

In my career as a graduate student, I have exercised teaching duties at all levels of undergraduate mathematics instruction. At the introductory level, I have lectured six semesters of calculus (both lower-level courses for humanities and social science students, and upper-level courses for math and science majors). At the intermediate level, I have taught fourth-hour problem sessions in multi-variable calculus and differential equations. And at the advanced level, I have served as grader for junior- and senior-level courses in abstract algebra, number theory, and advanced linear algebra. In all of my duties as an educator I have approached my task from a position profoundly influenced by my own undergraduate liberal arts education: namely, that the primary objective of a mathematics educator is to teach students to *think* and to *write*. The stuff of mathematics—algebra, analysis, topology—is just the medium the educator uses to achieve this goal.

By teaching students to think, I mean teaching them the abstract-reasoning, critical-thinking, and problem-solving skills necessary not only for solving homework exercises but for resolving any issue requiring creative and intelligent decisions. By teaching students to write, I mean teaching them the communication skills necessary to effectively convey their ideas to others. These two goals are inseparable: a student cannot think about abstract concepts if he or she lacks the ability to precisely frame those thoughts, either for personal recollection or for communication to others. Thus, my teaching activities are directed at simultaneously addressing these two important skills.

My objectives and methodologies vary with the level of the course and the backgrounds of the students. In an introductory course, I want students to appreciate mathematics as a body of powerful abstract concepts rather than as a collection of rote algorithms. I want them to express mathematical thoughts in complete sentences and to appreciate the difference between colloquial English and rigorous Mathematical English. In my freshman calculus courses, for instance, I assign projects directing students to apply concepts and techniques in ways beyond the scope of typical textbook exercises. For example, after a unit on power series, I directed the students to use power series to discover Euler's identity  $e^{\pi i} + 1 = 0$  and to find power series solutions to differential equations.

As part of each project, the students must write out explanations of how and why they reached their conclusions. To help students appreciate the level of detail and precision that must go into mathematical writing, after a very geometrically-oriented project on parametric curves of epicycloids, I instructed the students to write explanations so as to be understandable to students of ninth-grade geometry. In intermediate and advanced courses, I ask students to be more critical of their writing. Have they been explicit about their assumptions? Have they defined all of their notation? Is their logic easy to follow (nay correct)? Most of all, have they used language precisely? I devote considerable time to providing constructive feedback on these issues on each assignment I grade. I use the written responses of my students to assess their technical skills and weaknesses and to evaluate their progress in mastering the essential ideas of the course.

Another way I try to indoctrinate introductory students in abstract-reasoning skills is through extensive use of pictures, drawings, and diagrams. This, I hope, appeals to the visual learners and to the students with weaker mathematical backgrounds. I recognize that, for many of the students in my lower-level courses, this may be their terminal experience in formal mathematics education. Thus, I am not concerned with whether some years from now they remember, say, the formula for the surface area of a solid of revolution. Rather, I want them to learn and be able to recall the pictures showing how we found the surface area of the frustum of a cone and the pictures illustrating how a solid of revolution is approximated by frusta. I tell my students that if they can remember those pictures, then they will also remember the abstract leap of using limits to go from those approximations to the desired surface area equation.

Though most of my instructional experience to date has been at the introductory and intermediate levels, I have given thought to how I would specifically address the needs of advanced mathematics students. For example, I would like to work with advanced undergraduates either individually or in small groups on advanced reading projects or solving research problems, to give them an idea of what constitutes real mathematics research. In the course of conducting my dissertation work I have discovered numerous smaller problems that could be amenable to attack by interested undergraduate or graduate students, especially with the aid of computer algebra systems like GAP or Magma. I am eager to combine my experience from grading a course on computational algebra (which incorporated calculations in GAP) with my experience attending the AIM workshop on computational methods in representation theory, to devise projects suitable for student collaboration.

One of the challenges I face as an educator is to make the material of my courses interesting and relevant to the students. My first month of teaching calculus a student asked me, “What is the point of a piecewise-defined function?” At the time I was flummoxed. Now I might ask the student for his major subject, and then point out some phenomenon in that field most naturally modeled by a piecewise-defined function. This spring I plan to ask the students in my applied calculus course to give 3–5 minute presentations on interesting applications of calculus in their major fields. And this winter I have registered for a professional development course at the Joint Mathematics Meetings to learn new methods for increasing student interest in the course. I want my students to develop a genuine interest in mathematics because only then will they invest effort into the skills I teach.

My greatest challenge as an educator is to get my students actively involved in doing mathematics, that is, in doing problems. I believe that teaching problem-solving skills is primarily a task of teaching students to rely upon and have confidence in their own intellect. To get them started, I tell students my own problem-solving strategy: Understand the problem. Do an example. Try a similar or easier problem. Draw a picture. Most importantly, *use what you know*. For the students who are still stuck, I make myself as available as possible to answer questions and provide help during office hours. But I do not give away answers. Instead, I ask leading and suggestive questions, gently nudging my students along the problem-solving strategy, so that when they do eventually solve the problem the accomplishment is their own, and their confidence is correspondingly boosted. I try to provide my students with frequent and diverse opportunities to engage with the ideas and concepts of the material, and I believe I am successful. As one student wrote on a recent course evaluation, “I was impressed by the speed and rigor of the course. It was not so difficult as to deter me. Instead, it provided sufficient difficulty to keep me interested in the challenge.”