Solar System Formation Models

• Must explain...
  
  • Regular angular momentum features
    ➔ Lack of angular momentum in the Sun
  
  • Distribution of planetary composition / size with radius from Sun
    ➔ Explanation of stark compositional difference of Terrestrial planets from Solar abundances
  
  • Presence and structure of debris zones
  
  • History of Solar System bombardment
  
  • Lots of other details, particularly those preserved in Asteroids/Meteorites.

• Other Solar System architectures...
Star Formation – A Galactic Perspective
Molecular Clouds

- Concentrated regions of the interstellar medium with density a thousand to a million times that found in interstellar space in general – $10^6$/cm$^3$.
- Revealed by dust extinction but gas is still dominant by 100:1
- Ordinarily, ultraviolet radiation from hot stars keeps atoms in individual form rather than in molecules.
  - 4.5eV dissociates $H_2$
  - 11.1eV dissociates CO
- The concentration of dust shields atoms from starlight and permits molecule formation.
Molecular Clouds

- Hydrogen gas is in molecular form but largely undetectable.
  - $\text{H}_2$ is a symmetric molecule, and producing radiation is difficult.
- Carbon monoxide, though relatively rare, emits strongly at millimeter wavelengths and maps these regions.
  - CO is $10^{-4}$ times less abundant than $\text{H}_2$, but the significant dipole moment of the molecule makes its a strong emitter.

![Image of molecular cloud](image)
Molecular Clouds

- Molecules can emit radiation via rotation or vibration.
- The lowest energy is the rotational transition from J=1 to J=0 (one unit of h-bar).
- Given the moment of inertia of the CO molecule this transition corresponds to a photon of wavelength 2.6 mm (115 GHz).
- CO is a primary tool for mapping out the dense interstellar medium in the Milky Way and other galaxies.

→ Oxygen has three isotopes of increasing rarity O\(^{16}\), O\(^{17}\), O\(^{18}\); carbon has two C\(^{12}\), C\(^{13}\).
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  - Oxygen has three isotopes of increasing rarity $^{16}$O, $^{17}$O, $^{18}$O; carbon has two $^{12}$C, $^{13}$C.
  - Given the smaller “optical depth” these trace isotopologues can probe conditions in the densest regions.
  - Since molecules emit sharp spectral lines the Doppler shift can reveal detailed gas motions.
  - Under very dense conditions other molecules like HCN, H$_2$CO, ro... form and are tracers of cloud collapse and ultimately planet formation.
Molecular Clouds

**Figure 3.** Velocity channel maps of the $^{13}$CO (3–2) line in L1551 NE observed with the SMA. Contour levels are from 2σ in steps of 2σ ($1\sigma = 0.107 \text{ Jy beam}^{-1}$). Crosses indicate the positions of the protobinary, and a filled ellipse at the bottom-right corner in each panel indicates the synthesized beam ($0.95 \times 0.66$; P.A. = $-88^\circ$). A number at the top-left corner in each panel denotes the LSR velocity.
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Nearby Star Forming Regions

- The nearest region of low-mass (solar type) star formation is in Taurus-Auriga and is 140 parsecs away.
  - 1 arcsecond = 140 AU at this distance - 5 times the size of Neptune's orbit.
- The nearest massive star forming region is the Orion Nebula – 400 parsecs away.
Sizes of Star Forming Regions

- Given a density of $10^{12}$ H$_2$ per cubic meter, how big a region of a molecular cloud must collapse to make the Sun?
  - The mass of a hydrogen atom is $1.7 \times 10^{-27}$ kg.
Is a Molecular Cloud Stable?

- Gravity will try to collapse any distribution of interstellar medium.
- Countering gravity is thermal support – gas pressure.
- Gravity has a timescale dictated by the freefall time of a particle from the edge of the cloud down to the center.
- Gas pressure has a response timescale dictated by the sound speed in the cloud.
- If gas pressure is faster than gravity the cloud will be stable against collapse.
Freefall Time

- A spherical cloud collapsing under its own gravity will have a freefall time equal to half the orbital period of a high eccentricity particle on an elongated orbit falling in from the edge of the cloud.

- Consider a cloud of radius \( r_0 \)

- Apply Kepler’s 3\(^{rd}\) Law

\[
P^2 = \frac{4\pi^2}{GM} a^3
\]

- Where the freefall time

\[
2t_{ff} = P
\]

- Note:

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M = \frac{4}{3} \pi r_0^3 \rho_0
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- Note:

  $$M = \frac{4}{3} \pi r_0^3 \rho_0$$

  $$t_{ff} = \sqrt{\frac{3 \pi}{32 G \rho_o}}$$

$$a = \frac{r_0}{2}$$
Pressure Wave (Sound) Propagation Time

- A pressure disturbance at the cloud edge will propagate to the center at the sound speed.
- To first order the sound speed derives from equating average energy per particle, \( \sim kT \), with kinetic energy, \( \frac{1}{2} mv^2 \).

\[
c_s = \sqrt{\frac{\gamma kT}{\mu m_H}}
\]
Gas Pressure Resists Collapse

- Although it is difficult to conceive of the pressure in interstellar space, there is a well defined pressure described by the ideal gas law.

\[ P = \frac{\rho k T}{\mu m_H} \]

- A good way of representing whether the feeble local pressure can resist the weak gravitational pull is to consider the sound speed – how fast fair warning of gravity's action propagates.

- If collapse can occur before the sound pressure wave gives warning then gravity can do it's evil deed before the cloud notices.

\[ \frac{1}{2} m v^2 \approx k T \quad c_s \approx \sqrt{\frac{2k T}{m}} \]
Sound Speed

• To first order the sound speed is equal to the velocity of the individual particles in a gas.

• Equipartition suggests that the particle energy is $\frac{1}{2} k T$ per degree of freedom.
  
  • For particles behaving like ball bearings there are just 3 translational degrees of freedom.
  
  • More complex particles have additional degrees of freedom (e.g. internal vibration) putting energy into these modes as well as translation. In the formal expression of sound speed this is accounted for by the adiabatic index, $\gamma$.

• Formally

$$c_s = \sqrt{\frac{\gamma k T}{m_{\text{particle}}}}$$
Conditions Permitting Gravitational Collapse

• Gravity has to beat a warning propagated at the sound speed.
  
  • If the freefall time is shorter than the time for a sound wave to propagate from the edge to the center of the cloud \((r_0/c_s)\), the cloud can collapse.
The Jeans Length/Mass

• See section 17.1 for details

\[ t_{ff} = \left[ \frac{3 \pi}{32 G \rho_{cloud}} \right]^{1/2} \]

\[ t_{\text{sound}} = \frac{r_{\text{original}}}{c_s} = r_{\text{original}} \sqrt{\frac{m_{\text{particle}}}{\gamma k T}} \]

\[ r_{\text{Jeans}} = \sqrt{\frac{3 \pi \gamma k T}{32 G \rho_o m_{\text{particle}}}} \]

– A cloud has to be bigger than \( r_{\text{Jeans}} \) to collapse, which comes from requiring that \( t_{\text{sound}} > t_{ff} \)

– There is an associated “Jeans Mass” – simply the mass in a Jeans volume given the density.

\[ M_{\text{Jeans}} = \frac{4}{3} \pi r_{\text{Jeans}}^3 \rho_0 \]
The Jeans Length/Mass

- Using the typical values to create scaling relations

\[ r_{\text{Jeans}} = \sqrt{\frac{3 \pi \gamma k T}{32 G \rho_o m_{\text{particle}}}} \]

\[ r_{\text{Jeans}} = 2000 \text{ AU} \left( \frac{T}{10 \text{K}} \right)^{1/2} \left( \frac{\text{molecular hydrogens per cubic meter}}{10^{12}} \right)^{-1/2} \]

\[ M_{\text{Jeans}} = 0.2M_{\text{sun}} \left( \frac{T}{10 \text{K}} \right)^{3/2} \left( \frac{\text{molecular hydrogens per cubic meter}}{10^{12}} \right)^{-1/2} \]
Formation of a Star via Cloud Collapse

- Conservation of angular momentum guides an initially spherical collapse to form a disk around the forming star.

\[ m_o v_o r_o = m_f v_f r_f \quad v_f = \left( \frac{r_o}{r_f} \right) v_o \]

- Collapse is halted, to first order, when rotational speed at the edge of the cloud reaches orbital speed

\[ \sqrt{\frac{G M}{r_f}} = v_f = \left( \frac{r_o}{r_f} \right) v_o \]
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\[ \sqrt{\frac{GM}{r_f}} = v_f = \left(\frac{r_o}{r_f}\right) v_o \]

- Given typical 0.1 km/s motions in cloud cores, the centrifugal radius of the collapsed cloud (now disk) is a couple of hundred AU (also see section 17.1).

\[ r_f = r_{disk} = \frac{r_{o}^{2}v_{o}^{2}}{GM} = 200 \text{ AU} \left(\frac{v_{o}}{0.1 \text{ km/s}}\right)^{2} \left(\frac{r_{original}}{4000 \text{ AU}}\right)^{2} \left(\frac{M}{M_{sun}}\right)^{-1} \]
This process of disk formation and accretion is common to any forming star anywhere. Many, if not most, stars are likely to have planets around them.
Disks are Seen Directly

Orion nebula (at left) is 400 parsecs away.

Nearest regions of significant star formation are 140 parsecs away.
Disks are Seen Directly

Disks around Young Stars
PRC99-05b - STScI OPO
C. Burrows and J. Krist (STScI), K. Stapelfeldt (JPL) and NASA

Protoplanetary Disks
Orion Nebula
PRC95-45b - ST ScI OPO - November 20, 1995
M. J. McCaughrean (MPIA), C. R. O'Dell (Rice University), NASA
Edge-On Protoplanetary Disk
Orion Nebula
PRC95-45c · ST ScI OPO · November 20, 1995
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Infrared Observations Reveal Disks

- Warm dust emits thermal radiation at infrared wavelengths.
- Infrared observations of forming stars reveal excess infrared light coming from these stars.
  - The distribution of infrared light is consistent with the material being spread out in a disk.
What is the expected disk temperature profile with radius?

- If the disk were optically thin, that is, if every particle could see the Sun unhindered, then we already know temperature should fall off with radius inversely as the square root of the distance.

- However, now we are talking about an optically thick disk that is behaving more like a sheet of cardboard.
  - Incident flux needs to be calculated accounting for the Sun shining obliquely on each square centimeter of surface.
  - Whereas before flux dropped off as $R^2$, it now drops off as $R^2 \sin(\theta)$, where $\theta$ is the apparent angular radius of the sun (and thus it's altitude in the “sky”) for that square centimeter.

- For a “cardboard” disk $T \propto R^{3/4}$.

- Ultimately “spectral energy distributions” can reveal the mass distribution and structure of a circumstellar disk.
Flared Disks

Disks around Young Stars

HST - WFPC2

PRC99-05b - STScI OPO
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LOG N

AA TAU

0.1 1 10 100 1000 10^4

\lambda (\mu m)

-14 -12 -10

\log n_\lambda

Disks Should Fade As Planets Grow

- As planets grow the disk should begin to clear.
- Simulations/calculations say that within 10 million years most of the small dust should be gone.
  - This age corresponds well with observations of young stars showing the infrared excesses go away before an age of 10 million years.
Figure 9. $V$ versus $(V - I)$ CMD for the identified candidate YSOs. The PMS isochrones for 0.1, 1 and 5 Myr from Siess et al. (2000) and the isochrone for 1 Myr from Marigo et al. (2008) are drawn as the dotted-dashed and continuous curves, respectively. All the isochrones are corrected for the distance and reddening of S311. The average errors in $V$ and $(V - I)$ colours are shown on the left side of the figure. Evolutionary tracks for various masses are also shown as thin solid curves.
Reading the Tea Leaves

a) Weak-excess, anemic, or homologously depleted disk

b) Classical transition or cold disk

c) Cold disk

d) Cold disk or pre-transition disk
Faint Disks Should Linger

- Leftover comets and asteroids should continue to collide and generate faint dust signatures.
- We see it in our solar system and elsewhere.
Beta Pic is a star with 1.8 times the mass of the Sun (spectral type A6) located 20 parsecs away.

**Beta Pictoris**

COMPARISON WITH ZODIACAL DUST DISTRIBUTION  It is fascinating to compare the distribution of dust in $\beta$ Pic disk and its closest analog in the Solar System – zodiacal light (ZL). Interestingly, like $\beta$ Pic disk, ZL has a nearly exponential (sharply centrally peaked) vertical profile with scattering area proportional in one model to $\exp(-4.2(z/r)^{1.2})$ (Good et al 1986). It is also inclined to the ecliptic and the invariant plane by a few degrees and is slightly warped [due to inclined dust sources (asteroids producing so-called IRAS dust bands)], probably to adjust to local orbital planes of the planets. However, ZL forms a thicker structure since the vertical profile has a scale height as large as $z = 0.3r$, two to three time larger than in $\beta$ Pic. Zodiacal dust, unlike most of the $\beta$ Pic disk, has fairly uniform radial distribution (at least in the inner solar system) expressed as $\tau \sim r^{-0.3}$ to $\tau \sim r^{-0.1}$. Finally, the normalization of $\tau$ throughout the planetary region of our system is close to $\tau = 10^{-7}$, or $\leq 10^{-4}$ of typical $\beta$ Pic’s value (cf Equation 1). The differences likely mirror the different spatial density distribution of parent bodies in the respective systems, but not the basic nature of the phenomenon.
Fomalhaut (Alpha Piscis Austrini)

- Structure in debris disks suggests the gravitational influence of planets.

Go looking for Fomalhaut due South in the evening (declination -30).