Correlations in Few* Body Systems

EMMI Workshop: Cold dense nuclear matter: from short-range nuclear correlations to neutron stars

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* and heavier
Outline

• Introduction
• NN potential responsible
  • Implications
• Correlations exist
  • Some examples
• Two nucleon correlations
  • Inclusive
  • Exclusive – Few body systems
• 3N correlations
  • 4He/3He
  • 12C/4He etc
• $A_{zz}$: Scattering from a Tensor Polarized Deuteron
• Finish
Independent Particle Shell Model

- Independent particle states of a uniform potential – a mean field.

\[ S(\vec{p}, E) = \sum_a |\Phi_a(p)|^2 \delta(E + \xi_a) \]

- Enormous strong force acting
- So many nucleons to collide with
- How can nucleons possibly complete whole orbits \((10^{21}/s)\) without interacting?

Hard core is small part of the nuclear volume

\[
\frac{V_c}{V_{total}} = \left( \frac{c}{2r_o} \right)^3 \approx 1/100
\]
The nucleon-nucleon (NN) interaction is singularly repulsive at short distances.
Difficult to find two nucleons close to each other.
Loss in configuration space components signals an increase of high-momentum components.
Both the correlation hole and the high-k components are absent in IPMs.
Taken together the loss of configuration space and the strengthening of high-momentum components are "correlations".
The NN tensor force also provides high-momentum components; required to obtain the quadrupole moment of the deuteron and predicts a isospin dependence of SRCs.
Possible Two Nucleon states

<table>
<thead>
<tr>
<th>L</th>
<th>S</th>
<th>J</th>
<th>$\pi = -1^L$</th>
<th>$T(L+S+T)$ odd</th>
<th>$^{2S+1}L_J$</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
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<td>$^1S_0$</td>
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<td>$^3D_1$</td>
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</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
<td>+</td>
<td>0</td>
<td>$^5D_3$</td>
</tr>
</tbody>
</table>

Two-nucleon states

Symmetric triplet $T = 1$

- $^3(T)_1 = |p_1> |p_2>$ proton-proton state
- $^3(T)_1 = |n_1> |n_2>$ neutron-neutron state
- $^3(T)_0 = \frac{1}{\sqrt{2}} (|p_1> |n_2> + |p_2> |n_1>)$ neutron-proton state

Antisymmetric singlet $T = 0$

- $^1(T)_0 = \frac{1}{\sqrt{2}} (|p_1> |n_2> - |p_2> |n_1>)$ neutron-proton state

The Pauli principle requires that two-nucleon states be antisymmetric wrt to exchange of the nucleons’ space, spin, and isospin coordinates.

D-state nucleon flips spin

Deuteron S and D
- $^1S_0$, $S=0$, $T=1$
- $^1P_1$, $S=0$, $T=1$
- $^3P_0$, $S=1$, $T=1$
- $^3P_1$, $S=1$, $T=1$
- $^3P_2$, $S=1$, $T=1$
- $^3D_1$, $S=1$, $T=0$
- $^3D_2$, $S=1$, $T=0$
- $^3D_3$, $S=1$, $T=0$

Explain the SRC ratios, isospin asymmetry

Deuteron Wavefunction

- $L=0$, $S$ state minimum at $p \sim 0.45$ GeV
- $D$ state significant at $p > 0.3$ GeV
- $D$ state normally 4-6%
- Beyond 0.3 GeV dominant
- Region to study tensor force
- In $D$ state nucleon spins flip

AV$_{18}$

Deuteron 3D-14
- Generated on 2013-04-09 by Donal Day
How can two nucleons combine?

- The SR NN attraction dominated tensor interaction, which yields high momentum iso-singlet (np) pairs.

- Absent in the iso-triplet channel (pp, nn, np).

- The two-body distribution should be identical to the deuteron distribution, \( n_2(k) = n_D(k) \), and the ratio of scattering cross sections between a heavy nucleus A and the deuteron to yield \( a_2 (A, Z) \)

- Without the tensor contribution the deuteron would not be bound

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Two-nucleon states

\[
S_{1,2} = 3 (\vec{\sigma}_1 \cdot \vec{r}_{12}) (\vec{\sigma}_2 \cdot \vec{r}_{12}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2
\]
The tensor interaction causes a quadrupole type dependence as a function of the angle between the total spin direction (which we aligned along the z axis) and the direction of the distance vector $\mathbf{r}$. The main attraction is obtained when the spins of the nucleons are aligned with the distance vector $\mathbf{r}$ while almost no attraction exists in the x direction where the spins are orthogonal to $\mathbf{r}$.

\[
S_{1,2} = 3 (\vec{\sigma}_1 \cdot \hat{r}_{12}) (\vec{\sigma}_2 \cdot \hat{r}_{12}) - \sigma_1 \sigma_2
\]

The potential in the $M_s = 1$ states is attractive for parallel and repulsive for perpendicular.
$\rho^{(1)}(r_1) = \frac{1}{2J+1} \sum_{M} \langle \Psi; JM | \sum_{i=1}^{A} \delta^{3}(\hat{r}_i - r_1) | \Psi; JM \rangle$

$n^{(1)}(k_1) = \frac{1}{2J+1} \sum_{M} \langle \Psi; JM | \sum_{i=1}^{A} \delta^{3}(\hat{k}_i - k_1) | \Psi; JM \rangle$

Two-body densities in coordinate space for a pair of nucleons with $S = 1$, $M_S = 1$, and $T = 0$ in the ground states of $^3$H, $^3$H, and $^4$He and the 20.21 MeV excited state of $^4$He.

Where the potential is attractive, $r \approx (0,0,\pm 1 \text{ fm})$, the densities are $>>$ and in regions where the interaction is repulsive or close to zero the probability of finding the particle pair is small.

These correlations can not be represented in a shell model and the two-body densities have their maximum at relative distance $r = 0$.
Momentum Distributions

Figure 2.1: Nucleon momentum distribution for various nuclei [28], where dotted lines are from a mean field calculation, solid lines include SRC. Dots are from experimental data. The unit of the momentum is fm

The asymptotic form of momentum distribution can be broken down into several regions. At \( k > k_F \), the strength is mainly contributed by the mean field potential. At the momentum range \( 300 < k < 600 \text{ MeV/c} \), the contribution of the mean field is small and SRCs play a significant role. The tail of \( n(k) \) for different nuclei has a similar shape - reflecting that the NN interaction, common to all nuclei, is the source of these dynamical correlations.

Theory suggests a common feature for all nuclei: Isolate short range interactions (and SRC’s) by probing at high \( p_{\text{lab}} \): \((e,e'p)\) and \((e,e')\). The nucleus is a bottom-up system where the tail of \( n(k) \) is dominated by SRCs at large \( k \) and \( n(k) \) exhibits the same shape for all nuclei for \( k > k_{\text{Fermi}} \).

\( k > 250 \text{ MeV/c} \)
- 20% of nucleons
- 60% of KE

\( k < 250 \text{ MeV/c} \)
- 80% of nucleons
- 40% of KE

n(k) is dominated by SRCs at large \( k \) and \( n(k) \) exhibits the same shape for all nuclei for \( k > k_{\text{Fermi}} \).
Correlations and charge distributions

Central density is saturated - nucleons can be packed only so close together:
\[ \rho_{\text{ch}} \times (A/Z) = \text{constant} \]

Evidence of SRC

Charge density archive

IPM (full lines), LRC (long dashed lines), SRC (short dashed lines).
Evidence of SRC

Density difference between $^{206}$Pb and $^{205}$Tl: differ by a single $3s^{1/2}$ proton

Experiment – Cavedon et al (1982)
Theory: Hartree-Fock orbitals with adjusted occupation numbers describe the shape of the $3s^{1/2}$ orbit.

Occupation numbers scaled down by a factor $\sim 0.65$. 

Appearance of plateaus is $A$ dependent.

**Kinematics:** Heavier recoil systems do not require as much energy to balance momentum of struck nucleon – hence $p_{\text{min}}$ for a given $x$ and $Q^2$ is smaller.

**Dynamics:** Mean field part in heavy nuclei persist in $x$ to larger values.

Have to go to higher $x$ or $Q^2$ to insure scattering is not from mean-field nucleon.

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Wieslaw Czyż and Kurt Gottfried
Annals of Physics 21, 47 (1963)
Ratios, SRC's and $Q^2$ scaling

\[ \frac{2\sigma_A}{A\sigma_D} = a_2(A); \quad (1.4 < x < 2.0) \]

\[ A(e,e'), \quad 1.4 < Q^2 < 2.6 \]

\[ \times_{2N} \approx 20\% \]

\[ \times_{3N} \approx 1\% \]

CLAS data
Egiyan et al., PRL 96, 082501, 2006

$a_j(A)$ is probability of finding a j-nucleon correlation
Ratios and SRC

Dominance of np pairs in SRC region leads us to drop the isoscalar correction. We correct for COM motion of pair.

\( R_{2n} \): number of np pairs relative to the deuteron

<table>
<thead>
<tr>
<th>A</th>
<th>( R_{2n} ) (E02-019)</th>
<th>SLAC</th>
<th>CLAS</th>
<th>( F_{CM} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^3)He</td>
<td>1.93 ± 0.10</td>
<td>1.8 ± 0.3</td>
<td>⋯</td>
<td>1.10 ± 0.05</td>
</tr>
<tr>
<td>(^4)He</td>
<td>3.02 ± 0.17</td>
<td>2.8 ± 0.4</td>
<td>2.80 ± 0.28</td>
<td>1.19 ± 0.06</td>
</tr>
<tr>
<td>Be</td>
<td>3.37 ± 0.17</td>
<td>⋯</td>
<td>⋯</td>
<td>1.16 ± 0.05</td>
</tr>
<tr>
<td>C</td>
<td>4.00 ± 0.24</td>
<td>4.2 ± 0.5</td>
<td>3.50 ± 0.35</td>
<td>1.19 ± 0.06</td>
</tr>
<tr>
<td>Cu(Fe)</td>
<td>4.33 ± 0.28</td>
<td>(4.3 ± 0.8)</td>
<td>(3.90 ± 0.37) &amp; 1.20 ± 0.06</td>
<td></td>
</tr>
<tr>
<td>Au</td>
<td>4.26 ± 0.29</td>
<td>4.0 ± 0.6</td>
<td>⋯</td>
<td>1.21 ± 0.06</td>
</tr>
</tbody>
</table>

\( \langle Q^2 \rangle \) \( \approx 2.7 \) GeV\(^2\) \( \approx 1.2 \) GeV\(^2\) \( \approx 2 \) GeV\(^2\)

\( x_{min} \) 1.5 ⋯ 1.5

\( \alpha_{min} \) 1.275 1.25 1.22–1.26

Evidence of 2N-SRC at \( x>1.5 \)
Qualitative agreement with calculations; effect of tensor force. Huge violation of often assumed isospin symmetry.

Data show large asymmetry between np, pp pairs:

Two-nucleon knock-out experiment

High momentum(!!) strength in proton knockout in \( (e,e'p) \)

\(^2\text{H}(ee'p)n \text{ Mainz}\


\[ E = 0.855 \]
\[ \theta = 45 \]
\[ E' = 0.657 \]
\[ Q^2 = 0.33 \]
\[ x = 0.88 \]

Not the best place to look for
SRGs – \( \Delta s \), MECs FSI dominate
large IC+MEC

\[ 0 < \pi < 1 \]
High momentum strength in $A(e,e'p)$

$^3\text{He}(e,e'p)d$ E89-044, Hall A

Measured far into high momentum tail:
Cross section is $\sim$5-10x expectation
High momentum pair can come from SRC (initial state)
OR
Final State Interactions (FSI) and Meson Exchange Contributions (MEC), $\Delta$'s

M. M. Rvachev et al. PRL 94, 192302 (2005)
Arrows indicate expected location of correlated pair

- dotted line PWIA
- dash-dot: Laget (PWIA)
- FSI (long dashed line) to full calculation (solid line), including meson-exchange current and final-state interactions: Laget
- In the 620 MeV/c panel
  - short dashed curve is a calculation with PWIA + FSI only within the correlated pair.
Events with one leading nucleon and a spectator correlated NN pair
- The spectator nucleons each have less than 20% of the transferred energy
- Leading nucleon's momentum perpendicular to $\vec{q}$ be less than 0.3 GeV/c.
- The ratio of pp to pn pair cross sections for $0.3 < p_{rel} < 0.5$ GeV/c is very small at low $p_{tot}$ and rises to approximately 0.5 at large $p_{tot}$.

The pp pairs at low $p_{tot}$ are in an s-state, this ratio shows the dominance of tensor over central correlations.
E97-006 Correlated Spectral Function and $^{12}$C(e,e'p) Reaction Mechanism


Data suggests more strength at smaller $E$ - accessible at large $x$

Frick et al. PRC 70, 024309 (2004)
Self consistent Green’s Function (SCGF)
Integrated strength in the covered $E_m-p_m$ region:

$$Z_c = 4\pi \int_{130}^{670} dp \, p_m^2 \int dE_m S(E_m, p_m)$$

“correlated strength” in the chosen $E_m-p_m$ region:

In terms of # of protons in $^{12}$C

<table>
<thead>
<tr>
<th>$^{12}$C</th>
<th>exp.</th>
<th>CBF theory</th>
<th>G.F. 2.order</th>
<th>self-consistent G.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental area</td>
<td>0.61</td>
<td>0.64 ≈ 10%</td>
<td>0.46</td>
<td>0.61</td>
</tr>
<tr>
<td>in total (correlated part)</td>
<td></td>
<td>22%</td>
<td>12%</td>
<td>≈20%</td>
</tr>
</tbody>
</table>

Contribution from FSI: -4%

- ≈ 10% of the protons in $^{12}$C at high $p_m, E_m$ found
- first time directly measured

Comparing to theory leads to conclusion that

≈ 20% of the protons in Carbon are beyond the IPSM region.
3N Correlations

2N SRC (3N SRC)

- \( p > k_F \) i.e. its momentum exceeds characteristic nuclear Fermi momentum, \( (k_F \approx 250 \text{ MeV/c}) \)
- balanced by the momentum of a (two) correlated nucleon(s)
- In both cases the center of mass momentum of the SRC, \( p_{cm} \ll k_F \)

Where to look: \( \geq 600 \text{ MeV/c}, \; Q^2 > 4, \; x = 2 - 3 \)  

Misak’s talk
Good agreement in the 2N-SRC region

but potential difference in the 3N-SRC region
Data from SLAC

Rock et al, PRC 26, 1593 (1982)

FIG. 7. The inelastic structure function $vW_2$ as a function of the scaling variable $w_x = 1 + 2q^2 + 2M_y - Q^2/Q^2$ for several different values of $Q^2$. The curves at fixed $Q^2$ are to guide the eye and they seem to approach a common limit, (a) $^3$He and (b) $^4$He.

Ratio of sigma($4\text{He}/3\text{He}$)*$3/4$ - E121 Threshold data

Exponential fit to 4He (green) with 4He and 3He - E121 Threshold data

E121 threshold data nuW2

Ratio of nuW2($3\text{He}/4\text{He}$) - E121 Threshold data

generated on 2015-10-06 by Donal Day
More data at larger $Q^2$ needed, $x > 1$ at 12 GeV, E12-06-105, see Arrington, Fomin talks.
Ratio of $^{9}$Be, $^{12}$C, Cu, Au to $^{4}$He @ $Q^2 = 2.7$

From E02019, Fomin thesis data

Acceptance effects, windows

Probe short-range repulsion and tensor force in nucleon-nucleon interaction through tensor asymmetries from quasi-elastic and elastic deuteron scattering.

Tensor asymmetries are predicted to be sensitive to the D state probability as well as relativistic effects.

Conditionally Approved (44 days) on improving the tensor polarization and its measurement.

E.Long (UNH), DD, D. Keller, K. Slifer, P. Solvignon, D. Higinbotham
\[ A_{zz} = \frac{2}{fp_{zz}} \left( \frac{\sigma_p - \sigma_u}{\sigma_u} \right), \text{ is sensitive to WF admixtures} \]

\[ A_{zz} \propto \frac{1}{2} w^2(k) - u(k)w(k)\sqrt{2} \frac{u^2(k) + w^2(k)}{u^2(k) + w^2(k)} \]

\( A_{zz} \) can be used to discriminate between hard and soft wave functions. In impulse approximation \( A_{zz} \) is directly related to the S- and D-states which have very different r and p behavior.

Modern calculations indicate a large separation of hard and soft WFs begins above the quasi elastic peak at \( x > 1.4 \).
Is the deuteron wave function hard or soft?
- Hard like AV18 or softer like CD Bonn
- Unpolarized deuterons need to be probed at $k > 500$ MeV/c to distinguish between hard and soft WFs
  - Difficult, absolute cross sections
- At present no unambiguous evidence for hard/soft.
- Tensor polarization exposes the D-state, allowing hard and soft WFs to be distinguished at lower momenta

O. Hen, et al, arXiv:
Deuterons D-states differ at large $k$

\[ A_{zz} \propto \frac{\frac{1}{2} w^2(k) - u(k)w(k)\sqrt{2}}{u^2(k) + w^2(k)} \]

375 MeV/c

generated on 2015-10-02 by Donal Day
Measurement of Quasi Elastic $A_{zz}$

Sensitive to effects that are very difficult to measure with unpolarized deuterons
Huge 10-120% asymmetry
Measuring $A_{zz}$ over a range in $x$ and $Q^2$ provides insight to
- Nature of NN Forces
- Hard/Soft wf
- Relativistic NN Dynamics
- On-Shell/Off-Shell Effect

Decades of theoretical interest that we can only now probe with a high-luminosity tensor-polarized target
Vector and Tensor Polarization of the Deuteron

Deuteron in a magnetic field $H$, spin projection on $Z$ can only take values $m_d = +1, 0, -1$.

If $N_+, N_0, N_-$ are the relative numbers in the substates $m_d = +1, 0, -1$, $(N_+ + N_0 + N_- = 1)$, then vector $p_Z$ and tensor $p_{ZZ}$ polarizations of a deuteron target are:

**Vector:** $p_Z = N_+ - N_-$

**Tensor (Alignment)** $p_{ZZ} = N_+ + N_- - 2N_0 = 1 - 3N_0$. 
Deuteron has quadruple moment that interacts with the electric field gradient in the lattice (in ND₃ for example)

\[ \nu_D = \text{deut Lamour freq. (6.54 MHz/Tesla)} \]
\[ \nu_Q = \text{ND₃ quadrupole freq (335.6 kHZ)} \]
\[ = \frac{1}{\beta} \frac{e^2 q Q}{h} \]
\[ eQ = \text{deuteron quadrupole moment} \]
\[ eq = \text{electric field gradient} \]
\[ \theta = \text{angle between eq and B} \]

\[ E_m = -h\nu_D m + h\nu_Q [3 \cos \theta^2 - 1][3m^2 - l(l+1)] \]

NMR line shape

θ = π/2

θ = 0

θ = π/2

θ = 0
Vector $P_z = p_+ - p_-$

Tensor $P_{zz} = (p_+ + p_-) - 2p_0$

Positive $P_{zz}$: fill up the first two and minimize the $m_0$-state

$P_{zz} = 2 - \sqrt{4 - 3P_z^2}$

50% Vector $P_z \Rightarrow 20\%$ Tensor

5 Tesla at 1K
3cm target length
Tensor Polarization Progress

At UVa progress measuring $P_{zz}$ through NMR line-shape analysis advancing (Dustin Keller)

Solid state NMR $P_{zz}$ can be confirmed with elastic ($T_{20}$) Enhancement through RF hole burning

D Keller, PoS(PSTP 2013) 010
D Keller, HiX Workshop (2014)
UVA Tensor Enhancement on Butanol (2014)
D Keller, PSTP 2015, to be published

$P = 0.503 \rightarrow 0.447$
$P_{zz} = 0.196 \rightarrow 0.325$
$A_{zz}$ for $Q^2 > 1$ GeV$^2$ with $p_{zz} = 30\%$

$Q^2 = 1.5$ GeV$^2$

$Q^2 = 1.8$ GeV$^2$

$Q^2 = 2.9$ GeV$^2$

Solid = Quasi-elastic
Open = Elastic

Summary

• 2N SRC and their isospin dependence (anticipated by our understanding of the NN interaction, is now firmly established in multiple observables, experiments projectiles, final states and nuclei
• Relation of SRC to EMC established – only lacking are calculations that exposes the underlying connection
• Refined theory and calculation are needed incorporating SRC, FSI, and off-shell behavior will advance understanding
• SRC demand high densities (momenta, virtuality) and, if these rare fluctuations can be captured, they should expose, potentially large, medium modifications
• 3N SRC are as yet unseen in inclusive electron scattering – some sleuthing underway
• Approved experiments across labs with different focuses over next 5-7 years will reveal much
• Next big opportunity in inclusive scattering (in my view) is the transition from QES to DIS at $x > 1$ at very large momenta transfer
• Tensor polarized targets advances will allow exposure of deuteron wf through asymmetry measurements
  • Opportunities exist for experiments with electrons and photons on tensor polarized deuterons