Phys 521 Midterm Exam 16 October 2008

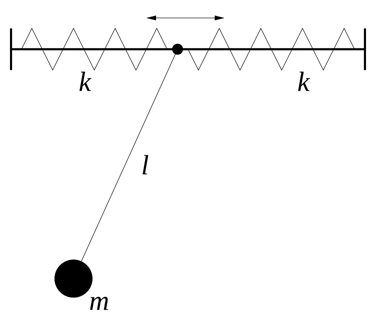
This is a closed book, closed notes exam, to be taken in a single 75 minute period. The problems should be worked on separate pages and attached to this sheet when completed. There are four problems, which will be weighted equally. For full credit, be sure to show and explain all your work.

Name: ______________________________

Signature: ______________________________

1. A particle of mass m moves under the influence of a central force with potential $V(r)$. Derive an integral expression for the orbit $\phi(r)$.

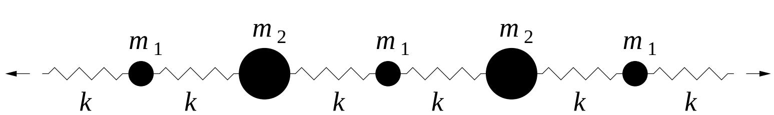
2. A particle of mass m forms a pendulum of length ℓ. The support point of the pendulum can slide without friction along a horizontal rod, but two horizontal springs with spring constant k restrict this motion, as shown. Considering only motions in the plane of the diagram, calculate the frequency for small oscillations of the mass near equilibrium.



3. Consider a linear chain of particles with alternating masses m_1 and m_2, all connected by springs with spring constant k and equilibrium length a. Normal modes for this system consist of motions

$$\eta_j(t) = Ae^{i\kappa aj}\cos(\omega t + \phi)$$

in which the amplitude A takes one value for j even and a different value for j odd. Determine the dispersion relation $\omega(\kappa)$.



4. A particle of mass m slides on a wire that is formed into the shape of a long helix, described in cylindrical coordinates by $z = b\phi$ and $\rho = a$. The helix is oriented vertically in a gravitational field g.

(a) What is the Lagrangian for the particle?

(b) Suppose the helix is rotating about its axis at angular frequency $\omega(t)$. What is the Lagrangian in the co-rotating frame?

(c) Derive the equation of motion in the rotating frame, and find the condition on $\omega(t)$ required to allow an equilibrium solution $\ddot{\phi} = 0$.