Assignment 5

4.11 N indentical particles with masses m are connected by $N+1$ identical massless springs with force constant k and fixed endpoints as shown in Fig. 24.1. In the original static configuration, each spring is stretched from its equilibrium length a_0 to a new length a. Construct the Lagrangian for general small oscillations in a plane, and show that the longitudinal and transverse modes *decouple*. Discuss how the frequencies depend on $a - a_0$. Discuss and interpret the behavior for $a < a_0$.

4.13 Consider a long chain of identical simple pendulums of mass m and length ℓ, coupled with springs of spring constant k. In equilibrium with the pendulums all vertical, the springs are unstretched and the separation between adjacent pendulums is a. Find the dispersion relation for the propagation of waves of small amplitude, and sketch the resulting function $\omega(q)$. If there are N pendulums, apply periodic boundary conditions to determine the allowed vibration frequencies. What is the frequency of the lowest mode?

4.15 Investigate the small-amplitude longitudinal oscillations of $N+1$ identical point masses m joined by N identical springs with spring constant κ.

(a) Use Newton's laws of motion to find the appropriate boundary conditions on the end particles $j = 0$ and N.

(b) Generalize the treatment of Eq. (24.29) to determine the allowed eigenfrequencies and eigenfunctions.

(c) (See hints.) Show that these solutions are either even (+) or odd (-) under the transformation $\eta_j(t) \leftrightarrow \eta_{N-j}(t)$ and that the corresponding allows wave numbers satisfy the equations

$$\tan \frac{ka}{2} = \pm \left(\cot \frac{kaN}{2} \right)^{\pm 1}$$

(d) (See hints.)

(e) Compare with the case of fixed ends.

4.17 A uniform string with fixed ends has length ℓ, mass density σ, and uniform tension τ. A point mass m is attached at its center.

(a) Assuming small transverse displacements in one plane, find the equations of motion for the two halves of the string and for the mass m.

(b) Show that the modes in which m moves have frequencies that satisfy the equation

$$\frac{2c}{\omega \ell} \cot \frac{\omega \ell}{2c} = \frac{m}{\sigma \ell}$$

where $c^2 = \tau/\sigma$. Discuss the solutions in the limiting cases $m \to 0$ and $m \to \infty$.

Packet #4 Consider a classical treatment of the small oscillations of the atoms about equilibrium in a linear tri-atomic molecule. Two of the masses are equal and at equilibrium are located a distance a from the third. Making a reasonable assumption about the functional form of the potential for small motions, obtain the frequencies of vibration. Describe the mode of vibration for each of the eigenfrequencies. You need only consider motion along the molecular axis.