Assignment 2

1.12 The orbit of the planet mercury has an eccentricity of 0.206 and a period of 0.241 year; moreover, the perihelion advances slowly at a rate of 43 seconds of arc per century. One possible explanation of this effect is that the potential energy around the sun has the form $V = -(mMG/r)(1 + \alpha GM/rc^2)$, where α is a dimensionless constant and $MG/c^2 \approx 1.475$ km characterizes the sun's gravitational field. Demonstrate that the resulting orbit indeed represents a precessing ellipse. Find the magnitude and sign of α needed to fit the observed data.

1.13 A rocket with velocity v_∞ and impact parameter b approaches a planet of radius R_0 and mass M. What is the condition that the rocket will strike the planet? If it just misses, what is its angle of deflection?

1.17 A uniform beam of particles with energy E is scattered by an attractive central potential

$$V(r) = \begin{cases} 0 & r > a \\ -V_0 & r < a \end{cases}$$

Show that the orbit of a particle is identical with that of a light ray refracted by a sphere of radius a and index of refraction $n = [(E + V_0)/E]^{1/2}$. Prove that the differential elastic cross section for $\cos \frac{1}{2}\theta > n^{-1}$ is

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{n^2 a^2}{4\cos\frac{1}{2}\theta} \frac{[n\cos(\frac{1}{2}\theta) - 1](n - \cos\frac{1}{2}\theta)}{(1 + n^2 - 2n\cos\frac{1}{2}\theta)^2}$$

What is the total cross section?

3.3 A simple pendulum of mass m_2 and length l is constrained to move in a single plane. The point of support is attached to a mass m_1 which can move on a horizontal line in the same plane. Find the lagrangian of the system in terms of suitable generalized coordinates. Derive the equations of motion. Find the frequency of small oscillations of the pendulum.

Packet #2 Protons, with mass m and charge e, are given (nonrelativistic) energy E and sent as a beam to scatter from much heavier nuclei of charge Ze. The experiment shows that the differential cross section agrees with the Rutherford cross section for scattering angles less than some critical angle θ_c, but departs rapidly from it for larger angles. This is due to the presence of a strong force between the incoming proton and the nucleus from which it scatters; this force is of short range and effectively only comes into play when the proton touches the nucleus. Assuming that the nucleus is spherical, find its effective radius in terms of the given parameters.

Hint: The equation of motion for the orbit of a particle of mass m and angular momentum L moving under the influence of a central force of magnitude k/r^2 is

$$\frac{1}{r} = \frac{mk}{L^2}\left[\left(1 + \frac{2EL^2}{mk^2}\right)^{1/2}\cos(\theta - \theta') + 1\right]$$