterministic steady state). The first-order component, $x_t(1)$, is linearly proportional to the innovations (or the standard deviations of model innovations). The second-order component, $x_t(2)$, is proportional to cross-products of model innovations (or the variance of model innovations), and so on.

The method distinguishes between the difference across countries in portfolio Euler equations and all “other equations” and similarly between the difference z_t^D across countries in portfolio allocation and all “other variables”. It first solves for the zero-order component of z^D and the first-order component of the “other variables” by jointly imposing the second-order component of the difference across countries in portfolio Euler equations and the first-order component of the “other equations”.

This step is subsequently repeated one order higher for all equations and variables in order to obtain the first-order component of z_t^D jointly with the second-order component of all “other variables”. But our solution focuses mainly on the first step, which is sufficient to compute the first-order component of *net* capital flows or the current account. The second step of the solution is needed to compute the first-order component of *gross* capital flows.

3 Current Account Response to Temporary Income Shocks

3.1 Dynamic Response of the Economy

The new rule is about the current account response to a temporary income shock. We analyze such a shock by considering the first-order response of the economy to a temporary income shock in the Home country in the form of a rise in ε_t^H . The first-order components of (9) and (10) imply that the expected returns are

⁶Specifically: $\eta = n(1 - \bar{c}^H) [n(1 - \bar{c}^H) + (1 - n)(1 - \bar{c}^F)]^{-1}$ where \bar{c}^i is the steady state ratio of the consumption of young agents to the wage in country i .

equalized across Home and Foreign assets: $E_t R_{t+1}^H(1) = E_t R_{t+1}^F(1) = E_t R_{t+1}(1)$. Taking expectations of (4) then implies that the response of the capital stock is the same in both countries. It is equal to

$$k_{t+1}^H(1) = k_{t+1}^F(1) = k_{t+1}(1) = \delta_k^k \left[k_t(1) + \frac{k(0)}{1-\omega} \varepsilon_t^A(1) \right] \quad (13)$$

where $0 < \delta_k^k < 1$, $\varepsilon_t^A(1) = \eta \varepsilon_t^H(1) + (1-\eta) \varepsilon_t^F(1)$, and $k(0)$ is the value of the capital to labor ratio in steady state (zero-order component).

The first-order component of net investment is then the same in both countries as well and equal to $i_t(1) = (1+g)k_{t+1}(1) - k_t(1)$. The per capita capital stock and investment are the same in the two countries because expected returns must be equalized to the first-order. Both are driven by ε_t^A . A temporary rise in world productivity raises world saving, which must lead to a rise in world investment. Since investment only depends on world productivity shocks, an income shock in a small country has little effect on domestic investment.

Result 1 *To the first-order, per capita investment is the same across countries and only reflects world productivity shocks. Investment in a small country therefore changes very little in response to an income shock in that country.*

These results imply that the current account is determined by the difference in saving across the two countries. To see that, we use that world saving is equal to world investment:

$$n s_t^H(1) + (1-n) s_t^F(1) = n i_t^H(1) + (1-n) i_t^F(1) = i_t(1) \quad (14)$$

The Home current account, given by the difference between saving and investment, is then:

$$ca_t^H(1) = s_t^H(1) - i_t(1) = (1-n)(s_t^H(1) - s_t^F(1)) \quad (15)$$

The current account therefore reflects cross-country differences in saving fluctuations.

The first-order component of saving in a country i depends on the dynamics of wages and consumption by the young:

$$s_t^i(1) = \tilde{\Delta} W_t^i(1) - \tilde{\Delta} C_{y,t}^i(1) \quad (16)$$

where the tilde is defined for a variable a as $\tilde{\Delta}a_t(1) = a_t(1) - a_{t-1}(1) / (1 + g)$. Intuitively, aggregate saving (16) reflects the saving by the young generation as well as the dis-saving by the old generation.

The first-order components of wages and consumption by young agents, which drive saving, are:

$$W_t^i(1) = W(0) \left(\varepsilon_t^i + (1 - \omega) \frac{k_t(1)}{k(0)} \right) \quad (17)$$

$$C_{y,t}^i(1) = \bar{c}^i W_t^i(1) - \frac{1 - \gamma}{\gamma} \bar{c}^i (1 - \bar{c}^i) W(0) \frac{E_t R_{t+1}(1)}{R(0)} \quad (18)$$

where $W(0)$ and $R(0)$ are the steady state values of wages and asset returns, which are the same for both countries. \bar{c}^i is the steady state ratio of consumption by young agents in country i and the wage.

The impact of a temporary income shock is most clearly illustrated by considering a log utility of consumption ($\gamma = 1$).⁷ Combining (16) and (18), saving is directly proportional to wages: $s_t^i(1) = (1 - \bar{c}^i) \tilde{\Delta}W_t^i(1)$. We consider a temporary increase in Home productivity at time zero: $\varepsilon_0^H = \varepsilon > 0$, assuming that the economy is initially in a steady state: $k_0(1) = 0$. Home saving then rises by

$$s_0^H(1) = (1 - \bar{c}^H) W(0) \varepsilon \quad (19)$$

in the immediate response to the shock, while Foreign saving does not change.

From (15) we get

$$ca_0^H(1) = (1 - n) s_0^H(1) > 0 \quad (20)$$

The current account therefore rises in response to the shock in a way that is unrelated to the steady state ratio of net foreign assets to wealth. It is clear that the new rule does not hold. Under the new rule, the current account changes by the rise in Home saving times the ratio of the Home net foreign assets position to wealth, which is unrelated to $1 - n$. In fact, for equal time discount rates ($\beta^H = \beta^F$) the net foreign asset position is simply zero and the new rule predicts no current account response at all. We focus on this case for the remainder of this section ($\bar{c}^H = \bar{c}^F = \bar{c}$).⁸

⁷While our results are easily illustrated in the case of a log utility, they do not qualitatively change when we allow saving to depend on the expected excess return ($\gamma \neq 1$). The general case is discussed in the Technical Appendix.

⁸Our results remain valid under different time discount rates ($\beta^H \neq \beta^F$).

Result 2 *Under a log utility of consumption the fraction of a first-order increase in saving that is invested abroad only reflects the size of the country. This fraction differs from the steady-state ratio of net foreign assets to wealth, which reflects international asymmetries in the time discount rate.*

While the shock is temporary, it has persistent effects through capital accumulation (13) and future dissaving by the workers that were young at the time of the shock. The increase in the Home saving equally boosts the capital intensity worldwide, leading to equal impacts on wages in both countries in period 1. In that period, the old agents dissave, implying the following aggregate saving:

$$\begin{aligned} s_1^H(1) &= (1 - \bar{c}) \tilde{\Delta} W_1^H(1) = (1 - \bar{c}) \left[(1 - \omega)n - \frac{1}{1 + g} \right] W(0)\epsilon \\ s_1^F(1) &= (1 - \bar{c}) \tilde{\Delta} W_1^F(1) = (1 - \bar{c}) (1 - \omega)n W(0)\epsilon \end{aligned}$$

The dissaving by old Home agents implies that aggregate Foreign saving exceeds aggregate Home saving and the Home country runs a current account deficit:

$$ca_1^H(1) = -\frac{1 - \bar{c}}{1 + g} W(0)\epsilon$$

Again this shows that the new rule does not hold, as there is a non-zero response of the current account to the income shock. In subsequent periods, the generation that was young at the impact of the shock has disappeared. While a first-order effect on wages remains, it solely reflects capital accumulation and is the same across countries, implying that the current account is zero.

3.2 Why Does the New Rule Not Hold?

It is useful to understand why exactly the new rule does not hold. In order to shed light on this, we focus on the response of the current account during the period of the shock itself. In addition we will continue to assume that the rates of time preference are the same in both countries, implying that the steady state net foreign asset to wealth ratio is zero and the new rule predicts a zero current account response to income shocks.

A useful starting point is the first-order component of the Home asset market clearing condition (11) across subsequent periods:

$$i_t^H(1) = z^H(0) s_t^H(1) + \frac{1 - n}{n} z^F(0) s_t^F(1) + \frac{1}{n} W(0) (1 - \bar{c}) \tilde{\Delta} z_t^A(1) \quad (21)$$

Intuitively, investment in the Home country must be equal to demand for Home capital by investors. There are two factors that shift demand for Home assets. The first is portfolio growth, which reflects changes in saving that are invested in Home capital at steady state portfolio shares. This is captured by the first two terms on the right hand side of (21). The second factor is changes in portfolio allocation between Home and Foreign assets. This is captured by the last term on the right hand side of (21). An increase in $z_t^A(1)$ captures a portfolio shift towards Home assets and therefore a greater demand for Home assets.

Consider first what happens when portfolio shares are held constant, so that $z_t^A(1) = 0$. Assuming again that $\gamma = 1$, so that only Home saving rises in response to the shock, we have:

$$i_t^H(1) = z^H(0) s_t^H(1)$$

The change in Home investment is equal to the steady state fraction invested in Home capital by Home agents times the change in their saving. The current account, which equals saving minus investment, is then

$$ca_t^H(1) = (1 - z^H(0))s_t^H(1)$$

An increase in saving is partially invested abroad, even though the steady state value of net foreign assets is zero. If $z^H(0) = 0.8$ for example, an increase in Home saving by 100 leads to capital outflows, and a current account, of 20. The new rule, which predicts that all saving is invested domestically, does not hold even though we specifically assumed that portfolio shares remain constant.

We now expand the analysis by considering the endogenous reaction of portfolio shares. In equilibrium there is a portfolio shift towards Foreign assets ($z_t^A(1) < 0$), leading to further net capital outflows. This is a second factor that pushes us even further away from the new rule. To see why portfolio shares change in equilibrium, notice that if assume that $z_t^A(1) = 0$, the Home and Foreign asset market clearing conditions (11)-(12) imply the following impact of a change in Home saving:

$$i_t^H(1) = z^H(0)s_t^H(1) \quad ; \quad i_t^F(1) = (1 - z^H(0))\frac{n}{1-n}s_t^H(1)$$

Under portfolio home bias ($z^D(0) > 0$), we have that $z^H(0) > n$, so that the investment rate is higher in the Home country: $i_t^H(1) > i_t^F(1)$. Intuitively, an increase in Home saving is disproportionately allocated to Home capital, and clearing the asset markets requires the supply of Home capital to increase, relative to the supply of Foreign capital. This however is inconsistent with portfolio arbitrage, which

implies that expected asset returns are equalized to the first order, so that $i_t^H(1)$ must equal $i_t^F(1)$.

As the relative supply of Home capital cannot increase, asset market clearing requires that the average investor must shift towards Foreign capital, so that $z_t^A(1) < 0$. Specifically:

$$\tilde{\Delta} z_t^A(1) = -\frac{n(1-n)}{W(0)(1-\bar{c})} z^D(0) [s_t^H(1) - s_t^F(1)] \quad (22)$$

Our discussion so far views $z_t^A(1)$ from the point of view of the supply of assets needed to clear the markets. We now turn to the perspective of the asset demand by investors. What induces them to shift their portfolio allocation? From the third-order component of the average of the portfolio Euler equations (9)-(10) we can show that (details are provided in a Technical Appendix available on request):

$$z_t^A(1) = \frac{R(0)}{\gamma} \frac{E_t(R_{t+1}^H(3) - R_{t+1}^F(3))}{[var(R_{t+1}^H - R_{t+1}^F)](2)} + m_t(1) \quad (23)$$

The first term on the right hand side shows how the average portfolio share depends on the third-order component of the expected excess return, scaled by the second-order variance of excess returns. The second term is $m_t(1)$, which is associated with time-varying risk that in equilibrium is proportional to $k_t(1)$.

This portfolio expression implies that a first-order portfolio shift towards Foreign assets is achieved by only a third-order drop in the expected return on Home capital, relative to Foreign capital. This is achieved through a third-order rise in Home investment relative to Foreign investment. To the first-order, investment is determined by equality between expected returns. In order for asset markets to be in equilibrium, there needs to be a first-order portfolio shift, which in turn only requires a third-order change in relative investment. This latter adjustment does not impact the current account to the first-order. We summarize our findings as follows:

Result 3 *The new rule does not hold, for two reasons. First, higher Home saving leads to a capital outflow even with constant portfolio shares. This "portfolio growth" component allocates saving in line with steady-state portfolio shares. Second, an endogenous shift in portfolio shares towards Foreign capital pushes the current account response even further away from the new rule.*

It is important to point out that our findings are not some artifact of our approximation method. This is shown below through an exact numerical solution of the model, but the intuition can be understood as follows. Consider the two factors driving the Home current account surplus. In the extreme case where there is constant returns to scale, portfolio returns have a constant expected return: $R_{t+1}^i = 1 - \delta + A_{t+1}^i$. As expected returns never change, portfolio shares do not either. This follows immediately from the portfolio Euler equations (9)-(10) without any need for approximation. In that case we know from the discussion above that

$$dca_t^H = (1 - z^H(0))ds_t^H$$

holds exactly, so that the change in the current account is positive as the steady state fraction invested abroad is positive. In the presence of Home bias, an increase in Home saving increases the demand for Home capital, relative to Foreign capital. Under constant returns to scale, this can be met by an increase in the relative supply of Home capital, as returns and investments are no longer linked.

As long as there are some decreasing returns to scale, returns and investment are linked and an increase in the relative supply of Home capital lowers the expected return on Home capital, relative to Foreign capital. Only the exact magnitude is subject to approximation, not the direction. This leads to a portfolio reallocation to Foreign capital and therefore an even larger current account surplus for the Home country.⁹

An alternative approach to our results is to consider the ratio of net foreign assets to wealth, denoted by x_t^H . In our setting, national wealth is equal to saving by young agents. Therefore $nfa_t^H = x_t^H s_{y,t}^H$, where $s_{y,t}^H = W_t^H - C_{y,t}^H$ is per capita saving by the young. To a first-order, this implies:

$$nfa_t^H(1) = x_t^H(0) s_{y,t}^H(1) + x_t^H(1) s_y^H(0)$$

The new rule requires that the ratio of net foreign assets to wealth does not change

⁹While unambiguously $dca_t^H > (1 - z^H(0))ds_t^H > 0$, so that the new rule does not hold, the numerical approximation discussed below shows that the first-order approximation becomes less accurate as we approach the extreme case of constant returns to scale. When ω is positive but very close to zero, the third-order rise in $i_t^H - i_t^F$ discussed above becomes substantial as the expected excess return becomes almost insensitive to $i_t^H - i_t^F$. As $\omega \rightarrow 0$ we approach the constant returns to scale case where $dca_t^H = (1 - z^H(0))ds_t^H$, which is less than the first-order approximation $ca_t^H(1) = (1 - n)s_t^H(1)$.

in response to a saving shock, so that $x_t^H(1) = 0$. In that case

$$ca_t^H(1) = nfa_t^H(1) = x^H(0) s_{y,t}^H(1)$$

and the new rule holds.

Our model however implies that not only x_t^H changes to a first-order, but that this change is not independent from movements in saving. By definition, x_t^H is given by:

$$x_t^H = \frac{n(1 - z_t^H) s_{y,t}^H - (1 - n) z_t^F s_{y,t}^F}{n s_{y,t}^H}$$

Focusing on the case where the rates of time preference are the same in both countries, and using (22), this is written to a first order as:

$$\begin{aligned} x_t^H(1) &= -\frac{1}{n} z_t^A(1) + \frac{1 - n}{n} \frac{z^F(0)}{W(0)(1 - \bar{c})} (s_{y,t}^H(1) - s_{y,t}^F(1)) \\ &= \frac{1 - n}{W(0)(1 - \bar{c})} [s_{y,t}^H(1) - s_{y,t}^F(1)] \end{aligned} \quad (24)$$

An increase in Home saving raises the ratio of net foreign assets to wealth. It does so both through a portfolio growth channel and through a portfolio shift towards Foreign capital.

3.3 Intertemporal versus Portfolio Theory of the Current Account

In order to further shed light on the results, it is useful to distinguish between the intertemporal versus portfolio theory of the current account. In the intertemporal theory of the current account, developed in the early 1980s, the current account is determined by a combination of intertemporal consumption smoothing (which generates saving) and equality between the marginal product of capital across countries (which generates investment). Subsequent DSGE open economy models continued to be characterized by this intertemporal view of the current account. This is particularly the case as the solution of these models usually involved linearization, which leads to a condition that equates the *expected* marginal product of capital across countries.

Even though we have argued that the new rule cannot plausibly hold in theory, the contribution by KV was broader in that it highlighted an alternative view of

the current account, which is a portfolio theory of the current account. This view is most clearly illustrated by assuming that there is constant returns to scale in production, so that the marginal product of capital does not depend on the capital stock itself. In that case investment cannot be derived from a condition that equates the marginal product of capital across countries as in the intertemporal view of the current account. The current account is now entirely determined by portfolio allocation, with investment simply accommodating whatever portfolio demand for the assets is.

Consider a shock that raises the expected return on Foreign capital or alternatively one that reduces the risk on Foreign capital relative to Home capital. This leads to a portfolio shift towards Foreign capital and net capital outflows from the perspective of the Home country. In this case investment in both countries changes endogenously to accommodate the portfolio shift. It drops in the Home country and rises in the Foreign country.

Which of these views is correct? In general the truth lies in between as long as there is some degree of decreasing returns to scale, so that the marginal product of capital depends on the capital stock itself. Portfolio allocation affects demand for capital, which drives investment. On the other hand, investment affects the marginal product of capital, which affects portfolio allocation.

When portfolios are very sensitive to expected return differences (e.g. low degree of risk), then expected returns need to be very close in equilibrium, so that we approach the intertemporal theory of the current account. When there is very little decreasing returns to scale, expected returns and therefore portfolio allocation are not much affected by changes in investment. In that case it is primarily portfolio allocation that drives investment rather than the other way around and we are close to the portfolio theory of the current account. It is for this reason that KV correctly pointed out that the portfolio view of the current account is the right one when there is limited decreasing returns to scale and there is high asset return risk.

In terms of order accounting, to the first-order the intertemporal theory of the current account is exactly correct. Expected returns have to be equal across countries to the first-order, which determines investment to the first-order. However, KV emphasize that the portfolio view is relevant for high levels of risk. This aspect is present in our model and shows up in the third-order component of investment, which affects the third-order expected excess return that drives the first-order com-

ponent of the average portfolio share in (23). It is possible that for high levels of risk such third-order effects are large and the portfolio view of the current account is important. This raises the possibility that our first-order approximation could be misleading as it neglects higher-order terms that are quantitatively relevant.

We address this issue by computing the exact numerical solution of the model, without relying on any approximations. For simplicity, we set $\gamma = 1$ and consider the same rate of time preference in both countries. (7) and (8) then imply that $s_{y,t}^H = \beta/(1 + \beta)W_t^H$. We consider the impact on the current account during the period of the income shock in the Home country. This raises Home saving by

$$ds_{y,t}^H = \frac{\beta}{1 + \beta} \omega (k_t^H)^{1-\omega} dA_t^H$$

and leaves Foreign saving unaffected. We assume that both countries are in a steady state prior to the shock, so that $k_t^i = k(0)$.

We set the standard deviation of productivity innovations, and their correlation across countries, in order to match the evidence in Solnik (1991) that shows an average stock return correlation of 0.39 across 17 industrialized countries, and 14.45% standard deviation of the world stock return.¹⁰ Note that this parameterization tilts the scale in favor of the portfolio theory of the current account by calibrating the model to the very risky equity returns. In reality there are many types of assets that all represent claims on capital, with equity only the most risky residual claim. In terms of the other parameters, we set $g = 0.02$, $\beta = 1$ and $\delta = 0.1$. We set the investment friction τ in order to generate an average home bias z^D of 0.73, in line with the results from Fidora, Fratzscher and Thimann (2007) for 5 industrialized countries over the 2001-2003 period.

Our analysis proceeds in two steps. First, we start from the case where there are no shocks in the current period. The assumption of identical rates of time preference ensures that the current account is zero. Given z_t^H and z_t^F , the capital stocks k_{t+1}^H and k_{t+1}^F (and therefore also investment) follow from the market clearing conditions (11)-(12). These in turn determine the asset returns R_{t+1}^H and R_{t+1}^F , which also depend on the stochastic productivities. The asset returns, together with the portfolio shares, determine the overall portfolio returns. We then solve for z_t^H and z_t^F from the portfolio Euler equations (9)-(10). This is done numerically

¹⁰We generalize the model a bit by allowing for a positive correlation of productivity innovations across countries, although none of the results discussed above depend on this.

by solving a fixed point problem in Gauss. In evaluating the expectations in the Euler equations we take 100,000 draws from uniform distributions of ε_{t+1}^H and ε_{t+1}^F .

In our second step, we recompute the equilibrium after introducing a one standard deviation Home productivity shock in the current period. We compute the impact of the shock on saving by comparing the level of saving to its value in the first step: $s_t^H - \bar{s}^H$. We analyze the allocation of the extra saving by taking the ratio of the current account in the second step to the extra saving: $ca_t^H / (s_t^H - \bar{s}^H)$. Figure 1 shows this ratio as a function of the share of labor in production, ω , for the case of two countries of equal size (left panel, $n = 0.5$) and for the case of a very small country (right panel, $n = 0.001$).

We contrast the exact actual current account response (thick solid line), the response under the new rule (thick dotted line, equal to zero), the response under the intertemporal theory of the current account (thin solid line), and under the portfolio theory (thin dotted line). For the intertemporal theory of the current account we set the expected marginal product of capital equal across countries, implying $i_t^H = i_t^F$, so that $ca_t^H = (1-n)(s_t^H - \bar{s}^H)$. This response is exactly the same as under the first-order solution discussed above. For the portfolio theory of the current account we hold portfolio shares constant, implying $ca_t^H = z^H(0)(s_t^H - \bar{s}^H)$.

Three points stand out from Figure 1. First, the new rule does not hold. The actual response of the current account is between that under the portfolio theory and intertemporal theory, which are both positive. Second, the change in the current account is virtually identical to the intertemporal theory (and therefore also our first-order solution) for all but extremely low values of ω . Under constant returns to scale ($\omega = 0$) the portfolio theory provides an exact description of the current account. When there are decreasing returns to scale ($\omega > 0$), the portfolio theory only shows up in the third- (and higher) order component of investment. Even for a calibration based on the very high equity return risk, this component is very small, unless one gets extremely close to constant returns to scale.

Third, in the very small country case the portfolio view is only relevant when ω is infinitesimally close to zero. When the Home country is infinitesimally small, and the large Foreign country can freely arbitrage between Home and Foreign assets, the arbitrage condition for the large country determines the expected returns for the small country. Even a tiny change in the expected return on Home capital leads to enormous capital flows from the large Foreign country that will be inconsistent with equilibrium. With an unchanged expected return, capital and investment will

not change when ω is (even barely) positive and the intertemporal theory holds. Our results can be summarized as follows.

Result 4 *The exact solution of the model shows that the new rule does not hold. The intertemporal theory of the current account accurately describes the current account response to saving unless decreasing returns to scale are nearly zero, in which case the portfolio theory is more accurate.*

These results suggest that the portfolio theory of the current account requires (nearly) constant returns to scale. However, we do not interpret this result as implying that the portfolio theory of the current account is without meaning. While in a standard production function ω measures the labor share, and $1 - \omega$ the capital share, there could be far lower decreasing returns to scale when allowing for externalities across sectors. When the capital stock and output in one firm has positive externalities on the output of other firms, the aggregate production function could be much closer to constant returns to scale than the individual one. In fact, there may even be increasing returns to scale, in which case the portfolio theory is very relevant.¹¹

One thing we can say with confidence though, which is that the new rule does not hold. It is inconsistent with both the intertemporal and portfolio theories, and does not correctly describe the current account response to a temporary income shock regardless the parameterization of the model.

4 What Accounts for the Cross-Section Evidence by Kraay and Ventura?

How can we reconcile our model with the empirical evidence in KV? They find a very tight cross-sectional link between the current account and the product of saving and the net foreign asset to wealth ratio, with a slope coefficient of 1.

Such a result comes naturally in our model. We show this by turning to the zero-order components of the variables, which can be interpreted as their steady

¹¹While this goes well beyond the scope of the paper, we should point out that the portfolio theory may explain the Felstein-Horioka puzzle (in its original cross-section form). Under the portfolio theory, and portfolio home bias, a country with high saving will invest a lot at Home. This is endogenously accommodated through high Home investment in the portfolio theory, which can account for the strong cross-section relationship between saving and investment rates.

state values (Devereux and Sutherland 2008). The ratio of the consumption of young agents to the wage in country i is inversely related to the time discount factor in that country:

$$\bar{c}^i = \frac{C_y^i(0)}{W(0)} = c(\beta^i, R(0)) \quad ; \quad \frac{\partial \bar{c}^i}{\partial \beta^i} < 0$$

This ratio is lower in the Home country ($\bar{c}^H < \bar{c}^F$) if we assume, without loss of generality, that Home agents are more patient ($\beta^H > \beta^F$). It immediately follows that national saving (net of depreciation) is larger in the Home country:

$$s^i(0) = \frac{gW(0)}{1+g} (1 - \bar{c}^i) \Rightarrow s^H(0) > s^F(0) \quad (25)$$

The portfolio Euler equations (9) and (10) imply that the zero-order components of asset returns must be the same across Home and Foreign capital. (4) then implies that the zero-order component of the capital stock is the same in the two countries, and so is investment. With investment rates equalized across countries, the current account simply reflects differences in saving rates, essentially as a zero-order version of (15). The patient Home country then runs a current account surplus and is a net creditor:

$$ca^H(0) = \frac{(1-n)gW(0)}{1+g} (\bar{c}^F - \bar{c}^H) > 0 \quad (26)$$

$$nfa^H(0) = (1-n)(\bar{c}^F - \bar{c}^H)W(0) > 0 \quad (27)$$

The steady state wealth of country i , per unit of labor, is simply the saving of the young generation: $(1 - \bar{c}^i)W(0)$. (25)-(27) then imply that the zero-order component of the current account is equal to the zero-order component of saving times the zero-order component of the ratio of the net foreign asset position to wealth:

$$ca^i(0) = \frac{nfa^i(0)}{(1 - \bar{c}^i)W(0)} s^i(0) = x^i(0)s^i(0) \quad (28)$$

This results is simply due to the fact that along a balanced growth path the ratio of flows (current account and saving) is equal to the ratio of corresponding stocks (net foreign assets and wealth). This is a quite general result that will hold in other models as well along a balanced growth path.¹² It is therefore to be expected that this relation shows up in the cross-sectional evidence presented by KV. To sum up, the steady state analysis shows that:

¹²This point was previously argued by one of us, in van Wincoop (2003).

Result 5 *In the steady state of the general equilibrium model the current account is equal to saving times the ratio of net foreign assets to wealth.*

It is important to note that this cross-sectional steady state relationship along a balanced growth path has absolutely nothing to do with the new rule, which is about the current account response to temporary income shocks. The balanced growth result is unrelated to the response to income shocks, or any other shocks for that matter, at any horizon. A criticism of our analysis may be that countries are never exactly on a balanced growth path. However, Guo and Jin (2008) show that it is the large cross-section variation in the ratio of net foreign assets to wealth that drives our result, even when this ratio is not on a balanced growth path.

5 Are There Different Models Where the New Rule Does Hold?

In this section we ask whether other models exist where the new rule does hold. The answer to this question is positive, but it would require a setup that is entirely counterfactual. Most critically, it requires that there is no distinction between gross and net foreign asset positions. This means that there are only one-way capital flows. There are either capital inflows or outflows, either external assets or liabilities, but not both. This is not consistent with the observed pattern of financial integration with large values of both external assets and liabilities that far out-shadow the net foreign asset position (Lane and Milesi-Ferretti 2007). In the United States for instance, the ratio of gross external liabilities to GDP more than tripled from 40 percent in 1990 to nearly 150 percent in 2007, with a similar pattern for gross external assets.

In order for the new rule to hold, the ratio of net foreign assets to total wealth should remain unchanged in response to a temporary income shock that raises Home saving. Denote the financial wealth of country i by a_t^i , and its gross external assets by gfa_t^i . The ratio of net foreign assets to wealth in the Home country is then:

$$nfa_t^H/a_t^H = (gfa_t^H - gfa_t^F)/a_t^H = gfa_t^H/a_t^H - gfa_t^F/a_t^F \cdot a_t^F/a_t^H \quad (29)$$

Even when portfolio shares gfa_t^H/a_t^H and gfa_t^F/a_t^F are constant, the ratio (29) increases when a temporary rise in Home saving reduces a_t^F/a_t^H . This is consistent

with our model, where saving affects the ratio of net foreign asset to wealth even when holding portfolio shares constant.

In order to get the new rule to hold, we must move away from two-way asset trade. One way to do so is to assume that the Home country can buy claims on Foreign capital but the Foreign country cannot but claims on Home capital. When claims on capital are the only assets, this implies $gfa_t^F/a_t^F = 0$ and (29) becomes:

$$nfa_t^H/a_t^H = gfa_t^H/a_t^H \quad (30)$$

(30) is clearly constant when gross foreign assets are a constant share of wealth in the Home country. This is the case if we assume constant returns to scale, as expected returns do not change, removing any incentive to adjust portfolio shares. This is a very peculiar setting though. In addition to being at odds with the evidence of large gross external positions, it implies that while the new rule holds for the Home country, it does not the Foreign country.¹³

Another approach that delivers one-way capital flows is to assume that all investors in the world trade a riskfree bond, while claims on capital are held by domestic investors only. In that case the model only has implications for net external positions and capital flows, not gross positions and capital flows. In this case we can denote gfa_t^H/a_t^H as the portfolio share by the Home country in the riskfree bond, which can be both positive or negative. This portfolio share is equal to the ratio of net foreign assets to wealth as in (30).

The absence of any impact of temporary income shocks on (30) requires two assumptions. First, the expected return on Home capital remains constant due to constant returns to scale in production. Second, the interest rate remains constant, implying that the Home country is infinitesimally small. If the country is large enough to impact the world interest rate, it affects the portfolio share in the bond, which in turn affects the ratio of net foreign assets to wealth. To put it another way, if the Home country is a creditor in the international bond, higher Home saving increases the Home country's bond holdings when portfolio shares are constant. This must be met by other countries through higher short positions in the bond through an adjustment in portfolio shares as their savings are constant. When the Home country is infinitesimally small, its higher demand for bonds is accomodated by tiny (but non zero) shifts in portfolio shares abroad (this can be seen from the

¹³To be precise, we have $nfa_t^F/a_t^F = -(gfa_t^H/a_t^H)(a_t^H/a_t^F)$, which changes due to a Home saving shock even when the portfolio share gfa_t^H/a_t^H is constant.

1997 working paper version of Kraay and Ventura (2000)). But most critically, the assumption of international asset trade taking place solely through a bond is not consistent with the evidence of large cross-border assets and liabilities in a broad range of instruments.¹⁴

We summarize our results as follows:

Result 6 *The new rule can only hold under two assumptions. First, production exhibits constant returns to scale. Second, there are either capital inflows or outflows, either external assets or liabilities, but not both. The second assumption is counterfactual.*

6 Conclusion

Recent years have seen a renewed interest in the role of portfolio choice as a determinant of international capital flows. The contributions by KV were pioneering in that line of research in several ways. They stressed a portfolio view of the current account nearly a full decade before the recent renewed interest, established clear and robust empirical patterns, and developed a theoretical model to account for them.

We have shown that no matter what view one takes on the importance of the portfolio theory of the current account versus the traditional intertemporal theory of the current account, the new rule for the current account established in the contribution by KV does not hold. This remains even the case under a portfolio theory of the current account, when there is very limited feedback from relative investment back to portfolio choice through changes in relative expected returns. The link between a portfolio view of the current account and the new rule that was established in the KV papers relies on a setup that makes no distinction between gross and net foreign asset positions. Once that is corrected, we have shown that the new rule cannot hold in a general equilibrium model with portfolio choice.

In addition we have shown that the cross-section evidence in favor of the new rule in the KV papers simply reflects cross-sectional variation in net foreign assets

¹⁴Kraay and Ventura (2000) illustrate the new rule in the context of a model that is in the intersection of the two examples given above. There is trade in a riskfree bond. In addition the Home country can buy claims on both Home and Foreign capital, but the Foreign country can only buy claims on Foreign capital.

positions that applies in almost any model along a balanced growth path. This is conceptually distinct from the new rule, which is about the response to a temporary income shock.

Appendix

A General Equilibrium Model

This Appendix presents the zero-order solution and the first-order expansions of the variables around their zero order values for the general equilibrium model.

Zero-order component of variables

The zero-order components of all equations other than the difference across countries in portfolio Euler equations gives is the zero-order component of all variables other than the zero-order component of the difference across countries in portfolio shares. Variables that grow at rate g are scaled by the country's labor supply.

From (2) we have $A^i(0) = 1$ for $i = H, F$. From the portfolio Euler equations (9)-(10) we have

$$R^H(0) = R^F(0) \equiv R(0)$$

It then follows from (4) and (3) that

$$\begin{aligned} k^H(0) &= k^F(0) \equiv k(0) \\ R(0) &= 1 - \delta + (1 - \omega)k(0)^{-\omega} \\ W_H(0) &= W_F(0) = \omega k(0)^{1-\omega} \end{aligned}$$

Gross investment follows from the capital accumulation:

$$i^H(0) = i^F(0) = (g + \delta)k(0)$$

(7) and (9) are written as:

$$\begin{aligned} C_o^i(0) &= (W(0) - C_y^i(0))R(0) \\ C_o^i(0) &= C_y^i(0)(\beta^i)^{1/\gamma}R(0)^{1/\gamma} \end{aligned}$$

The solution for the young' consumption then follows as:

$$C_y^i(0) = \frac{W(0)R(0)}{R(0) + (\beta^i)^{1/\gamma}R(0)^{1/\gamma}} \equiv \bar{c}^i W(0)$$

The sum of the asset market clearing conditions (11)-(12) gives

$$(1 + g)k(0) = n(W(0) - C_y^H(0)) + (1 - n)(W(0) - C_y^F(0)) \quad (31)$$

Substituting the expressions for $W(0)$, $C_y^H(0)$ and $C_y^F(0)$ above then yields an implicit solution for $k(0)$. Finally, from the Home asset market clearing condition (11) we have

$$(1 + g)k(0) = (W(0) - C^H(0))z^H(0) + (W(0) - C^F(0))z^F(0)\frac{1 - n}{n} \quad (32)$$

This gives a solution for a weighted average of portfolio shares, namely $z^A(0) = n$.

We now turn to the implied zero-order components of saving and the current account. Saving is equal to income minus consumption. Aggregate consumption at time t in the Home country is

$$N_t^H C_{y,t}^H + (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H R_t^{p,H} + \tau R_t^F (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H (1 - z_{t-1}^H)$$

The three components are consumption by the young, the old and the brokers that immediately consume the revenues from the fee τ on foreign returns. Aggregate income (in net terms) is

$$W_t^H N_t^H + (W_{t-1}^H - C_{y,t-1}^H)N_{t-1}^H (z_{t-1}^H (R_t - 1 + \delta) + (1 - z_{t-1}^H)(R_t^F - 1 + \delta) - \delta)$$

Therefore national saving, which is income minus consumption, is

$$S_t^H = N_t^H (W_t^H - C_{y,t}^H) - N_{t-1}^H (W_{t-1}^H - C_{y,t-1}^H)$$

Dividing by N_t^H and taking the zero-order component, we have

$$s^H(0) = \frac{g}{1 + g}(W(0) - C_y^H(0))$$

Since the current account is saving minus investment (net of depreciation), the previous results imply

$$ca^H(0) = \frac{g}{1 + g}(W(0) - C_y^H(0)) - gk(0)$$

Substituting (31), this becomes

$$ca^H(0) = \frac{g}{1 + g}(1 - n)(C_y^F(0) - C_y^H(0))$$

Finally we compute the steady state net foreign asset position of the Home country. We have

$$NFA_t^H = (W_t^H - C_{y,t}^H)N_t^H(1 - z_t^H) - (W_t^F - C_{y,t}^F)N_t^F z_t^F$$

Dividing by N_t^H and taking the zero-order component, we have

$$nfa^H(0) = (W(0) - C_y^H(0))(1 - z^H(0)) - (W(0) - C_y^F(0))z^F(0)\frac{1-n}{n}$$

Substituting (32), this becomes

$$nfa^H(0) = (W(0) - C_y^H(0)) - (1+g)k(0)$$

which together with (31) becomes

$$nfa^H(0) = (1-n)(C_y^F(0) - C_y^H(0))$$

Wealth per young agent is $W(0) - C_y^H(0)$, so that the ratio of net foreign assets to wealth is:

$$x^H(0) = \frac{nfa^H(0)}{W(0) - C_y^H(0)} = (1-n)\frac{C_y^F(0) - C_y^H(0)}{W(0) - C_y^H(0)}$$

First-Order Solution of the other variables

From (2) we have $A_t^i(1) = \varepsilon_t^i$ for $i = H, F$. From a first-order expansion of the portfolio Euler equations, and the definition of the portfolio return, we have

$$E_t R_{t+1}^H(1) = E_t R_{t+1}^F(1) = E_t R_{t+1}^{p,i}(1) \equiv E_t R_{t+1}(1)$$

It then follows from (4) and (3) that

$$\begin{aligned} k_{t+1}^H(1) &= k_{t+1}^F(1) \equiv k_{t+1}(1) \\ E_t R_{t+1}(1) &= -\omega(1-\omega)k(0)^{-\omega-1}k_{t+1}(1) \\ W_t^i(1) &= W(0)\varepsilon_t^i + (1-\omega)\frac{W(0)}{k(0)}k_t(1) \end{aligned}$$

The sum of the first-order components of (11) and (12) implies:

$$k_{t+1}(1)(1+g) = n(W_t^H(1) - C_{y,t}^H(1)) + (1-n)(W_t^F(1) - C_{y,t}^F(1))$$

Using (8) and the expected value of (7), the consumption of young agents is:

$$C_{y,t}^i(1) = \bar{c}^i W_t^i(1) - \frac{1-\gamma}{\gamma}\bar{c}^i(1-\bar{c}^i)W(0)\frac{E_t R_{t+1}(1)}{R(0)}$$

Combining our results, we solve for the first-order capital stock as:

$$k_{t+1}(1) = \delta_k^k \left[k_t(1) + \frac{k(0)}{1-\omega} \varepsilon_t^A(1) \right] \quad (33)$$

where $\varepsilon_t^A(1) = \eta \varepsilon_t^H(1) + (1-\eta) \varepsilon_t^F(1)$, and:

$$\begin{aligned} \delta_k^k &= \frac{n(1-\bar{c}^H) + (1-n)(1-\bar{c}^F) \frac{W(0)}{k(0)}}{\Omega_k} (1-\omega) \\ \Omega_k &= 1 + g + \frac{1-\gamma}{\gamma} \left[n\bar{c}^H(1-\bar{c}^H) + (1-n)\bar{c}^F(1-\bar{c}^F) \right] \omega \frac{W(0)}{k(0)} \frac{R(0) - (1-\delta)}{R(0)} \end{aligned}$$

We now have the dynamics of all state variables, namely $k_t(1)$, $\varepsilon_t^A(1)$, and $\varepsilon_t^D(1) = \varepsilon_t^H(1) - \varepsilon_t^F(1)$.

The consumption of young agents in the Home country is computed as:

$$C_{y,t}^H(1) = \delta_k^{cy,H} k_t(1) + \delta_{eA}^{cy,H} \varepsilon_t^A(1) + \delta_{eD}^{cy,H} \varepsilon_t^D(1)$$

where:

$$\begin{aligned} \delta_k^{cy,H} &= \bar{c}^H (1-\omega) \frac{W(0)}{k(0)} \left[1 + \frac{1-\gamma}{\gamma} (1-\bar{c}^H) \frac{\omega k(0)^{-\omega}}{R(0)} \delta_k^k \right] \\ \delta_{eA}^{cy,H} &= \bar{c}^H W(0) \left[1 + \frac{1-\gamma}{\gamma} (1-\bar{c}^H) \frac{\omega k(0)^{-\omega}}{R(0)} \delta_k^k \right] \\ \delta_{eD}^{cy,H} &= \bar{c}^H W(0) (1-\eta) \end{aligned}$$

Similarly, the consumption in the foreign country is:

$$C_{y,t}^F(1) = \delta_k^{cy,F} k_t(1) + \delta_{eA}^{cy,F} \varepsilon_t^A(1) + \delta_{eD}^{cy,F} \varepsilon_t^D(1)$$

where:

$$\begin{aligned} \delta_k^{cy,F} &= \bar{c}^F (1-\omega) \frac{W(0)}{k(0)} \left[1 + \frac{1-\gamma}{\gamma} (1-\bar{c}^F) \frac{\omega k(0)^{-\omega}}{R(0)} \delta_k^k \right] \\ \delta_{eA}^{cy,F} &= \bar{c}^F W(0) \left[1 + \frac{1-\gamma}{\gamma} (1-\bar{c}^F) \frac{\omega k(0)^{-\omega}}{R(0)} \delta_k^k \right] \\ \delta_{eD}^{cy,F} &= -\bar{c}^F W(0) \eta \end{aligned}$$

The average portfolio share is computed from asset market clearing:

$$z_t^A(1) = -\eta(1-\eta) z^D(0) \left[\varepsilon_t^D(1) - (\bar{c}^H - \bar{c}^F) \frac{1-\gamma}{\gamma} \frac{\omega(1-\omega)k(0)^{-\omega-1}}{R(0)} k_{t+1}(1) \right]$$

Net saving of country i is computed as:

$$\begin{aligned} s_t^i(1) &= (W_t^i(1) - C_{y,t}^i(1)) - \frac{1}{1+g}(W_{t-1}^i(1) - C_{y,t-1}^i(1)) \\ &= (1 - \bar{c}^i)\tilde{\Delta}W_t^i(1) - \frac{1-\gamma}{\gamma}\bar{c}^i(1 - \bar{c}^i)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1}\tilde{\Delta}k_{t+1}(1) \end{aligned}$$

where:

$$\tilde{\Delta}x_t = x_t - \frac{1}{1+g}x_{t-1}$$

The current account represents the gap between net saving and investment:

$$\begin{aligned} ca_t^H(1) &= (1 - \bar{c}^H)\tilde{\Delta}W_t^H(1) \\ &\quad - \left[1 + g + \frac{1-\gamma}{\gamma}\bar{c}^H(1 - \bar{c}^H)\frac{W(0)}{R(0)}\omega(1-\omega)k(0)^{-\omega-1} \right] \tilde{\Delta}k_{t+1}(1) \end{aligned}$$

Difference in portfolio shares

The zero-order difference in portfolio shares, $z^D(0)$, is computed by taking a difference between the second-order components of (9) and (10):

$$0 = 2\tau - \gamma \left(\frac{R_{t+1}^H(1)}{R(0)} - \frac{R_{t+1}^F(1)}{R(0)} \right) \left(\frac{R_{t+1}^{p,H}(1)}{R(0)} - \frac{R_{t+1}^{p,F}(1)}{R(0)} \right)$$

Using the first-order solution for the returns on portfolio, this leads to:

$$z^D(0) = \frac{\tau}{\gamma\sigma_a^2} \left[\frac{R(0)}{(1-\omega)k(0)^{-\omega}} \right]^2 \quad (34)$$

The first-order difference in portfolio shares, $z_t^D(1)$, is computed by taking a difference between the third-order components of (9) and (10). Following steps detailed in the Technical Appendix, this implies:

$$\begin{aligned} 0 &= -\gamma z_t^D(1) \left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)} \right]^2 2\sigma_a^2 \\ &\quad - 2\gamma z^D(0) \frac{R(0) - (1-\delta)}{R(0)} 2\sigma_a^2 E_t R_{t+1}(1) \frac{1}{R(0)} \\ &\quad + 2(1-\gamma)\tau \frac{E_t R_{t+1}(1)}{R(0)} \\ &\quad + \gamma(1+\gamma)z^D(0) \left[\frac{(1-\omega)k(0)^{-\omega}}{R(0)} \right]^2 2\sigma_a^2 \frac{E_t R_{t+1}(1)}{R(0)} \end{aligned}$$

Using our earlier results, this implies

$$z_t^D(1) = \frac{2\omega(1-\delta)}{R(0)k(0)} z^D(0) k_{t+1}(1)$$

Average in portfolio shares

The first order average portfolio share, $z_t^A(1)$, is computed by taking a weighted average of the third-order components of (9) and (10), leading to:

$$z_t^A(1) = \frac{R(0)}{\gamma} \frac{E_t(R_{t+1}^H(3) - R_{t+1}^F(3))}{E_t(R_{t+1}^H(1) - R_{t+1}^F(1))^2} + \Omega \frac{E_t R_{t+1}(1)}{R(0)}$$

where Ω is a zero-order coefficient:

$$\Omega = \frac{(R(0))^2}{\gamma} \frac{(2\eta - 1)\tau}{E_t(R_{t+1}^H(1) - R_{t+1}^F(1))^2} + \frac{(1-\delta)^2 - (R(0))^2}{E_t(R_{t+1}^H(1) - R_{t+1}^F(1))^2} (2n - 1) \sigma_a^2$$

Drivers of capital flows

The gross capital outflows and inflows from the perspective of the Home country are given by:

$$\begin{aligned} out_t^H(1) &= s_t^H(1) (1 - z^H(0)) - W(0) (1 - \bar{c}^H) \tilde{\Delta} z_t^H(1) \\ in_t^H(1) &= \frac{1-n}{n} \left[s_t^F(1) z^F(0) + W(0) (1 - \bar{c}^F) \tilde{\Delta} z_t^F(1) \right] \end{aligned}$$

The first term in each line corresponds to the portfolio growth component, while the second term is the portfolio reallocation component. The net capital flows are:

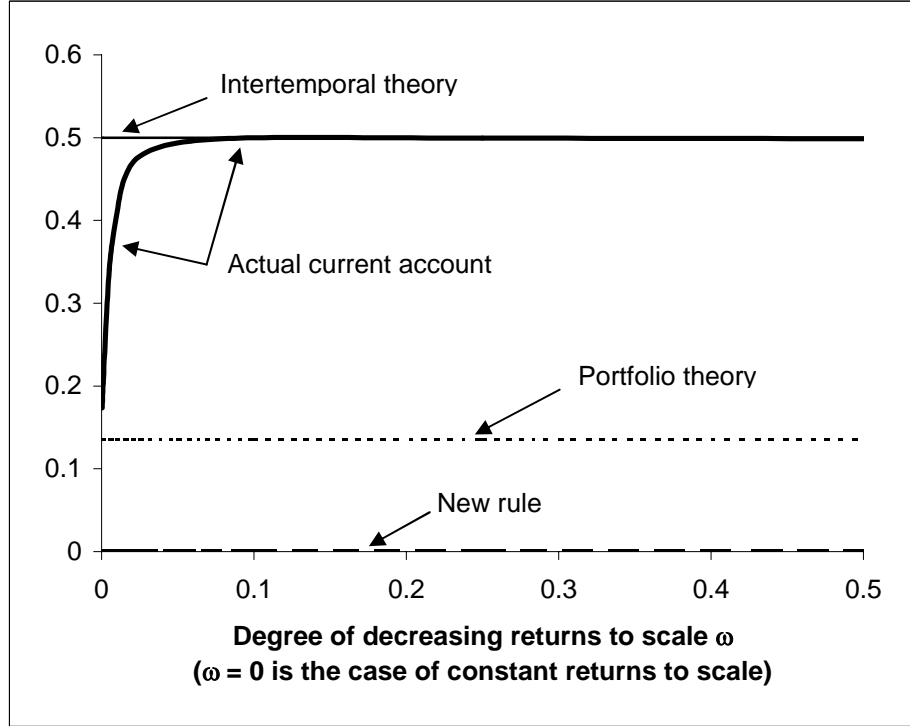
$$\begin{aligned} net_t^H(1) &= (1-n) (1 - z^D(0)) [s_t^H(1) - s_t^F(1)] \\ &+ \frac{(1-n)(\bar{c}^F - \bar{c}^H)z^D(0)}{n(1 - \bar{c}^H) + (1-n)(1 - \bar{c}^F)} [ns_t^H(1) + (1-n)s_t^F(1)] \\ &- \frac{1}{n} W(0) [n(1 - \bar{c}^H) + (1-n)(1 - \bar{c}^F)] \tilde{\Delta} z_t^A(1) \end{aligned}$$

References

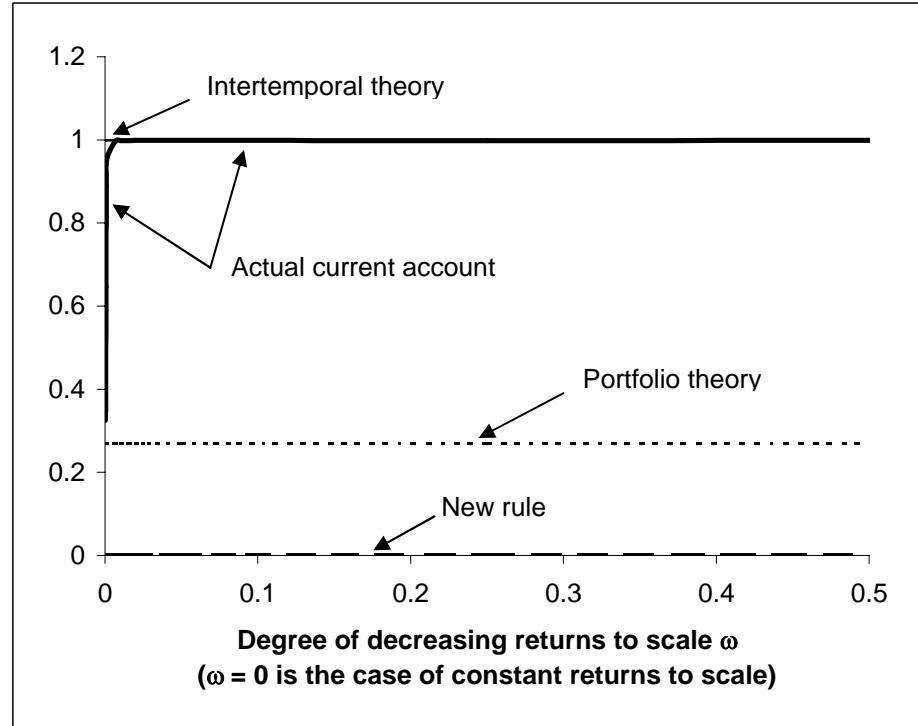
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Figure 1: Current account response to a temporary income shock

**Panel A: Home country represents
50% of the world (n=0.5)**



**Panel B: Home country represents
0.1% of the world (n=0.001)**



The figures show the response of the current account scaled by the change in savings: $ca_H / (s_H - \bar{s}_H)$. ω is the degree of decreasing returns to scale. $1 - \omega$ is the share of capital in the production function, so $\omega = 0$ corresponds to the case of constant returns to scale. The "actual current account" line is the actual response in a numerical simulation without any approximation. The "intertemporal theory" line is the response under the standard intertemporal view of the current account where the share of extra savings invested abroad is one minus the share of the country to the world. The "new rule" line is the response under the new rule where the share of extra savings invested abroad is the steady state ratio of net foreign assets to wealth, which we set to zero. The "portfolio theory" line is the response when the share of extra savings invested abroad is equal to the average share of foreign assets to wealth.