SRC and $x > 1$ at 12 GeV
What can we learn?

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Short Range Correlations in Nuclei and Hard QCD Phenomena

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Prologue

Inclusive electron scattering has not fallen out of fashion even in the presence of cw accelerators). Why?

Because it still provides a rich, albeit complicated, mixture of physics that has yet to be fully exploited.

- Momentum distributions and the spectral function $S(k,E)$.
- Short Range Correlations and Multi-Nucleon Correlations
- FSI
- Scaling ($x$, $y$, $\varphi'$, $x$, $\xi$), and scale breaking
- Medium Modifications -- tests of EMC; 6-quark admixtures
- Duality
- Superfast quarks $\Rightarrow$ partons that have obtained momenta $x > 1$

The inclusive nature of these studies make disentangling all the different pieces a challenge but experiments over a range of $Q^2$ and with different $A$ will help.

Interpretation demands theoretical input
Outline

• Short range Correlations
• Inelastic Electron Scattering
• Do FSI obstruct us from gleaning information about SRCs in inclusive electron scattering?
• Ratios and FSI
• Transition from the study of correlations in QES to correlations in DIS
• New experiments
How do we know short range correlations exist?

Central density is saturated - nucleons can be packed only so close together: \( p_{\text{ch}} \times (A/Z) = \text{constant} \)

Spatial correlations

J.W. Negele RMP 54 (1982)

What else - Occupation Numbers?

Density difference between $^{206}_{\text{Pb}}$ and $^{205}_{\text{Tl}}$.

Experiment - Cavedon et al (1982)
Theory: Hartree-Fock orbitals with adjusted occupation numbers is given by the curve.

The shape of the $3s^{1/2}$ orbit is very well given by the mean field calculation.

Occupation numbers scaled down by a factor $\sim 0.65$. 


Elastic Scattering

$(e,e')$ at NIKHEF

$p_m$ [$\text{MeV/c}$]

Relative spectroscopic factor (%)

Mass number $A$
Realistic many body calculations of the spectral function contain correlated strength and it is significant.

$k < k_F$: single-particle contribution dominates

$k \approx k_F$: SRC already dominates for $E > 50$ MeV

$k > k_F$: single-particle negligible
What many calculations indicate is that the tail of $n(k)$ for different nuclei has a similar shape - reflecting that it is the short distance part of the NN interactions, common to all nuclei, is the source of these dynamical correlations.

Search for SRC in inclusive (e,e') experiments
Inclusive Electron Scattering from Nuclei

Two dominant and distinct processes

- Quasielastic from the nucleons in the nucleus
- Inelastic (resonances) and DIS from the quark constituents of the nucleon.

Quasielastic:
\[ k + q, \quad W^2 = M^2 \]

Inelastic:
\[ W^2 \geq (M_n + m_\pi)^2 \]

Inclusive final state means no separation of two dominant processes

\[ x = Q^2/(2m\nu) \]

\( \nu, \omega = \text{energy loss} \)

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The two processes share the same initial state

\[
\frac{d\sigma^2}{dQe'dEe'} = \frac{d^2}{Q^4} \frac{E'_e}{E_e} L_{\mu\nu} W_{\mu\nu}
\]

QES in IA

\[
\frac{d^2\sigma}{dQd\nu} \propto \int dk \int dE \sigma_{ei} S_i(k, E) \delta() \quad \text{Spectral function}
\]

DIS

\[
\frac{d^2\sigma}{dQd\nu} \propto \int dk \int dE \ W_{1,2}^{(p,n)} S_i(k, E) \quad \text{Spectral function}
\]

However they have very different \(Q^2\) dependencies

\[\sigma_{ei} \propto \text{elastic (form factor)}^2 \approx 1/Q^4\] \(W_{1,2}\) scale with \(\ln Q^2\) dependence

Exploit this dissimilar \(Q^2\) dependence
The shape of the low $\nu$ cross section is determined by the momentum distribution of the nucleons.

As $Q^2 \gg$ inelastic scattering from the nucleons begins to dominate

We can use $x$ and $Q^2$ as knobs to dial the relative contribution of QES and DIS.

The quasielastic peak (QE) is broadened by the Fermi-motion of the struck nucleon.

The quasielastic contribution dominates the cross section at low energy loss ($\nu$) even at moderate to high $Q^2$. 

$8^\circ$, E from 3-15 GeV

$^3$He SLAC (1979)
A dependence: higher internal momenta broadens the peak

\[ \Delta \omega = \sqrt{(\vec{k_f} + \vec{q})^2 + m^2} - \sqrt{(\vec{k_f} + \vec{q})^2 + m^2} \]

But... plotted against x, the width gets narrower with increasing q -- momenta greater than k\textsubscript{f} show up at smaller values of x (x > 1) as q increases
Inelastic contribution increases with $Q^2$

DIS begins to contribute at $x > 1$

Convolution model

We expect that as $Q^2$ increases to see evidence for $x$-scaling - and $Q^2$ independence.
Correlations are accessible in QES and DIS at large $x$ (small energy loss).

For example, the spectral function $\delta(k, E)$ in QES and DIS can be described by integrals involving the differential cross sections $d^2\sigma/dQd\nu$ and $d^2\sigma/dQd\nu$.

In QES, $\sigma_{ei}(k, E)$ represents the elastic scattering cross section, while in DIS, $W_{1,2}^{(p,n)}(k, E)$ represents the inelastic scattering cross section.

The graphs show the $F_2(x)$ distribution for different nuclei, with $Q^2 = 50$ GeV$^2$ and various $x$ values.

References:
- CdA, Day, Liuti, PRC 46 (1045) 1992
- Rozynek & Birse, PRC 38 (2201) 1988
- L. Conci and M. Traini, UTF 261/92.

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Deuteron $F(y)$

and calculations based on NN potentials

$S(k,E=2.2\text{MeV}) = n(k)$

Assumption: scattering takes place from a quasi-free proton or neutron in the nucleus.

$y$ is the momentum of the struck nucleon parallel to the momentum transfer: $y \approx -q/2 + mv/q$

$F(y) = \frac{\sigma^{\text{exp}}}{(Z\sigma_p + N\sigma_n)} \cdot K$

$n(p) = -\frac{1}{2\pi y} \frac{dF(y)}{dy}$

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What role FSI?

In \((e,e')\) flux of outgoing protons strongly suppressed: 20-40% in C, 50-70% in Au

In \((e,e')\) the failure of IA calculations to explain \(d\sigma\) at small energy loss

Meier-Hadjuk NPA 395, 332 1983
Failure of the spectral function or of PWIA indicating role of FSI?

One can make arguments ...

- RSC – lead to smaller correlation effects
- Redistribution of strength in E can account for the difference
- Such a proposal has been made by:
  - C. degli Atti, E. Pace, and G. Salme, Phys. Lett. B127 (1983) 303 and
- Data from JLab suggest as much
JLab data on $^{12}$C (e,e'p) of Rohe et al.

Frick et al. PRC 70, 024309 (2004)

Self consistent Greens Function (SGGF)

Data suggests more strength at smaller E - accessible at large x

Integration limits (e,e') Carbon 18°
FSI in QES.

Take $V$ and $W$ to be the real and imaginary parts of the optical potential of a nucleon in nuclear matter.

$V$ can be ignored ($\sim 20$ MeV) compared to the 100s of MeV here.

$$W^A_{\mu\nu}(q, \omega) = \int_0^\infty d\omega' F(\omega - \omega') W^A_{\mu\nu,IA}(q, \omega' - V(q))$$

Folding function $F(\omega - \omega') = \frac{1}{\pi} \Re \int_0^\infty dt e^{i(\omega - \omega')t} e^{-W(q,t)t}$

If $W = 0$ then $F(\omega - \omega')$ becomes $\delta$ function and $W^A_{\mu\nu} \Rightarrow W^A_{\mu\nu,IA}$

Imaginary part of optical potential

$$W(p') = \frac{\hbar}{2} \rho v(p') \sigma_{NN}(p')$$

$\Delta t = 1/W$
Rescattering depends on joint probability of finding the struck particle at position $r_i$ and a spectator at position $r_j$ 

$$\rho^{(2)}(r_i, r_j) = \rho_A(r_i)\rho_A(r_j)g(r_i, r_j)$$

\[\tilde{\rho}^{(2)}(r_{i,j}) = \frac{1}{A} \int d^3R_{ij}\rho^{(2)}(r_i, r_j)\]

If density is 0, the motion is undamped

**SRC suppress FSI**
FSI has two effects: energy shift and a redistribution of strength from QEP to the tails, just where correlation effects contribute.

Benhar et al. PRC 44, 2328
Benhar, Pandharipande, PRC 47, 2218
Benhar et al. PLB 3443, 47
O. Benhar private comm.
O. Benhar, NMBT and CGA for FSI

Carbon 5.766, 18°, $Q^2 = 2.5 \text{ (GeV/c)}^2$

Benhar et al. PRC 44, 2328
Benhar, Pandharipande, PRC 47, 2218
Benhar et al. PLB 3443, 47
O. Benhar private comm.

Carbon 5.766, 32°, $Q^2 = 5.2 \text{ (GeV/c)}^2$
FIG. 13. Sensitivity of the inclusive cross section to the $N-N$ pair distribution function at $\epsilon = 3.6$ GeV and $\theta = 25^\circ$. 

Sensitivity to $g(r)$ 

$\epsilon=3.6$ GeV $\theta=25^\circ$ 

$\frac{1}{A} \frac{d^2 \sigma}{d \omega d \Omega}$ ($\mu b$/sr GeV) 

energy loss $\omega$ (GeV) 

$x = 3$ 

$x = 2.0$ 

$x = 1$ 

$g(r)$, 

$g(0.8r)$
Issues about FSI

- Extreme sensitivity to hole size
- On-shell cross sections: nucleon is off-shell by in E by $\hbar/\Delta t = \hbar W$
- Total cross section?
- Unitarity? Folding function is normalized to one.
- Role of momentum dependent folding function (Petraki et al, PRC 67 014605) has lead to a quenching of the tails.
  - Comparison to data with this new model would be useful
- Reasonable but what is the error band on the results?
What I do not understand about FSI in QES.

Every nucleon has a 'hole' around it

Exclusion zone surrounds every nucleon

electron is sensitive to a region \( r \sim 1/q \) around the vertex

for \( q = 1 \text{ GeV}/c \), \( r \sim 0.2 \text{ fm} \)

the 'hole' is about that large

What is the range of the FSI?

If FSI are restricted to the region of the hole then \( FSI^{AA} = FSI^{A=2} \)
CS Ratios and SRC

In the region where correlations should dominate, large $x$,

$$\sigma(x, Q^2) = \sum_{j=1}^{A} \frac{1}{j} a_j(A) \sigma_j(x, Q^2)$$

$$= \frac{A}{2} a_2(A) \sigma_2(x, Q^2) + \frac{A}{3} a_3(A) \sigma_3(x, Q^2) + \cdots$$

$a_j(A)$ are proportional to finding a nucleon in a $j$-nucleon correlation. It should fall rapidly with $j$ as nuclei are dilute.

$$\sigma_2(x, Q^2) = \sigma_{eD}(x, Q^2) \quad \text{and} \quad \sigma_j(x, Q^2) = 0 \quad \text{for} \quad x > j.$$ 

$$\Rightarrow \quad \frac{2 \sigma_A(x, Q^2)}{A \sigma_D(x, Q^2)} = a_2(A) \bigg|_{1 < x \leq 2}$$

$$\frac{3 \sigma_A(x, Q^2)}{A \sigma_{A=3}(x, Q^2)} = a_3(A) \bigg|_{2 < x \leq 3}$$

In the ratios, off-shell effects and FSI largely cancel.

$a_j(A)$ is proportional to probability of finding a $j$-nucleon correlation.

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Knocking out a nucleon in a two-nucleon pair

$\alpha_{tn}$: light cone variable for interacting nucleon belonging to correlated nucleon pair

$$a_{tn} = 2 - \frac{q_- + 2m}{2m} \left( 1 + \frac{\sqrt{W^2 - 4m^2}}{W} \right) \rightarrow x \quad (Q^2 \gg)$$

$F_2(a_{tn})$

Ratios

Accounts for $Q^2$ dependence
Ratios, SRC's and $Q^2$ scaling

$$\frac{2 \sigma_A}{A \sigma_D} = a_2(A); \ (1.4 < x < 2.0)$$

$A(e,e')$, $1.4 < Q^2 < 2.6$

CLAS data
Egiyan et al., PRL 96, 082501, 2006

$a_j(A)$ is probability of finding a j-nucleon correlation
As can be seen the ratio at $x = 1$ ($\alpha = 1$) increases with $Q^2$ because of inelastic processes, spoiling the scaling with $\alpha$

$$a_{tn} = 2 - \frac{q_- + 2m}{2m} \left( 1 + \frac{\sqrt{W^2 - 4m^2}}{W} \right) \underset{(Q^2 \gg)}{\longrightarrow} x$$

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N. Fomin just showed these

Emphatic arguments have been made that these ratio values are an artifact – can not be interpreted as the ratio of correlated in strength in heavy to light nuclei

The plateaus, remarkable as they appear, are a result of FSI (and the role of SRC in FSI)
Ratios are NOT a measure of $a_2$ - the relative number of SRC pairs in nuclei

- The hand-waving argument that FSI effects might cancel in the A/d cross section ratios is contradicted by the quantitative calculations.
- The idea that the FSI could be the same as in the deuteron is also conceptually wrong: if the nuclear medium affects via Initial State Interaction the correlated 2-nucleon system --- it does as the high-k tail is (say) 4 times higher in a nucleus than in the deuteron --- then the nuclear medium also increases the FSI by a comparable factor.
- Indeed, in the standard Glauber-type calculations the FSI effects are explicitly proportional to the nuclear density.
At $\omega = 2000$ MeV ($y \approx -0.50$ and $x = 1.75$) FSI are not responsible for most of the strength.

“Despite these complications the data at large $x$ are sensitive to the properties of $P(k,E)$ at large $k$.... The reduction of the high-$k$ components by a factor of two, with the corresponding change of the short-range FSI has significant effect at $x > 1.3$.”

Benhar et al, PLB 343 (1995) 47-52

It would be useful to have new calculations that uses a spectral function that has the high $k$ strength moved to match data.
If $a_2$ is an artifact or complicated from FSI then does it destroy this relation?

It can only survive if the FSI from $^3$He to Au have a sympathetic relationship in the plateau region.
Ratios predictions from $n(k)$

E02019, $18^\circ$, Cu/$^2$H


no CM motion, QES only
What is an experimentalist to do?

Encourage theorists to examine FSI with respect to the message the data is trying to send.

- Direct ratios to $^2\text{H}$, $^3\text{He}$, $^4\text{He}$ out to large $x$ and over wide range of $Q^2$

- Study $Q^2$, $A$ dependence (FSI)

- Absolute Cross section to test exact calculations and FSI

- Extrapolation to NM
Experiments

• 6 GeV (completed in Spring 2011)
  • E-08-014: Three-nucleon short range correlations studies in inclusive scattering for $0.8 < 2.8 \text{(GeV/c)}^2$ [Hall A]

• 12 GeV
  • E12-06-105: Inclusive Scattering from Nuclei at $x > 1$ in the quasielastic and deeply inelastic regimes [Hall C], approved.
Motivation for E08-014

- Study onset of scaling, ratios as a function of $\alpha_{2n}$ for $1 < x < 2$
- Verify and define scaling regime for 3N-SRC
- 3N-SRC over a range of density: $^{40}$Ca, $^{12}$C, $^4$He ratios
- Test $\alpha_{3n}$ for $x > 2$
- Absolute cross sections: test FSI, map out IMF distribution $\rho_A()$
- Isospin effects on SRCs: $^{48}$Ca vs. $^{40}$Ca

2N SRC
Kin 3.1: $21.0^o$, 2.905 GeV/c
$^2$He, $^3$He, $^4$He, $^{12}$C, $^{40,48}$Ca

Kin 4.1: $23.0^o$, 2.855 GeV/c
$^3$He, $^{12}$C, $^{40,48}$Ca

Kin 5.1: $25.0^o$, 2.795 GeV/c
$^2$H, $^3$He, $^4$He, $^{12}$C, $^{40,48}$Ca

3N SRC
Kin 3.2: $21.0^o$, 3.055 GeV/c
$^3$He, $^4$He, $^{12}$C, $^{40,48}$Ca

Kin 4.2: $23.0^o$, 3.035 GeV/c
$^3$He, $^4$He, $^{12}$C, $^{40,48}$Ca

Kin 5.2: $25.0^o$, 2.995 GeV/c
$^3$He, $^4$He, $^{12}$C, $^{40,48}$Ca

Kin 6.5: $28.0^o$, 2.845 GeV/c
$^3$He, $^{12}$C
Can we make an connection to quark distributions at $x > 1$?

Two measurements (very high $Q^2$) exist so far:

CCFR ($\nu$-C): $F_2(x) \propto e^{-sx}$ \hspace{1cm} $s = 8$

BCDMS ($\mu$-Fe): $F_2(x) \propto e^{-sx}$ \hspace{1cm} $s = 16$

Poor resolution, limited x range

Low statistics

CCFR results suggested large contribution from SRC or other exotic effects

We can, but first we must account for the fact that none of these measurements are at the asymptotic limit.
How well does this work?

The comparison to the world data set is good and can be used to extract the behavior of the SF at large $x$.

- At $\xi \leq 0.75$ where the high $Q^2$ data dominates our data the agreement is good down to about $Q^2 = 3$ GeV$^2$.
- As $\xi$ increases the dependence on $Q^2$ grows continually.
- Agreement is still good except at low $Q^2$ where there is a QES contribution and HT must play a role.
- Finally note that the BCDMS data fails to display a dependence on momentum transfer above $\xi$ about 0.65.
Compare to the very high $Q^2$ BCDMS and CCFR data

Fit our $F_2^0$ (over a limited range of $\xi$) with the functional form $F_2^0 = \text{Constant} \times e^{-s\xi}$

CCFR – ($Q^2 = 125$ GeV$^2$) $s=8.3\pm0.7$

BCDMS – ($Q^2: 52 - 200$ GeV$^2$) $s=16.5\pm0.5$

Our results contradict those of CCFR and support BCDMS
Sensitivity to SRC

We want to be able to isolate and probe two-nucleon and multi-nucleon SRCs

Dotted = mean field approx.
Solid = +2N SRCs.
Dashed = +multi-nucleon.

11 GeV can reach $Q^2 = 20(13) \text{ GeV}^2$ at $x = 1.3(1.5)$
- very sensitive, especially at higher $x$ values
Inclusive Scattering from Nuclei at $x > 1$ in the quasielastic and deeply inelastic regimes

- Moderate $Q^2$ and large $x$
  - Two and multi-nucleon correlations
    - $A$-dependence of strength, density dependence, non-isoscalarity
  - Provide tests of ‘exact’ calculations [$S(k,E)$] through $\sigma$, expose role of FSI
- Very high $Q^2$ and $1 < x < 1.5$
  - Extraction of SF and underlying quark distributions at $x > 1$
  - Provide insight into origin of EMC effect
  - Provide extreme sensitivity to non-hadronic components
• Inclusive \((e,e')\) at large \(Q^2\) scattering and \(x>1\) is a powerful tool to explore long sought aspects of the NN interaction
  • Considerable body of data exists
• Provides access to SRC and high momentum components through scaling, ratios of heavy to light nuclei and allows systematic studies of FSI
• Scaling in \(\xi\) appears to work well even in regions where the DIS is not the dominate process
  • DIS is does not dominate over QES at 6 GeV but should at 11 GeV and at \(Q^2 > 10 - 15\) (GeV/c)\(^2\). We can expect that any scaling violations will vanish as we go to higher \(Q^2\)
• Once DIS dominates it will allow another avenue of access to SRC and to quark distribution functions
• New experiments have been approved to push these investigations into heretofore unexplored regions
If it was only this easy.

The correlation between the FTSE and the DOW for the last 6 months.