Assignment 2

2.1 Prove that for fixed N,

(a)

$$\left(\frac{\partial E}{\partial X}\right)_T = Y - T\left(\frac{\partial Y}{\partial T}\right)_X$$

(b)

$$\left(\frac{\partial E}{\partial Y}\right)_T = T\left(\frac{\partial X}{\partial T}\right)_Y + Y\left(\frac{\partial X}{\partial Y}\right)_T$$

(c)

$$TdS = C_X\left(\frac{\partial T}{\partial Y}\right)_X dY + C_Y\left(\frac{\partial T}{\partial X}\right)_Y dX$$

2.2 Take N fixed throughout this problem.

(a) By equating two different expressions for dE, show that $TdS = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$.

(b) Using a similar method with H, show that $TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$.

(c) By holding appropriate variables constant in the above relations, derive

$$\frac{C_P}{C_V} = -\frac{(\partial V/\partial T)_P}{(\partial P/\partial T)_V}\left(\frac{\partial P}{\partial V}\right)_S = \frac{\kappa_T}{\kappa_S}.$$

(d) and $C_P - C_V = \frac{TV\alpha_P^2}{\kappa_T}$.

2.3 An isotropic magnetic substance satisfies Curie's law, $M = \mathcal{C}\mathcal{H}/T$, and has a constant heat capacity $C_M = C$. Find the internal energy, entropy, enthalpy, Helmholtz free energy, and Gibbs free energy as functions of M and T.

2.4 (a) Calculate the Joule-Thompson coefficient μ_J for a van der Waals gas with arbitrary interaction strength. Express your answer in terms of the volume per particle v, the temperature T, and the heat capacity C_P. Check that the low density limit gives

$$\mu_j = \frac{N}{C_P}\left(\frac{2a}{k_B T} - b\right)$$

as claimed in class.

(b) Nitrogen gas approximately satisfies the van der Waals equation of state, with coefficients $a = 1.408$ liter^2 bar/mol^2 and $b = 0.03913$ liter/mol. At $P = 1$ atm and $T = 300$ K, the specific heat capacity c_P is approximately 0.2470 cal/g K. Numerically evaluate μ_J for these conditions.