Assignment 9

9.1 (Pathria 7.4) Show that for an ideal Bose gas

$$\frac{1}{z}\left(\frac{\partial z}{\partial T}\right)_P = -\frac{5}{2T}\frac{g_{5/2(z)}}{g_{3/2}(z)};$$

see for instance Eq. (7.1.36). Hence show that

$$\gamma \equiv \frac{C_P}{C_V} = \frac{(\partial z/\partial T)_P}{(\partial z/\partial T)_v} = \frac{5}{3}\frac{g_{5/2}(z)g_{1/2}(z)}{g_{3/2}(z)^2},$$

as in Eq (7.1.48b). Note that as $z \to 1$, $g_{1/2}(z) \to \infty$ (Eq. D.8), so γ and C_P both diverge as $(T - T_c)^{-1}$.

*H*int: I got the first part straight from Eq. (7.1.7). In the second part, it is the middle equality that takes some thought. It can be derived from the fact that the entropy (Eq. (7.1.44a) depends only on z.

9.2 (Pathria 7.13) Consider an ideal Bose gas confined to a region of area A in *t*wo dimensions. Express the number of particles in the excited states, N_e and the number of particles in the ground state N_0, in terms of z, T, and A, and show that the system does not exhibit Bose-Einstein condensation unless $T \to 0$.

Refine your argument to show that, if the area A and the total number of particles N are held fixed and we require both N_e and N_0 to be of order N, we do achieve condensation when

$$T \sim \frac{h^2}{mk\ell^2}\frac{1}{\log N}$$

where $\ell \sim \sqrt{A/N}$ is the mean interparticle distance in the system. Of course, if both A and $N \to \infty$, keeping ℓ fixed, then the desired T does go to zero.

9.3 (Pathria 7.18) The sun may be regarded as a black body at a temperature of 5800 K. Its diameter is about 1.4×10^9 m while its distance from the earth is about 1.5×10^{11} m.

(a) Calculate the total radiant intensity (in W/m^2) of sunlight at the surface of the earth.

(b) What pressure would it exert on a perfectly absorbing surface placed normal to the rays of the sun?

(c) If a flat surface on a satellite, which faces the sun, were an ideal absorber and emitter, what equilibrium temperature would it ultimately attain?

9.4 (Pathria 8.10) Consider an ideal Fermi gas with energy spectrum $\epsilon = ap^s$, contained in a box of "volume" V in a space of n dimensions. (Here a is a constant.) Show that for this system

(a)

$$PV = \frac{s}{n}U$$

(b)

$$\frac{C_V}{Nk} = \frac{n}{s}\left(\frac{n}{s}+1\right)\frac{f_{(n/s)+1}(z)}{f_{n/s}(z)} - \left(\frac{n}{s}\right)^2\frac{f_{n/s}(z)}{f_{(n/s)-1}(z)}$$