Assignment 8

5.2 Foucault gyrocompass A gyroscope in the form of a symmetric top is mounted with no gravitaional torque, and its symmetry axis is constrained to move only in the horizontal plane parallel to the earth's surface. The gyroscope is set sppinning about its symmetry axis with an angular velocity $\mathbf{\Omega}$.

(a) If $\Omega \gg \omega$, where ω is the angular velocity of the earth's rotation, show that the Coriolis force exerts the following torque on the gyroscope about its center of mass:

$$\mathbf{\Gamma} = -2 \sum_p m_p \left\{ \mathbf{r}_p \times \left[\omega \times (\mathbf{\Omega} \times \mathbf{r}_p) \right] \right\}$$

where the sum runs over all the particles in the gyroscope with position $\mathbf{r}_p$ relative to the center of mass.

(b) The gyroscope is located at polar angle (colatitude) θ. Consider its angular motion about the vertical 1 (1^0) axis. Show that the symmetry axis (3) of the gyroscope will oscillate about the northerly direction according to

$$\ddot{\phi} = -\left(\frac{I_3}{I_1} \Omega \omega \sin\theta \right) \sin\phi$$

and can thus be used as a compass.

5.5 (a) Verify that Eqs. (28.32) and (28.33) are indeed first integrals of Euler's equations (28.27) for torque-free motion.

(b) Eliminate ω_1 and ω_2 in terms of E, L, and ω_3, and hence reduce the resulting motion for $\omega_3(t)$ to a definite integral. Obtain the formal solution for $\vec{\omega}(t)$.

5.6 An asymmetric top ($I_1 < I_2 < I_3$) executes torque-free motion with $2EI_2 = L^2$. If $\vec{\omega}$ initially lies in the plane of $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_3$ (as in Fig. 28.4), integrate Euler's equations to obtain the solution

$$\omega_1(t) = \omega_\infty \left[\frac{I_2(I_3 - I_2)}{I_1(I_3 - I_1)} \right]^{1/2} \operatorname{sech} \frac{t}{\tau}$$

$$\omega_2(t) = \omega_\infty \tanh \frac{t}{\tau}$$

$$\omega_3(t) = \omega_\infty \left[\frac{I_2(I_2 - I_1)}{I_3(I_3 - I_1)} \right]^{1/2} \operatorname{sech} \frac{t}{\tau}$$

where $\omega_\infty = 2E/L$ and $\tau^{-1} = \omega_\infty \left[(I_3 - I_2)(I_2 - I_1)/I_3 I_1 \right]^{1/2}$. Discuss the time dependence of ω^2 and sketch the motion of $\hat{\omega}$ as seen in the body-fixed frame.

Packet #7 Consider a uniform solid cylinder of mass M, length L, and radius R, as shown. Let P denote the point at the center of the top face, and consider some point Q which is on the side of the cylinder a distance R from the top (where R is the radius of the cylinder and you may assume $L > R$). A narrow hole is drilled through the cylinder along the line PQ, and this hole is then slipped over a greased rod which is fixed to be horizontal, as shown. Find the frequency of small oscillations of the cylinder about this rod, assuming it oscillates without friction, and expressing your answer in terms of g, M, L, and R.

[Hint: One way to do this problem is to first compute the components of the moment of inertia tensor about some conveniently chosen origin (state clearly what origin you choose) with some convenient choice of the three axis of your coordinate system. Then find the moment of inertia about the axis PQ that goes through P and Q. Then use that to find the frequency of small oscillations.]

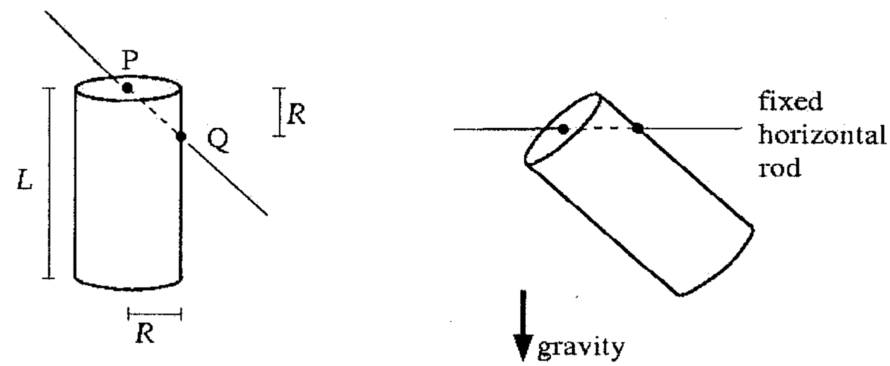

Fig. A   Fig. B

Problem #6