Phys 831 Midterm Exam 2 Due 2 April 2009

This is a take-home exam. You may refer to Pathria's textbook, the class notes, and the homework solutions. (You may use both your own notes and solutions, and the ones that I posted.) You may consult a table of integrals and other mathematical data, or you may use a computer program for the same purpose. No other references are allowed, and you may not discuss the problems with anyone (except the instructor). Violation of these instructions would constitute an honor offense.

You have one week to complete the exam. There are four problems which will be weighted equally. The problems should be worked on separate pages and attached to this sheet when completed. For full credit, be sure to show and explain all your work. It would also be a good idea to neatly write up and check over your solutions before turning them in.

The exam is due at 11:00 AM on Thursday, April 2. Late exams will be accepted by prior arrangement only.

Name: ______________________________

Signature: ______________________________

One handy formula, which might be otherwise hard to find:

$$\sum_{n=n_1}^{n_2} [n(n+1)\ldots(n+k-1)] = \left.\frac{n(n+1)\ldots(n+k)}{k+1}\right|_{n_1-1}^{n_2}$$

1. (a) Calculate the phase space available to a (non-relativistic) particle with energy less than or equal to E, which is moving in a potential $V(\mathbf{r}) = -\gamma/r$. Take $E < 0$ and $\gamma > 0$.

(b) The quantum mechanical eigenstates for this system have energies

$$\epsilon_n = -\frac{m\gamma^2}{2\hbar^2 n^2}$$

for $n = 1, 2, \ldots$. The degeneracy of each level is n^2. Calculate the number of states available with energy less than E, and compare to the result of part (a).

2. In atom-trapping experiments, we often study a system of N non-relativistic and non-interacting atoms at temperature T that are confined in a potential of the form

$$V(x, y, z) = c\sqrt{x^2 + y^2 + 4z^2}.$$

Calculate the average energy per particle and the total entropy for this system, assuming it is in the classical limit.

3. A two-level system consists of a quantum entity with only two energy states available. This might represent a spin-1/2 particle in a magnetic field, a molecule with two nearly degenerate states, or a qubit in a quantum computer. We ignore any other degrees of freedom, such as translational motion. Suppose one state has energy zero and the other energy ϵ. Consider a set of N distinguishable entities that do not interact.

(a) Using the canonical ensemble, calculate the total energy of the system, the entropy, and the heat capacity as a function of the temperature T.

(b) Evaluate the energy, entropy, and heat capacity in the limits of high and low T, and explain why the results obtained make sense physically.

4. Use the micro-canonical ensemble to analyze a set of N two-level systems as in problem 3. For a total energy of E, determine the entropy and temperature of the system. Show that the results obtained here are consistent with those of problem 3. (You can assume $N \gg 1$ and $E \gg \epsilon$.)