TOPICS
IN
HARMONIC ANALYSIS

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PREFACE

Our purpose in writing this book was to supplement the 1962 book of Rudin, *Fourier Analysis on Groups*, for our harmonic analysis course and seminar at the University of Virginia. The first part (Chapters 1–6) is about locally compact abelian groups and it includes a complete discussion of the maximal ideal space of $M(G)$ and the new proof of the Cohen idempotent theorem by Itô and Amemiya. The second part (Chapters 7–10) is an invitation to harmonic analysis on compact non-abelian groups. It contains a discussion of the algebra $A(G)$ (the non-abelian analogue of $L^1(G)$), spherical harmonics, the Poisson integral, and analytic functions in the $n$-complex ball. Appendix A contains Bredon’s proof of the existence and uniqueness of Haar measure. Appendix B discusses integration algebras and the Hausdorff-Young-Kunze theorem. Appendix C describes some current research on compact groups.

Our topics clearly do not include all the significant new results since 1962; for example, the work of Varopoulos on $M(G)$ has not been included because of space and time limitations.

We use $\square$ to indicate the end of a proof. The symbol $\subset$ denotes containment whereas $\subsetneq$ denotes proper containment. The paragraphs which compose the text are numbered consecutively. For example, Theorem 7.2.8 is the eighth paragraph in Section 2 of Chapter 7. In Chapter 7 this theorem is referred to as Theorem 2.8. To refer to a reference book we simply invoke a symbol; for example, $[R]$ denotes Rudin’s book, *Fourier*
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Analysis on Groups. To cite a particular paper of Rudin we would write, for example, Rudin [4]. We caution the reader to be aware that the definition of a Fourier-Stieltjes transform of a measure on a locally compact abelian group involves no inverse whereas the inverse is employed in the compact nonabelian definition.

Since we use a great amount of notation we have provided an index of special symbols which refers the reader to the respective definitions. The historical notes scattered throughout the book give the basic references for the various theorems. Our references are not meant to be a complete listing of all works in the field.

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C.F.D.
D.E.R.
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