Manipulating Differentials

A differential relation $dF = Adx + Bdy$ contains a great deal of information, and can be manipulated much like an algebraic relation. For instance, the following derivations are valid:

(a) Set $dy = 0$ and divide by dx:

$$\left(\frac{\partial F}{\partial x}\right)_y = A.$$

(b) Set $dF = 0$ and divide by dy:

$$A\left(\frac{\partial x}{\partial y}\right)_F + B = 0$$

(c) Introduce a new variable $z = z(x, y)$. Hold z constant and divide by dx:

$$\left(\frac{\partial F}{\partial x}\right)_z = A + B\left(\frac{\partial y}{\partial x}\right)_z$$

(d) Set $dx = 0$ and divide by dz:

$$\left(\frac{\partial F}{\partial z}\right)_x = B\left(\frac{\partial y}{\partial z}\right)_x$$

(e) Introduce another new variable $w = w(x, y)$. Hold w constant and divide by dz:

$$\left(\frac{\partial F}{\partial z}\right)_w = A\left(\frac{\partial x}{\partial z}\right)_w + B\left(\frac{\partial y}{\partial z}\right)_w$$

In fact, all of the preceeding relations can be considered as special cases of (e).

These relations can also be derived more conventionally using the partial derivative relations given in lecture 1. However, the above derivations may be easier to remember.