Assignment 1

1.1 A gasoline engine uses an approximation to the Otto cycle, in which an ideal gas with initial pressure, volume, and temperature (P_1, V_1, T_1) is first adiabatically compressed to volume V_2, then heated at constant volume to temperature T_3, then adiabatically expanded back to volume V_1, and finally cooled at constant volume back to temperature T_1.

(a) Draw the curve for an Otto cycle in the PV plane.

(b) For a working gas with energy $E = \alpha N k_B T$, calculate the efficiency of the engine in terms of the compression ratio (V_1/V_2). The efficiency here is defined as W/Q_{23}, since Q_{23} is the heat taken up in the hot stage of the cycle. Evaluate the efficiency for a typical compression ratio of 10:1 and $\alpha = 3$.

1.2 Experimentally, a rubber band is found to satisfy

$$\left(\frac{\partial J}{\partial L}\right)_T = \frac{aT}{L_0}\left[1 + 2\left(\frac{L_0}{L}\right)^3\right]$$

and

$$\left(\frac{\partial J}{\partial T}\right)_L = \frac{aL}{L_0}\left[1 - \left(\frac{L_0}{L}\right)^3\right]$$

where J is the tension, $a = 1.0 \times 10^{-2}$ N/K is a constant, and $L_0 = 0.5$ m is the length of the band with no tension applied. The heat capacity at constant length C_L is determined to be 1.0 J/K, independent of L and T.

(a) Compute $(\partial L/\partial T)_J$ and discuss its physical meaning.

(b) Show that dJ is an exact differential and find the equation of state.

(c) Suppose that at $T = 0$, the internal energy of the band E is independent of the length. (This is plausible since at $T = 0$, $J = 0$ for any L.) Show that $(\partial E/\partial L)_T = 0$ at any T.

(d) If the band is initially unstretched at a temperature of 300K, find the change in temperature resulting if it is stretched reversibly and adiabatically to a length of 1 m. How much work does this require?

1.3 A paramagnetic substance obeying Curie's law, $M = \mathcal{C}\mathcal{H}/T$, sits in a magnetic field $\mathcal{H}_0$ at temperature T_0. The heat capacity C_M is constant, and the internal energy does not depend on the field. If the field is turned off adiabatically, find the new temperature that results. Explain physically why the temperature decreases.

1.4 An isotropic magnetic material satisfying Curie's law has a constant heat capacity C_M and an internal energy which is independent of the magnetization M. The substance is used to create a Carnot engine operating between temperatures T_h and T_c with $T_h > T_c$. The heat taken up at T_h is Q_h and the heat released at T_c is Q_c.

(a) Determine qualitatively (the sign is sufficient) how the magnetization and field must change during each stage, and give a qualitative plot in the $\mathcal{H}M$ plane of the curve for one complete cycle.

(b) Find relationships between the magnetizations at the beginning and end of each stage. By combining the four relations, verify that $Q_h/T_h = Q_c/T_c$.

(c) Calculate the work done by the engine in each stage and verify that the total work done is $Q_h - Q_c$.