

Assignment 7

S1 A particle with charge q in an electromagnetic field experiences the Lorentz force

$$\mathbf{F} = q\mathbf{E} + \frac{q}{c}\dot{\mathbf{r}} \times \mathbf{B}$$

for electric field \mathbf{E} , magnetic field \mathbf{B} , particle velocity $\dot{\mathbf{r}}$ and speed of light c .

(a) Show that the corresponding equation of motion can be derived from the Lagrangian

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 - q\phi + \frac{q}{c}\mathbf{A} \cdot \dot{\mathbf{r}}$$

where $\mathbf{A}(\mathbf{r}, t)$ is the vector potential and $\phi(\mathbf{r}, t)$ is the scalar potential, related to the fields by

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

(b) Calculate the canonical momentum and the Hamiltonian for this system.

S2 Consider a rigid body consisting of a cube with side $2a$ that is missing one corner section consisting of a cube of side a . (See figure.) The total mass of the object is M and the density is uniform.

(a) Calculate the inertia tensor for this object relative to the center of the large cube, and using axes aligned to the faces of the large cube.

(b) Find the center of mass of the object, and calculate the inertia tensor relative to the center of mass (using the same axes as in (a)).

(c) Find the principle axes and principle moments of inertia for this object (relative to the center of mass).

Don't be surprised if you get some messy fractions in your answers.

